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CHEMICAL PROCESS SYSTEMS
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Drop Breakup in Stirred-Tank Contactors
Part III: Correlations for Mean Size and
Drop Size Distribution

by

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Drop Breakup in Turbulent Stirred-Tank Contactors

Part III: Correlations for Mean Size and Drop Size Distribution

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In Part II, the extent to which dispersed-phase viscosity and interfacial tension influence equilibrium mean drop size and drop size distribution was determined for dilute suspensions produced in baffled cylindrical tanks of standard geometry equipped with six-blade Rushton turbines. Low to moderate viscosity ($\mu_d \leq 0.5$ Pa·s) dispersed-phase systems behaved similarly in that Sauter mean diameter could be correlated using the mechanistic arguments of Part I, and drop sizes, normalized with respect to D_{32}, could be correlated by a normal distribution in volume. Limited moderate viscosity data were reported in Part I but were not used to develop the correlations of Part II. The objective of this study is to combine the low to moderate viscosity data of Parts I and II with those obtained by other investigators to obtain correlations of broader utility, and to extend these via mechanistic arguments so that they apply to nondilute systems.

Dilute Dispersions

Several investigators have studied the behavior of dilute liquid-liquid systems in the geometry of Parts I and II (see Figure 1 of Part I). Chen and Middleman (1967) conducted a detailed study of surface forced stabilized (low μ_d) dispersions encompassing a broad range of operating conditions. They considered dispersed phases with viscosities up to about 0.025 Pa·s but did not account for viscous resistance to breakage when correlating their data. Sprow (1967) obtained limited data for inviscid dispersed phases, while Arai et al. (1977) acquired limited data showing the effect of dispersed-phase viscosity on maximum stable drop size at constant interfacial tension. The apparatus

employed in these studies differed from that of Parts I and II in only two respects. Baffles were mounted flush to tank walls and bottom. Chen and Middleman varied the ratio of impeller to tank diameter (L/T) and Sprow obtained most of his data for $L/T = 0.29$.

For dilute suspensions, coalescence rates are negligible. Equilibrium drop sizes are determined by breakup that occurs primarily in the impeller region. Small modifications in baffle placement should be relatively unimportant and the L/T ratio should be of secondary importance. Chen and Middleman found that the effect of L/T fell within the scatter in their data for the range $0.21 \leq L/T \leq 0.73$. However, it should be noted that Okamoto et al. (1981) report that energy dissipation rates become more uniform as L/T increases. Therefore, the ratio of the maximum to mean energy dissipation rate per unit mass increases as L/T decreases. For a given $\bar{\epsilon}$, drops will experience higher local turbulent energy as L/T decreases, indicating that D_{32} should decrease with L/T.

Mean drop size correlations

The low to moderate viscosity data reported in Parts I and II and by the cited investigators were fit to models developed in Part II. The range of variables investigated in each study is summarized in Table 1. The table provides information for dilute, low to moderate viscosity dispersed phases, only, since these are the systems to which the correlations of Part II apply. For instance, higher viscosity data reported in Parts I and II and by Arai et al. are not included. Data reported by Sprow for nondilute dispersions are not given. Arai et al. reported maximum stable drop size. Based on the data of Parts I and II and of Chen and Middleman, it was assumed that $D_{32} \approx 0.6 \, D_{max}$.

Two models were fitted to the 349 data sets via nonlinear least-squares regression. These are the semitheoretical model
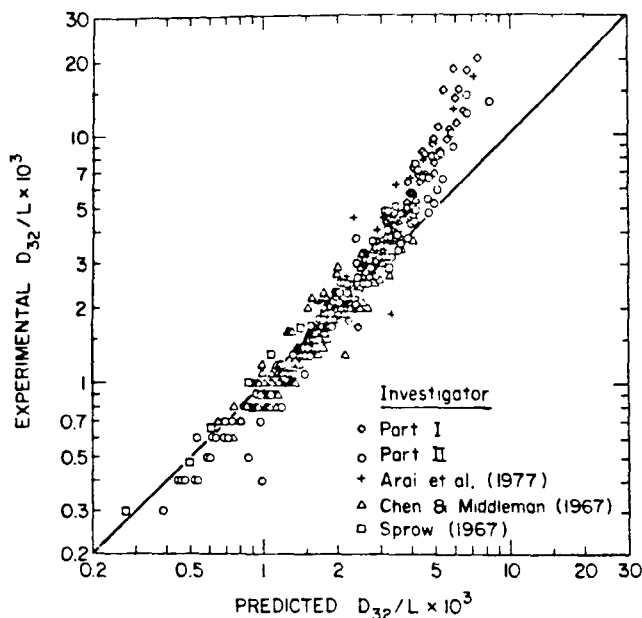
Figure 2. Goodness of fit of Eq. 4 to mean drop size data; $\sigma_{rms} = 23.1\%$.

Correlation for drop size distribution

The results of Part II show that for low to moderate viscosity dispersed phases, drop size distributions are normally distributed in volume and can be correlated by normalization with D_{32}. Furthermore, the correlation developed in Part II is essentially the same as that of Chen and Middleman for low-viscosity drops. Therefore these (Eqs. 23 and 25 of Part II) can be combined to yield

$$F_v\left(\frac{D}{D_{32}}\right) = 0.5\left[1 + \text{erf}\left(\frac{D/D_{32} - 1.07}{0.23\sqrt{2}}\right)\right] \qquad (5)$$

The cumulative volume frequency, F_v, is related to the volume probability density function, $P_v(D/D_{32})$ through Eqs. 18 to 22 of Part II. Therefore Eq. 5 corresponds to

$$P_v\left(\frac{D}{D_{32}}\right) = \frac{1}{0.23\sqrt{2\pi}}\exp\left[-9.1\left(\frac{D}{D_{32}} - 1.07\right)^2\right] \qquad (6)$$

Chen and Middleman provide a method for extracting the distribution of interfacial area from Eq. 6.

Nondilute Dispersions

The most limiting feature of the above correlations is that they apply only in the limit $\phi \rightarrow 0$. The presence of a greater amount of dispersed phase affects equilibrium drop size in two ways. When drops are in close proximity, the small-scale structure of the continuous phase is altered, thereby decreasing the turbulent energy acting to disrupt the drop. Furthermore, coalescence is promoted due to increased probability of drop collisions. If the dispersed-phase volume fraction is not too large, the presence of stabilizing agents can inhibit coalescence. In the absence of such agents coalescence will be primarily confined to more quiescent regions of the tank. A model can be developed for low to moderate ϕ which considers only the effect of ϕ on

disruptive energy. The result should apply to drop sizes leaving the impeller region or to the entire tank for noncoalescing (stabilized) dispersions. It is important to note that at large ϕ some coalescence will even occur in the impeller region, particularly for $\rho_d < \rho_c$. It is well-known that air bubbles are captured by the centrifugal fields of the trailing vortex system behind turbine blades to form stable gas cavities (van't Riet and Smith, 1973). At large ϕ, any model that ignores coalescence may not even apply to the impeller region.

Mean drop size correlation

Doulah (1975) argued that the effect of ϕ on the turbulent energy available to disrupt a drop could be quantified by considering its effect on the local energy dissipation rate. He showed that the energy dissipation rate per unit mass for finite ϕ (ϵ_ϕ) is related to that for negligible ϕ (ϵ) by

$$\epsilon_\phi = \epsilon\left(\frac{\rho_c}{\rho_c}\right)^3(1 + 2.5\,\phi)^{-3} \qquad (7)$$

Equation 7 can be used to replace ϵ by ϵ_ϕ in the equations of Part I (used to derive Eq. 1) to yield the following result:

$$\frac{D_{32}}{L} = A\left(\frac{\rho_c}{\rho_c}\right)^{-6/5}(1 + 2.5\,\phi)^{6/5}\,We^{-3/5}$$
$$\cdot\left[1 + B_1\left(\frac{\rho_c}{\rho_c}\right)(1 + 2.5\,\phi)^{-1}\,Vi\left(\frac{D_{32}}{L}\right)^{1/3}\right]^{3/5} \qquad (8)$$

The emulsion density, ρ_e, is given by $\rho_e = \rho_c(1 + \phi\Delta\rho/\rho_c)$ when $\Delta\rho = \rho_d - \rho_c$. For most liquid-liquid systems $\Delta\rho$ is small. If ϕ is also not too large, then $\rho_e \approx \rho_c$. Furthermore, the terms containing ϕ can be approximated by a series expansion. If terms of order ϕ^2 and greater are ignored, then Eq. 8 yields

$$\frac{D_{32}}{L} = A(1 + 3\phi)$$
$$\cdot We^{-3/5}\left[1 + B_1(1 - 2.5\,\phi)\,Vi\left(\frac{D_{32}}{L}\right)^{1/3}\right]^{3/5} \qquad (9)$$

In the limit as $Vi \rightarrow 0$ (negligible viscous resistance to breakage), Eq. 9 reduces to

$$\frac{D_{32}}{L} = A(1 + \gamma\phi)\,We^{-3/5} \qquad (10)$$

where $\gamma = 3$. Several researchers (Brown and Pitt, 1970; Calderbank, 1958; Mlynek and Resnick, 1972) have correlated low-viscosity dispersed-phase data obtained in the geometry considered here by Eq. 10. Their values for the geometric coefficient A (0.051 to 0.060) do not differ substantially from $A = 0.054$ given by Eq. 3 .or dilute dispersions. Brown and Pitt measured drop sizes, via photography, at the tip of the impeller and found that data for $\phi < 0.3$ were well-correlated by Eq. 10. Their data yielded $\gamma = 3.14$, in close agreement with the predicted value of $\gamma = 3$. A linear dependency of D_{32} on ϕ was not found at $\phi = 0.4$. Brown and Pitt suggested that the value of $\gamma = 9$ obtained by Calderbank, using a light transmittance technique, was due to measuring drop sizes remote from the impeller region, where

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