TECHNICAL RESEARCH REPORT

Performance Analysis of Coherent TCM Systems with Diversity Reception in Slow Rayleigh Fading

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T.R. 96-44

INSTITUTE FOR SYSTEMS RESEARCH

Sponsored by
the National Science Foundation
Engineering Research Center Program,
the University of Maryland,
Harvard University,
and Industry

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Abstract

Coherent trellis coded modulation (TCM) systems employing diversity combining are analyzed. Three different kinds of combining are considered: maximal ratio, equal gain, and selection combining. For each combining scheme, the cutoff rate parameter is derived assuming transmission over a fully-interleaved channel with flat, slow, Rayleigh fading; in addition, tight upper bounds on the pairwise error probabilities are derived. These upper bounds are expressed in product form to permit bounding of the BER via the transfer function approach. In each case it is assumed that the diversity branches are independent and that the channel state information (CSI) can be recovered perfectly.

Also included is an analysis of maximal ratio combining when the diversity branches are correlated; the cutoff rate and a tight upper bound on the pairwise error probability are derived. It is shown that, with double diversity, a branch correlation coefficient as high as 0.5 results in only slight performance degradation.

Submitted May 1996 to IEEE Transactions on Vehicular Technology

This research was supported in part by National Science Foundation grant NCR-8957623; also by the NSF Engineering Research Centers Program, CDR-8803012.

1 Introduction

Diversity combining is a well-known and effective method for improving the performance of digital communication systems over fading channels [1]– [3]. The basic principle of M-fold diversity is to use M independent channels so that the probability of a "deep fade" on *all* channels is low. These independent channels can be created in a number of ways, including frequency, time, and/or polarization diversity; if multiple antennas are used to receive multiple versions of the received signal, the approach is called *spatial* diversity. A combining circuit is used to form a single resultant signal from the M different "branch" signals. There are (at least) three different methods for combining.

- The optimal combining scheme is called *maximal ratio combining* (MRC) [2, 3]. In such a scheme, the matched filter output of each diversity path is weighted by the fading attenuation of that path. The resultant SNR at the output of the combiner is the sum of the SNR's of the M branches.

- In *equal gain combining* (EGC) the resultant signal is simply the unweighted sum of the signals from the M branches.

- In *selection combining* (SC) the resultant signal is the one with highest SNR among the M received signals; in practice, the signal with the strongest received signal – i.e., signal plus noise – is selected.

Error probability expressions for uncoded systems with different combining schemes in Rayleigh fading are presented in [4, 5]

Bandwidth efficient coding such as trellis-coded modulation (TCM) [6] also provides a form of diversity – *time diversity.* The performance of TCM schemes may be evaluated by computing the pairwise error probability and using the transfer function approach to upper-bound the bit error rate [7]-[9].

Recently, the combined use of bandwidth efficient codes with diversity reception has been investigated [10]-[14]. In [10], trellis coded 16-QAM with maximal ratio combining was proposed for use in a TDMA digital cellular system. Upper bounds on the bit error probability for TCM with different combining schemes were presented in [11, 12]. These expressions use the Chernoff bound to establish an upper limit on the pairwise error probability and are loose. A tighter upper bound on the bit error probability for maximal ratio combining has recently been developed by Ventura-Traveset, Caire, Biglieri, and Taricco [14]; the upper bound in [14] requires numerical evaluation of the pairwise error probability and the use of a truncated transfer function.

In this paper, the use of trellis-coded modulation with diversity reception is investigated. Cutoff rate expressions for the Rayleigh distributed channel with diversity reception and the three different combining schemes are presented, as are tight upper bounds on the pairwise error probability. The same system configurations in [11, 12] are used; the new bounds are shown to be tighter than those presented in [11, 12].

The next section describes the system model and the combining metrics. In Section 3, expressions for the cutoff rates for the three combining schemes are derived and compared. Tight upper bounds on the bit error probability for trellis coded systems with the three combining schemes are derived and analyzed in Section 4. Section 5 analyzes the effect of branch correlation on maximal ratio combining. Finally, Section 6 gives conclusions.

2 System Model

The underlying system can be described as follows. Suppose the complex signal x_i is transmitted at time i and M corresponding signals $\underline{y}_i = \{y_{i,1}, y_{i,2}, \ldots, y_{i,M}\}$ are received; i.e.,

$$\begin{aligned} y_{i,1} &= a_{i,1}x_i + n_{i,1} \\ y_{i,2} &= a_{i,2}x_i + n_{i,2} \\ &\vdots \\ y_{i,M} &= a_{i,M}x_i + n_{i,M} \end{aligned} \tag{1}$$

where $\underline{a}_i = \{a_{i,1}, a_{i,2}, \ldots, a_{i,M}\}$ are the fading amplitudes, assumed to be Rayleigh-distributed and normalized so $E(a_{i,j}^2) = 1$; we assume ideal interleaving and independent diversity branches, so $\{a_{i,j}\}$ are i.i.d. Rayleigh. Here also, $\{n_{i,j}\}$ are complex-valued noise samples with independent real and imaginary components, each Gaussian distributed with mean zero and variance $N_0/2$.

The transmitter produces a sequence of signals $\mathbf{x}_N = \{x_1, x_2, \ldots, x_N\}$. At the receiver, the sequence of received M-tuples $\mathbf{y}_N = \{\underline{y}_1, \underline{y}_2, \ldots, \underline{y}_N\}$ and the channel fade amplitudes $\mathbf{a}_N = \{\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_N\}$ are the inputs to a TCM decoder which performs maximum likelihood (ML) decoding assuming ideal channel state information – i.e., the assumption that $\mathbf{a}_N$ is available to the decoder means that the receiver can ascertain the severity of the fading during each signaling interval. Techniques such as pilot symbol insertion [17] or decision feedback coupled with adaptive linear prediction [18] can be employed to recover $\mathbf{a}_N$.

The decoder selects as its estimate of the transmitted sequence the one minimizing the decoding metric

$$m(\mathbf{x}_N, \mathbf{y}_N; \mathbf{a}_N) = \sum_{i=1}^{N} m(x_i, \underline{y}_i; \underline{a}_i). \tag{2}$$

Here the symbol metric $m(x_i, \underline{y}_i; \underline{a}_i)$ depends on which form of signal combining is used.

- For maximal ratio combining, the assumption of CSI means that the signal metric is given by

$$m(x_i, \underline{y}_i; \underline{a}_i) = -\sum_{l=1}^{M} |y_{i,l} - a_{i,l}x_i|^2.$$

- For equal gain combining,

$$m(x_i, \underline{y}_i; \underline{a}_i) = -\left|\sum_{l=1}^{M} (y_{i,l} - a_{i,l}x_i)\right|^2.$$

- For selection combining,

$$m(x_i, \underline{y}_i; \underline{a}_i) = -|y_{i,j^\star} - a_{i,j^\star}x_i|^2$$

  where

$$j^\star = \arg\max\{a_{i,j}, j = 1, \ldots, M\}.$$

It should be noted that, in [11], Rasmussen and Wicker refered to the metric in (2) as the "interleaved code combining" (ICC) metric. ICC is a diversity combining technique in which each of the M received K-dimensional diversity signals is regarded as a component of a single MK-dimensional signal. Without CSI at the receiver, this technique may be regarded (as Rasmussen and Wicker did in [11]) to be a form of equal-gain combining; however, with the assumption of CSI at the receiver, this approach becomes equivalent to maximal ratio combining.

3 Cutoff Rate for Diversity Reception

The pairwise error probability $P(\mathbf{x}_N \rightarrow \hat{\mathbf{x}}_N)$ is the conditional probability that the metric associated with the coded sequence $\hat{\mathbf{x}}_N$ exceeds that of $\mathbf{x}_N$, given $\mathbf{x}_N$ was in fact transmitted. It can be upper bounded using the Chernoff bound as follows

$$\begin{aligned} P(\mathbf{x}_N \rightarrow \hat{\mathbf{x}}_N) &= P(m(\hat{\mathbf{x}}_N, \mathbf{y}_N; \mathbf{a}_N) - m(\mathbf{x}_N, \mathbf{y}_N; \mathbf{a}_N) \geq 0) \\ &\leq E[\exp(\lambda\{m(\hat{\mathbf{x}}_N, \mathbf{y}_N; \mathbf{a}_N) - m(\mathbf{x}_N, \mathbf{y}_N; \mathbf{a}_N)\})] \\ &= C(\mathbf{x}_N, \hat{\mathbf{x}}_N, \lambda) = \textstyle\prod_{i=1}^{N} C(x_i, \hat{x}_i, \lambda) \end{aligned} \tag{3}$$

where

$$C(x_i, \hat{x}_i, \lambda) = E[\exp(\lambda\{m(\hat{x}_i, \underline{y}_i; \underline{a}_i) - m(x_i, \underline{y}_i; \underline{a}_i)\})], \tag{4}$$

and the expectation is taken with respect to the noise $\underline{n}_i$ and the fading $\underline{a}_i$.

The cutoff rate R_o in bits/transmitted signal can be expressed as [19]

$$R_o = 2\log_2(|A|) - \log_2\left(\sum_{x_i \in A}\sum_{\hat{x}_i \in A} C(x_i, \hat{x}_i)\right) \tag{5}$$

where A is the signal set and $C(x_i, \hat{x}_i) = \min_\lambda C(x_i, \hat{x}_i, \lambda)$.

For maximal ratio combining, the cutoff rate is given by [14]

$$R_o = 2\log_2(|A|) - \log_2\left(\sum_{x_i \in A}\sum_{x_j \in A} \frac{1}{\left(1 + \frac{|x_i - \hat{x}_i|^2}{4N_o}\right)^M}\right). \tag{6}$$

Figure 1 shows the cutoff rate values of the 16-QAM signal constellation and maximal ratio combining with diversity orders of $M = 1, 2, 3, 4$. It is clear that the largest incremental gain is obtained in going from single to double diversity. The coding gains diminish as the order of diversity increases. For example, the curves show that reliable communication at a rate of 2 bits/symbol can be achieved at $E_s/N_o = 11$ dB (or $E_b/N_o = 8$ dB) for single channel reception. However, the required SNR can be reduced to $E_b/N_o = 6.1$ dB (or $E_b/N_o = 3.1$ dB) if double diversity with maximal ratio combining is used.

In equal gain combining the tightest conditional Chernoff bound is $C(x_i, \hat{x}_i, \lambda|\underline{a}_i)$

$$C(x_i\hat{x}_i|\underline{a}_i) = \min\{C(x_i, \hat{x}_i, \lambda|\underline{a}_i)\} = \exp(-\mu\frac{|x_i - \hat{x}_i|^2}{4MN_o}) \tag{7}$$

where

$$\mu = T^2 = (\sum_{l=1}^{M} a_{i,l})^2. \tag{8}$$

However, no closed form expression for the sum of Rayleigh distributed random variables is available for the case of $M > 2$, so an approximate expression is used. This expression is based on the small argument approximation [2, 3]; Beaulieu [20] showed that this expression is very accurate for $M \leq 8$. The approximation to the pdf of T is given by

$$f_T(t) = \frac{t^{(2M-1)}\exp(-t^2/2b_o)}{2^{(M-1)}b_o^M(M-1)!} \tag{9}$$

for $t \geq 0$, where

$$b_o = [(2M-1)!!]^{1/M} = [(2M-1)\cdot(2M-3)\cdots 3\cdot 1]^{1/M}. \tag{10}$$

Recognizing that $f_\mu(\mu) = f_T(\sqrt{\mu})/2\sqrt{\mu}$, we obtain the approximation

$$f_\mu(\mu) = \frac{\mu^{(M-1)}\exp(-\mu/b_o)}{b_o^M(M-1)!} \tag{11}$$

for $\mu \geq 0$. So μ has an M-Erlang distribution with parameter $1/b_o$. The last step is to perform the intergration

$$C(x_i, \hat{x}_i) = \frac{1}{(2M-1)!!\cdot(M-1)!}\int_0^\infty \mu^{M-1}\exp(-\frac{\mu}{b_o})\exp(-\mu\frac{|x_i - \hat{x}_i|^2}{4MN_o})d\mu. \tag{12}$$

Therefore, the cutoff rate for equal gain combining receivers is expressed as

$$R_o = 2\log_2(|A|) - \log_2\left(\sum_{x_i\in A}\sum_{x_j\in A}\frac{1}{\left(1 + \frac{[(2M-1)!!]^{1/M}}{M}\frac{|x_i - \hat{x}_i|^2}{4N_o}\right)^M}\right). \tag{13}$$

Comparing equation (13) with (6), we see that maximal ratio combining always has a greater cutoff rate than equal gain combining because $M/[(2M-1)!!]^{1/M}$ is greater than one and monotonically increases with M. Figure 2 shows the cutoff rate values of the 16-QAM signal constellation and equal gain combining with a diversity order of $M = 1, 2, 3, 4$. Similar to the maximal ratio combining case, the largest incremental gain is obtained in going from single to double diversity and coding gains diminish as the order of diversity increases.

In the selection combining case, the Chernoff bound can be written as

$$\begin{aligned} C(x_i, \hat{x}_i, \lambda | \underline{a}_i) &= E_{\underline{n}_i}[\exp(\lambda\{m(\hat{x}_i, \underline{y}_i; \underline{a}_i) - m(x_i, \underline{y}_i; \underline{a}_i)\})] \\ &= E_{n_{i,j^\star}}[\exp(\lambda\{|y_{i,j^\star} - a_{i,j^\star}x_i|^2 - |y_{i,j^\star} - a_{i,j^\star}\hat{x}_i|^2\})]. \end{aligned} \tag{14}$$

Again, it can be simplified to

$$\begin{aligned} C(x_i, \hat{x}_i, \lambda | \underline{a}_i) &= E_{n_{i,j^\star}}[\exp(\lambda\{-a_{i,j^\star}^2|x_i - \hat{x}_i|^2 - -2(a_{i,j^\star})\Re(n_{i,j^\star} \cdot (x_i - \hat{x}_i)^*)\})] \\ &= \exp(-\lambda a_{i,j^\star}^2|x_i - \hat{x}_i|^2) E_{n_{i,j^\star}}[\exp\{-2\lambda a_{i,j^\star}\Re(n_{i,j^\star} \cdot (x_i - \hat{x}_i)^*)\}] \\ &= \exp(-\lambda a_{i,j^\star}^2|x_i - \hat{x}_i|^2) \exp\{\lambda^2 a_{i,j^\star}^2 N_o |x_i - \hat{x}_i|^2\}. \end{aligned} \tag{15}$$

Therefore,

$$C(x_i, \hat{x}_i, \lambda | \underline{a}_i) = \exp(-\lambda (a_{i,j^\star})^2 (1 - N_o\lambda)|x_i - \hat{x}_i|^2). \tag{16}$$

$C(x_i, \hat{x}_i, \lambda | \underline{a}_i)$ is minimized by choosing $\lambda = 1/2N_o$. Therefore, the tightest conditional Chernoff bound can be written as

$$C(x_i \hat{x}_i | a_{i,j^\star}) = \min_{\lambda}\{C(x_i, \hat{x}_i, \lambda | a_{i,j^\star})\} = \exp(-\nu \frac{|x_i - \hat{x}_i|^2}{4N_o}) \tag{17}$$

where

$$\nu = a_{i,j^\star} = \max\{a_{i,1}^2, a_{i,2}^2, \cdots a_{i,M}^2\}. \tag{18}$$

Since $\{a_{i,1}, a_{i,2} \ldots a_{i,M}\}$ are independent, ν is just the maximum of M independent exponential random variables, each with mean one, so its pdf is given by

$$f_\nu(\nu) = M[1 - \exp(-\nu)]^{(M-1)} \exp(-\nu), \tag{19}$$

for $\nu \geq 0$, which can be rewritten using the binomial expansion as

$$f_\nu(\nu) = \sum_{k=1}^{M} (-1)^{k+1} M \begin{pmatrix} M-1 \\ k-1 \end{pmatrix} \exp(-k\nu). \tag{20}$$

Performing the integration, the Chernoff factor simplifies to

$$C(x_i,\hat{x}_i) = \sum_{k=1}^{M}(-1)^{k+1}M\begin{pmatrix} M-1 \\ k-1 \end{pmatrix}\frac{1}{k+\frac{|x_i-\hat{x}_i|^2}{4N_o}}. \tag{21}$$

Therefore, the cutoff rate for selection combining receivers is given by

$$R_o = 2\log_2(|A|) - \log_2\left(\sum_{x_i\in A}\sum_{x_j\in A}\left[\sum_{k=1}^{M}(-1)^{k+1}M\begin{pmatrix} M-1 \\ k-1 \end{pmatrix}\frac{1}{k+\frac{|x_i-\hat{x}_i|^2}{4N_o}}\right]\right). \tag{22}$$

Figure 3 shows the cutoff rate values of the 16-QAM signal constellation and selection combining with diversity orders of $M = 1,2,3,4$. The "diminishing returns" effect is more obvious in this case.

4 Pairwise Error Probability

In this section, tight upper bounds on the pairwise error probability are derived for the three combining schemes. Moreover, the pairwise error probability expressions are expressed in product form – i.e.,

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N|\mathbf{a}_N) = K_c \times \prod_{l=1}^{N} W(x_l,\hat{x}_l), \tag{23}$$

where K_c is a constant that does not depend on the length of the error sequence, and $W(x_l,\hat{x}_l)$ is the error weight profile between x_l and $\hat{x}_l$. Expressing the pairwise error probability in this form allows the use of the transfer function technique of trellis codes to be used in bounding the bit error rate.

4.1 Maximal Ratio Combiner

The conditional pairwise error probability for maximal ratio combining can be expressed as

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N|\mathbf{a}_N) &= P(m(\hat{\mathbf{x}}_N,\mathbf{y}_N;\mathbf{a}_N) - m(\mathbf{x}_N,\mathbf{y}_N;\mathbf{a}_N) \geq 0|\mathbf{a}_N) \\ &= P(\textstyle\sum_{i=1}^{N}\sum_{l=1}^{M}(|y_{i,l}-a_{i,l}x_i|^2 - |y_{i,l}-a_{i,l}\hat{x}_i|^2) \geq 0|\mathbf{a}_N) \end{aligned} \tag{24}$$

which can be simplified to

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N|\mathbf{a}_N) &= P(\textstyle\sum_{i=1}^{N}\sum_{l=1}^{M}[-a_{i,l}^2|x_i-\hat{x}_i|^2 - 2a_{i,l}\Re\{n_{i,l}(x_i-\hat{x}_i)^*\}] \leq 0|\mathbf{a}_N) \\ &= P(z_N \geq \textstyle\sum_{i=1}^{N}\sum_{l=1}^{M}a_{i,l}^2|x_i-\hat{x}_i|^2|\mathbf{a}_N), \end{aligned} \tag{25}$$

where z_N is zero-mean Gaussian with variance $2N_o \sum_{i=1}^{N} \sum_{l=1}^{M} a_{i,l}^2 |x_i - \hat{x}_i|^2$. This probability can be expressed as

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N | \mathbf{a}_N) = \frac{1}{2} \mathrm{erfc}\left(\sqrt{\sum_{i=1}^{N} \gamma_i d_i}\right) \tag{26}$$

where $\mathrm{erfc}(x) = (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$, $d_i = |x_i - \hat{x}_i|^2 / 4N_o$ and $\gamma_i = \sum_{l=1}^{M} a_{i,l}^2$. Since the $a_{i,l}$'s are i.i.d. Rayleigh distributed random variables with $E(a_{i,l}^2) = 1$, their squares are i.i.d. exponentially distributed random variables with a mean equal to one. Hence, γ_i will have an M-Erlang distribution with parameter one – i.e., its pdf is

$$f_\gamma(\gamma_i) = \frac{1}{(M-1)!} \gamma_i^{(M-1)} e^{-\gamma_i}, \quad \gamma_i \geq 0. \tag{27}$$

The unconditional pairwise error probability is thus

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) = \frac{1}{2} \int_0^\infty \cdots \int_0^\infty \mathrm{erfc}\left(\sqrt{\sum_{i=1}^{N} \gamma_i d_i}\right) f_\gamma(\gamma_1) \cdots f_\gamma(\gamma_N) d\gamma_1 \cdots d\gamma_N. \tag{28}$$

Define

$$\delta_i = \frac{d_i}{1 + d_i} \qquad \text{and} \qquad \omega_i = \gamma_i (1 + d_i). \tag{29}$$

Then the unconditional pairwise error probability can be represented as

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) &= \tfrac{1}{2} \prod_{i \in \eta} \tfrac{1}{(1+d_i)^M} \int_0^\infty \cdots \int_0^\infty \mathrm{erfc}\left(\sqrt{\sum_{i=1}^{N} \delta_i \omega_i}\right) \\ &\quad \times \exp\left[\sum_{i=1}^{N} \delta_i \omega_i\right] f(\omega_1) \cdots f(\omega_N) d\omega_1 \cdots d\omega_N \end{aligned} \tag{30}$$

where $\eta = \{i : x_i \neq \hat{x}_i\}$ and $L_\eta = |\eta|$. Note that

$$\sum_{i \in \eta} \delta_i \omega_i \geq \delta_m \sum_{i \in \eta} \omega_i \tag{31}$$

where $\delta_m = \min\{\delta_i, i \in \eta\}$. Since $\mathrm{erfc}(x) e^{x^2}$ is monotonically decreasing for $x \geq 0$, the pairwise error probability can be upper bounded by

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) \leq \frac{1}{2} \prod_{i \in \eta} \frac{1}{(1 + d_i)^M} \int_0^\infty \mathrm{erfc}\left(\sqrt{\delta_m \Omega}\right) \times e^{(\delta_m \Omega)} f_\Omega(\Omega) d\Omega, \tag{32}$$

where $\Omega = \sum_{i \in \eta} \omega_i$. Since the ω_i's are independent M-Erlang distributed random variables each with parameter one, Ω will have an (ML_η)-Erlang distribution with parameter one:

$$f_\Omega(\Omega) = \tfrac{1}{(ML_\eta - 1)!} \Omega^{(ML_\eta - 1)} e^{-\Omega}, \quad \Omega \geq 0. \tag{33}$$

To evaluate the integral, we use the following equality [8]

$$\frac{1}{2(K-1)!}\int_0^\infty \operatorname{erfc}(\sqrt{xy})e^{-y(1-x)}y^{(K-1)}dy = \frac{1}{2^{2K}}\sum_{j=1}^{K}\begin{pmatrix} 2K-j-1 \\ K-1 \end{pmatrix}\left(\frac{2}{1+\sqrt{x}}\right)^j \tag{34}$$

which is valid for $x < 1$. Integration yields

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) \leq \left[\frac{1}{2^{2ML_\eta}}\sum_{j=1}^{ML_\eta}\begin{pmatrix} 2ML_\eta-j-1 \\ ML_\eta-1 \end{pmatrix}\left(\frac{2}{1+\sqrt{\delta_m}}\right)^j\right] \times \prod_{i\in\eta}\frac{1}{(1+d_i)^M}. \tag{35}$$

Consider the special case of uncoded BPSK modulation; in this case, the error event length is $L_\eta = 1$, so the pairwise error probability is equal to the bit error probability P_b. Also, since $\delta_m = \delta_i$, the upper bound is satisfied with equality. Therefore,

$$\delta_m = \delta_i = \frac{E_s/N_o}{1+E_s/N_o} \tag{36}$$

and

$$\frac{1}{1+d_i} = 1-\delta_i = (1-\sqrt{\delta_i})(1+\sqrt{\delta_i}). \tag{37}$$

Thus bit error probability of uncoded BPSK can be expressed as

$$\begin{aligned} P_b &= \tfrac{1}{2^{2M}}\textstyle\sum_{j=1}^{M}\begin{pmatrix} 2M-j-1 \\ M-1 \end{pmatrix}\left(\tfrac{2}{1+\sqrt{\delta_i}}\right)^j\left(\tfrac{1}{1+d_i}\right)^M \\ &= \left(\tfrac{1-\sqrt{\delta_i}}{2}\right)^M\textstyle\sum_{j=1}^{M}\begin{pmatrix} 2M-j-1 \\ M-j \end{pmatrix}\left(\tfrac{1+\sqrt{\delta_i}}{2}\right)^{M-j}. \end{aligned} \tag{38}$$

If we define $k = M-j$, then P_b can be written as

$$P_b = \left(\frac{1-\sqrt{\delta_i}}{2}\right)^M\sum_{k=0}^{M-1}\begin{pmatrix} M+k-1 \\ k \end{pmatrix}\left(\frac{1+\sqrt{\delta_i}}{2}\right)^k. \tag{39}$$

This is exactly the same expression that appears in Proakis' text [24].

Let L be the minimum time diversity of the code – i.e., the minimum Hamming distance, in signal symbols, between any two valid sequences. Then, $L \leq L_\eta$ and we can further upper bound the pairwise error probability by

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) \leq \left[\frac{1}{2^{2ML}}\sum_{j=1}^{ML}\begin{pmatrix} 2ML-j-1 \\ ML-1 \end{pmatrix}\left(\frac{2}{1+\sqrt{\delta_m}}\right)^j\right] \times \prod_{i\in\eta}\frac{1}{(1+d_i)^M}. \tag{40}$$

Note that the upper bound in [8] is a special case of this bound ($M = 1$). The upper bound in (40) is in a product form, allowing us to use the transfer function approach to yield the following bound on the bit error probability:

$$P_b \leq \frac{1}{k}\left[\frac{1}{2^{2ML}}\sum_{j=1}^{ML}\begin{pmatrix} 2ML-j-1 \\ ML-1 \end{pmatrix}\left(\frac{2}{1+\sqrt{\delta_m}}\right)^j\right]\frac{\partial T(\bar{D},I)}{\partial I}\mid_{I=1,D=e^{-E_s/4N_o}} \tag{41}$$

where

$$\bar{D}\mid_{D=e^{-E_s/4N_0}} = \frac{1}{(1+\frac{E_s}{4N_o})^M}. \tag{42}$$

Comparing this with the Chernoff bound, we note that the extra term on the left of the transfer function is at most one, so this bound is at least as tight as the Chernoff bound. To see the tightness of the bound, an 8-state I-Q TCM code employing 16-QAM is used as an example. Its bandwidth efficiency is 2 bits/sec/Hz. I-Q TCM codes are trellis codes in which the in-phase and quadrature components of the transmitted signal are encoded independently; Al-Semari and Fuja [21, 22] have shown that this approach yields better performance over Rayleigh fading channels than codes designed using the "traditional" approach. This particular code has a minimum time diversity of $L = 4$, and its performance is superior to that of the comparable 8-state conventional TCM code using 8-PSK in Rayleigh fading. (See [22] for details.) For this specific coding/modulation

$$\delta_m = \frac{0.8E_s/N_o}{1+0.8E_s/N_o} \tag{43}$$

Figure 4 compares the newly derived bound for dual diversity (M=2) and maximal ratio combining with the Chernoff bound for the same code; the new bound is slightly more than 1 dB tighter than the Chernoff bound at a bit error rate of 10^{-5}.

The performance of the 16-QAM 8-state code with MRC and different orders of diversity is shown in Figure 5. It is clear that the largest gain is obtained in going from single to double diversity; i.e., the coding gains diminish as the order of diversity increases. This confirms the conclusions obtained from the cutoff rate curves.

4.2 Equal Gain Combining

In the case of equal gain combining the conditional pairwise error probability is given by

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N|\mathbf{a}_N) &= P(m(\hat{\mathbf{x}}_N, \mathbf{y}_N; \mathbf{a}_N) - m(\mathbf{x}_N, \mathbf{y}_N; \mathbf{a}_N) \geq 0|\mathbf{a}_N) \\ &= P(\textstyle\sum_{i=1}^{N}(|\sum_{l=1}^{M}(y_{i,l} - a_{i,l}x_i)|^2 \\ &\quad -|\textstyle\sum_{l=1}^{M}(y_{i,l} - a_{i,l}\hat{x}_i)|^2) \geq 0|\mathbf{a}_N). \end{aligned} \tag{44}$$

It can be simplified to

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N|\mathbf{a}_N) &= P(\textstyle\sum_{i=1}^{N}\{-(\sum_{l=1}^{M} a_{i,l})^2|x_i - \hat{x}_i|^2 \\ &\quad -2(\textstyle\sum_{l=1}^{M} a_{i,l})\Re[\sum_{l=1}^{M} n_{i,l}(x_i - \hat{x}_i)^*]\} \leq 0|\mathbf{a}_N) \\ &= P(z_N \geq \textstyle\sum_{i=1}^{N}(\sum_{l=1}^{M} a_{i,l})^2|x_i - \hat{x}_i|^2|\mathbf{a}_N), \end{aligned} \tag{45}$$

where z_N is a Gaussian random variable variance $2N_oM\sum_{i=1}^{N}(\sum_{l=1}^{M} a_{i,l})^2|x_i - \hat{x}_i|^2$ and zero mean This probability can be expressed as

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N|\mathbf{a}_N) = \frac{1}{2}\text{erfc}\left(\sqrt{\sum_{i=1}^{N} \mu_i\tilde{d}_i/b_o}\right) \tag{46}$$

where $\tilde{d}_i = b_o|x_i - \hat{x}_i|^2/4MN_o$ and $\mu_i = (\sum_{l=1}^{M} a_{i,l})^2$. Here, μ_i has pdf

$$f_\mu(\mu_i) = \frac{\mu_i^{(M-1)}\exp(-\frac{\mu_i}{b_o})}{b_o^M(M-1)!}, \quad \mu_i \geq 0. \tag{47}$$

Define $\Gamma_i = \mu_i/b_o$. Then

$$f_{\Gamma_i}(\gamma_i) = \frac{\gamma_i^{(M-1)}\exp(-\gamma_i)}{(M-1)!}, \quad \gamma_i \geq 0. \tag{48}$$

Then the unconditional pairwise error probability can be expressed as

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) &= \tfrac{1}{2}\textstyle\int_0^\infty \cdots \int_0^\infty \text{erfc}\left(\sqrt{\sum_{i=1}^{N}\gamma_i\tilde{d}_i}\right) \\ &\quad \times f_\Gamma(\gamma_1)\cdots f_\Gamma(\gamma_N)d\gamma_1\cdots d\gamma_N. \end{aligned} \tag{49}$$

Similarly, define

$$\tilde{\delta}_i = \frac{\tilde{d}_i}{1+\tilde{d}_i} \qquad \text{and} \qquad \tilde{\omega}_i = \gamma_i(1+\tilde{d}_i). \tag{50}$$

Therefore, the unconditional pairwise error probability can be represented as

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) &= \tfrac{1}{2}\textstyle\prod_{i\in\eta}\frac{1}{(1+\tilde{d}_i)^M}\int_0^\infty \cdots \int_0^\infty \text{erfc}\left(\sqrt{\sum_{i=1}^{N}\tilde{\delta}_i\tilde{\omega}_i}\right) \\ &\quad \times e^{(\sum_{i=1}^{N}\tilde{\delta}_i\tilde{\omega}_i)}f(\tilde{\omega}_1)\cdots f(\tilde{\omega}_N)d\tilde{\omega}_1\cdots d\tilde{\omega}_N \end{aligned} \tag{51}$$

where $\eta = \{i : x_i \neq \hat{x}_i\}$ and $L_\eta = |\eta|$. Note that

$$\sum_{i\in\eta} \tilde{\delta}_i \tilde{\omega}_i \geq \sum_{i\in\eta} \delta_e \tilde{\omega}_i \tag{52}$$

where $\delta_e = \min\{\tilde{\delta}_i, i \in \eta\}$. Hence, the pairwise error probability can be upper bounded as

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) \leq \frac{1}{2} \prod_{i\in\eta} \frac{1}{(1+\tilde{d}_i)^M} \int_0^\infty \operatorname{erfc}\left(\sqrt{\delta_e \tilde{\Omega}}\right) \times e^{(\delta_e \tilde{\Omega})} f(\tilde{\Omega}) d\tilde{\Omega} \tag{53}$$

where $\tilde{\Omega} = \sum_{i\in\eta} \tilde{\omega}_i$ and $\tilde{\Omega}$ is distributed as

$$f(\tilde{\Omega}) = \frac{1}{(ML_\eta - 1)!} \tilde{\Omega}^{(ML_\eta - 1)} e^{-\tilde{\Omega}}, \quad \tilde{\Omega} \geq 0. \tag{54}$$

Finally performing the integration yields

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) \leq \left[\frac{1}{2^{2ML_\eta}} \sum_{j=1}^{ML_\eta} \binom{2ML_\eta - j - 1}{ML_\eta - 1} \left(\frac{2}{1+\sqrt{\delta_e}}\right)^j\right] \times \prod_{i\in\eta} \frac{1}{(1+\tilde{d}_i)^M}. \tag{55}$$

For the uncoded BPSK system, $\tilde{d}_i = b_o E_s / M N_o$, and so $\tilde{\delta}_e = \tilde{\delta}_i = \tilde{d}_i/(1+\tilde{d}_i)$ and

$$\frac{1}{1+\tilde{d}_i} = 1 - \tilde{\delta}_i = (1 - \sqrt{\tilde{\delta}_i})(1 + \sqrt{\tilde{\delta}_i}). \tag{56}$$

Hence, the bit error probability of uncoded BPSK can be expressed as

$$P_b = \frac{1}{2^{2M}} \sum_{j=1}^{M} \binom{2M - j - 1}{M - 1} \left(\frac{2}{1+\sqrt{\tilde{\delta}_{min}}}\right)^j \left(\frac{1}{(1+\tilde{d}_i)}\right)^M, \tag{57}$$

which can be written as

$$P_b = \left(\frac{1-\sqrt{\tilde{\delta}_i}}{2}\right)^M \sum_{k=0}^{M-1} \binom{M + k - 1}{k} \left(\frac{1+\sqrt{\tilde{\delta}_i}}{2}\right)^k. \tag{58}$$

Again, since $L \leq L_\eta$ we can further upper bound $P(\mathbf{x}_N \to \hat{\mathbf{x}}_N)$ by

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) \leq \left[\frac{1}{2^{2ML}} \sum_{j=1}^{ML} \binom{2ML - j - 1}{ML - 1} \left(\frac{2}{1+\sqrt{\delta_e}}\right)^j\right] \times \prod_{i\in\eta} \frac{1}{(1+\tilde{d}_i)^M}. \tag{59}$$

Therefore, the bit error probability can be tightly upper bounded by

$$P_b \leq \frac{1}{k} \left[\frac{1}{2^{2ML}} \sum_{j=1}^{ML} \binom{2ML - j - 1}{ML - 1} \left(\frac{2}{1+\sqrt{\delta_e}}\right)^j\right] \frac{\partial T(\bar{D}, I)}{\partial I} \Big|_{I=1, D=e^{-E_s/4N_o}}. \tag{60}$$

where

$$\bar{D}\mid_{D=e^{-E_s/4N_0}}=\frac{1}{(1+\frac{b_oE_s}{4MN_o})^M}. \tag{61}$$

For the same 8-state 16-QAM IQ-TCM scheme previously considered, δ_e is given by

$$\delta_e=\frac{0.8b_oE_s/MN_o}{1+(0.8b_oE_s/MN_o)}.$$

The performance of the previous code with equal gain combining and different orders of diversity is shown in Figure 6.

4.3 Selection Combining

With selection combining, the conditional pairwise error probability can be expressed as

$$\begin{aligned} P(\mathbf{x}_N\to\hat{\mathbf{x}}_N|\mathbf{a}_N) &= P(m(\hat{\mathbf{x}}_N,\mathbf{y}_N;\mathbf{a}_N)-m(\mathbf{x}_N,\mathbf{y}_N;\mathbf{a}_N)\geq 0|\mathbf{a}_N) \\ &= P(\textstyle\sum_{i=1}^{N}(|y_{i,l}-a_{i,j^\star}x_i|^2-|y_{i,l}-a_{i,j^\star}\hat{x}_i|^2)\geq 0|\mathbf{a}_N). \end{aligned} \tag{62}$$

This expression can be simplified to

$$\begin{aligned} P(\mathbf{x}_N\to\hat{\mathbf{x}}_N|\mathbf{a}_N) &= P(\textstyle\sum_{i=1}^{N}(-a_{i,j^\star}^2|x_i-\hat{x}_i|^2-2a_{i,j^\star}\Re\{n_{i,l}(x_i-\hat{x}_i)^*\})\leq 0|\mathbf{a}_N) \\ &= P(z_N\geq\textstyle\sum_{i=1}^{N}\nu_i|x_i-\hat{x}_i|^2|\mathbf{a}_N). \end{aligned} \tag{63}$$

where $\nu_i=a_{i,j^\star}^2$ and z_N is a Gaussian random variable with zero mean and a variance of $2N_o\sum_{i=1}^{N}\nu_i|x_i-\hat{x}_i|^2$. This probability can be expressed as

$$P(\mathbf{x}_N\to\hat{\mathbf{x}}_N|\mathbf{a}_N)=\frac{1}{2}\mathrm{erfc}\left(\sqrt{\sum_{i=1}^{N}\nu_id_i}\right) \tag{64}$$

where $d_i=|x_i-\hat{x}_i|^2/4N_o$. The pdf of ν_i is

$$\begin{aligned} f_\nu(\nu_i) &= M(1-e^{-\nu_i})^{(M-1)}e^{-\nu_i} \\ &= \textstyle\sum_{k_i=1}^{M}M(-1)^{k_i+1}\begin{pmatrix} M-1 \\ k_i-1 \end{pmatrix}e^{-k_i\nu_i}. \end{aligned} \tag{65}$$

The unconditional pairwise error probability can be expressed as

$$\begin{aligned} P(\mathbf{x}_N\to\hat{\mathbf{x}}_N) &= \tfrac{1}{2}\textstyle\sum_{k_1=1}^{M}\cdots\sum_{K_N=1}^{M}\prod_{i=1}^{N}\left\{M(-1)^{k_i+1}\begin{pmatrix} M-1 \\ k_i-1 \end{pmatrix}\right\} \\ &\quad\times\textstyle\int_0^\infty\cdots\int_0^\infty\mathrm{erfc}\left(\sqrt{\sum_{i=1}^{N}\nu_id_i}\right)e^{-k_1\nu_1}\cdots e^{-k_N\nu_N}d\nu_1\cdots d\nu_N. \end{aligned} \tag{66}$$

Define

$$\delta_{i,k_i}=\frac{d_i}{k_i+d_i}\qquad\text{and}\qquad\omega_{i,k_i}=\nu_i(k_i+d_i). \tag{67}$$

Hence, the pairwise error probability can be expressed as

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) &= \tfrac{1}{2}\sum_{k_1=1}^{M}\cdots\sum_{K_N=1}^{M}\prod_{i=1}^{N}\{M(-1)^{k_i+1}\begin{pmatrix} M-1 \\ k_i-1 \end{pmatrix}\left(\tfrac{1}{k_i+d_i}\right)\} \\ &\times \int_0^\infty\cdots\int_0^\infty \operatorname{erfc}\left(\sqrt{\sum_{i=1}^{N}\delta_{i,k_i}\omega_{i,k_i}}\right) e^{(\sum_{i=1}^{N}\delta_{i,k_i}\omega_{i,k_i})} \\ &\times e^{-\omega_{1,k_1}}\cdots e^{-\omega_{N,k_N}}\,d\omega_{1,k_1}\cdots d\omega_{N,k_N}. \end{aligned} \tag{68}$$

Defining $\Gamma = \sum_{i\in\eta}\omega_{i,k_i}$, we obtain Therefore,

$$f_\Gamma(\gamma) = \tfrac{1}{(L_\eta-1)!}\gamma^{(L_\eta-1)}e^{-\gamma}, \quad \gamma \geq 0. \tag{69}$$

Also, observe that

$$\sum_{i\in\eta}\delta_{i,k_i}\omega_{i,k_i} \geq \sum_{i\in\eta}\delta_s\omega_{i,k_i} \tag{70}$$

where

$$\delta_s = \min\{\delta_{i,k_i}, i \in \eta, k_i \in \{1,\ldots M\}\} = \frac{\min\{d_i\}}{M+\min\{d_i\}}. \tag{71}$$

Using the above expressions, the pairwise error probability can be expressed as

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) &\leq \tfrac{1}{2}\prod_{i\in\eta}\left[\sum_{k=1}^{M} M(-1)^{k+1}\begin{pmatrix} M-1 \\ k-1 \end{pmatrix}\tfrac{1}{(k+d_i)}\right]\times \\ &\int_0^\infty \operatorname{erfc}\left(\sqrt{\delta_s\gamma}\right)e^{(\delta_s\gamma)}f_\Gamma(\gamma)d\gamma. \end{aligned} \tag{72}$$

The final step is to evaluate the integral and replace L_η by L. Doing so yields

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) &\leq \left[\tfrac{1}{2^{2L}}\sum_{j=1}^{L}\begin{pmatrix} 2L-j-1 \\ L-1 \end{pmatrix}\tfrac{2^j}{(1+\sqrt{\delta_s})^j}\right] \\ &\times\prod_{i\in\eta}\left[\sum_{k=1}^{M} M(-1)^{k+1}\begin{pmatrix} M-1 \\ k-1 \end{pmatrix}\tfrac{1}{(k+d_i)}\right]. \end{aligned} \tag{73}$$

For the uncoded BPSK systems:

$$d_i = E_s/N_o \quad \text{and} \quad \delta_{i,k_i} = \frac{d_i}{k_i+d_i} \qquad \text{and} \qquad \frac{k_i}{k_i+d_i} = (1-\sqrt{\delta_{i,k}})(1+\sqrt{\delta_{i,k}}) \tag{74}$$

Therefore,

$$P_b = \frac{1}{2}\sum_{k_i=1}^{M} M(-1)^{k_i+1}\begin{pmatrix} M-1 \\ k_i-1 \end{pmatrix}\frac{1}{(k_i+d_i)}\frac{1}{(1+\sqrt{\delta_{i,k_i}})}, \tag{75}$$

which can be finally written as

$$P_b = \frac{1}{2}\sum_{k=1}^{M}(-1)^{k+1}\begin{pmatrix} M \\ k \end{pmatrix}(1-\sqrt{\frac{d_i}{k+d_i}}). \tag{76}$$

For trellis coded systems, the bit error probability can now be expressed as

$$P_b \leq \frac{1}{k}\left[\frac{1}{2^{2L}}\sum_{j=1}^{L}\left(\begin{array}{c}2L-j-1\\ L-1\end{array}\right)\left(\frac{2}{1+\sqrt{\delta_s}}\right)^j\right]\frac{\partial T(\bar{D},I)}{\partial I}\,|_{I=1,D=e^{-E_s/4N_o}}, \tag{77}$$

where

$$\bar{D}\,|_{D=e^{-E_s/4N_0}}=\sum_{k=1}^{M} M(-1)^{k+1}\left(\begin{array}{c}M-1\\ k-1\end{array}\right)\frac{1}{k+\frac{E_s}{4N_o}}. \tag{78}$$

For the same previous coding/modulation scheme, δ_s is expressed as

$$\delta_s = \frac{0.8E_s/N_o}{M+0.8E_s/N_o}. \tag{79}$$

The performance of the previous code with selection combining and different orders of diversity is shown in Figure 7.

Comparing the three combining schemes shows that MRC achieves the best performance. EGC error performance is within 1 dB from MRC. As M increases, the difference between the three schemes increases. Also, it is obvious that the upper bound is very tight and gives very accurate BER values, especially at bit error rates less than 10^{-3}.

5 The Effect of Branch Correlation

In the previous analysis we have assumed that the fading in the different diversity branches are independent. In some cases, this is difficult to achieve due to improper antennae positioning or receiver space limitations. Therefore, it is important to examine the possible performance degradation due to correlated branch signals. The effect of branch correlation on the distribution of the received signal was studied by Schwartz et. al. in 1966 [2]. Recently, the effect of correlation on non-coherent orthogonal digital modulation was studied [23]. They derived upper bounds for binary convolutional codes and non-coherent orthogonal digital modulation.

In this section the pairwise error probability for maximal ratio combining with correlated branch signals is derived. Recall from Eqn. 26 that the conditional pairwise error probability may be expressed as

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N|\mathbf{a}_N) = \frac{1}{2}\text{erfc}\left(\sqrt{\sum_{i=1}^{N}\sum_{l=1}^{M} a_{il}^2 d_i}\right) \tag{80}$$

where $d_i = |x_i - \hat{x}_i|^2/4N_o$. Also, $a_{il}^2 = |h_{il}|^2$ where h_{il} is a complex Gaussian random variable with zero mean and variance of 1/2 for both the real and imaginary parts. Observe that

$$\sum_{l=1}^{M} a_{il}^2 = \underline{h}_i \underline{h}_i^* \tag{81}$$

where $\underline{h}_i = \{h_{i1}, \ldots, h_{iM}\}$ and $(\cdot)^*$ denotes the Hermitian transpose. The probability density function of $\underline{h}_i$ is expressed as

$$f(\underline{h}_i) = \frac{1}{\pi^M \det K_{\underline{h}_i}} \exp(-\underline{h}_i K_{\underline{h}_i}^{-1} \underline{h}_i^*), \tag{82}$$

where $K_{\underline{h}_i}$ is an $M \times M$ covariance matrix with entries $(K_{\underline{h}_i})_{lk} = E(h_{il} h_{ik}^*)$. It is assumed that the real parts of h_{il}'s are independent of the imaginary parts – i.e., the cross-covariances are zero.

The first step is to uncorrelate the random variables using a linear transformation. We are interested in generating a new vector $\underline{g}_i = \{g_{i1}, \ldots, g_{iM}\}$ with a diagonal covariance matrix. Let U be the transformation - i.e.,

$$\underline{g}_i = U \underline{h}_i. \tag{83}$$

Using this transformation, the covariance matrix of $\underline{g}_i$, denoted by $K_{\underline{g}_i}$ is expressed as

$$K_{\underline{g}_i} = U K_{\underline{h}_i} U^t, \tag{84}$$

where U^t is the transpose of U. Since $K_{\underline{h}_i}$ is a symmetric matrix, it can be represented as

$$K_{\underline{h}_i} = Q \Lambda Q^t, \tag{85}$$

where Λ is a diagonal matrix that consists of the eigenvalues of $K_{\underline{h}_i}$ and Q is a matrix whose columns are the orthonormal eigenvectors of $K_{\underline{h}_i}$. The last equation can be rewritten

$\Lambda = Q^t K_{\underline{h}_i} Q$. Therefore, if we let $U = Q^t$ then the Gaussian random variables $\underline{g}_i$ are independent and $K_{\underline{g}_i} = \Lambda$ – i.e.,

$$K_{\underline{g}_i} = \begin{cases} \lambda_j & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \tag{86}$$

The second step is to make another transformation so that the covariance matrix becomes the identity matrix. This is achieved via the transformation

$$\underline{g}_i = \sqrt{K_{\underline{g}_i}} \underline{p}_i. \tag{87}$$

This makes the Gaussian random variables $\underline{p}_i$ independent with same variance. Using this transformation, the unconditional pairwise error probability can be expressed as

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) = \tfrac{1}{2} \textstyle\int_0^\infty \cdots \int_0^\infty \operatorname{erfc}\left(\sqrt{\sum_{i=1}^N \sum_{l=1}^M \lambda_l q_{il} d_i}\right) \times \\ f_q(q_{11}) \cdots f_q(q_{1M}) \cdots f_q(q_{NM}) dq_{11} \cdots dq_{1M} \cdots dq_{NM}, \end{aligned} \tag{88}$$

where $q_{il} = |p_{il}|^2$. Define

$$\tilde{\delta}_{il} = \frac{\lambda_l d_i}{1 + \lambda_l d_i} \qquad \text{and} \qquad \tilde{\omega}_{il} = q_{il}(1 + \lambda_l d_i). \tag{89}$$

Hence, the unconditional pairwise error probability can be expressed as

$$\begin{aligned} P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) = \tfrac{1}{2} \textstyle\prod_{i \in \eta} \prod_{l=1}^M \left(\frac{1}{1+\lambda_l d_i}\right) \int_0^\infty \cdots \int_0^\infty \operatorname{erfc}\left(\sqrt{\sum_{i=1}^N \sum_{l=1}^M \tilde{\delta}_{il} \tilde{\omega}_{il}}\right) \times \\ f_\omega(\tilde{\omega}_{11}) \cdots f_\omega(\tilde{\omega}_{1M}) \cdots f_\omega(\tilde{\omega}_{NM}) d\tilde{\omega}_{11} \cdots d\tilde{\omega}_{1M} \cdots d\tilde{\omega}_{NM} \end{aligned} \tag{90}$$

where $\eta = \{i : x_i \neq \hat{x}_i\}$ and L_η is its cardinality. However,

$$\sum_{i \in \eta} \sum_{l=1}^{M} \tilde{\delta}_{il} \tilde{\omega}_{il} \geq \sum_{i \in \eta} \sum_{l=1}^{M} \delta_c \tilde{\omega}_{il}, \tag{91}$$

where $\delta_c = \min\{\tilde{\delta}_{il}, i \in \eta, l = 1, \cdots M\}$ Hence, the pairwise error probability can be upper bounded as

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) \leq \frac{1}{2} \prod_{i \in \eta} \prod_{l=1}^{M} \frac{1}{(1 + \lambda_l d_i)} \int_0^\infty \operatorname{erfc}\left(\sqrt{\delta_c \Phi}\right) \times e^{(\delta_c \Phi)} f_\Phi(\Phi) d\Phi, \tag{92}$$

where $\Phi = \sum_{i \in \eta} \sum_{l=1}^M \tilde{\omega}_{il}$. Since the $\tilde{\omega}_{il}$'s are independent exponentially distributed random variables each with parameter one, Φ will have an (ML_η)-Erlang distribution with parameter one – i.e.,

$$f_\Phi(\Phi) = \tfrac{1}{(ML_\eta - 1)!} \Phi^{(ML_\eta - 1)} e^{-\Phi}, \quad \Phi \geq 0. \tag{93}$$

Finally performing the integration yields

$$P(\mathbf{x}_N \to \hat{\mathbf{x}}_N) \leq \left[\frac{1}{2^{2ML_\eta}} \sum_{j=1}^{ML_\eta} \left(\begin{array}{c} 2ML_\eta - j - 1 \\ ML_\eta - 1 \end{array} \right) \left(\frac{2}{1+\sqrt{\delta_c}} \right)^j \right] \times \prod_{i\in\eta} \prod_{l=1}^{M} \frac{1}{(1+\lambda_l d_i)}. \tag{94}$$

Therefore, the bit error probability can be expressed as

$$P_b \leq \frac{1}{k} \left[\frac{1}{2^{2ML}} \sum_{j=1}^{ML} \left(\begin{array}{c} 2ML - j - 1 \\ ML - 1 \end{array} \right) \left(\frac{2}{1+\sqrt{\delta_c}} \right)^j \right] \frac{\partial T(\bar{D}, I)}{\partial I} \,|_{I=1, D=e^{-E_s/4N_o}}, \tag{95}$$

where

$$\bar{D} \,|_{D=e^{-E_s/4N_0}} = \prod_{j=1}^{M} \frac{1}{(1+\lambda_j \frac{E_s}{4N_o})}. \tag{96}$$

Similarly, the cutoff rate can be expressed as

$$R_o = 2\log_2(|A|) - \log_2 \left(\sum_{x_i \in A} \sum_{x_j \in A} \left[\prod_{l=1}^{M} \frac{1}{\left(1+\lambda_l \frac{|x_i - \hat{x}_i|^2}{4N_o}\right)} \right] \right). \tag{97}$$

As an example, dual diversity is used in many practical systems. The covariance matrix $K_{\underline{h}_i}$ is represented as

$$K_{\underline{h}_i} = \left[\begin{array}{cc} 1 & \rho \\ \rho & 1 \end{array} \right]. \tag{98}$$

So ρ is the correlation coefficient between the two antenna elements; the eigenvalues of $K_{\underline{h}_i}$ are $(1-\rho)$ and $(1+\rho)$. A 4-state I-Q TCM 16-QAM scheme (2 bits/s/Hz) is used as an example. For this configuration, δ_c will be

$$\delta_c = \frac{(1-\rho)(0.8E_s/N_o)}{1+(1-\rho)(0.8E_s/N_o)}. \tag{99}$$

A comparison between the bound and simulations are shown in Figure 8 for the case of $\rho = 0.5$. Clearly, the bound is very tight. Figure 9 shows the bit error probability upper bound curves for this code with different values of ρ. It is noted that values as large as $\rho = 0.5$ degrade the performance slightly. The effect of space correlation is not as severe as time correlation (which can be minimized via interleaving).

Finally, a comparison between three schemes is shown in Figure 10. The first scheme uses a 4-state code and maximal ratio combining of two branches. The branch correlation ρ

is assumed to be 50%. The second scheme uses a 64-state code but no diversity combining. Both schemes are 16-QAM. The third scheme employs diversity only. It uses uncoded QPSK with Gray mapping; the diversity order is $M = 3$ and independent branches are assumed. All systems have a bandwidth efficiency of 2 b/s/Hz. Simulation results are plotted for the first two systems and analytical values are shown for the third system. Clearly, the first scheme outperforms the other schemes at BER $< 4 \times 10^{-3}$ even though moderate branch correlation (50%) exists. This suggests that a combination of simple channel coding and double diversity might yield in general better performance than using complex channel coding schemes or several diversity receivers. Moreover, increased delay and interleaving for complicated channel codes is avoided. This results in less system delay, which is favorable in mobile and personal communications.

6 Conclusions

In this paper, cutoff rate expressions of coherent systems with maximal ratio, equal gain, and selection combining schemes have been evaluated using Chernoff bounds. Moreover, tight upper bounds on the pairwise error probability have been derived. These upper bounds were used to evaluate a variety of system configurations, including uncoded and coded systems. The upper bounds were expressed in product form to allow the use of the transfer function approach for evaluating the performance of trellis coded systems. Simulations of different systems show that the derived bounds are very tight.

For the case of branch correlation, the cutoff rate and a tight upper bound on the pairwise error probability were derived for maximal ratio combining. Again, the pairwise error probability was expressed in product form so that the transfer function approach could be used. Branch correlation with correlation coefficients less than 0.5 result in a slight performance loss. The results indicate that the joint use of simple coding and diversity results in a substantial improvement in Rayleigh fading over the use of a separate more complex codes (without diversity) or a higher degree of diversity (without coding).

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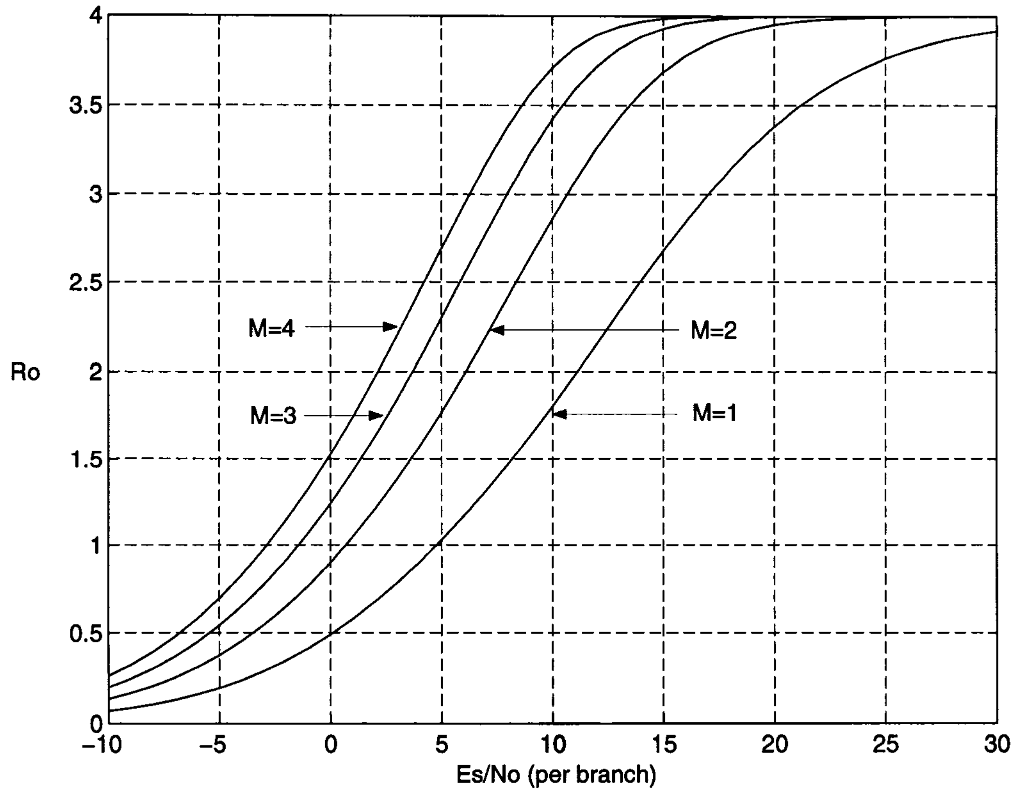

Figure 1: The cutoff rate of 16-QAM with maximal ratio combining and different diversity orders

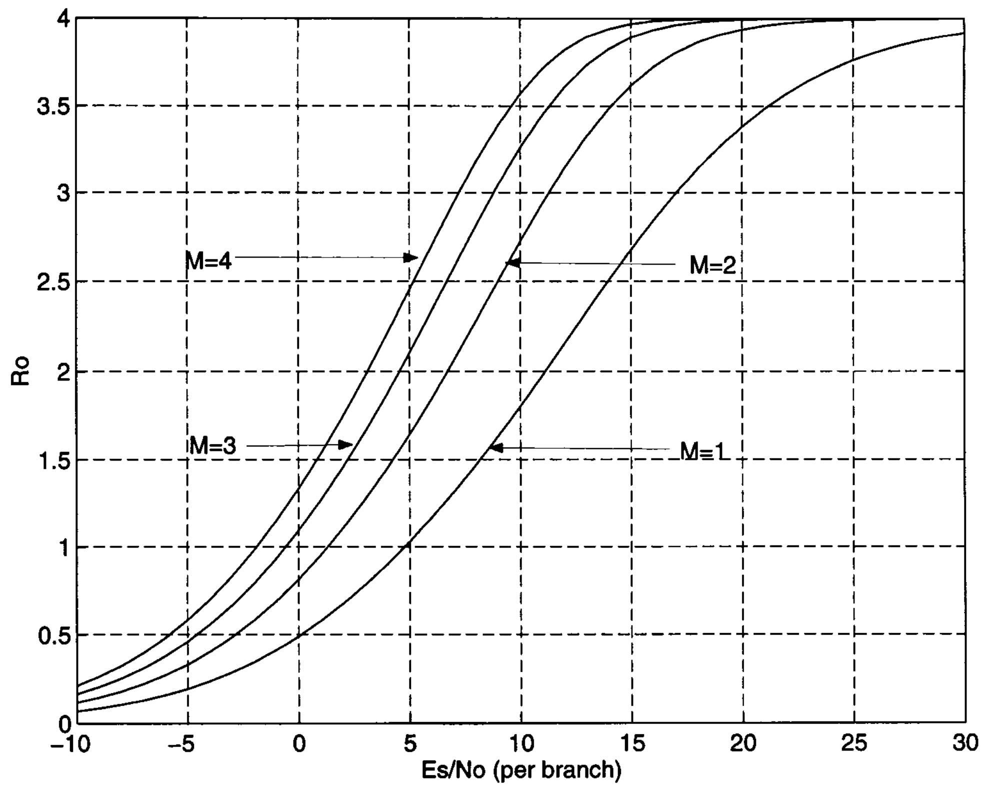

Figure 2: The cutoff rate of 16-QAM with equal gain combining and different diversity orders

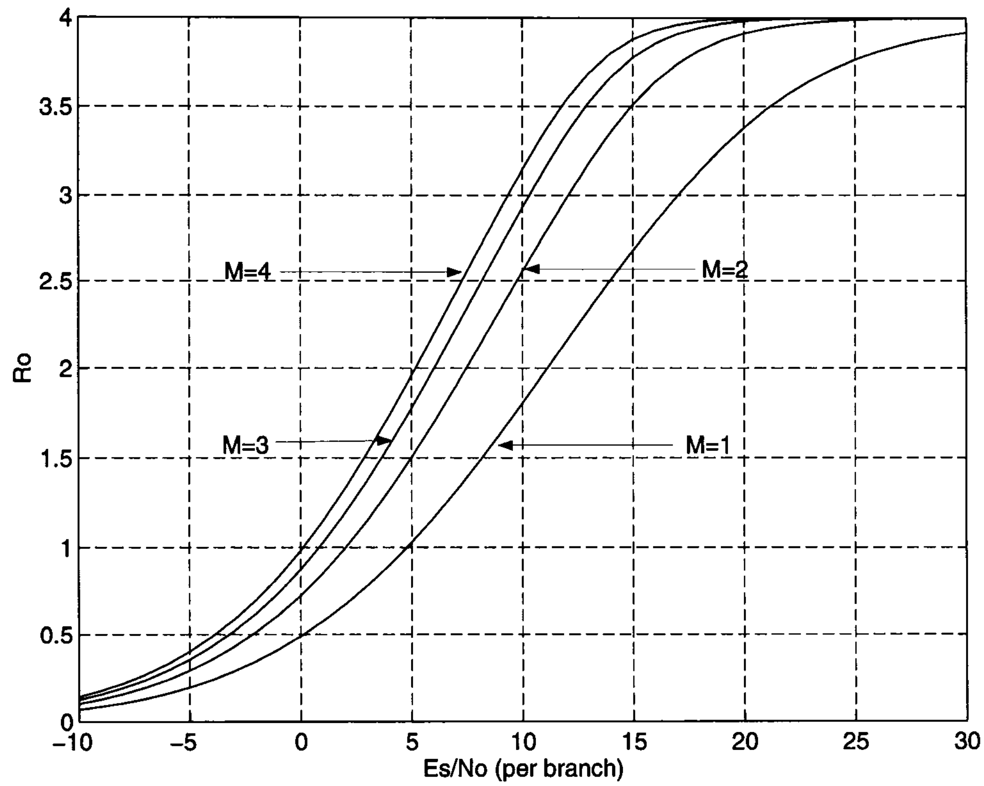

Figure 3: The cutoff rate of 16-QAM with selection combining and different diversity orders

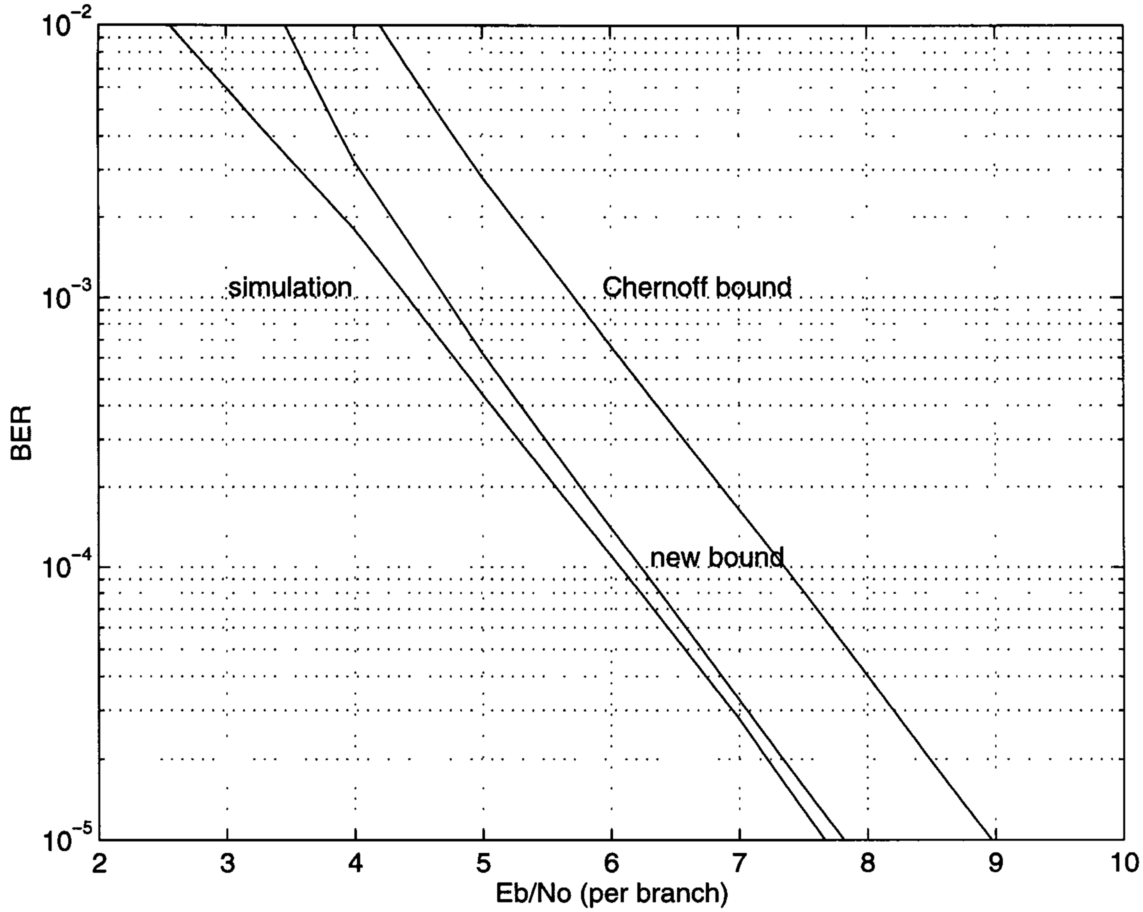

Figure 4: A comparison between the new bound and the Chernoff bound for the 16-QAM I-Q TCM 8-state code with maximal ratio combining and double diversity.

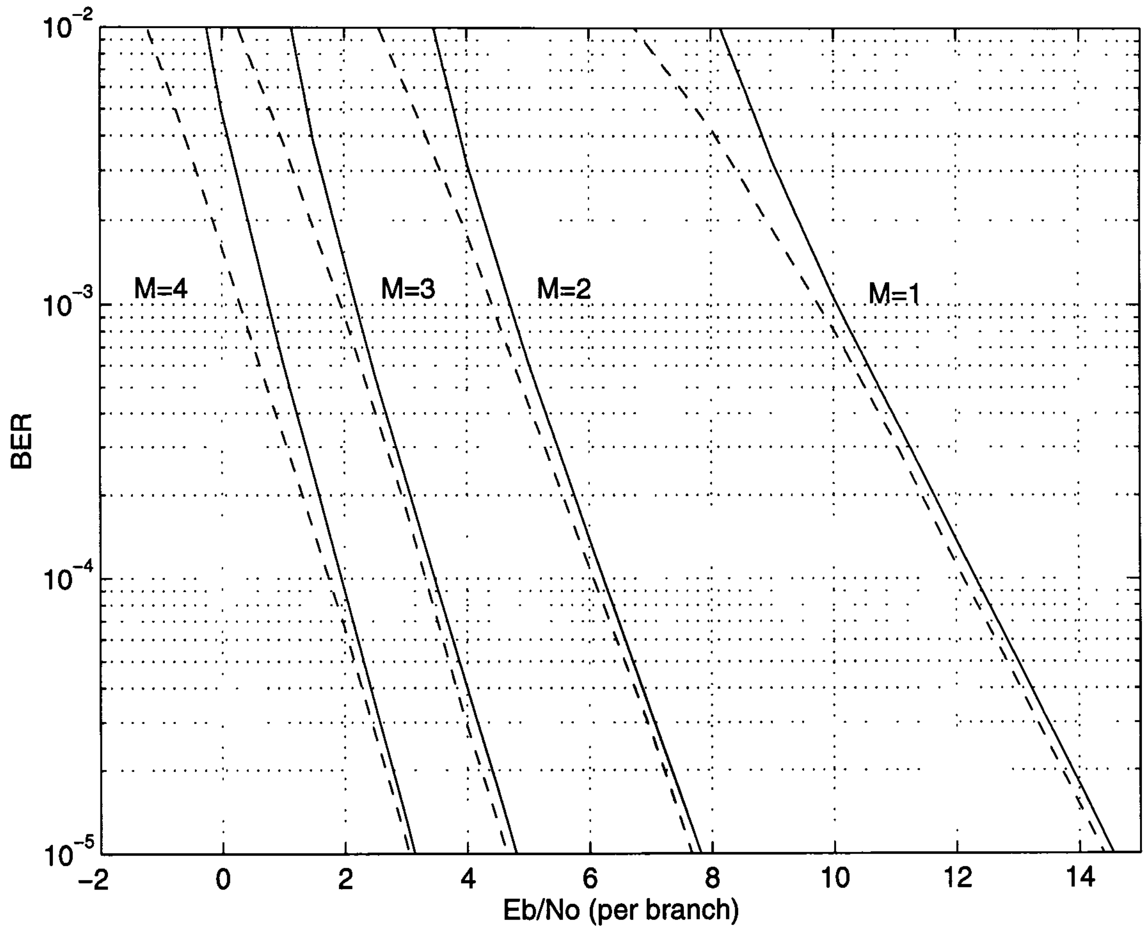

Figure 5: BER of the 16-QAM I-Q TCM 8-state code with maximal ratio combining and different diversity orders. solid(bound), dashed(simulation)

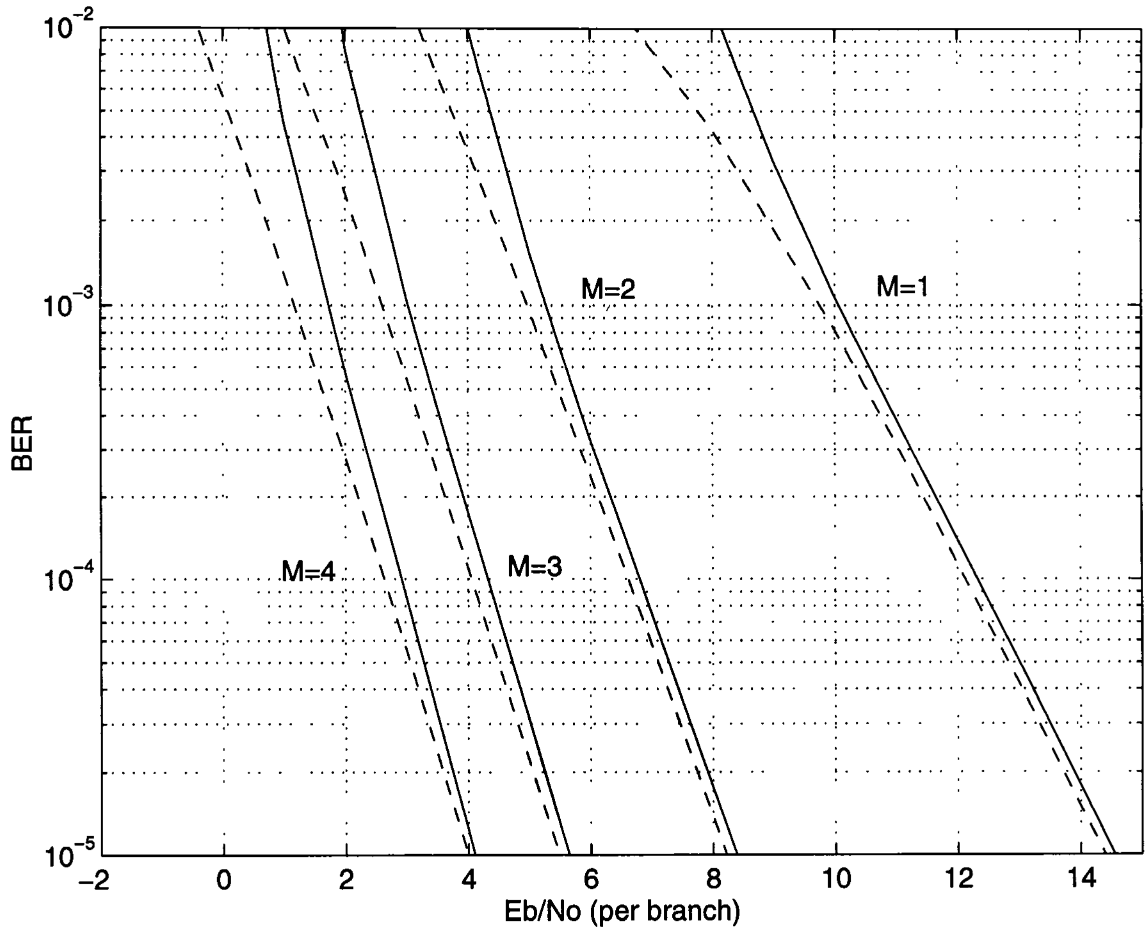

Figure 6: BER of the 16-QAM I-Q TCM 8-state code with equal gain combining and different diversity orders. solid(bound), dashed(simulation)

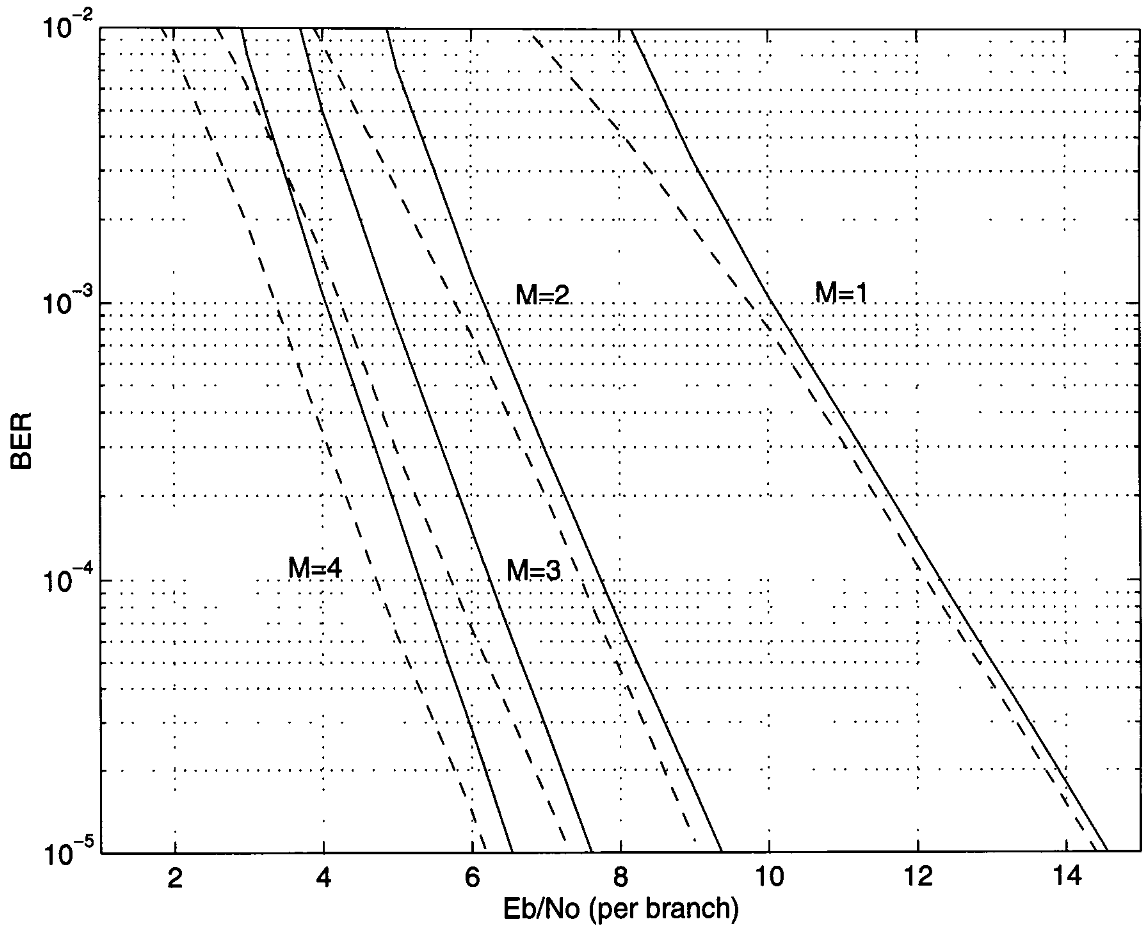

Figure 7: BER of the 16-QAM I-Q TCM 8-state codes with selection combining and different diversity orders. solid(bound), dashed(simulation)

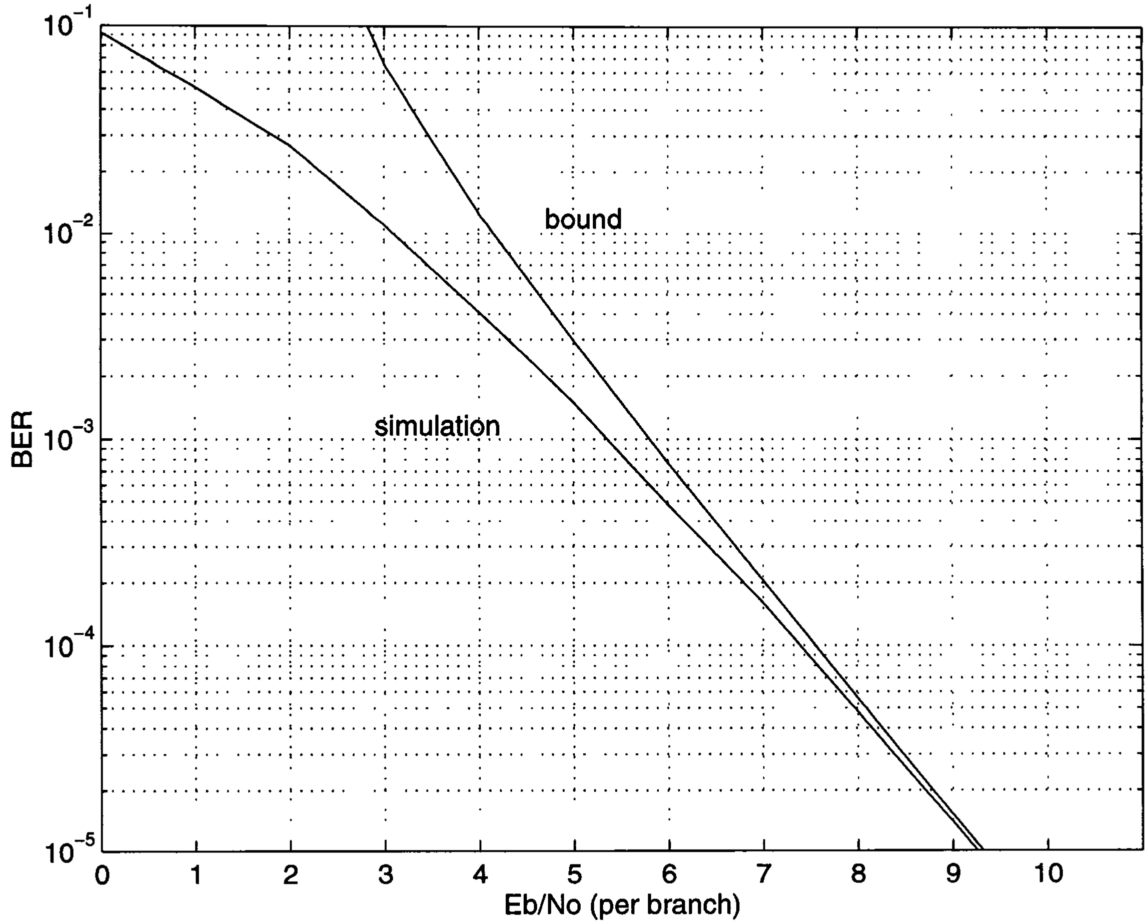

Figure 8: A comparison between the upper bound and simulated BER of 16-QAM I-Q TCM 4-state code with MRC dual diversity and $\rho = 0.5$.

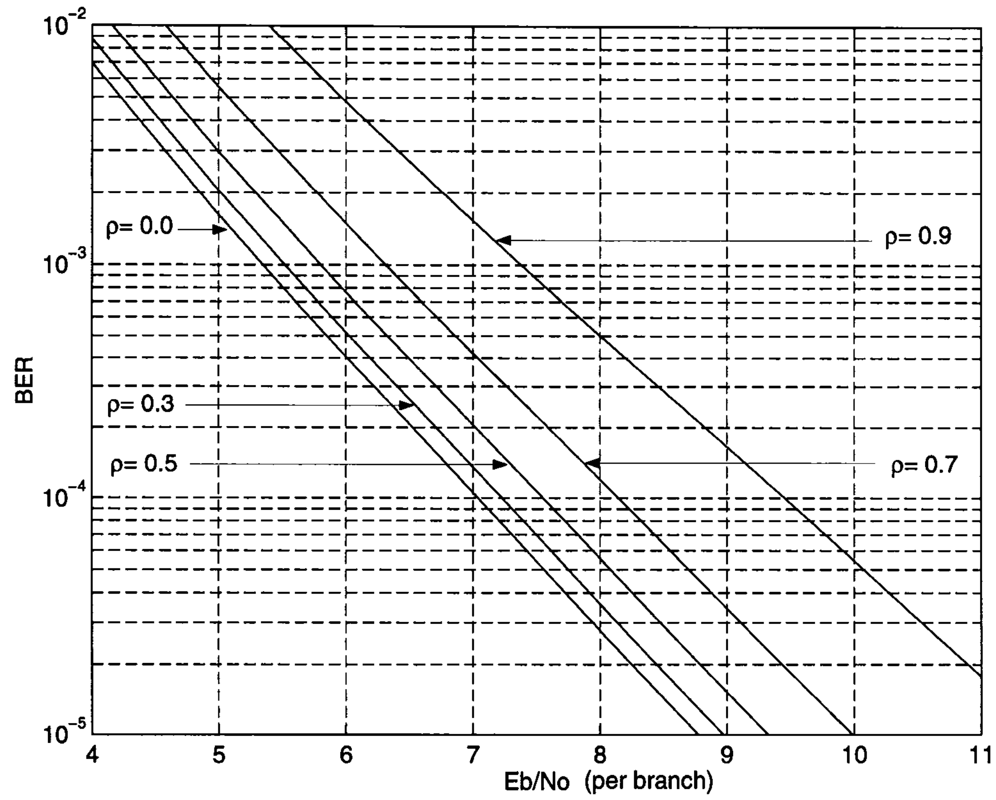

Figure 9: Analytical BER of 16-QAM I-Q TCM 4-state code with MRC dual diversity and different correlation values.

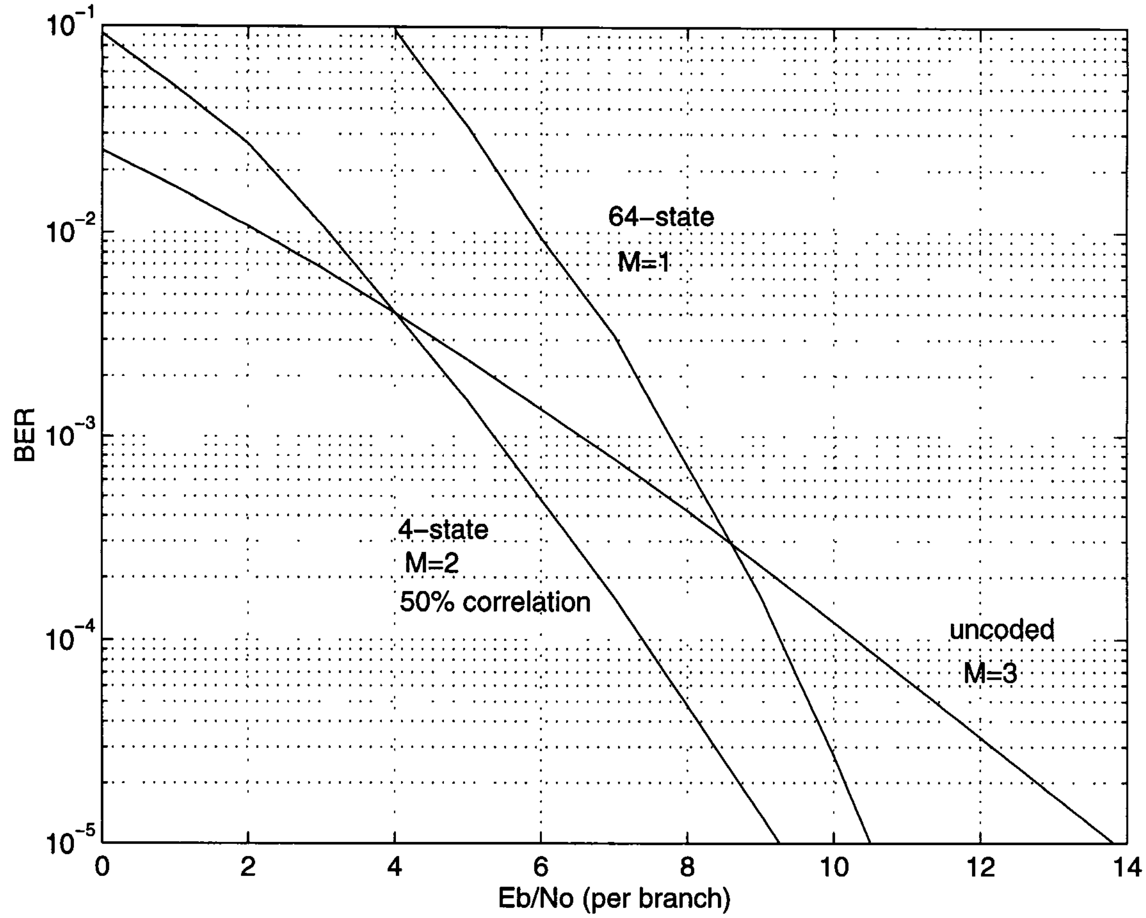

Figure 10: Copmarison between the 16-QAM I-Q TCM 4-state code (with double diversity, MRC and 50% branch correlation) and the 64-state code (with no diversity).