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Minimizing Work-in-Process and Material Handling in the Facilities Layout Problem

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Abstract

We consider the plant layout problem for a job shop environment. This problem is generally treated as the quadratic assignment problem with the objective of minimizing material handling costs. In this paper we investigate conditions under which the quadratic assignment solution also minimizes average work-in-process. To get some initial insights, we model the plant as an open queueing network and show that under a certain set of assumptions, the problem of minimizing work-in-process reduces to the quadratic assignment problem. We then investigate via simulation the robustness of this result. We found that the relationship between material handling costs and average work-in-process holds under much more general conditions than are assumed in the analytical model. For those cases where the result does not hold, we have refined the model by developing a simple secondary measure which appears to work well in practice.

Keywords: plant layout, quadratic assignment problem, queueing network models for manufacturing systems.

1 Introduction

The facilities layout problem has generally been tackled using one of two approaches. The first approach seeks a solution that minimizes the total material handling costs between all pairs of facilities and is known as the quadratic assignment problem (QAP) (see, e.g., Burkard [3]). The second approach is known as the adjacency requirements formulation and the objective is to maximize the sum of closeness ratings (Foulds [10]). These ratings may be subjective, and contribute to the objective function only if the two facilities in question are adjacent in a solution. Both formulations lead to an NP-hard problem ([3, 10]), and heuristic solution methods have to be applied to most real-size problems. The QAP model is preferred for most manufacturing and other operations where the total material handling cost is the appropriate criterion.

The most successful optimal solution procedures for the QAP are implicit enumeration algorithms based on the lower bound proposed by Gilmore [11] and Lawler [15]. A good implementation of this bound is given by Burkard and Derigs [5]. Linearizations of the QAP have been proposed, e.g., by Bazaraa and Sherali [2] and Kaufman and Broeckx [13], but have not been very successful computationally.

Heuristic procedures for the QAP can be classified as constructive, improvement, limited enumeration, or heuristic solutions to linearized problems. Hybrid procedures combining improvement with one of the other three approaches have been very successful, e.g, Burkard and Bonniger [4], Bazaraa and Kirca [1], and Kaku, Thompson, and Morton [12].

Attempts at solving the facilities layout problem in a multi-criteria context have been limited, mainly to work with objective functions that combine the QAP and the adjacency requirements objectives. Examples of such an approach can be found in Rosenblatt [16], Dutta and Sahu [8], and Fortenberry and Cox [9]. More recently, Kouvelis and Kiran [14] have incorporated throughput requirements into the layout design problem for automated manufacturing systems. They model the system as a closed queueing network based on the assumption that the number of pallets and hence the number of jobs in the system is fixed. Given some required throughput, they minimize the sum of transportation costs and the costs of work-in-process (WIP) inventory as measured by the number of jobs in the system necessary to maintain that throughput.

In this paper we consider queueing effects in a general job shop where the throughput is fixed, but WIP is not. Thus an open queueing network is the more appropriate model (see, for example, Buzacott and Shanthikumar [6]). We formulate an open queueing network model that includes layout considerations in the material handling system, which is considered to have a fixed capacity (e.g. a fixed number of forklifts). Under the analogous assumptions used by Kouvelis and Kiran [14] for the closed queueing network model, we show that the QAP formulation is equivalent to the goal of minimizing WIP. This implies that, as a rough approximation, the layout which minimizes WIP also minimizes material handling.

However, some of the assumptions on the material handling system used in the analytical queueing model may not be realistic. Even so, the approximate equivalence between the QAP and the problem of minimizing WIP derived from the analytical model allows us to limit the search space for a more refined simulation model. The simulation model handles the complexities in the material handling system not captured in the analytical queueing network model. The simulation results indicate that the result holds more generally, but there are exceptional situations where a refinement of the model is necessary. We characterize situations where such an adjustment is necessary, and develop a secondary measure to incorporate an appropriate adjustment.

The rest of the paper is organized as follows. In Section 2, we introduce the analytical open queueing network model and the necessary assumptions to establish our result. The simulation results and secondary measure are reported in Section 3. Conclusions are given in Section 4.

2 The Open Queueing Network Model

The problem can be stated as follows. There are a number of potential locations and a number of departments, and the objective is to match the departments to locations in such a way as to minimize the average work-in-process, given the set of part types and respective demand rates to be handled by the system. The material handling system is assumed to consist of a fixed number of transporting vehicles, which we will call forklifts for convenience.

We define some parameters of the system:

$$
\begin{aligned}
N &= \text{number of departments, number of locations} \\
\mathcal{P} &= \text{set of part types produced by the system} \\
P &= \text{number of part types} \\
m_0 &= \text{number of forklifts} \\
m_i &= \text{number of machines in department } i, \quad i = 1, \ldots, N \\
v &= \text{average velocity of a forklift} \\
d_{ij} &= \text{distance between location } i \text{ and } j, \quad i, j = 1, \ldots, N \\
\mathcal{R}_k &= \text{process route for part type } k, \quad k = 1, \ldots, P \\
r_k &= \text{demand rate for part type } k, \quad k = 1, \ldots, P
\end{aligned}
$$

For any two departments, say i and j, we model the system as follows. We model the processing at the departments i and j as multi-server queues. It follows that both queueing time and actual

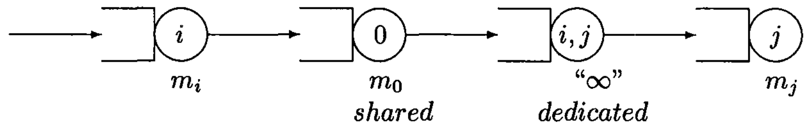

Figure 1: Queueing Model of the Link Between Two Departments

processing time are included in the model. Movement between the two departments involves the use of a shared resource, the material handling system, which in our model consists of a set of forklifts. The time for a batch of parts to travel from one department to another will be the sum of two times:

1. the time waiting for a forklift to become available at the first department, and
2. the actual transportation time between the two departments.

The first time consists of the potential waiting time for a forklift to become available anywhere in the plant (e.g., if all forklifts are in use when one is needed) plus a travel time for the forklift to come to the department.

We model the first part as a shared multi-server queue to take into account the potential contention for resources, and the second part as an infinite-server queue with mean service time equal to the distance between the two departments divided by the average velocity over all the forklifts. The modeling of the actual transportation time by means of an infinite server queue is an approximation, because in theory it allows for a larger number of batches in transit than there are forklifts available. However, the service times used in modeling the waiting for a forklift should compensate for this.

We use the index i ($i = 1, ..., N$) for the departments and reserve the index 0 for the material handling "waiting" station. We will use i, j to denote the actual transportation route from department i to j. This queueing model is shown in Figure 1.

In our model, for example, the process route $\mathcal{R} = (1, 5, 3)$ would be represented as a queueing route in the network as $1 \rightarrow 0 \rightarrow (1,5) \rightarrow 5 \rightarrow 0 \rightarrow (5,3) \rightarrow 3$.

We define the following:

$$L_0 \; = \; \text{average work-in-process waiting for a forklift}$$

$$L_i \; = \; \text{average work-in-process in department } i, \quad i = 1, \ldots, M$$

$$L_{i,j} \; = \; \text{average work-in-process in transportation from } i \text{ to } j, \quad i, j = 1, \ldots, M$$

The work-in-process can be located either (i) at the machines in the departments (in buffer or in process), (ii) waiting for transportation from the material handling system, or (iii) in transit from one department to another department. Thus, our objective is to determine the assignment of departments to locations so as to minimize

$$L_0 + \sum_{i=1}^{M} L_i + \sum_{i=1}^{M} \sum_{j=1}^{M} L_{i,j} \tag{1}$$

A commonly accepted set of assumptions for manufacturing systems which leads to a product-form solution is given by Buzacott and Shanthikumar [7]:

1. external part type arrival processes into the system are Poisson;
2. processing times at a department are i.i.d. exponential;
3. travel times of forklifts to a department are exponential and independent of the department;
4. buffer sizes are sufficiently large such that blocking is negligible;
5. service discipline is first-come, first-served (FCFS).

It is not necessary to assume exponential travel times *between* departments since we model these travel times as infinite server queues. We also note that multiple visits to a department are allowed in our formulation.

We will now show, under the usual product-form assumptions, that the first two terms in Equation (1) are independent of the layout. To demonstrate this, we define the following:

$$\lambda_0 \; = \; \text{rate of part flow into (out of) the material handling system}$$

λ_i = flow into (out of) department i, $i = 1, \ldots, M$

λ_{ij} = flow from department i to j, $i, j = 1, \ldots, M$

x_0 = mean waiting time at the material handling system

x_i = mean processing time in department i, $i = 1, \ldots, M$

τ_{ij} = mean transportation time from department i to j, $i, j = 1, \ldots, M$

Note that x_0, which is the mean waiting time at the material handling system, is the mean "service" time for node 0 in the queueing network model.

We define the following visit-counting functions of the routes:

$|\mathcal{R}|$ = total # of visits to departments in route $\mathcal{R}$,

$|\mathcal{R}|_i$ = # of visits to department i in route $\mathcal{R}$, $i = 1, \ldots, M$

$|\mathcal{R}|_{ij}$ = # of department i to department j routings in route $\mathcal{R}$ $i, j = 1, \ldots, M$

The flows λ_i and λ_{ij} can be found from the demand rates given by r_k and the routings $\mathcal{R}_k$ via the usual flow balance equations for open queueing networks (see Wolff [17]:

$$\lambda_0 = \sum_{k=1}^{P} r_k(|\mathcal{R}_k| - 1) \tag{2}$$

$$\lambda_i = \sum_{k=1}^{P} r_k |\mathcal{R}_k|_i, \quad i = 1, \ldots, M \tag{3}$$

$$\lambda_{ij} = \sum_{k=1}^{P} r_k |\mathcal{R}_k|_{ij}, \quad i, j = 1, \ldots, M \tag{4}$$

We assume x_i, $i = 1, \ldots, N$, can be estimated, and we have $\tau_{ij} = d_{i'j'}/v$, where i' is the location to which department i is assigned and j' is the location to which department j is assigned. Thus, if the system has enough production capacity to meet demand ($\lambda_i x_i / m_i < 1$ for all $i = 0, 1, \ldots, N$), then standard queueing theory results (Wolff [17] yield the following:

$$L_{i,j} = \lambda_{ij}\tau_{ij} = \lambda_{ij} d_{i'j'}/v \quad i, j = 1, \ldots, N \tag{5}$$

and

$$L_i = \sum_{n=1}^{\infty} n p_{n,i} \quad i = 0, 1, \ldots, N \tag{6}$$

where $p_{n,i}$ represents the steady-state probability of n parts at department i given by

$$p_{n,i} = \begin{cases} \frac{p_{0,i}(m_i \rho_i)^n}{n!} & \text{for } n \leq m_i \\ \frac{p_{0,i} m_i^{m_i} \rho_i^n}{m_i!} & \text{for } n \geq m_i \end{cases} \tag{7}$$

$$p_{0,i} = \left[\frac{(m_i \rho_i)^{m_i}}{(1-\rho_i) m_i!} + \sum_{j=0}^{m_i - 1} \frac{(m_i \rho_i)^j}{j!} \right]^{-1} \tag{8}$$

where

$$\rho_i = \frac{\lambda_i x_i}{m_i}$$

is the traffic intensity.

The average WIP in a department ($L_i; i > 0$) and the average WIP waiting for a forklift (L_0) depend only on demand rates, process routes, process times, and the number of machines; both are independent of the layout and thus of the distances between departments. This implies that to minimize WIP in the system, we need to determine the assignment of departments to locations that minimizes the third term in (1), i.e., the WIP in the material handling system. This term can be rewritten as:

$$\sum_{i=1}^{M} \sum_{j=1}^{M} L_{i,j} = \sum_{i=1}^{M} \sum_{j=1}^{M} \lambda_{ij} \tau_{ij} = \frac{1}{v} \sum_{i=1}^{M} \sum_{j=1}^{M} \lambda_{ij} d_{i'j'}, \tag{9}$$

Given that v is a constant, namely the average velocity over all forklifts, the problem has been reduced to the quadratic assignment problem. This model indicates that if the departments operate independently (except for the layout-dependent, material handling interaction), then the QAP formulation also serves to minimize work-in-process. Of course, this requirement of independence makes the model inappropriate for tightly coupled manufacturing systems such as those found in cellular manufacturing.

Whereas the optimal solution is independent of the actual material handling system capacity, the WIP level itself is not. By assigning costs to different levels of material handling capacity and to different WIP levels, it would be possible to investigate the question of what capacity should be provided in the material handling system to minimize the sum of these two costs.

3 The Simulation Experiment

In an attempt to build a simple model that can provide some insight into the facility layout decision, we included assumptions that may not always be satisfied in practice. Of particular concern are the following items:

1. The processing times in a department are distributed i.i.d. exponential over all part types.
2. The time it takes for a forklift to arrive to pick up a batch of material is independent of the layout. This arises due to the shared multi-server queue used to account for waiting time.
3. The number of batches in transit may be greater than the number of forklifts, due to the use of an infinite-server queue to model actual transportation time between two departments.

The first item is related more to analytic tractability of the system, and we believe that it is not a critical factor in the result. The next two items involve modeling simplifications, in particular the assumption that the time spent by parts in the material handling systems can be decomposed into a waiting time *independent* of the layout plus a travel time. This is a plausible assumption if the number of forklifts is not overutilized and the distribution of number of batches processed in departments is "uniform" in some sense. To test the extendibility of the results from the model to more general conditions, we conducted simulation experiments as described below.

3.1 Generating the test problems

We randomly generated 10 problems with 8 facilities each. Two of these facilities are in fixed locations, one at each end of the manufacturing facility. One is considered to be the raw material storage area and serves as an "entry" station; and the other is considered to be the finished goods storage and shipping area and serves as an "exit" station. The number of locations is also 8, arranged in a 2-by-4 rectangular grid, and distances are measured as rectilinear center-to-center.

To provide the flow pattern a structure similar to one that may be found in practice, we randomly generated demands and process routes for 10 part types for each problem, and then translated this information into the familiar flow matrix of QAPs. The step-by-step procedure for each part type is as follows:

1. Part-type demand in units is 40 times some multiple between 1 and 100, randomly generated from a uniform distribution.

2. Part-type batch size is 5, 10, 20 or 40, with the probability of larger batch sizes increasing as the demand increases. For example, if part demand is less than 1/4 of the maximum possible demand, then one of the four batch sizes is chosen with equal probability, whereas if part demand is larger than 3/4 of the maximum possible demand, then the batch size is set at 40.

3. The number of departments visited by the part type is randomly generated from a uniform distribution between 1 and 5, excluding the first and last visit which are fixed for all part types as the entry and exit stations, respectively. This number of departments is then chosen at random with limited backtracking allowed; a part type may return to one department that it has already visited, but only after visiting one or more departments in-between.

4. Parts move through the system in batches, with the number of batches determined as the demand divided by the batch size. Information on the process route and number of batches is translated into flows between specific departments for the part type.

This is repeated for all part types with flows between departments being aggregated into the flow matrix. Other data that need to be generated are:

1. The processing time per unit for the part type in a department is randomly chosen as an integer number of minutes from the range (1, 10), providing an average processing time of 5.5 minutes.

2. The setup time for each batch of the part type in a department is chosen similarly as an integer number of minutes from the range (R, $10R$). Two values were used for R: 5 and 25, such that the average setup time for the two cases is 27.5 and 137.5 minutes, respectively.

Finally, the following parameters are adjusted:

1. The speed of the forklifts is set at 40, 45, 50, or 55 feet per minute, based on the optimal cost of the solution. This adjustment was made to provide sufficient forklift capacity as total flows increased.

2. The number of machines in a department is adjusted so that utilization in the department does not exceed some prespecified level.

The ten test problems lead to 12 cases each through different settings for three parameters. These are the two values of R mentioned above; three values for the maximum utilization levels (90, 70 and 50%) in the departments; and two values for the number of forklifts (2 or 3).

To test the relationship between the QAP solution cost and the average WIP in the system we compare the simulation results for three solutions to each problem. These are an optimal solution, a "good" solution, and a poor solution. The optimal solution is found by the procedure published by Burkard and Derigs [5]. The good solution is found by a procedure that is a simplified version of the heuristic developed by Kaku, Thompson and Morton [12]. We construct 9 distinct solutions, improve them through pairwise interchange, and choose the best solution with cost greater than the optimal (in other words, alternate optima are not considered). The poor solution is obtained from the optimal solution by making pairwise interchanges that *worsen* the solution, and choosing the solution whose cost is closest to 30% above the cost of the optimal. Details on the 10 test problems and the 3 solutions to each are given in Table 1.

3.2 The simulation model and results

Unlike the analytical queueing network model of the previous section, the simulation model we employed explicitly takes into account the queueing effects of a limited number of forklifts and the possible additional lags due to travel of the forklift when empty to the next place where it is needed from its previous drop-off point. A forklift is assumed to remain at the station at which it unloads until another request is made. When a request is made, if more than one forklift is available, then the closest available one to the requesting station is used. These characteristics were modeled using the constructs available in the SIMAN simulation language.

Table 1: Solutions for the 10 test problems

Problem	Solution	Cost	% Deviation	Assignments[a]							
1	Optimal	3958	0.00	1	3	6	7	2	5	4	8
	Near-opt	4062	2.63	1	6	3	7	2	4	5	8
	Poor	4966	25.47	1	7	6	2	3	5	4	8
2	Optimal	5094	0.00	1	7	3	2	6	4	5	8
	Near-opt	5264	3.34	1	6	7	3	2	.4	5	8
	Poor	6756	32.63	1	5	2	4	3	7	6	8
3	Optimal	5578	0.00	1	4	3	6	7	5	2	8
	Near-opt	5672	1.69	1	4	5	3	7	2	6	8
	Poor	6880	23.34	1	4	3	7	5	2	6	8
4	Optimal	3186	0.00	1	2	3	5	4	7	6	8
	Near-opt	3192	0.19	1	6	3	5	7	4	2	8
	Poor	4296	34.84	1	3	4	5	6	2	7	8
5	Optimal	3620	0.00	1	6	7	4	2	3	5	8
	Near-opt	3712	2.54	1	3	7	4	5	6	2	8
	Poor	4752	31.27	1	6	5	4	2	3	7	8
6	Optimal	4144	0.00	1	7	6	2	3	5	4	8
	Near-opt	4252	2.61	1	7	3	2	6	4	5	8
	Poor	5612	35.42	1	3	6	7	2	5	4	8
7	Optimal	3354	0.00	1	5	6	4	2	3	7	8
	Near-opt	3474	3.58	1	4	3	5	2	6	7	8
	Poor	4498	34.11	1	5	6	4	2	7	3	8
8	Optimal	3932	0.00	1	3	4	7	6	2	5	8
	Near-opt	4288	9.05	1	5	4	7	3	6	2	8
	Poor	5044	28.28	1	3	6	7	2	4	5	8
9	Optimal	4790	0.00	1	3	2	4	5	7	6	8
	Near-opt	4796	0.13	1	5	3	4	2	7	6	8
	Poor	6194	29.31	1	3	2	5	7	6	4	8
10	Optimal	5396	0.00	1	5	4	6	7	3	2	8
	Near-opt	5754	6.63	1	4	7	3	6	5	2	8
	Poor	7304	35.36	1	5	7	6	2	3	4	8

[a] The assignments are stated for departments to locations. For example, the optimal solution to Problem 9 has department 1 in location 1, department 2 in location 3, department 3 in location 2, and so on.

The simulation model incorporates batches, meaning the decomposition of service time at a department into a set-up time per batch and an actual per part processing time. In the SIMAN implementation, we took the set-up times as exponential and the processing times as deterministic. We also took each batch as the entity in the system in order to reduce storage requirements and make the simulation more efficient. We ran each case for 6 weeks of production, with a warm-up period of 1 week in which no statistics were taken, where a week comprised 5 days of 3 shifts, i.e., 120 hours. Flow times, department WIP and utilization, and forklift WIP and utilization, were recorded, all in terms of batches, and statistics based on 40 independent replications were calculated. These statistics included individual 95% confidence intervals around the estimated mean, and paired-t 95% confidence intervals for each pair of differences in WIP and flow times.

In all 120 instances (12 cases, 10 problems), the poor solution does decidedly worse than the other two solutions, i.e., in general, better solutions to the QAP problem lead to lower levels of average WIP in the system. However, in the case of two solutions that are very close to each other in terms of QAP costs, the WIP results are not clearly predictable in terms of these costs. These results seem to be consistent across all 12 cases. Table 2 shows a snapshot summary of a comparison between the optimal and good solutions based on a paired-t comparisons of the mean flow times, which by Little's Law serves as a surrogate for the *total* batch WIP. The notation used in this table indicates the direction of the comparison and the statistical significance. For example "≪" indicates the optimal solution had a lower average flow time and the difference is statistically significant, whereas "<" indicates the optimal solution had a lower average flow time, but the difference is not statistically significant.

In the case of test problems 3, 4, and 9, the QAP costs of the good solutions are very close to the optimal costs — with a difference of less than 2% — and the good solutions may be termed as near-optimal solutions. For these three problems, the near-optimal solution sometimes has lower WIP than the optimal solution. A breakdown of the total WIP into its two components, department WIP and forklift WIP, shows further that the department WIP is practically identical; the difference lies in the forklift WIP. To understand this result, a detailed comparison of the placement of departments in the two solutions is required. For this purpose, we make a distinction between the 4 central locations

Table 2: Comparison of Optimal and Near-Optimal Solutions at the 95% level

	Case											
Problem	1	2	3	4	5	6	7	8	9	10	11	12
1	$\ll$	$<$	$<$	$<$	$\ll$	$\ll$	$<$	$<$	$<$	$<$	$\ll$	$<$
2	$\gg$	$<$	$<$	$\ll$	$\ll$	$\ll$	$>$	$\ll$	$<$	$<$	$\ll$	$<$
3	$\gg$	$<$	$>$	$<$	$\gg$	$<$	$<$	$<$	$>$	$>$	$\gg$	$<$
4	$>$	$<$	$>$	$>$	$\gg$	$\ll$	$<$	$<$	$<$	$>$	$<$	$>$
5	$>$	$<$	$>$	$\ll$	$<$	$<$	$>$	$>$	$<$	$>$	$<$	$>$
6	$\ll$	$\ll$	$<$	$\ll$	$\ll$	$\ll$	$>$	$<$	$\ll$	$<$	$\ll$	$<$
7	$<$	$<$	$<$	$<$	$\ll$	$\ll$	$>$	$>$	$<$	$<$	$\ll$	$<$
8	$\ll$	$\ll$	$<$	$\ll$	$\ll$	$\ll$	$\ll$	$<$	$\ll$	$\ll$	$\ll$	$\ll$
9	$\gg$	$>$	$\ll$	$>$	$\gg$	$\gg$	$<$	$>$	$>$	$<$	$\gg$	$>$
10	$\ll$	$<$	$\ll$	$\ll$	$\ll$	$\ll$	$<$	$<$	$\ll$	$<$	$\ll$	$<$

and the 4 corner locations in the 2-by-4 grid since the former have shorter average distances to other locations than the latter. The optimal and near-optimal solutions for test problem 9 are shown in Figure 2 to facilitate this discussion. Comparing the optimal and near-optimal solutions, we see that departments 3, 6, and 7 remain in central locations, while departments 1, 4, and 8 remain in corner locations. The only difference is in the case of departments 5 and 2, which exchange positions with respect to central and corner locations. Department 5, which is in a corner location in the optimal solution, has a larger flow through it (856 batches) than department 2 (456 batches). Thus the forklift is more likely to be at a corner location in the optimal solution at any given point in time and, therefore, will take longer to respond to the next service call, contributing to higher forklift WIP. However, this is not a critical factor when there is a fairly clear QAP cost difference between the optimal and the good solution, because the in-transit forklift WIP dominates the WIP waiting for a forklift. If the two solutions are very close to each other, then the in-transit WIP for both is nearly the same, and the waiting WIP becomes a factor.

A similar effect can be observed in Problems 3 and 4; however, this type of visual analysis is not reliable for all situations, e.g., changes in locations for more than two departments, or larger problems with more classes of locations than the two in our example problems. To overcome this limitation, we have devised an easily calculated measure to capture the effect of layout on the time spent waiting for material handling service in the form of the expected travel time for an empty

Total flow through department 2 = 456

Total flow through department 5 = 856

1	3	2	4
5	7	6	8

Optimal

1	5	3	4
2	7	6	8

Near-optimal

Figure 2: Optimal and near-optimal solutions for test problem 9

forklift to arrive at the station demanding service. This time is given approximately by

$$\tilde{d}_i/\nu \quad \text{w.p. } \lambda_i/\lambda, \quad \text{where } \lambda = \sum_{i=1}^{M} \lambda_i,$$

and $\tilde{d}_i$ is the distance from department i to where the forklift is at present, which is d_{ij} w.p. λ_j/λ. Thus, an estimate for the expected travel time for an empty forklift is

$$\sum_{i=1}^{M-1} \sum_{j=2}^{M} \frac{d_{ij}}{\nu} \frac{\lambda_j}{\lambda} \frac{\lambda_i}{\lambda} = \frac{1}{\nu\lambda^2} \sum_{i=1}^{M-1} \sum_{j=2}^{M} d_{ij}\lambda_i\lambda_j,$$

so our secondary measure is given by

$$\sum_{i=1}^{M-1} \sum_{j=2}^{M} d_{ij}\lambda_i\lambda_j.$$

In essence, the refined model treats x_0 as layout-dependent, whereas in the original model, it was considered to be layout independent. Our basic algorithm to minimize WIP is as follow:

1. Find the best QAP solutions. If one is significantly better than all the others, choose it.

2. If near-optimal solutions exist, check the secondary measure to discard solutions whose measures are considerably higher than others.

3. Pick one of the finalists, possibly based on other additional considerations.

We ran another set of simulations to test the usefulness of this secondary measure. Ten new problems were generated, each having the characteristic that there is at least one near-optimal QAP

Table 3: Solutions for the second set of 10 test problems

Problem	Solution	Cost	% Deviation	Sec. Measure	Assignments							
1	Optimal	5362	0.00	1.712845	1	3	6	7	2	5	4	8
	Near-opt	5386	0.45	1.731464	1	6	3	7	2	4	5	8
2	Optimal	5224	0.00	1.919127	1	7	3	2	6	4	5	8
	Near-opt	5244	0.38	1.957579	1	6	7	3	2	4	5	8
3	Optimal	4135	0.00	1.771174	1	4	3	6	7	5	2	8
	Alt.-opt	4135	0.00	1.769924	1	4	5	3	7	2	6	8
	Near-opt	4153	0.44	1.780842	1	4	3	7	5	2	6	8
4	Optimal	2612	0.00	1.790738	1	2	3	5	4	7	6	8
	Near-opt	2622	0.38	1.823147	1	6	3	5	7	4	2	8
5	Optimal	4078	0.00	1.923231	1	6	7	4	2	3	5	8
	Near-opt	4094	0.39	1.794920	1	3	7	4	5	6	2	8
6	Optimal	3682	0.00	1.819641	1	7	6	2	3	5	4	8
	Near-opt	3684	0.05	1.827726	1	7	3	2	6	4	5	8
7	Optimal	2450	0.00	1.838522	1	5	6	4	2	3	7	8
	Alt.-opt	2450	0.00	1.882022	1	4	3	5	2	6	7	8
	Near-opt	2458	0.33	1.842749	1	5	6	4	2	7	3	8
8	Optimal	4811	0.00	1.720258	1	3	4	7	6	2	5	8
	Near-opt	4815	0.08	1.712955	1	5	4	7	3	6	2	8
9	Optimal	4643	0.00	1.966118	1	3	2	4	5	7	6	8
	Near-opt	4647	0.09	1.967324	1	5	3	4	2	7	6	8
10	Optimal	4770	0.00	2.046220	1	5	4	6	7	3	2	8
	Near-opt	4776	0.13	2.049782	1	4	7	3	6	5	2	8
	Near-opt	4778	0.17	2.119964	1	5	7	6	2	3	4	8

solution within 0.5% of the optimal. This time alternate optimal solutions were accepted, provided that they were not simply mirror images of the optimal. (A mirror image is obtained by rotating any solution around its horizontal or vertical axis.) For the ratio of setup to processing times we set $R = 10$; maximum machine or assembly workstation utilization at 70%; and the number of forklifts at 2. These problems and their solutions are presented in Table 3. We see that 2 of the problems had alternate optima and 3 had two additional solutions within 0.5% of the optimal.

The results are shown in Table 4. The secondary measure consistently picks the solution with the lower WIP, though the results are not always statistically significant.

Table 4: Output Statistics for Case 13

		Department		Forklift	
Problem	Flow Time	WIP	utilization	WIP	utilization
	600± 4	32.5± 0.4	0.65±0.01	2.03±0.04	0.67±0.01
1	601± 4	32.5± 0.4	0.65±0.01	2.07±0.04	0.68±0.01
	499± 4	27.2± 0.3	0.65±0.01	2.19±0.05	0.69±0.01
2	501± 4	27.2± 0.3	0.65±0.01	2.33±0.07	0.69±0.01
	593± 4	24.7± 0.3	0.60±0.00	2.41±0.06	0.72±0.01
3	594± 4	24.8± 0.3	0.60±0.00	2.40±0.06	0.72±0.01
	593± 4	24.7± 0.3	0.60±0.00	2.43±0.07	0.72±0.01
	699± 7	19.1± 0.3	0.62±0.01	1.03±0.01	0.45±0.00
4	699± 7	19.0± 0.3	0.62±0.01	1.04±0.01	0.45±0.00
	552± 4	25.8± 0.3	0.62±0.01	2.89±0.10	0.76±0.01
5	546± 4	25.8± 0.3	0.62±0.01	2.53±0.07	0.73±0.01
	510± 3	21.7± 0.3	0.60±0.01	1.99±0.04	0.67±0.01
6	512± 3	21.7± 0.3	0.60±0.01	2.04±0.05	0.67±0.01
	544± 5	17.3± 0.2	0.64±0.01	1.03±0.01	0.45±0.00
7	545± 5	17.3± 0.2	0.64±0.01	1.04±0.01	0.45±0.00
	545± 4	17.3± 0.2	0.64±0.01	1.04±0.01	0.45±0.00
	560± 4	28.9± 0.3	0.62±0.01	1.99±0.04	0.66±0.01
8	560± 4	28.9± 0.4	0.62±0.01	1.98±0.04	0.66±0.01
	474± 6	21.0± 0.3	0.66±0.01	2.81±0.11	0.76±0.01
9	476± 6	21.1± 0.3	0.66±0.01	2.82±0.11	0.76±0.01
	533± 5	28.5± 0.3	0.63±0.01	3.12±0.11	0.78±0.01
10	534± 5	28.6± 0.3	0.63±0.01	3.14±0.12	0.78±0.01
	538± 5	28.5± 0.3	0.63±0.01	3.41±0.14	0.80±0.01

3.3 The effect of transfer batches

We next investigated the effect of transfer batches on our results[2]. In the usual sense, a transfer batch is the quantity transported between departments. It has been shown that using transfer batches smaller than the production batch size leads to reduced work in process. For this set of experiments, we set the size of the transfer batch to be half the size of the production batch, in effect doubling the number of batches that have to be transported. To keep forklift utilization at approximately the same level as in the previous experiments, we doubled the number of forklifts in the material handling system. Many policies on selecting the next waiting transfer batch to be processed at a department are possible. In our study, we assumed a first-come, first-served queue discipline. Of course, set ups would not be incurred if transfer batches of the same class follow each other on the same machine. We ran Case 5 from the first set of problems (see Table 2) under this new scenario. The results remained essentially the same, with the optimal QAP solution doing better at minimizing WIP for the same problems as before.

4 Conclusions

We have used a simple analytical queueing network model as a starting point to investigate the relationship between material handling cost and work-in-process. This model is used to show that under certain conditions in a job shop environment, the QAP with an objective of minimizing a weighted average of material flows also serves to minimize average work-in-process. A simulation study allowed us to relax some of the assumptions of the analytical model as well as vary the levels of some important parameters. The result was found to hold under more general conditions, within the bounds of these parameters. In particular, department WIP seems to be fairly insensitive to layout considerations, so the layout dependence enters primarily through the material handling system. This result appears to hold even when transfer batches are smaller than the production batches. As explained in the previous section, for situations where two or more QAP solutions are very close, we need a means to discriminate between candidate solutions in terms of work-in-process. We have developed a simple secondary measure for this purpose and simulation results indicate

[2]We thank one of the anonymous referees for suggesting this line of investigation.

that this combination of two measures works well in selecting the layout which minimizes average work-in-process levels.

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