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Decomposition of a Multiple Access Network into a Number of Interconnected Subnetworks

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Abstract

A random access system with a finite number of buffered terminals is considered. The problem of how to decompose this system into a number of interconnected subsystems in order to maximize its throughput is studied. Two different interconnection topologies are considered, a directional ring topology and a fully connected network of subsystems. For each of these two topologies the problem of maximizing the throughput is reduced to a simple maximization problem. We apply the attained results to a slotted ALOHA and a CSMA/CD system. For the former a significant improvement in the throughput performance is reported, while simulation analysis shows an expected deterioration in the delay performance, for low intensity traffic.

1 Introduction

Given a set of terminals that must communicate over shared (multiple access) channels, the question arises of how to organize this set in order to obtain optimal performance from the resulting system. One option is to allow the entire set of terminals to access a single, common channel according to some protocol (such as ALOHA, CSMA or other similar ones). Most studies of multiaccess protocols generally adopt this option although there is no indication about its optimality. Furthermore, in many cases it is not realistic to assume that any two terminals can communicate with each other without the use of intermediate tranceivers. It was this last consideration that gave rise to a second option, namely to separate the set into groups, each of which may operate as a single system under a given protocol, while intergroup traffic can be accommodated via dedicated links between the subsets. This option leads to the study of a multihop system.

The aim of this paper is to compare these two options. The motivation for such a comparison was provided by the work of Merakos et. al. , who in [1] showed that by splitting a multiple access system with an infinite number of unbuffered users and operating under the CSMA protocol into two subnetworks increases the maximum stable throughput. Here we explore this question further by considering to what degree further subdivision improves the maximum stable throughput. Our attention is focused on multiple access channels with a finite number of buffered terminals operating under a variety of protocols.

It should be noted that traditional approaches of multihop analysis rely on heavy approximations and/or simulations (see for example [2, 3, 4, 5]). In this study we try to employ as general and detailed models as possible. Precise analysis was employed for the throughput performance, for the delay performance however we had to rely on simulation methods.

One problem, similar to the one considered here, was studied by Kleinrock and Silvester, who, in [6], considered communication range models and calculated the optimal transmission range of a radio user network for maximum overall throughput. The assumptions made in that study are completely different from ours and thus it is impossible to relate their conclusions to ours.

Clearly, as soon as we subdivide the set of terminals into more than two subsets, the question arises how to interconnect these subsets, i. e. the question of topological connectivity. Depending on the choice of such topology, the effect of frequency reuse, that the subdivision principle ensures, will be mitigated differently for each topology, by the need for routing multi-hop traffic.

In the following, after a precise definition of the model, we consider two different topologies and study the trade-off involved in the decomposition.

2 The Model

We assume a number of M users that can be connected by means of a single multiple access channel of given overall bandwidth W (packets/sec). We furthermore assume that the users are identical in the sense that each of them generates packets of constant bit length X at an average rate λ (packets/sec) and that the destination of the packets is uniformly distributed among the users, i. e. each packet is intended for any of the remaining $M-1$ users with the same probability, $1/(M-1)$. At this point nothing is assumed about what protocol is used, other than that it has a known maximum stable throughput [1] given by: $S_p(\omega, n)$ packets/packet-length, where ω is the used bandwidth and n is the number of users. The throughput function, in packets per second, given by $\omega \cdot S_p(\omega, n)$ is asummed to be a continuous and nondecreasing function of ω while a decreasing function of n. These assumptions do not heart the generality of our model. One expects that an increase in the number of users will have as a result an increase of the contention, thus a decrease of the MST. On the other hand an increase of the available bandwidth will have as a result faster transmission, which in turn cannot deteriorate the performance of the system. The above assumptions are actually met in practically all known protocols.

We consider a partition of the M users into k subsets, each with N_i users. Obviously we have

$$\sum_{i=1}^{k} N_i = M$$

[1] We use the standard definition of maximum stable throughput as the supremum of the packet arrival rates for which the system is stable

Communication within each subset is accomplished by means of a fixed, given protocol, which is the same for all subsets. Communication among different subsets is achieved by means of special purpose nodes, the bridge nodes. We assume that there are two bridge nodes in each subsystem:

1. The Receiving Bridge Node (RBN), which receives packets transmitted from neighboring subnetworks and broadcasts them to the users within the subnetwork or to the Transmitting Bridge Node of the subnetwork, depending on whether their destination belongs or not to the subnetwork.

2. The Transmitting Bridge Node (TBN), which receives packets broadcasted from within the subnetwork or from the subnetwork's RBN, that are intended for users in other subnetworks, and broadcasts them to the RBN 's of neighboring subsystems.

For an observer inside a subnetwork the corresponding TBN operates as an output port that broadcasts outside of the subnetwork packets generated by nodes inside it. Similarly the corresponding RBN operates as an input port that receives packets generated outside and intended for nodes inside the subnetwork. On the other hand for an observer outside a subnetwork the corresponding pair of TBN – RBN operates as an intermediate hop in the communication between nodes belonging to different subnetworks.

With respect to the separation of the subsystems there are two possibilities, logical separation or physical separation. In the first case the separation of the sub–networks is achieved by bandwidth assignment, i. e. each subnetwork uses a distinct portion of the overall bandwidth. In the second case the separation of the sub–networks is achieved by space separation. For Packet Radio Networks this can be achieved by geographical dispersion and transmission power control. In the case of cable LAN s each subnetwork can use a different cable channel.

Following a construction similar to the one in [1] we consider three queueing systems that can describe the network of each subgroup of users:

1. The system with the combined aggregate queue $Q_D(i)$ of the packets in the buffers of all users in the i-th subnetwork (this is a conceptual "queue" and consists of the packets generated by all the users in this subsystem). We denote its arrival rate by $A_D(i)$, its departure rate by $D_D(i)$ and its service rate by $T_D(i)$. It is allocated a portion $u_i W$ of the overall bandwidth.

2. The system with the queue $Q_T(i)$ of packets in the buffer of the TBN in subsystem i. We denote its arrival rate by $A_T(i)$, its departure rate by $D_T(i)$ and its service rate by $T_T(i)$. It is allocated a portion $x_i W$ of the overall bandwidth.

3. The system with the queue $Q_R(i)$ of packets in the buffer of the RBN in subsystem i. We denote its arrival rate by $A_R(i)$, its departure rate by $D_R(i)$ and its service rate by $T_R(i)$. It is allocated a portion $v_i W$ of the overall bandwidth.

In general these queueing systems are not "decoupled" and do not operate independently of one another. For each case we need to obtain precise relationships among them.

As a measure of the performance of the system we use the maximum stable throughput (MST), which we define as the supremum of the total packet generation rate (given by $M\lambda$ in our case) for which the above described system is stable. We denote this quantity by S. We characterize this system as stable if for each constituent subset all three queues that describe it are stable. For the stability of a queue we use the traditional definition, i. e. that a queue is stable if it possesses a steady state queue size distribution that yields a finite average delay. In [7] it was shown that the following proposition is true.

Proposition 1 *A queue with arrival rate $\mathcal{A}$ and service rate $\mathcal{T}$ is stable if $\mathcal{A} < \mathcal{T}$ and unstable if $\mathcal{A} > \mathcal{T}$, provided that the interarrival as well as the service times form two stationary and metrically transitive (but not necessarily independent) sequences.*

In a stationary queueing system in steady state (as above) the arrival rate is never less than the departure rate; they are equal if the system is stable.

<u>Note:</u> The assumptions for stationarity and metrical transitivity are certainly very mild ones. In stable systems they are due to the steady state behavior of the system. (The stationarity follows directly by the definition of the steady state behavior, while the metrical transitivity follows by the strongly mixing property that such systems exhibit.) It is also of interest to observe that, using a simple contradiction-type argument, we can obtain the following corollary to Proposition 1:

Corollary 1 *A queueing system with arrival rate $\mathcal{A}$ and service rate $\mathcal{T}$ is stable if $\mathcal{A} < \mathcal{T}$ and unstable if $\mathcal{A} > \mathcal{T}$, provided that stability of it implies that the inter-arrival as well as the service times form two stationary and metrically transitive sequences.*

3 The Directional Ring Topology (DRT)

The first topology to be considered is the Directional Ring Topology, in which all subsets are cyclically arranged on a ring so that any RBN hears transmissions from the TBN of the previous subnetwork only (with subnetwork k preceding subnetwork 1).

We assume that the transmission ranges within a subnetwork and between two successive subnetworks on the ring are such that TBN i can be heard by nodes only in subnetworks $i, i+1$ and $i-1$ (but not by RBN $i-1$). In order to avoid undesired interference between the intra– and inter– subnet traffic we require that the following conditions hold true.

$$(u_i W)\ ,\ (v_i W)\ ,\ (x_i W)\ ,\ (x_{i-1} W)\ :\ \text{non-overlapping} \qquad (1.\ \text{a})$$

$$(u_1 W)\ ,\ (v_1 W)\ ,\ (x_1 W)\ ,\ (x_k W)\ :\ \text{non-overlapping} \qquad (1.\ \text{b})$$

The following inequalities are necessary for satisfaction of the above non-overlapping requirement. They are not sufficient, however.

$$u_i + v_i + x_i + x_{i-1} \leq 1 \quad \text{for i} = 2, \ldots, \text{M}$$
$$\text{and} \quad u_1 + v_1 + x_1 + x_k \leq 1$$

For our purposes the decomposition of the system is completely specified by (k, N, u, v, x) and our objective is to choose those parameters so that the MST of the system is maximized. Note that $\mathbf{u}, \mathbf{v}, \mathbf{x}$ denote the vectors whose components are the bandwidth portions u_i, v_i and x_i respectively while $\mathbf{N}$ denotes the vector $(N_1, \ldots, N_k)$.

In Appendix A it is shown that the MST of the system is given by

$$S = \min_i \; \{ u_i W \frac{M}{N_i} S_p(u_i W, N_i) \; , \; MG(\mathbf{N}) v_i W \; , \; MG(\mathbf{N}) x_i W \} \qquad (2)$$

where

$$G(\mathbf{N}) \triangleq \frac{2(M-1)}{M^2 - \sum_j N_j^2}$$

We can now formulate our objective in terms of a maximization problem, namely

$$\text{(P. 1)} \qquad \max_{(k,\mathbf{N},\mathbf{u},\mathbf{v},\mathbf{x})} S \quad \text{subject to (1)} \; \sum_i N_i = M \; ; \; (2) \; u_i, v_i, x_i \in [0,1] \; ;$$
$$(3) \; \text{Constraint (1) is satisfied}$$

Observe that, given any decomposition $(k, \mathbf{N}, \mathbf{u}, \mathbf{v}, \mathbf{x})$ satisfying the restrictions of problem (P. 1), we can replace x_i by $\hat{x}_i$ and v_i by $\hat{v}_i$, where

$$\hat{x}_i = \hat{v}_i = x = \min_i \{x_i, v_i\}$$

and this replacement will neither violate any of the restrictions in problem (P. 1), nor decrease the Maximum Stable Throughput.

Furthermore for any feasible solution of problem (P. 1) the u_i's must satisfy

$$u_i + 3x \leq 1$$

Observe, however, that, if k is even, then we can replace u_i by $\hat{u}_i = 1 - 3x$, thus adopting the bandwidth allocation of Fig. 2(a). If now k is odd the bandwidth allocation of Fig. 2(a) is not feasible, since it leads to the overlapping of the bands $x_k W$ and $x_1 W$. In this case one may adopt the allocation of Fig. 2(b), that also allows to replace u_i by $\hat{u}_i = 1 - 3x$.

Since now

$$\hat{u}_i \geq u_i \quad \forall \ i$$

it becomes clear that replacing u_i by $\hat{u}_i = 1 - 3x$ will neither decrease the Maximum Stable Throughput nor violate any of the restrictions in problem (P. 1), as it is obvious from Fig. 2(a) and 2(b).

Then problem (P. 1) can be written as

$$\text{(P. 2)} \qquad \max_{(k,\mathbf{N},\mathbf{u},\mathbf{v},\mathbf{x})} S \quad \text{subject to (1)} \sum_i N_i = M \ ; \ (2) \ u_i = u \ \in \ [0,1] \ ; \quad (3) \ v_i = x_i = (1-u)/3$$

where S is given by (2).

Observe now that in problem (P. 2) neither the cost function nor the constraints depend on the order of the subnetworks in the system. Hence the following condition can be added to these constraints without affecting the solution.

$$N_1 \geq N_2 \geq \ldots \geq N_k \tag{3}$$

After having added condition (3) to the restrictions of problem (P. 2) one can verify that the following condition (4) can also be added to the constraints of problem (P. 2) without changing its solution.

$$uW \frac{M}{N_1} S_p(uW, N_1) = MG(\mathbf{N}) \frac{1-u}{3} W \tag{4}$$

The verification is straightforward, by observing that for any decomposition (k, N, $u_i = u$, $v_i = x_i = (1-u)/3$) that satisfies the constraints of problem (P. 2), but not condition (4), there exists a decomposition ($\tilde{k} = k$, $\tilde{\mathbf{N}} = \mathbf{N}$, $\tilde{u}_i = \tilde{u}$, $\tilde{v}_i = \tilde{x}_i = (1-\tilde{u})/3$ that yields a higher Maximum Stable Throughput and that satisfies both the constraints of problem (P. 2) and condition (4).

The following Lemma 1 concerns the construction of the optimal vector **N**.

Lemma 1 *The vector* **N** *that solves the problem, resulting from problem (P. 2) after the addition of conditions (3) and (4) has the following structure:*

$$N_1 = \ldots = N_{k-1} \geq N_k$$

__Proof of Lemma 1:__ To show the optimality of the above construction of the vector **N** we observe that for any partition $\mathbf{N} = (N_1, \ldots, N_k)$ of the users, for which N_1 is fixed and given, [2] we can increase the maximum stable throughput by increasing a larger N_i at the expense of a smaller N_j (which can be easily shown by considering a partition in which N_i and N_j (with $N_i > N_j$) have been respectively replaced by $N_i + 1$ and $N_j - 1$ while the remaining N_k 's, including N_1, do not change and showing that the maximum stable throughput increases). Thus N_2 should be made as close to N_1 (the largest element that is fixed) as possible. If the total number of users M is greater than $2N_1$, indeed N_2 will be equal to N_1. Then we repeat the argument for N_3 and so on. Thus the first $k-1$ elements N_i end up being equal to each other and to the given N_1, while N_k may possibly be less depending on the values of M and N_1.

The proof of Lemma 1 is thus completed.

In view of Lemma 1, the original problem reduces to

$$\text{(P. 3)} \qquad \max_{N_1 \in \{1,\ldots,M\}} uW\frac{M}{N_1}S_p(uW, N_1) \quad \textit{where}$$

(1) **N** constructed as above ;

(2) u is the unique solution in $(0, 1]$ to equation (4) [3]

Problem (P. 3) is a constrained maximization problem over one variable that takes values in a finite set. An exhaustive search therefore suffices to completely characterize the solution, once the function $S_P(\cdot,\cdot)$ is specified. We do not address here the question of computational complexity and of whether this is the most

[2]Recall that N_1 is by definition greater than or equal to all other N_i s ($i > 1$).

[3]It is straightforward to show that indeed equation (4) has a unique solution in $(0, 1]$

efficient way of solving the original maximization problem. It certainly has the attractive feature of linear complexity to M.

4 The Fully Connected Network (FCN)

We now consider a Fully Connected topology of subnetworks, in which every TBN can be heard by the RBN s of all other subnetworks (see Fig. 3). In this case intercommunication between any two subsystems is accomplished in a single hop (in contrast to the case of the DRT).

In order to avoid undesirable interference, and similarly to the case of the Directional Ring Topology, we require that, for all i, $u_i W$ and $v_i W$ do not overlap either among themselves or with any of the $x_j W$'s (for $j = 1, \dots, k$). Furthermore no two $x_j W$'s may overlap (see Fig. 3). The following condition is necessary for such a bandwidth allocation:

$$u_i + v_i + \sum_n x_n \leq 1 \tag{5}$$

Obviously superior throughput can be achieved if inequality (5) is satisfied with equality. Therefore throughout the remaining of this section we will require that condition (5) is satisfied with equality.

Following the same procedure as in the case of the DRT we can express the MST of the FCN as

$$S = \min_i \ \{ u_i W \frac{M}{N_i} S_p(u_i W, N_i) \ , \ \frac{M(M-1)}{N_i(M-N_i)} v_i W \ , \ \frac{M(M-1)}{N_i(M-N_i)} x_i W \}$$

Our objective is to choose the parameters $k, \mathbf{N}, \mathbf{u}, \mathbf{v}, \mathbf{x}$ of the FCN in order to maximize its maximum stable throughput; this maximization can be formulated in terms of the following problem:

$$\text{(P. 4)} \qquad \max_{(k,\mathbf{N},\mathbf{u},\mathbf{v},\mathbf{x})} S \quad \text{subject to (1)} \ \sum_i N_i = M \ ;$$

$$\text{(2)} \ u_i, v_i, x_i \in [0,1] \ ; \ \text{(3)} \ u_i + v_i + \textstyle\sum_n x_n = 1$$

Using arguments similar with the ones in the previous section we can show that the following conditions can be added to the constraints of problem (P. 4) without changing its solution:

$$N_1 \geq N_2 \geq \ldots \geq N_k$$

$$v_i = x_i$$

$$u_i W \frac{M}{N_i} S_p(u_i W, N_i) \ \geq \ \frac{M(M-1)}{N_i(M-N_i)} x_i W$$

$$\frac{M(M-1)}{N_i(M-N_i)} x_i W : \text{ constant for all } i$$

Combining the above conditions with the restrictions of problem (P. 4) we can reduce the problem to:

$$\text{(P. 5)} \qquad \max_{\mathbf{N},x} \frac{M(M-1)}{M^2 - \sum_n N_n^2} xW \quad \text{subject to } (1) \sum_i N_i = M \ ;$$

$$(2)\ N_1 \geq N_2 \geq \ldots \geq N_k \ ; \qquad (3)\ x\phi_i \in [0,1] \ \forall i \ ;$$

$$(4)\ \frac{M(M-1)}{M^2 - \sum_n N_n^2} xW \ \leq \ (1-\phi_i x) W S_p(W(1-\phi_i x), N_i) \ \forall i$$

where

$$\phi_i \triangleq 1 + \frac{N_i(M-N_i)}{M^2 - \sum_n N_n^2} \tag{6}$$

By a straightforward comparison it follows that for all $x, \mathbf{N}$, satisfying the restrictions of problem (P. 4), we have

$$\min_i f_i(x, \mathbf{N}) = f_1(x, \mathbf{N}) \tag{7}$$

where

$$f_i(x, \mathbf{N}) \triangleq (1-\phi_i x) W S_p(W(1-\phi_i x), N_i) \frac{M}{N_1} \tag{8}$$

and ϕ_i is given by equation (6).

Furthermore it is straightforward to show that

$$\max_i \phi_i = \phi_1 \tag{9}$$

Equations (7) and (9) imply that constraints (3) and (4) of problem (P. 5) need only be examined for $i = 1$. Observe furthermore, that, for $i = 1$, we can replace the inequality with equality in constraint (4) of problem (P. 5), thus obtaining the following equation

$$\frac{M(M-1)}{M^2 - \sum_n N_n^2} xW = f_1(x, \mathrm{N}) \tag{10}$$

where $f_1(x, \mathrm{N})$ is given by equation (8) (for $i = 1$). This replacement cannot decrease the maximum stable throughput. Additionally the resulting equation has a unique solution in $(0, 1/\phi_1]$. In view of the above discussion we can further reduce problem (P. 5) to:

(P. 6) $\quad \max_{\mathbf{N},x} \frac{M(M-1)}{M^2 - \sum_n N_n^2} xW \quad$ subject to

(1) $\sum_i N_i = M$; $\qquad$ (2) $N_1 \geq N_2 \geq \ldots \geq N_k$;

(3) x is the unique solution in $(0, 1/\phi_1]$ to equation (10)

The final step in reducing the initial optimization problem to a simple form is to determine the optimal form of the vector **N**. In Appendix B it is shown that for given $N_1 \in \{1, \ldots M-1\}$ the vector that solves problem (P. 6) can be uniquely constructed as follows:

N_1 users are assigned to each LAN until they are either exhausted or reduced to a final remainder of $0 < n < N_1$. In this case these n users are assigned to one (last) LAN.

In view of this last assertion problem (P. 6) reduces to:

(P. 7) $\quad \max_{N_1 \in \{1,\ldots,M-1\}} \frac{M(M-1)}{M^2 - \sum_n N_n^2} xW$ where : (1) $N_1 = N_2 = \ldots = N_{k-1} \geq N_k$;

(2) x is the unique solution in $(0, 1/\phi_1]$ to equation (10)

Problem (P. 7) is a constrained maximization problem over one variable that takes values in a finite set. A complete characterization of its solution can be obtained if the function $S_p(\cdot, \cdot)$ is given. Again the complexity question is not addressed.

In Appendix C a comparison between the DRT and the FCN is provided. In particular it is shown that for any M and W the optimal value of the maximum stable throughput for a Network operating as a FCN of LAN s is greater than the one achieved under the DRT. We may also observe that

- In the DRT the utilization of the bandwidth assigned to a single TBN or RBN is inefficient since these nodes carry all the transit traffic in addition to that intended for the LAN in which they belong, in contrary to the FCN, in which the TBN s and RBN s carry only the traffic intended for the subnetwork in which they belong.

- In the DRT the utilization of the bandwidth assigned to all TBN s is more efficient than the one in the FCN, since in the former overlapping of bandwidths assigned to different TBN s is allowed.

We can therefore explain the performance deficit of the DRT as caused by the inefficient routing of the packets that cannot be balanced by the surplus that the frequency reuse induces.

Under the reasonable assumption that

$$S_p(\omega, 1) \equiv 1 \quad \text{packets/packet-length}$$

it can be shown that for $N_1 = 1$, hence $\mathbf{N} = (1, \ldots, 1)$, the value of x that satisfies the restrictions of problem (P. 6) is

$$x = \frac{M}{M+2}$$

which leads to

$$S(N_1 = 1) = \frac{M}{M+2} W \quad \text{packets/sec}$$

(The proof is omitted since it only involves straightforward computations based on problem (P. 7) after having fixed $N_1 = 1$.)

Hence we have that

$$\lim_{M \to \infty} S(N_1 = 1) = W \text{ packets/sec}$$

which implies that as M becomes large enough the optimal choice of N_1 is 1, since the value of W pack/sec for the MST is the supremum over all possible values of MST. As a result it is obvious that when considering only the MST as performance measure and when M is large enough the performance of the FCN is superior to the performance of any other topology. This is not surprising, since in that case the system approaches a single FDMA-like set of users for which the maximum stable throughput is known to be equal to W.

5 Applications

We apply the previous results to the case of two systems, namely a symmetric slotted ALOHA and a CSMA/CD system. For the former we evaluate the optimal decomposition for various numbers of users and present simulation results for the delay performance of the decomposed systems. For the latter we show how the bandwidth affects the optimal decomposition. As the numerical results indicate an increase of the throughput to very large values results in an extremely poor performance of the baseline system (see also [10]), however it does not affect the performance of the optimally decomposed one.

5.1 Symmetric Slotted ALOHA

Under ALOHA each user transmits with the same probability p, which is optimally chosen according to the number n of terminals in the subnetwork (i. e. $p = 1/n$). The MST function of this protocol can be easily calculated to be:

$$S_p(\omega, n) = (1 - \frac{1}{n})^{n-1} \text{ packets/packet-length}$$

We assume a number of M terminals communicating as described in section 2 over a multiple access channel of total bandwidth 1 packets/packet-length. For both topologies it can be easily shown that for all values of M the unique optimal number of subnetworks is equal to M. The extremity of this result can be attributed to the very fast deterioration in the throughput performance of the slotted ALOHA system as the number of users increases.

The actual value of the optimal MST for these two topologies as well as of the baseline system is shown in Fig. 4 as a function of M. As expected, $\lim_{M\to\infty} S_{FCN} = 1$ and $\lim_{M\to\infty} S_{BLS} = 1/e$ We can also see that for the ALOHA protocol under the DRT configuration we have $\lim_{M\to\infty} S_{DRT} = 2/3$. (This is easy to be shown, in view of the fact that the optimal decomposition of the M-user slotted ALOHA system, following the DRT topology, assigns one user to each subnetwork. However we cannot give any physical interpretation to it.)

The extremity in the optimal solution, for the case of th M-user slotted ALOHA system, renders the delay analysis of the decomposed system necessary. For this purpose we have considered to systems, a small one, with 6 users, and a larger one, with 15 users. Using simulation we have obtained the throughput–delay characteristics for both the baseline system and for the two topologies (FCN and DRT), for various numbers of subnets. The results are shown in figures 5 and 6. As we can see, in addition to exhibiting a better throughput performance, the FCN topology also exhibits a better delay performance, than the DRT topology with the same number of subnets. We can also observe that, for each topology, the delay performance shows an expected deterioration, for low intensity traffic, as the number of subnets increases.

5.2 CSMA/CD System

We consider here a slotted CSMA/CD system with M identical users and make the following assumptions:

Packets have constant length of T kbits.

The distance between two consecutive terminals is constant and equal to b m.

The detection of the energy level in the channel requires time a sec.

Then the duration of an idle period or a collision will be:

$$\gamma = a + \frac{b(n-1)}{c} \text{ sec}$$

where $c = 3 \cdot 10^5$ km/sec, is the speed of light. Expressed in slots γ becomes:

$$\gamma = (a + \frac{b(n-1)}{c})\frac{\omega}{T} \text{ slots}$$

where ω is the available bandwidth.

It can be shown that the throughput function for this protocol is given by

$$\S_p(\omega, n) = \frac{(1 - 1/n)^{n-1}}{\gamma + (1-\gamma)(1 - 1/n)^{n-1}}$$

For different values of the parameters M, T, b and a of this system we solved the optimization problems, and obtained the optimal DRT and FCN decompositions. The results are illustrated in figures 7 to 12. In particular, in figures 7 and 8 we show how the optimal number of subnets, N_{opt} and the optimal throughput vary with the total bandwidth W, for the case of Directional Ring and Fully Connected Network respectively, for a system with 60 users. The remaining parameters were chosen to be $T = 1$ $kbit$, $b = 300$ m and $a = 1$ μ sec. As we can see for very small bandwidth (100 $kbits$) $N_{opt} = 1$ and as the bandwidth increases N_{opt} increases as well. In figures 9 and 10 similar curves are shown for the case of a system with 6 users. In figures 11 and 12 curves of the optimal throughput vs. the total number of the users are shown for the DRT and the FCN respectively and for different values of the parameters W, b and a. As we expect, for all values of the parameters, the throughput of the Fully Connected Network converges to 1.

6 Conclusions

We considered the problem of improving the throughput performance of a multiaccess channel with a finite set of buffered terminals operating under a given protocol by partitioning that set into smaller subsets that are linked by dedicated links. An important question concerns the topology of the subnetwork interconnection as well as the consequent routing policy. Two topologies were considered, namely the DRT and the FCN. The reasons behind these choices can be found in the simplicity of both topologies that renders the throughput analysis tractable. Certain applications might require more complex topologies as well as additional constraints, regarding for example the number of subnets or the number of users in each subnet. Such considerations are beyond the scope of this paper. It should be noted however that constraints, like the ones mentioned above can be easily incorporated in the optimization problems, that were considered here. Physical rather than logical separation among the users was assumed, that can be achieved by suitable control of the transmission power of each node.

For the two topologies under consideration exact expressions for the MST were derived and the problem of the optimal choice of the system parameters was formulated and analyzed. The analysis led to numerical maximization problems over finite sets. Furthermore it was shown that as the number of users approached infinity the performance of the FCN approached the performance of a FDMA system, which, in terms of maximum stable throughput, is superior to all others. This last observation shows that the MST is not a totally satisfactory measure for the performance of a multiaccess system. Instead delay and other performance measures should also be considered. For the case of a symmetric slotted ALOHA system we have obtained a delay performance evaluation of the decomposed system, using simulation methods.

A Derivation of the Maximum Stable Throughput for the DRT

Here we prove the following Proposition

Proposition 2 *The maximum stable throughput of the DRT is given by*

$$S = \min_i \ \{ u_i W \frac{M}{N_i} S_p(u_i W, N_i) \ , \ MG(\mathrm{N}) v_i W \ , \ MG(\mathrm{N}) x_i W \}$$

Proof of Proposition : We start by showing that the following inequalities hold for all i

$$A_R(i) \leq \frac{\lambda}{2(M-1)} (M^2 - \sum_i N_i^2) \tag{11}$$

$$A_T(i) \leq \frac{\lambda}{2(M-1)} (M^2 - \sum_i N_i^2) \tag{12}$$

and that they are satisfied with equality if the system is stable. (Recall that $A_T(i)$ is the arrival rate in the queueing system of the TBN in subnet i, $Q_T(i)$, while $A_R(i)$ is the arrival rate in the queueing system of the RBN in subnet i, $Q_R(i)$.)

In steady state we have

$$D_D(i) \leq A_D(i) = N_i \lambda \tag{13}$$

A packet exiting $Q_D(j)$ will enter $Q_T(i)$ with the following probability:

$$\begin{aligned} & \Pr\{ \text{ packet enters } Q_T(i) \ / \ \text{ it exited } Q_D(j) \} \\ = \ & \Pr\{ \text{ packet generated by user in subnet j must go through subnet i} \} \\ = \ & \begin{cases} 0 & \text{if } j = i+1 \\ (N_{i+1} + \ldots + N_{j-1})/(M-1) & \text{if } j = i+2, \ldots, M \\ (N_{i+1} + \ldots + N_M)/(M-1) & \text{if } j = 1 \\ (N_1 + \ldots + N_{j-1} + N_{i+1} + \ldots + N_M)/(M-1) & \text{if } j = 2, \ldots, i \end{cases} \end{aligned} \tag{14}$$

The reason is that a packet generated in subnet j will enter $Q_T(i)$ iff its destination is in a subnet positioned after subnet i and before subnet j in the clockwise direction.

Let $a_i(j)$ be the flow rate of packets that enter $Q_T(i)$ and that have started from $Q_D(j)$ $(j = 1, \ldots, k)$. We then have:

$$a_i(j) = D_D(j) \Pr\{ \text{ packet enters } Q_T(i) \ / \ \text{ it has exited } Q_D(j) \} \tag{15}$$

Substituting for $D_D(j)$ in equation (16) the expression given in inequality (14) and for the quantity Pr{ packet enters $Q_T(i)$/ it has exited $Q_D(j)$} the expression given in Eq. (15) we obtain

$$\left.\begin{array}{lcl} a_i(i+1) & = & 0 \\ a_i(j) & \geq & \begin{cases} \lambda N_j(N_{i+1}+\ldots+N_{j-1})/(M-1) & \\ \qquad \text{if } j=i+2,\ldots,M & \\ \lambda N_j(N_{i+1}+\ldots+N_M)/(M-1) & \\ \qquad \text{if } j=1 & \\ \lambda N_j(N_1+\ldots+N_{j-1}+N_{i+1}+\ldots+N_M)/(M-1) & \\ \qquad \text{if } j=2,\ldots,i & \end{cases} \end{array}\right\} \quad (16)$$

However we have that

$$A_T(i) = \sum_j a_i(j) \quad (17)$$

Using inequality (17) in equation (18) we obtain

$$A_T(i) \leq \frac{\lambda}{2(M-1)}\sum_n \sum_{m\neq n} N_n N_m = \frac{\lambda}{2(M-1)}(M^2 - \sum_j N_j^2)$$

Furthermore

$$A_R(i) = D_T(i-1) \leq A_T(i-1) \leq \frac{\lambda}{2(M-1)}(M^2 - \sum_j N_j^2)$$

If the system is stable we shall have that $A_T(i) = D_T(i)$ and $A_D(i) = D_D(i)$. Then all inequalities in the previous analysis will be satisfied with equalities and so will inequalities (12) and (13) .

We can now proceed to derive an expression for the maximum stable throughput. Using inequalities (12) and (13) we can show that the system is stable iff the following conditions are satisfied for all i:

$$N_i\lambda < u_i W S_p(u_i W, N_i) \quad (18)$$

$$\frac{\lambda}{2(M-1)}(M^2 - \sum_j N_j^2) < v_i W \quad (19)$$

$$\frac{\lambda}{2(M-1)}(M^2 - \sum_j N_j^2) < x_i W \quad (20)$$

We first assume that the system is stable. Then inequalities (12) and (13) are satisfied with equality and together with Proposition 1 they imply conditions (20) and (21). Condition (19) obviously follows from the definition of the maximum stable throughput.

We assume now that conditions (19), (20) and (21) are satisfied for all i. Then it is clear that the condition for stability of Proposition 1 is satisfied for queues $Q_T(i)$ and $Q_R(i)$. Furthermore by the definition of the maximum stable throughput it follows immediately that queues $Q_D(i)$ are also stable. Hence the system is stable.

The maximum stable throughput of the DRT can then be computed using conditions (19), (20) and (21) in a straightforward fashion.

Inequalities (12) and (13) imply the following corollary

Corollary 2 *The transmitting as well as the receiving bridge nodes carry the same amount of traffic when the system is stable.*

B The optimal construction of the vector N for the FCN

We prove here the following Proposition

Proposition 3 *For given $N_1 \in \{1, \ldots, M\}$ the vector* N*, that solves problem (P. 5) can be uniquely constructed as follows:*
N_1 users are assigned to each subnet until they are either exhausted or reduced to a final remainder $0 < n < N_1$. In this case these n users are assigned to one (last) subnet.

<u>**Proof of Proposition :**</u> Fix $N_1 \in \{1, \ldots, M-1\}$ [4]

Define

$$z \triangleq \frac{M(M-1)}{M^2 - \sum_n N_n^2} \tag{21}$$

The quantity z takes values in the set

$$\mathcal{Z}_{N_1} \triangleq \{ z \in \mathcal{R} : z = \frac{M(M-1)}{M^2 - \sum_n N_n^2} \,;\, \sum_n N_n = M \,;\, N_1 \geq N_2 \geq \ldots \geq N_k \,;$$

$$N_1 \in \{1, \ldots, M-1\} : fixed\}$$

We have then that

$$\frac{N_1(M-N_1)}{M^2 - \sum_n N_n^2} = \frac{N_1(M-N_1)}{M(M-1)} \frac{M(M-1)}{M^2 - \sum_n N_n^2} = cz \tag{22}$$

where

$$c \triangleq \frac{N_1(M-N_1)}{M(M-1)} \tag{23}$$

and c is fixed since both N_1, M are fixed.

Then by substituting from equations (23) and (24) in equation (8) we obtain

$$f_1(x, \mathbf{N}) = [1 - x - cxz] W \frac{M}{N_1} S_p([1 - x - cxz]W, N_1) \tag{24}$$

Substituting now from equations (25) and (24) in equation (10) we obtain

$$[1 - x - cxz] W \frac{M}{N_1} S_p([1 - x - cxz]W, N_1) = xzW \tag{25}$$

Differentiating equation (26) and after some computations we obtain

$$\frac{dx}{dz} = -\frac{x}{z} \; \frac{cB + W}{Bz^{-1} + (cB + W)} \tag{26}$$

where

$$\begin{aligned} B \triangleq \quad & W \frac{M}{N_1} S_p([1 - x - cxz]W, N_1) + \\ + \quad & W^2 (1 - x - cxz) \frac{M}{N_1} \frac{d}{da} S_p(a, N_1)|_{a=(1-x-cxz)W} \\ > \quad & 0 \end{aligned} \tag{27}$$

[4] We do not consider the case $N_1 = M$ since the vector $\mathbf{N}$ is then uniquely defined

But since $z > 0$ and $B > 0$ we have

$$\frac{cB + W}{Bz^{-1} + (cB + W)} < 1$$

and by equation (27) we have

$$\frac{dx}{dz} > -\frac{x}{z}$$

which leads to

$$\frac{d}{dz}(Wxz) > 0 \qquad (28)$$

Observe now that the constraints of (P. 7) actually specify x uniquely as a function of z . Hence (P. 7) can be written as

$$\text{(P. 10)} \qquad \max_{z \in \bigcup_{N_1=1}^{M-1} \mathcal{Z}_{N_1}} Wzx(z)$$

By fixing $N_1 \in \{1, \ldots, M-1\}$ problem (P. 10) becomes:

$$\text{(P. 11)} \qquad \max_{z \in \mathcal{Z}_{N_1}} Wzx(z)$$

Equation (29) implies that for fixed N_1 and for $z \in \mathcal{Z}_{N_1}$ the function $Wzx(z)$ is strictly increasing in z . Hence (P. 11) is solved by

$$\max_{z \in \mathcal{Z}_{N_1}} z$$

which can be written as

$$\text{(P. 12)} \qquad \max_{\mathbf{N}} \frac{M(M-1)}{M^2 - \sum_n N_n^2} \quad \text{subject to} \quad \begin{cases} \sum_n N_n = M \\ N_1 \in \{1, \ldots, M-1\} : \text{ given} \end{cases}$$

The remainder of the proof is identical to the proof for the optimal construction of the vector **N** for the DRT.

C Comparison between the DRT and the FCN

We prove here the following proposition

Proposition 4 *For any M and W the optimal value of S for a network operating as a FCN of subnets is greater than the one achieved under the DRT.*

Proof of Proposition: We start by observing that if in problem (P. 3) we replace the construction of the vector N by the construction used in problem (P. 7) the optimal value of S will not decrease. Furthermore restriction (2) of problem (P. 3) implies that the cost function can be replaced by

$$\frac{M}{3}G(\mathrm{N})(1-u)W$$

Thus, instead of comparing the solutions of problems (P. 3) and (P. 7), we can compare the solution of problem (P. 7) with the solution of the following one:

$$\text{(P. 13)} \qquad \max_{N_1 \in \{1,\ldots,M-1\}} \frac{2M(M-1)}{M^2 - \sum_n N_n^2} yW \text{ subject to}$$

(1) $N_1 = N_2 = \ldots = N_{k-1} \geq N_k$

(2) y is the unique solution in $(0,1]$ to

$$(1-3y)W\frac{M}{N_1}S_p((1-3y)W, N_1) = \frac{2M(M-1)}{M^2 - \sum_n N_n^2} yW$$

It can be shown (after some simple calculations that we omit here) that if x, y are the *unique* values that satisfy restriction (2) of problems (P. 7) and (P. 13) respectively then

$$x \geq 2y \tag{29}$$

where inequality (30) is satisfied with equality iff $k \leq 2$.

Inequality (30) implies that

$$\frac{M(M-1)}{M^2 - \sum_n N_n^2} xW \geq \frac{2M(M-1)}{M^2 - \sum_n N_n^2} yW \; ; \quad \forall\, k \tag{30}$$

where inequality (31) is satisfied with equality iff $k \leq 2$.

The proof of Proposition 4 then follows immediately.

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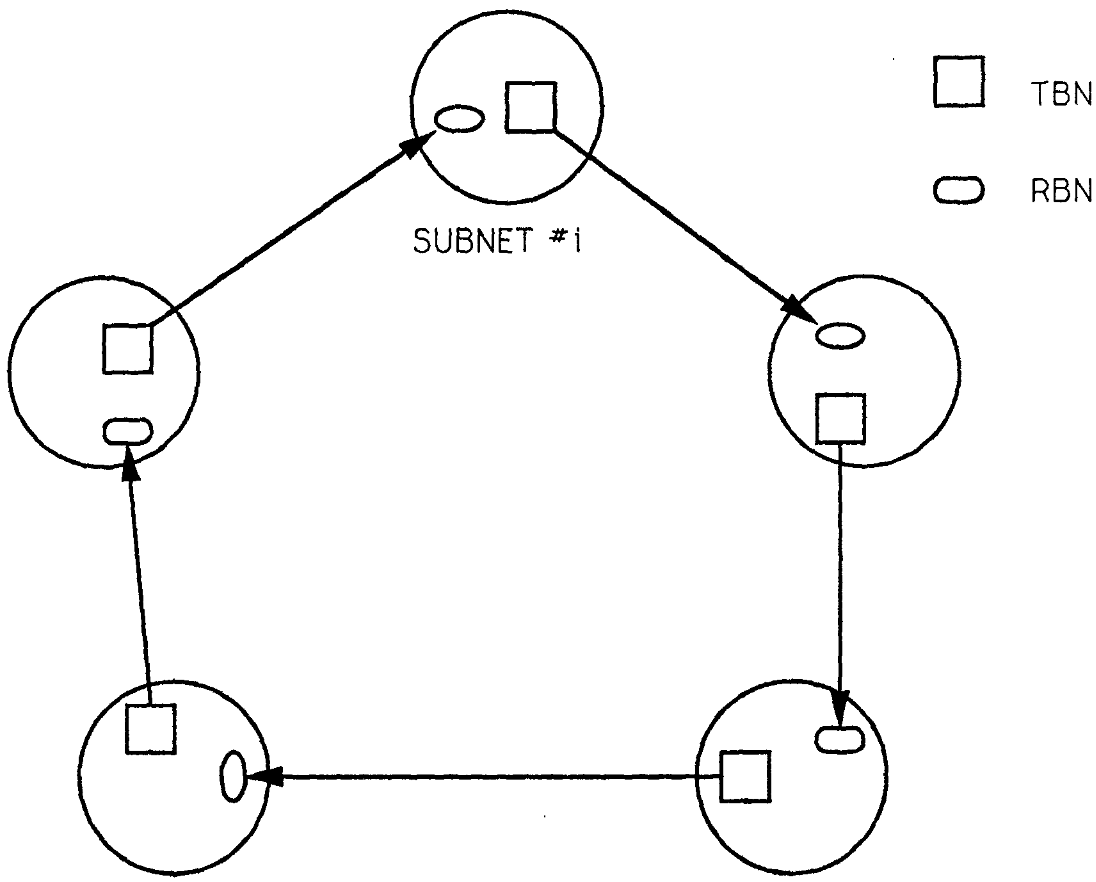


Figure 1.: The Directional Ring Topology

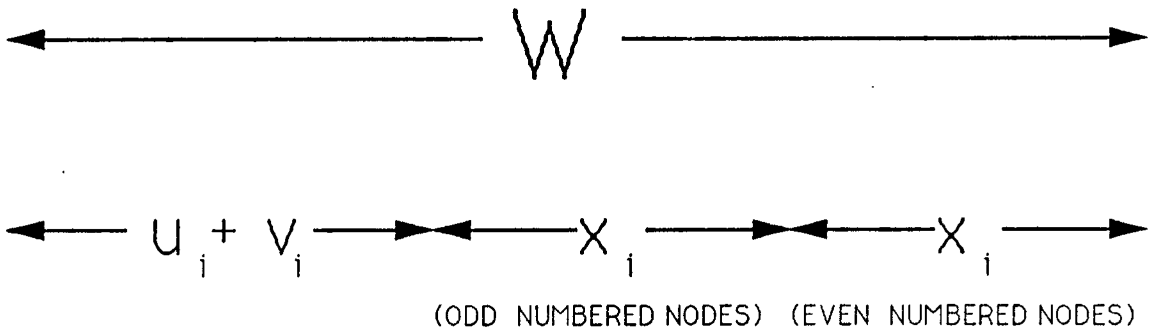


Figure 2(a): Band allocation for DRT with even number of subnets

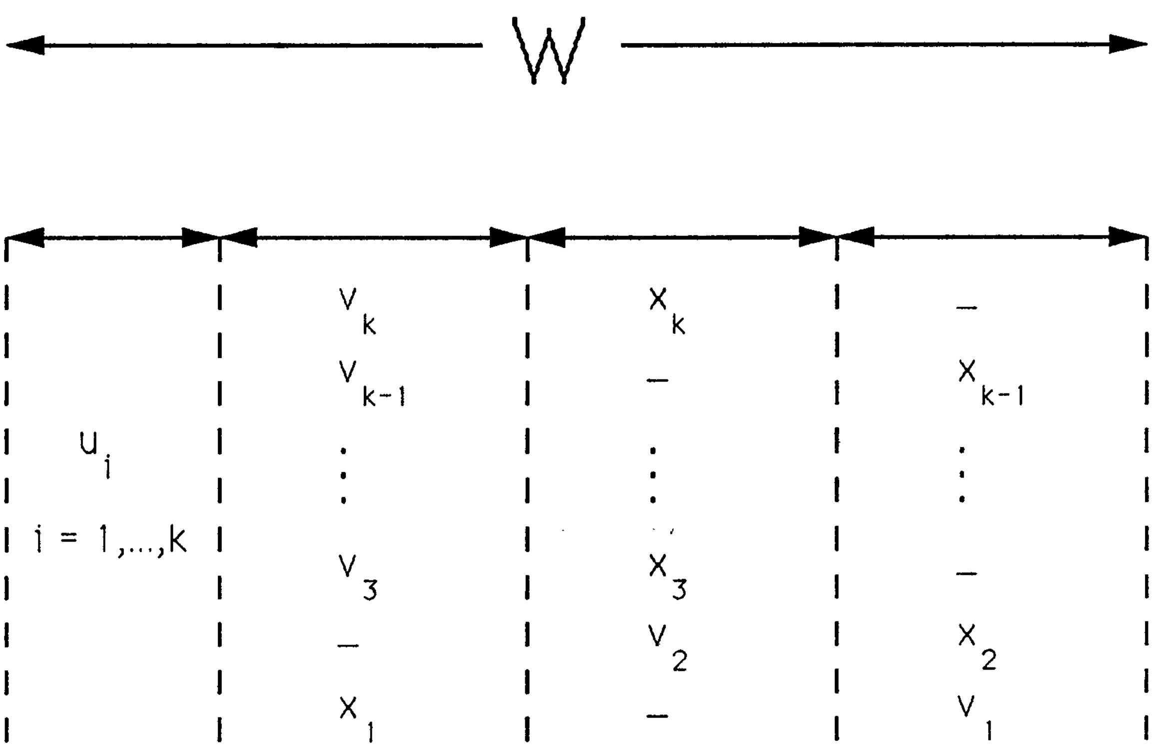


Figure 2(b): Band allcation for DRT with odd number of subnets

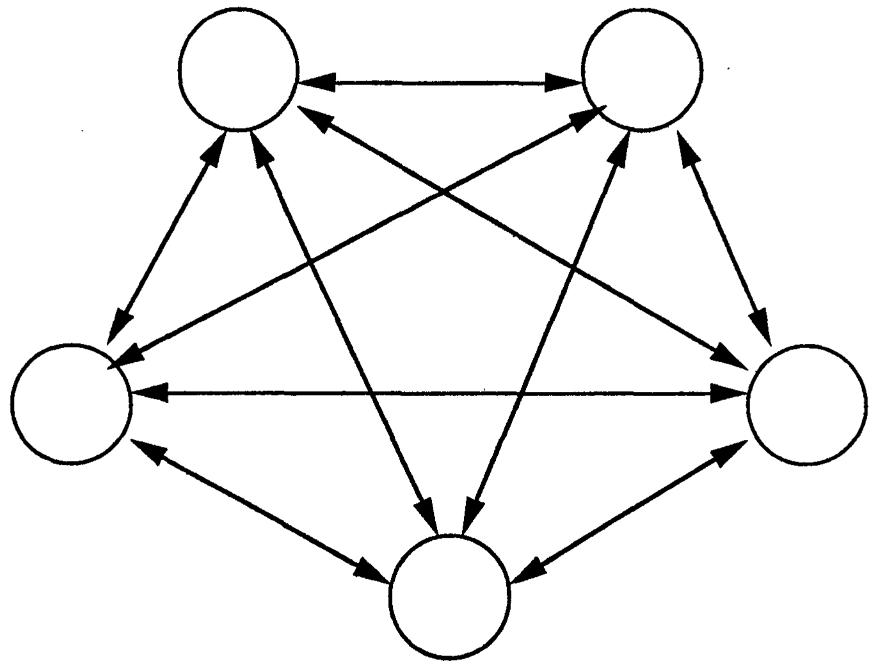


Figure 3(a): The Fully Connected Network

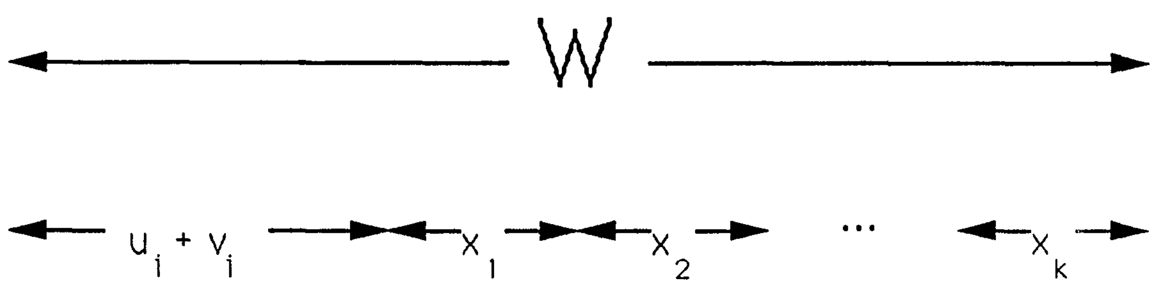


Figure 3(b): Band allocation for FCN

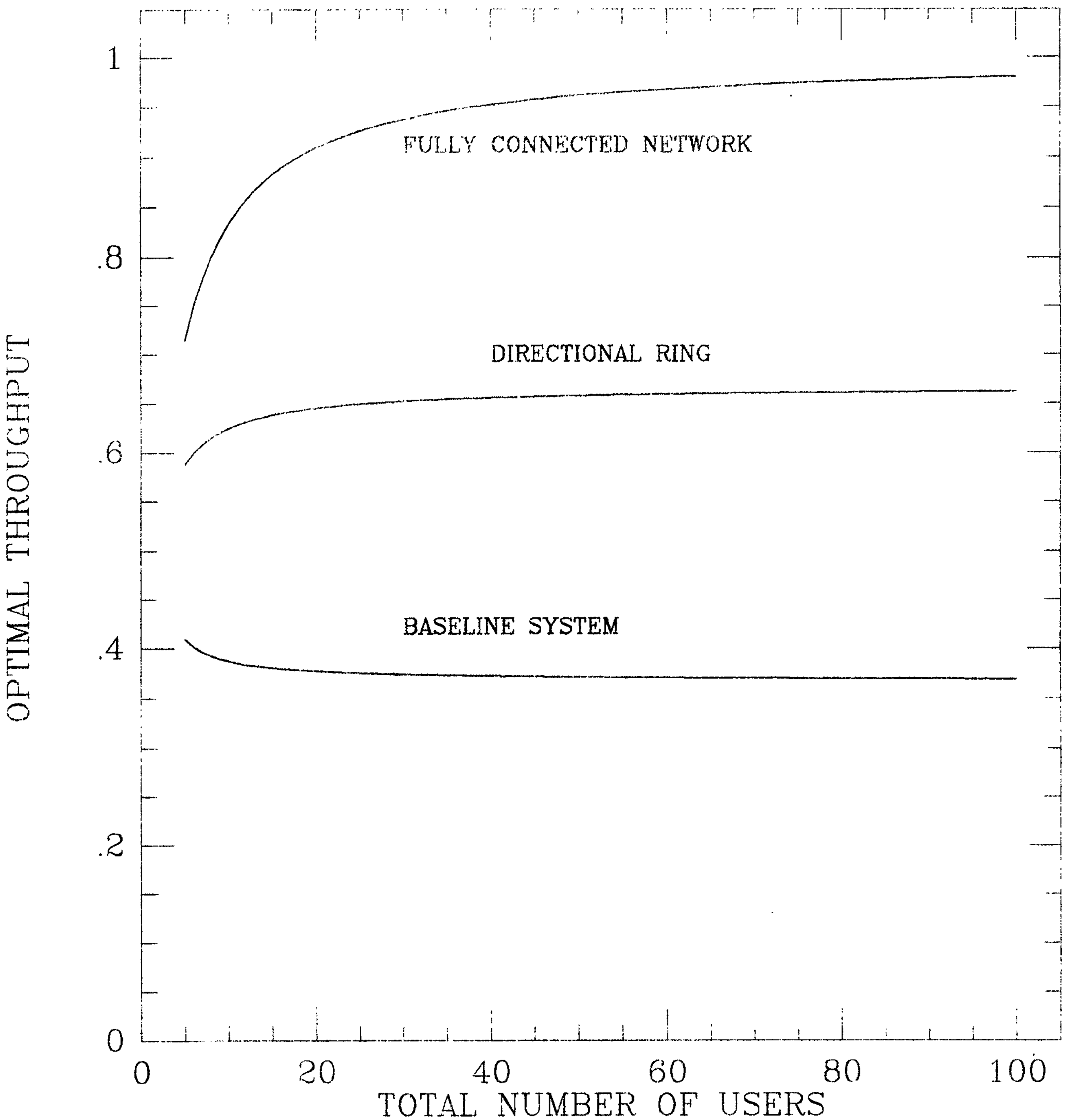


Figure 4: Optimal Throughput vs. total number of Users for slotted ALOHA

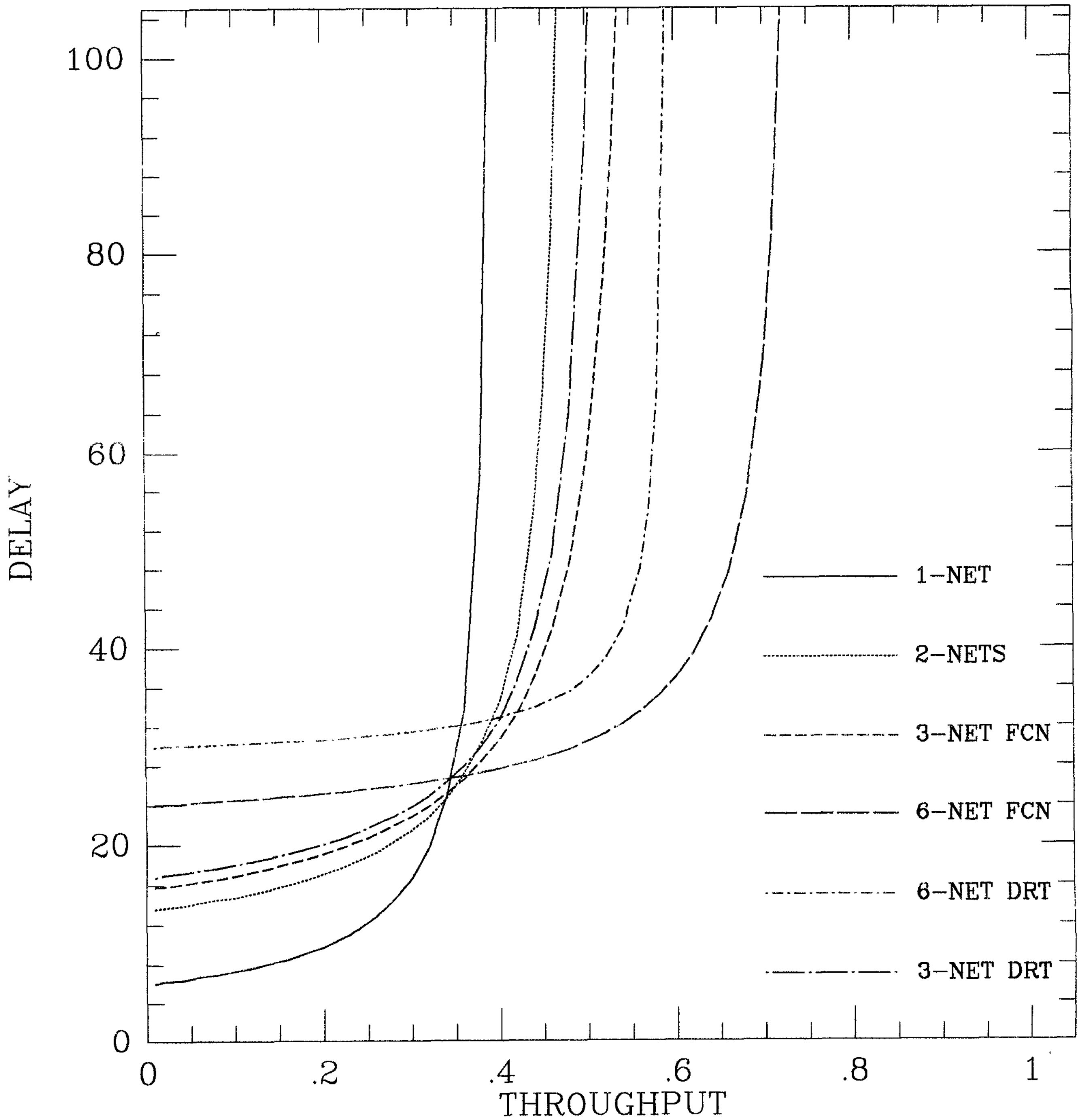


Figure 5: Throughput – Delay Caracteristics for 6–user slotted ALOHA system.

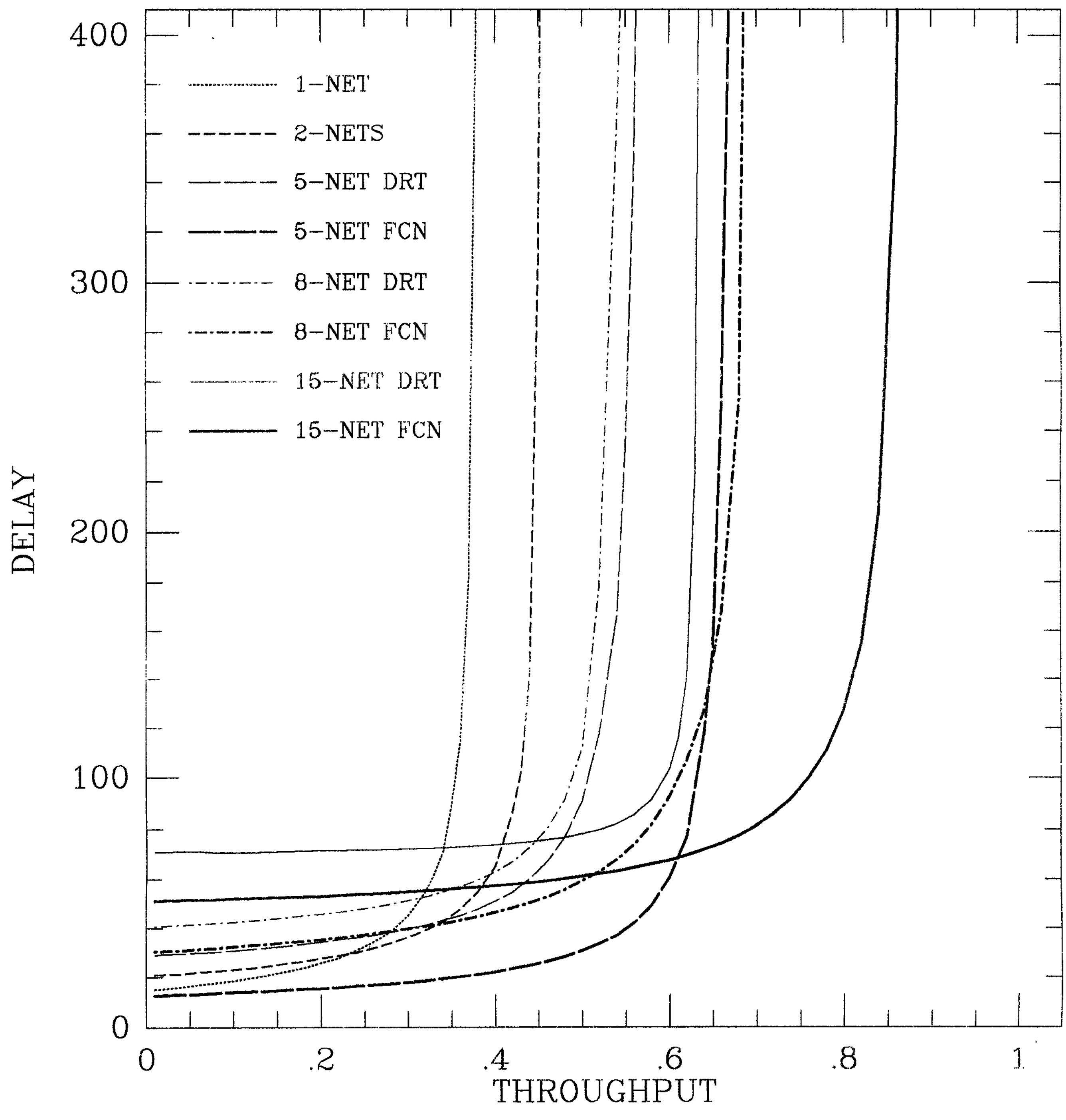


Figure 6: Throughput – Delay Caracteristics for 15-user slotted ALOHA system.

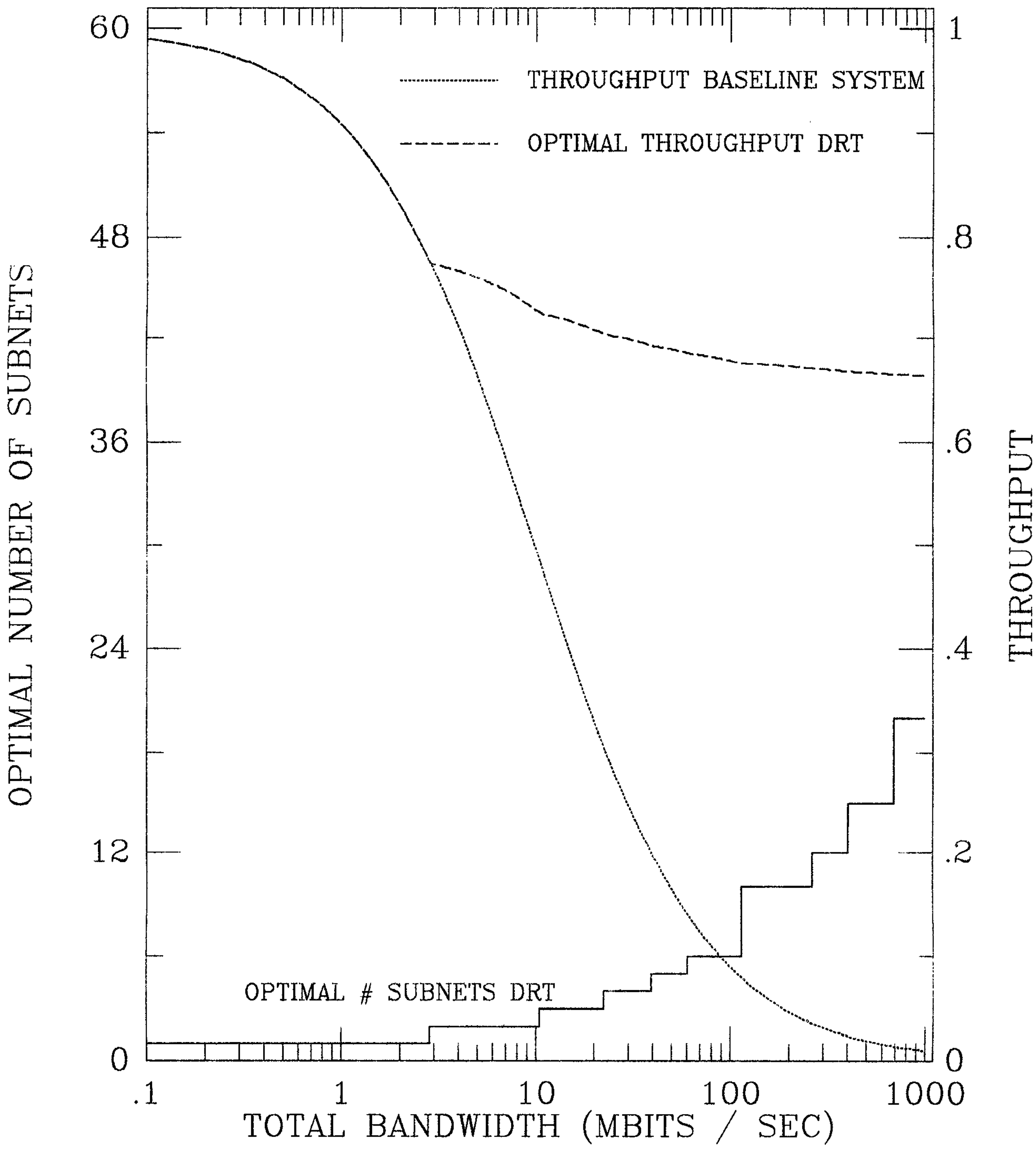


Figure 7: Optimal Number of Subnets and Optimal Throughput vs. Total Bandwidth for CSMA/CD System with 60 users.

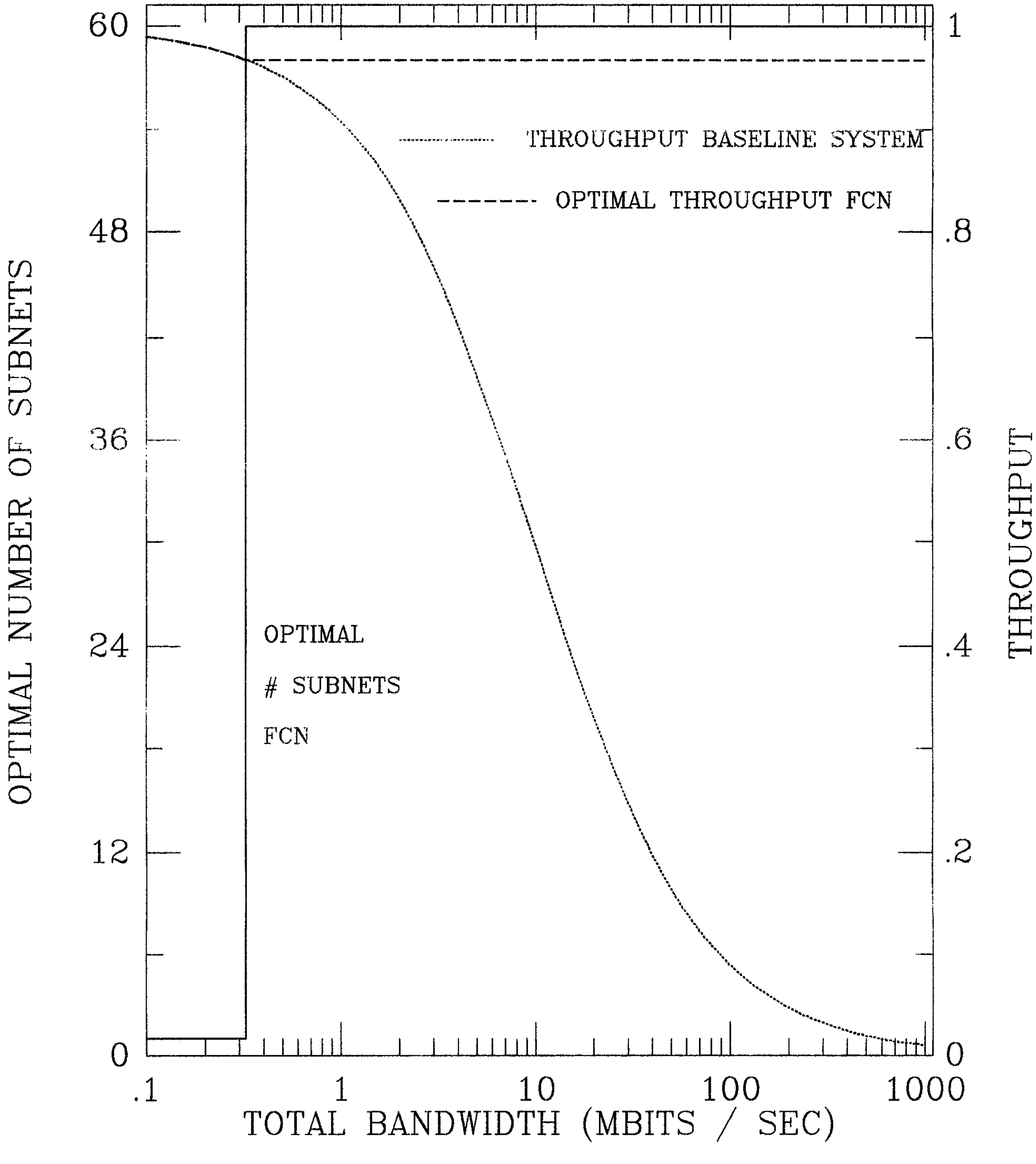


Figure 8: Optimal Number of Subnets and Optimal Throughput vs.

Total Bandwidth for CSMA/CD System with 60 users.

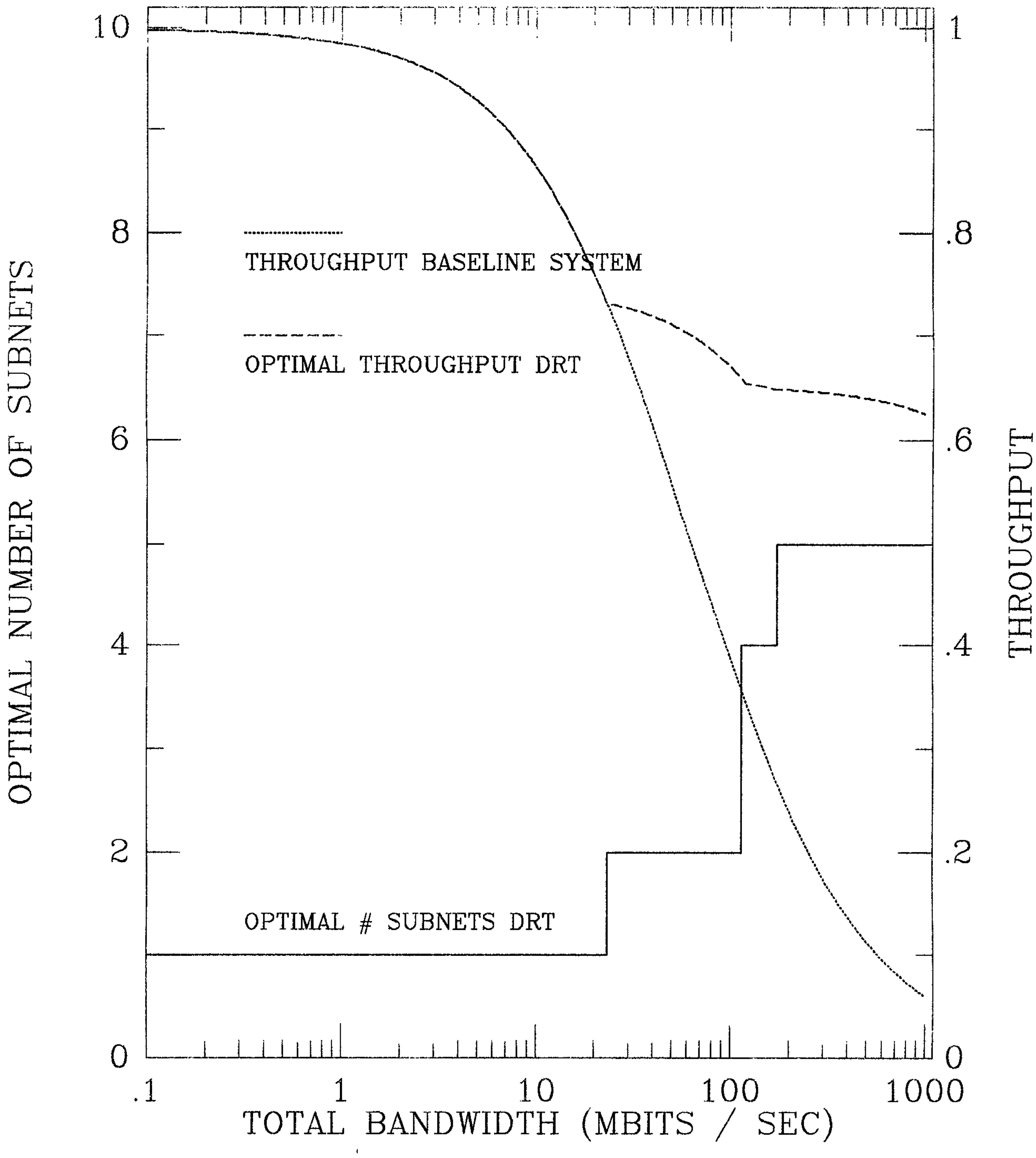


Figure 9: Optimal Number of Subnets and Optimal Throughput vs.

Total Bandwidth for CSMA/CD System with 10 users.

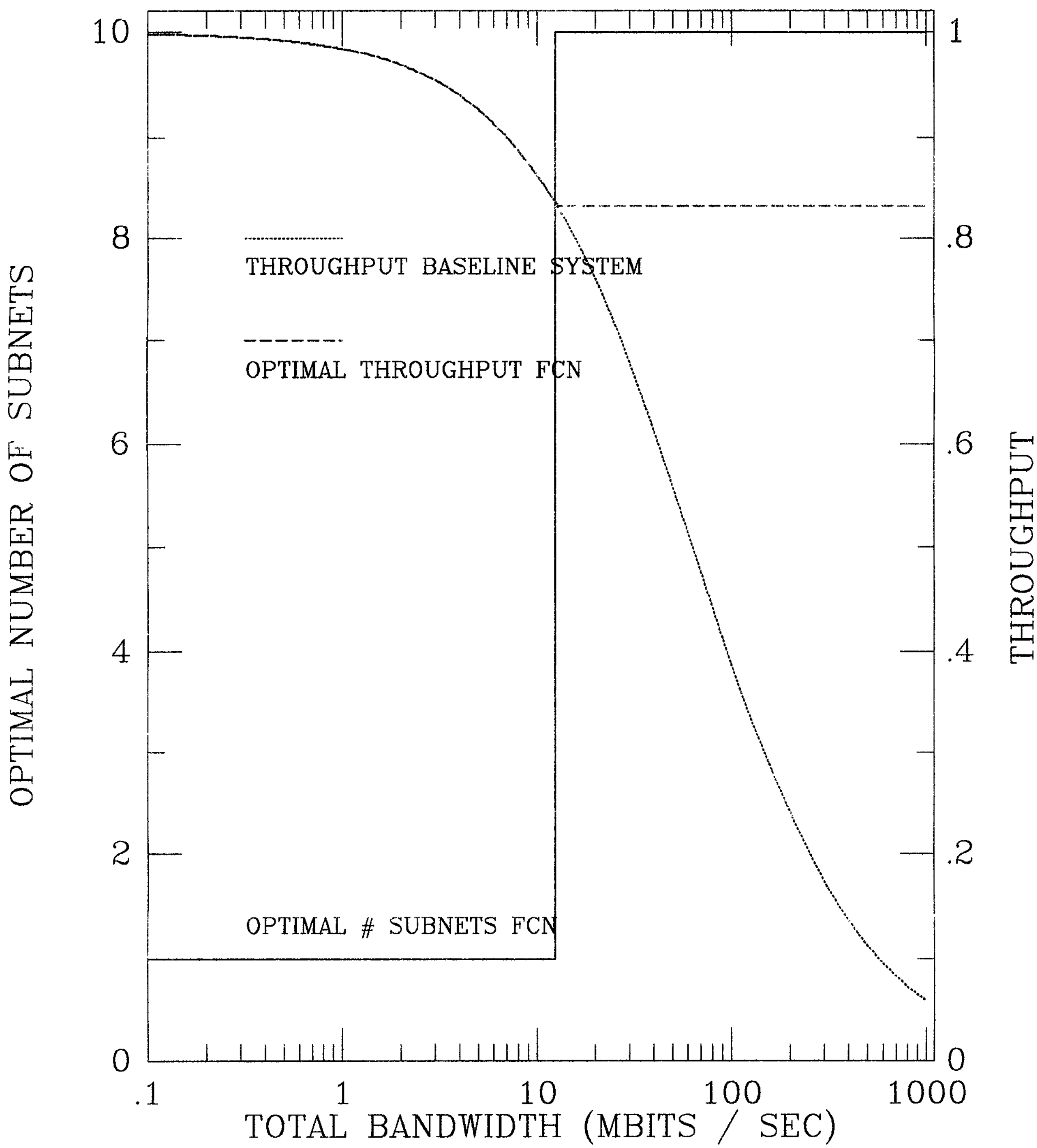
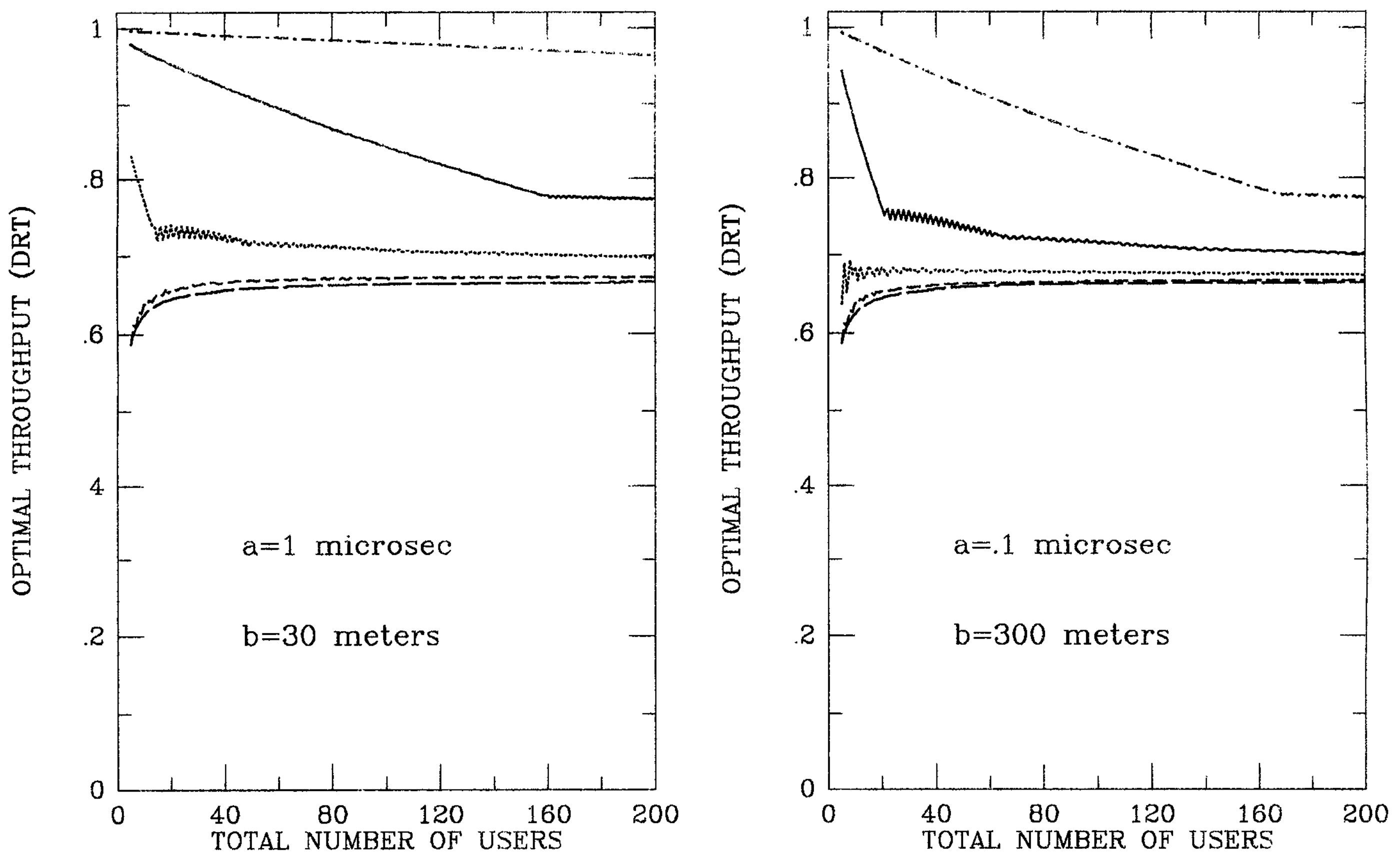
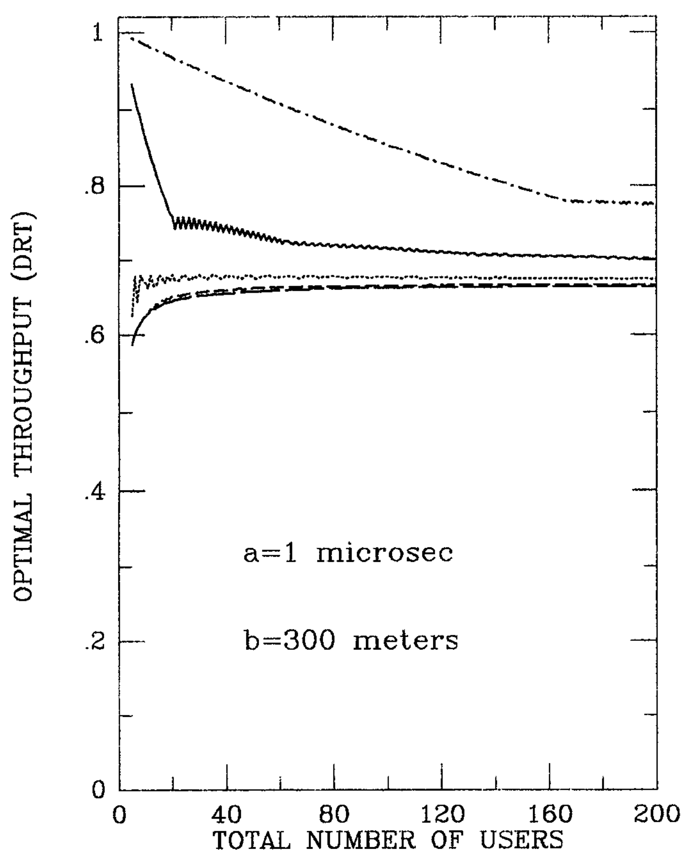


Figure 10: Optimal Number of Subnets and Optimal Throughput vs. Total Bandwidth for CSMA/CD System with 10 users.





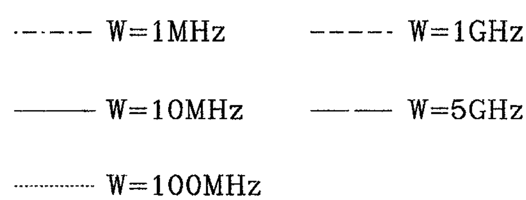
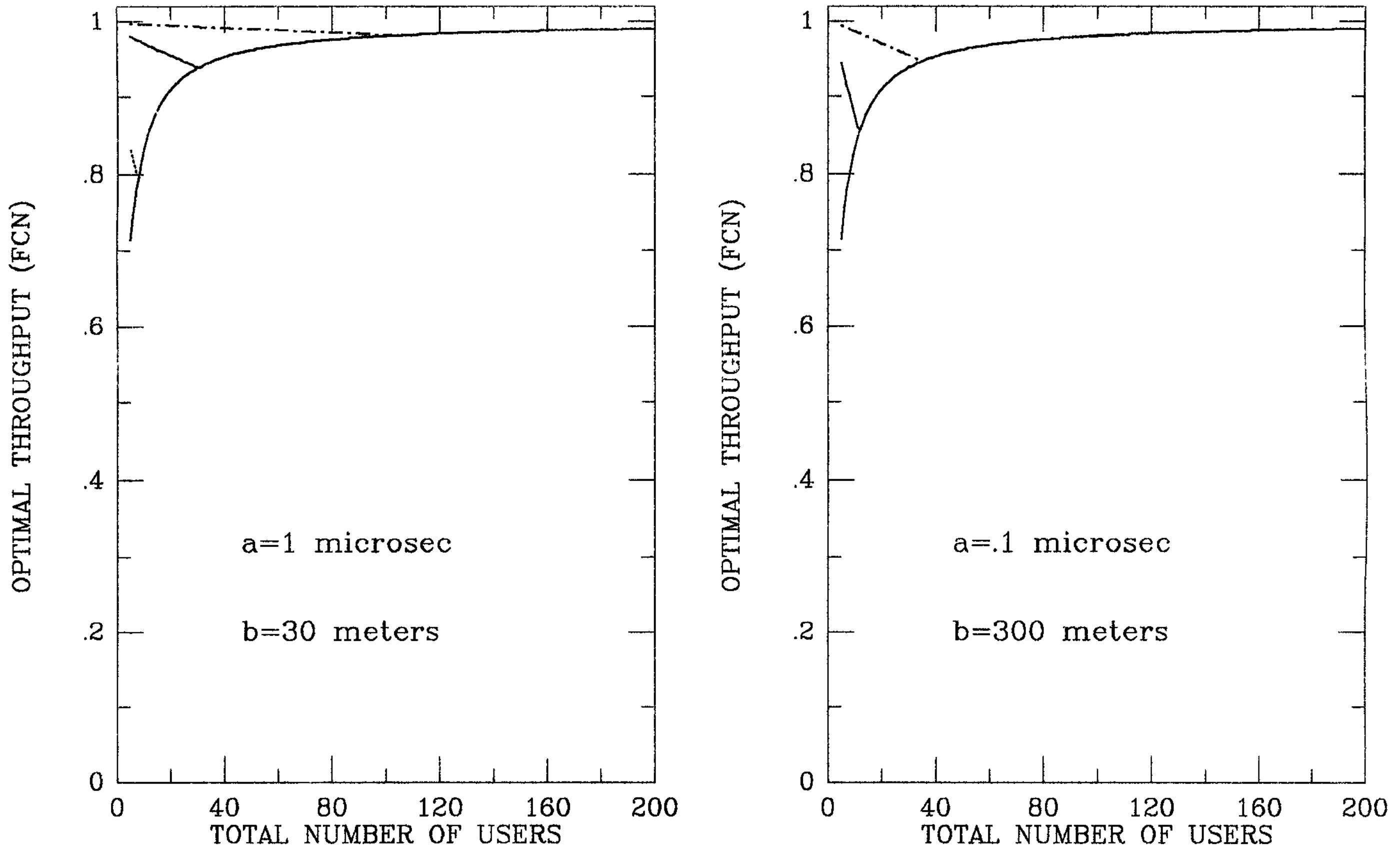


Figure 11:

Throughput of optimal DRT vs. total number of users for different values of the parameters a,b and W.



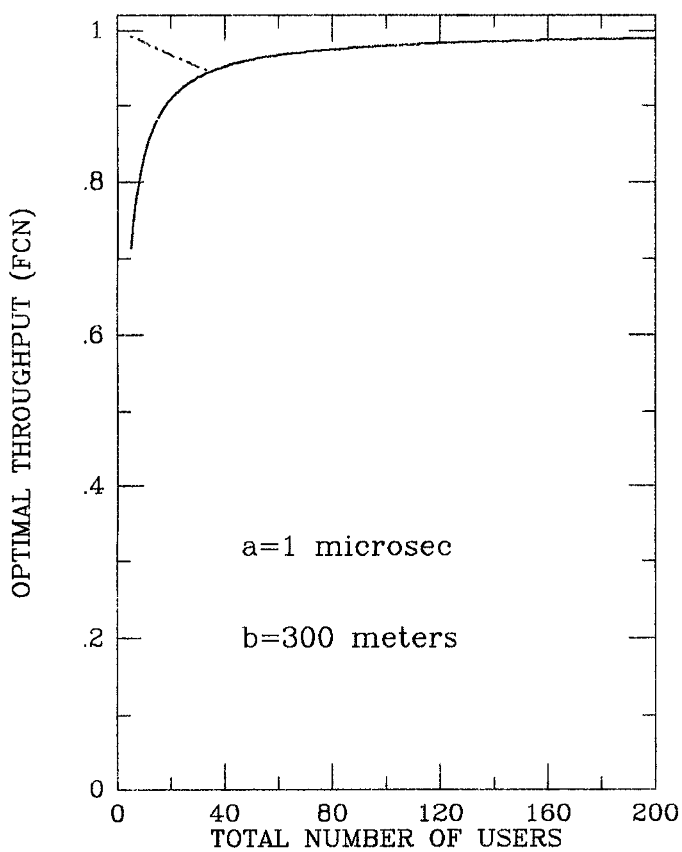
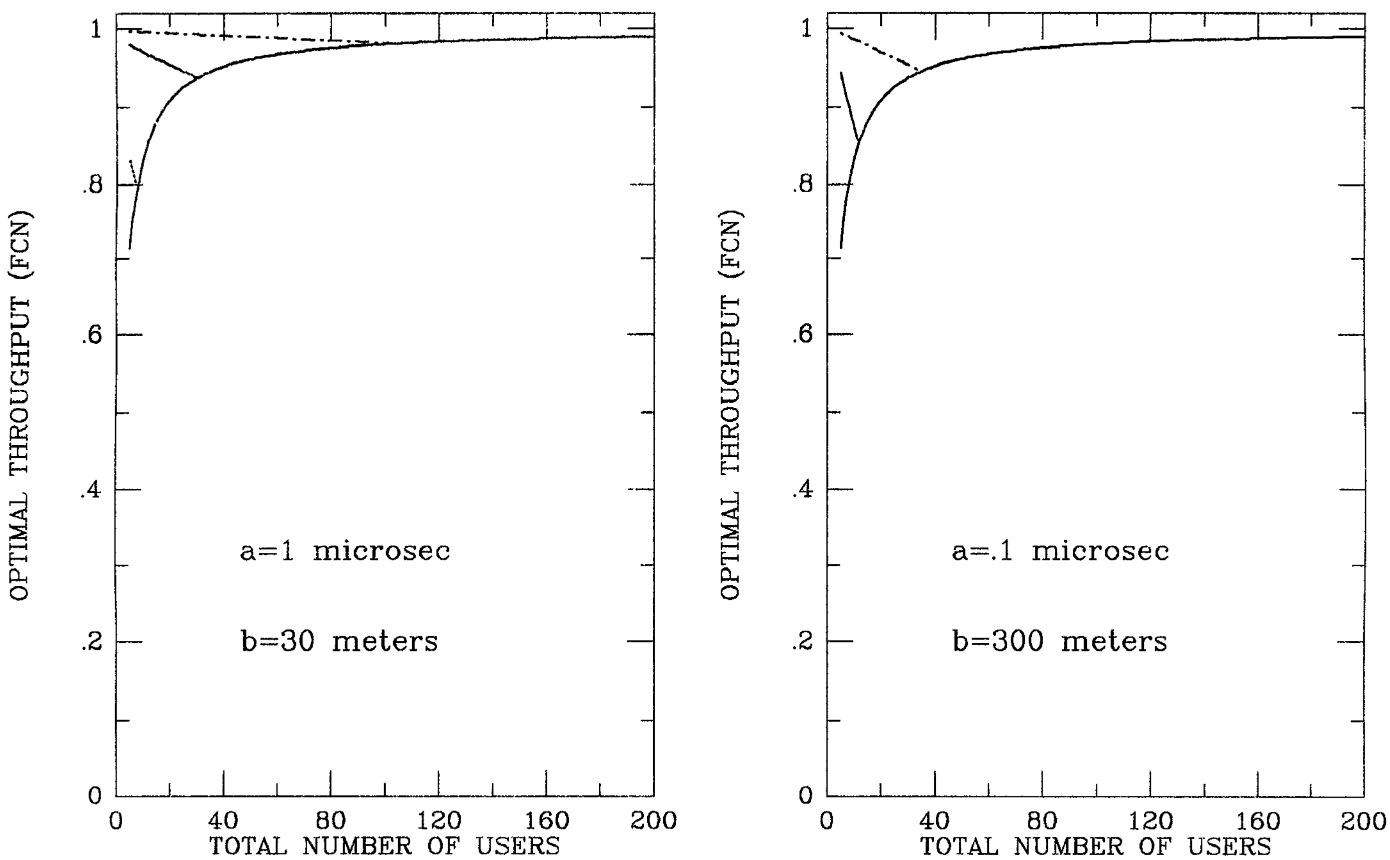
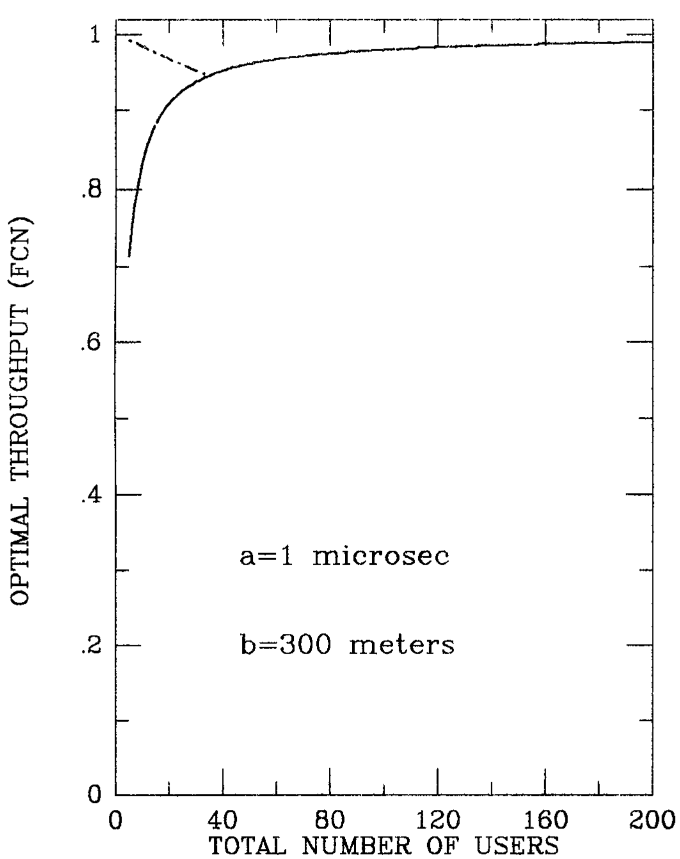


Figure 11:

Throughput of optimal FCN vs. total number of users for different values of the parameters a,b and W.





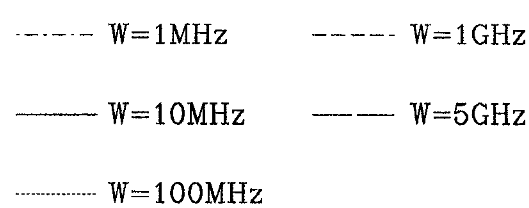


Figure 12:

Throughput of optimal FCN vs. total number of users for different values of the parameters a,b and W.