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A Geometric Theorem for Adaptable Log-Periodic Broadband Antenna

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Abstract- This communication presents a new geometric theorem for application to log-periodic antenna. Based on the new geometric theorem, two log-periodic antennas can be generated together so that the directive gain control and beam-form scanning can be done for log-periodic antenna. We will also show that the conventional log-periodic antenna is a special case of the result presented in this paper.

1 Introduction

The log-periodic antenna has been well-known in the literatures for its broad bandwidth capability. This broad bandwidth property comes majorly from the special geometric structure of the log-periodic antenna [1, 2]. The basic structures of a log-periodic antenna can be of any form as shown in Fig. 1. For either case, the following relation must be satisfied,

$$\frac{R_{n+1}}{R_n} = \frac{L_{n+1}}{L_n} = \tau, \quad n = 0, 1, 2, \cdots \tag{1}$$

where R_n and L_n are defined as in Fig. 1 and τ is a constant. We will show in this paper that in fact, such a structure is a special case of a geometric theorem called *Parallel Line in Triangle* (PLT) that will be proved later on. Based on the PLT, we will see that two log-periodic antennas can be co-existent in a given triangular with common feed points. Therefore, the beamform can be adaptively controlled.

The results in this show a more general structure for log-periodic antenna than the conventional one.

We will first prove the PLT theorem and consider its properties in section 2. The applications to log-periodic antenna will then be presented in section 3.

2 The Parallel Line in Triangle Theorem (PLT)

Before we consider the PLT theorem, let us describe first how to construct the desired geometric structure. As shown in Fig. 2, the construction procedure is given belows

- For any arbitrary triangle $\triangle$ABC, from vertex B we draw an arbitrary line $\overline{Bm_1}$ (L_1) to point m_1. From vertex C, we draw an arbitrary line $\overline{Cm_2}$ (L_2) to point m_2.

- From point m_2, draw line $\overline{m_2m_3}$ (L_3) which parallel to $\overline{Bm_1}$ (L_1), From point m_1, draw line $\overline{m_1m_4}$ (L_4) which parallel to $\overline{Cm_2}$ (L_2), and consecutively draw parallel lines L_5, L_6, and so on from points $m_3, m_4, m_5, m_6, ...$ et al., as in Fig. 2.

Parallel Line in Triangle Theorem: Those infinite number of points p_i, q_i, $i = 1, 2, 3, \cdots$, intersected by parallel lines L_i, $i = 1, 2, 3, \cdots$ as defined in Fig. 2 are collinear into two lines P and Q which cross on the vertex A.

Proof: The geometric proof is given in the Appendix.$\square$

In fact, this structure can be viewed as the projection of conical spiral structures, from which the conventional log-periodic antennas were derived, with two exactly the same shapes of equiangular spirals, but with some displacement.

Define a coordinate system $S = (0; \tilde{e_1}, \tilde{e_2})$ such that $A = (0,0), B = (\beta, 0), C = (0, \alpha), m_1 = (0, \gamma)$, and $m_2 = (\delta, 0)$. We then have the following line equations for L_i:

$$L_{2n-1} : \gamma x + \beta y = (\beta\gamma)(\frac{\delta}{\beta})^{(\lfloor \frac{n}{2} \rfloor)}(\frac{\gamma}{\alpha})^{(\lfloor \frac{n-1}{2} \rfloor)}$$

$$L_{2n} : \alpha x + \delta y = (\alpha\delta)(\frac{\gamma}{\alpha})^{(\lfloor \frac{n}{2} \rfloor)}(\frac{\delta}{\beta})^{(\lfloor \frac{n-1}{2} \rfloor)}$$

where $n = 1, 2, 3, \cdots$ as given in Fig.2 and $\lfloor x \rfloor$ is the largest integer that is less than or equal to x. From these equations, we can solve for the coordinates of m_i. Define $\tau = \frac{\alpha\beta}{\gamma\delta}$, we got

$$m_{4n-1} = (0, \tau^{n-1}(\frac{\delta}{\beta})\gamma)$$

$$m_{4n-2} = (\tau^{n-1}\delta, 0)$$

$$m_{4n-3} = (0, \tau^{n-1}\gamma)$$

$$m_{4n} = (\tau^{n-1}(\frac{\gamma}{\alpha})\delta, 0)$$

where $n = 1, 2, 3, \cdots$. It follows that

$$\overline{m_{4n-1}A} = (\frac{\delta}{\beta})\overline{m_{4n-3}A}$$

$$\overline{m_{4n-2}A} = (\frac{\alpha}{\gamma})\overline{m_{4n}A}. \quad (2)$$

It can then be shown that

$$\frac{\overline{m_{4n+1}m_{4n+3}}}{\overline{m_{4n-3}m_{4n-1}}} = \frac{\overline{m_{4n+2}m_{4n+4}}}{\overline{m_{4n-2}m_{4n}}} = \tau \quad (3)$$

for $n = 1, 2, 3, \cdots$.

3 Adaptable Log-Periodic Antenna

From the above discussion, we can see that the PLT gives a geometric structure consisting of a geometric progression of "cells" such that each cell is identical to its neighbor except for a constant expansion factor τ. Thus an expansion by the factor τ brings the structure back onto itself. Consequently, the performance remains unchanged when the frequency is increased by the factor τ [1]. This is the most fundamental property required by the structure of log-periodic antenna.

The major differences between the PLT structure and the conventional log-periodic structure are: (1) there are two log-periodic structures co-existent in a given triangular structure; (2) the two co-existent log-periodic structures are not symmetric.

The first property together with the fact that both structures share the same points p_n and q_n (see Fig.2) that can be used as common feed points for both log-periodic antenna implies that both beam patterns can be controlled simultaneously so that the resultant beampattern can be adaptable. Specifically, the coordinates of p_n and q_n are given by

$$p_n = \frac{\tau^{n-1}}{1-\tau}(\delta(1-\gamma/\alpha), \gamma(1-\delta/\beta)),$$

$$q_n = \frac{\tau^n}{1-\tau}(\beta-\delta, \alpha-\gamma),$$

where $n = 1, 2, 3, \cdots$. And $\overline{p_n A} = \tau \overline{p_{n-1} A}$, $\overline{q_n A} = \tau \overline{q_{n-1} A}$. The principle is simply to excite the elements of the array so that all radiate in phase in the desired direction. Since both have the same characteristic length L_n, they have almost the same frequency range. Therefore, the directive gain control and beamform scaning is possible in the new structure.

If the triangle $\triangle$ ABC is an isosceles triangle, say $\overline{AB} = \overline{AC}$, and let the angles

θ_1 equal to θ_2 (in this case, lines P and Q become one single line) as shown in Fig.3, then we have a special case of the PLT that produces the conventional log-periodic structures. The log-periodic structures given in Fig.1 can be easily obtained from the structure in Fig.3. Obviously, the PLT gives a more general structure for generating log-periodic type antennas.

4 Conclusion

In this paper, the author has presented a new geometric theorem, the PLT, that gives a more general structure for the log-periodic type antennas. It is shown that the conventional log-periodic structures are special cases of the newly proposed structure. Furthermore, the new strucutre consists of two log-periodic structure in a given triangle and can be simultaneously. Therefore, the beamform can be adaptively controlled from the common feed points. Since the PLT can be easily coded in computer program, it can be a good candidate for CAD design. The author, a systems researcher, hopes that the result presented in this paper may stimulate new research in adaptable log-periodic antenna with asymmetric structure.

References

[1] V.H. Rumsey, Frequency Independent Antenna, Academic Press, 1966.

[2] W.L. Stutzman and G.A. Thiele, Antenna Theory and Design, John Wiley & Sons, 1981.

5 Appendix

Proof of Parallel Line in Triangle Theorem:

Supposed that those p_i, q_i,i=1,2,3,..., are collinear respectedly and the two lines P and Q intersect on vertex A. Let's construct line $\overline{m_3m_4}$ and $\overline{m_1m_2}$ (see Fig.2), we have in △ABC that

$$\frac{\overline{Am_2}}{\overline{BM_2}} \cdot \frac{\overline{BD}}{\overline{CD}} \cdot \frac{\overline{Cm_1}}{\overline{Am_1}} = 1 \tag{4}$$

and in $\triangle Am_2m_1$

$$\frac{\overline{Am_4}}{\overline{m_4m_2}} \cdot \frac{\overline{Xm_2}}{\overline{Xm_1}} \cdot \frac{\overline{m_3m_1}}{\overline{Am_3}} = 1 \tag{5}$$

and in $\triangle Am_4m_3$

$$\frac{\overline{Am_6}}{\overline{m_6m_4}} \cdot \frac{\overline{Hm_4}}{\overline{Hm_3}} \cdot \frac{\overline{m_5m_3}}{\overline{Am_5}} = 1 \tag{6}$$

As $\overline{Cm_2}//\overline{m_1m_4}$, $\overline{Bm_1}//\overline{m_2m_3}$, we have

$$\frac{\overline{Am_4}}{\overline{m_4m_2}} = \frac{\overline{Am_1}}{\overline{Cm_1}} \qquad \frac{\overline{m_3m_1}}{\overline{Am_3}} = \frac{\overline{Bm_2}}{\overline{Am_2}}$$

Substituting to equation (5),

$$\frac{\overline{Am_1}}{\overline{Cm_1}} \cdot \frac{\overline{Xm_2}}{\overline{Xm_1}} \cdot \frac{\overline{Bm_2}}{\overline{Am_2}} = 1 \tag{7}$$

Multiplying equation (7) with equation (4), we got

$$\frac{\overline{BD}}{\overline{CD}} \cdot \frac{\overline{Xm_2}}{\overline{Xm_1}} = 1. \tag{8}$$

For $\frac{\overline{Am_4}}{\overline{Am_2}} = \frac{\overline{Am_1}}{\overline{AC}}$ and $\frac{\overline{Am_2}}{\overline{AB}} = \frac{\overline{Am_3}}{\overline{Am_1}}$, it follwos that

$$\frac{\overline{Hm_4}}{\overline{Hm_3}} = \frac{\overline{BD}}{\overline{CD}} \tag{9}$$

Substituting equation (9) to equation (8),

$$\frac{\overline{Hm_4}}{\overline{Hm_3}} = \frac{\overline{Xm_2}}{\overline{Xm_1}} = 1 \tag{10}$$

Now multiplying equation (6) with equation (5)

$$\frac{\overline{Am_4}}{\overline{m_4m_2}} \cdot \frac{\overline{Xm_2}}{\overline{Xm_1}} \cdot \frac{\overline{m_3m_1}}{\overline{Am_3}} \cdot \frac{\overline{Am_6}}{\overline{m_6m_4}} \cdot \frac{\overline{Hm_4}}{\overline{Hm_3}} \cdot \frac{\overline{m_5m_3}}{\overline{Am_5}} = 1 \tag{11}$$

Substituting equation (10) to equation (11), we got

$$\frac{\overline{Am_4}}{\overline{m_4m_2}} \cdot \frac{\overline{m_3m_1}}{\overline{Am_3}} \cdot \frac{\overline{Am_6}}{\overline{m_6m_4}} \cdot \frac{\overline{m_5m_3}}{\overline{Am_5}} = 1 \tag{12}$$

Since $\overline{m_4m_5}$ parallel to $\overline{m_2m_3}$, equation (12) is reduced to

$$\frac{\overline{m_3m_1}}{\overline{Am_3}} \cdot \frac{\overline{Am_6}}{\overline{m_6m_4}} = 1 \tag{13}$$

Therefore, we show that $\overline{m_3p_2}//\overline{m_1m_4}$. And with the same reason, $\overline{m_5q_2}//\overline{m_3m_6}$. This proves the PLT.

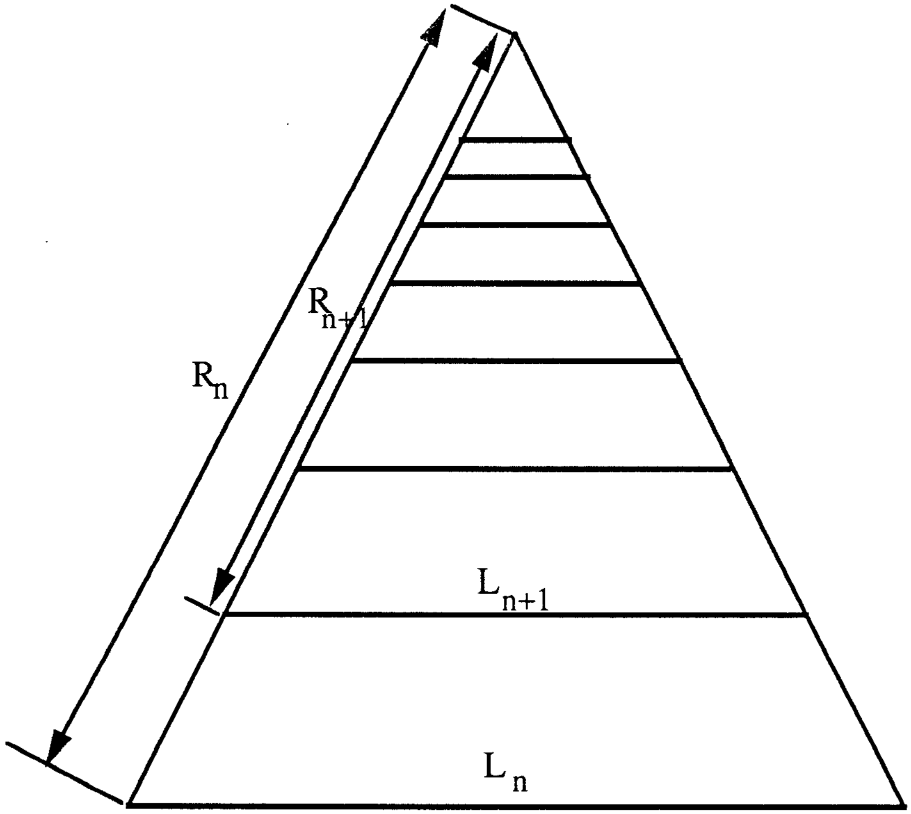

Fig.1 (a)

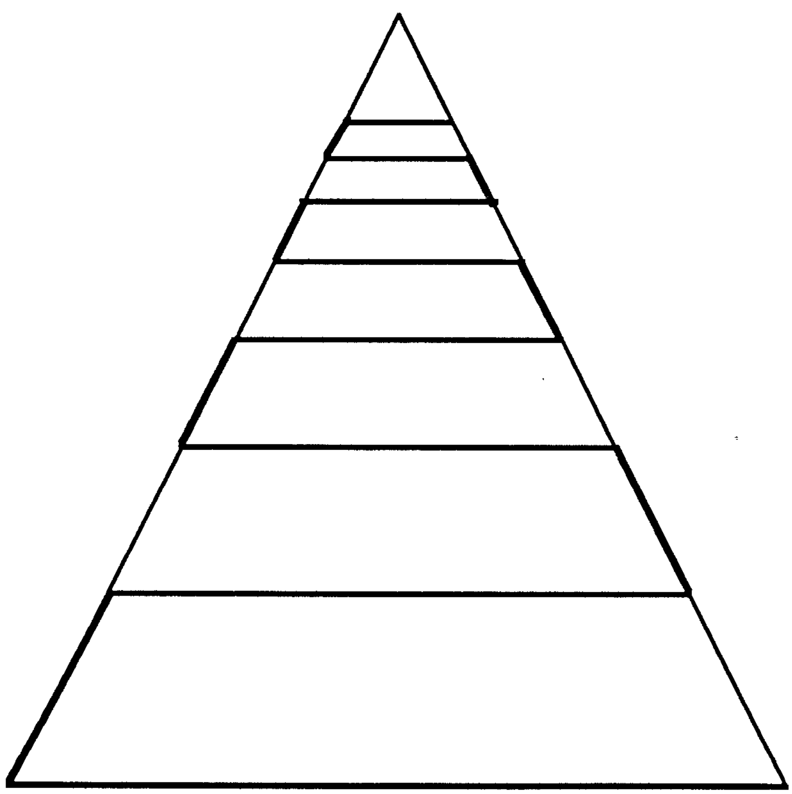

Fig.1 (b)

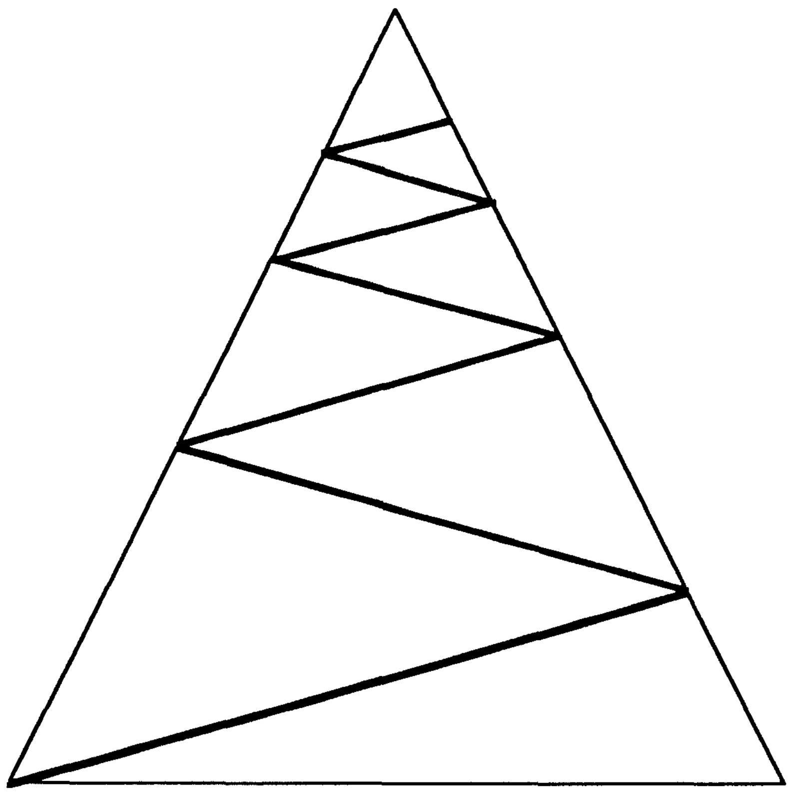

Fig. 1 (c)

Fig. 1 Some basic structures of conventional log-periodic antenna.

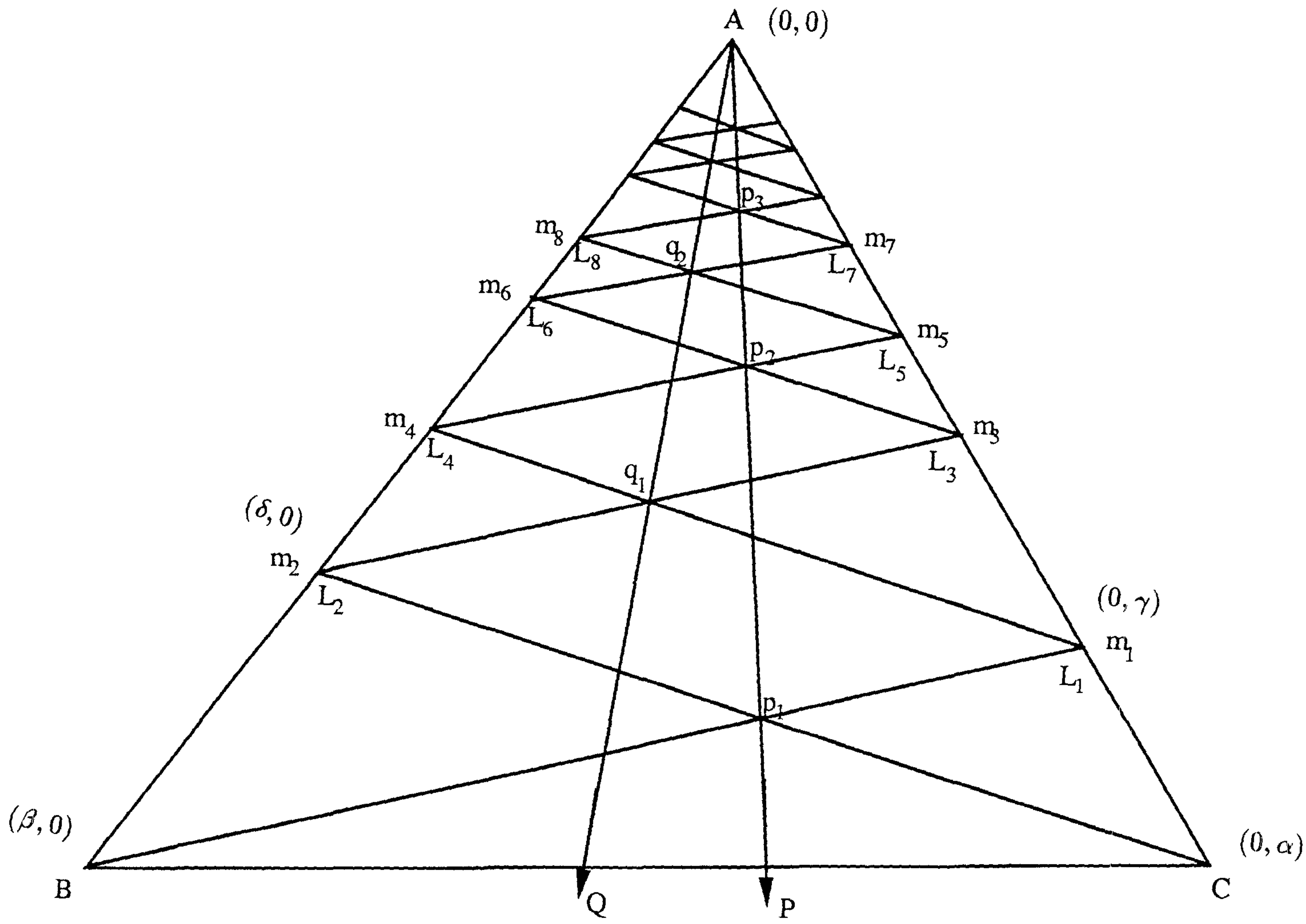

Fig.2 The structure of "Parallel Lines in Triangle Theorem".

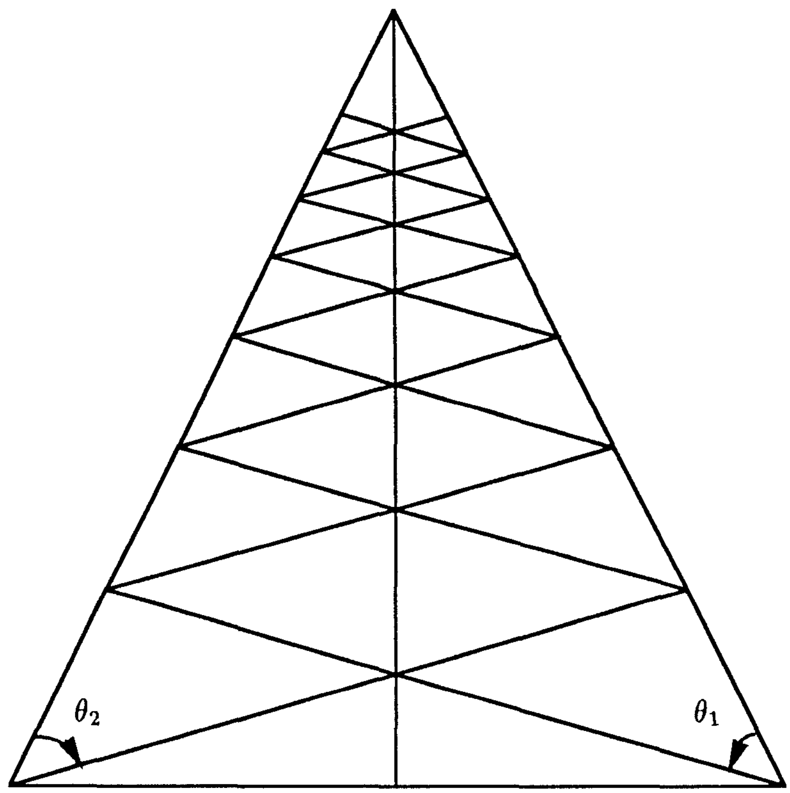

Fig. 3 A special structure of the PLT when the triangular is isosceles. It becomes the basic structure of the conventional log-periodic antenna.