ABSTRACT

Title of dissertation: ESSAYS ON MONETARY POLICY AND HOUSEHOLD INEQUALITY

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A central question in monetary economics is how changes in monetary policy affect economic activity. Three major changes in the U.S. economy are currently reshaping the academic debate surrounding this classic question. First, inequality in the United States has risen significantly in recent decades. Second, the Great Recession and the recent pandemic crisis pushed the nominal policy rate to effective lower bound (ELB), and consequently, the Federal Reserve resorted to unconventional monetary policies, such as quantitative easing (QE). Third, a secular decline has pushed real interest rates to levels close to zero, and in response, academia proposed an increase in the inflation target. Each of these developments motivates a chapter of my dissertation.

The first chapter of my dissertation develops a state-of-the-art heterogeneous agent monetary model and studies the transmission mechanisms and the distributional consequences of conventional monetary policy. Compared to the existing models in the literature, the model exhibits the responses of real wages and profits

to monetary policy shocks that are consistent with the existing empirical evidence. With the model parameterized to match the distribution of wealth and income in the United States, I first examine how an unexpected decrease in the nominal interest rate leads to an increase in aggregate consumption. My analysis shows that most of the consumption response is due to general equilibrium income effects rather than inter-temporal substitution, consistent with the existing results in the literature. However, when wages are rigid and profits are procyclical, as in the data, consumption responses are stronger at both ends of the wealth distribution, unlike in the standard model with flexible wages. Importantly, an expansionary monetary policy shock can increase inequality by inducing higher profits and equity prices, though it benefits households at the bottom significantly by reducing unemployment risks, as opposed to the existing results in the literature.

This second chapter studies how quantitative easing (QE) affects household welfare across the wealth distribution. To this end, the model developed in the first chapter is extended to feature frictional financial intermediation, an effective lower bound (ELB) on the policy rate, forward guidance, and QE. Moreover, to quantify the contribution of the various channels through which monetary policy affects inequality, I estimate the model using Bayesian methods, explicitly taking into account the occasionally binding ELB constraint and the QE operations undertaken by the Federal Reserve during the 2009-2015 period. I find that QE program unambiguously benefited all households by stimulating economic activity. However, it had non-linear distributional effects. On the one hand, it widened the income and consumption gap between the top 10% and the rest of the wealth distribution, by boosting profits and equity prices. On the other hand, QE shrank inequality within the lower 90% of the wealth distribution, primarily by lowering unemployment. On net, it reduced overall wealth and income inequality, as measured by the Gini index. Surprisingly, QE has weaker distributional consequences compared with conventional monetary policy. Lastly, forward guidance and an extended period of zero policy rates amplified both the aggregate and the distributional effects of QE.

The third chapter of my dissertation investigates the aggregate and the distributional consequences of raising the inflation target from 2% to 4% using the model developed and estimated in the second chapter. I find that, during the transition towards the 4% inflation target, the economy experiences substantial expansion because of the forward-looking Phillips curve and the Taylor rule that features a significant degree of interest rate smoothing. Also, the transition reduces the overall degree of inequality by lowering the unemployment rate and the real interest rate though it leaves bigger welfare gains for the top than for the middle. During the simulation around the new steady-state with the 4% inflation target, both frequency and duration of the ELB episodes are lower. Importantly, the average unemployment rate and its standard deviation are significantly lower with the higher inflation target, which leads to higher aggregate demand and lower overall inequality on average.

ESSAYS ON MONETARY POLICY AND HOUSEHOLD INEQUALITY

by

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Dedication

To my wife Hanna Park

and my children Sarang Lee and Aiden Yujun Lee

without whom this dissertation would not have been completed

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1.1 Introduction

The distributional consequences of monetary policy have received little attention until recently. Traditionally, central banks have focused on the aggregate implications of their actions, such as the effects of monetary policy on the nation's output, inflation rate, unemployment rate, and so on. Theoretical models used to examine these aggregate implications of monetary policy have mostly utilized a representative agent framework, which abstracts from any meaningful differences across agents. As a result, these models could not evaluate heterogeneity in the effects of monetary policy across agents. Recently, however, interactions between agent heterogeneity and monetary policy have started to gain attention in the literature, as rising inequality has become central in economic debates in the U.S. since the Great Recession. In this chapter, I examine the implications of monetary policy for the distribution of wealth and income, by linking the transmission mechanisms and the distributional consequences of conventional monetary policy.

To generate an accurate theoretical evaluation of the transmission mechanisms and the distributional consequences of monetary policy, a model must be able to produce plausible responses of factor prices and quantities to monetary policy shocks since households are heterogeneous in terms of their income and portfolio composition. According to the Survey of Consumer Finance (SCF hereinafter), relatively poor households rely almost exclusively on labor income. By contrast, asset and profit income are much more important for wealthy households' income. Regarding portfolio composition, the SCF reveals that wealthy households' portfolios consist mostly of equity, which is a claim to profits in the economy. Differences in households' income and portfolio composition imply that the responses of wages and profits to monetary policy shocks are of great importance in determining the transmission mechanisms and the distributional consequences of monetary policy.

Christiano et al. (2005) and Christiano et al. (2016) show that the real wage responds little to monetary policy shocks, while the unemployment rate responds substantially. Meanwhile, profits increase significantly after an expansionary monetary policy shock in the VAR analysis of Christiano et al. (2005). Similarly, Coibion et al. (2017) find that business income falls substantially in response to a contractionary monetary policy shock, i.e., an increase in the federal funds rate, while real wages and salaries remain relatively unchanged. An important goal of the model that I develop here is to generate empirically plausible responses of wages and profits to monetary policy shocks.

In many New Keynesian models, profits fall after a decrease in the policy rate because that the real marginal cost of production (or equivalently the mark-up) responds too flexibly to changes in aggregate demand. This is mainly because factor prices are flexible, while goods prices are rigid. When aggregate demand increases because of a lower real interest rate, firms want to hire more labor and utilize more capital to produce more goods. Such a higher demand for production factors increases the marginal cost of production, which leads to a smaller mark-up. As a result, profits can fall, even though firms produce and sell more goods. Thus, a key to generating a pro-cyclical response of profits to demand shocks is to dampen the response of real marginal cost. The first ingredient for such a dampened response is wage rigidity.[1] In particular, I introduce nominal wage rigidity, based on the empirical literature finding that such rigidity is prevalent in various types of data.[2] Assuming that the nominal wage is rigid has distributional consequences that work against poor households which rely almost solely on labor income. As Gornemann et al. (2016) emphasize, however, poor households can benefit significantly from expansionary monetary policy even if wages are rigid, if unemployment risk decreases substantially in response to higher aggregate demand. Thus, I also introduce search and matching labor market frictions into a heterogeneous agent New Keynesian (HANK) model. Households supply labor via labor agencies. Firms rent labor services from labor agencies. While the wage paid to households is rigid, the rental rate of labor services is assumed to be flexible. This set-up enables firms to increase labor input without a significant increase in the labor rental rate, by creating a substantial increase in labor agencies' mark-up after an expansionary monetary

[1] Broer et al. (2019) also rely on wage rigidity to generate pro-cyclical responses of profits to monetary policy shocks. They also discuss the importance of profit responses in determining the aggregate consequences of monetary policy shocks, and the resulting distributional consequences in a simple HANK model.

[2] See, for instance, Akerlof et al. (1996), Kahn (1997), Elsby (2009), Barattieri et al. (2014), and Kurmann and McEntarfer (2019) among others.

policy shock.[3] In response to an increase in this mark-up, labor agencies post more vacancies. Thus, aggregate labor supply increases, and the unemployment rate falls. This mechanism dampens the response of marginal cost to demand shocks.

Another factor that contributes to a significant increase in marginal cost (or a fall in the mark-up) after an expansionary monetary policy shock is firms' reliance on new production in meeting changes in demand. In reality, firms adjust not only production but also their inventory stock in response to demand shocks. If firms rely in part on inventories rather than new production, the increase in factor demand is limited, and so is the response of marginal cost to monetary policy shocks. Existing empirical evidence has shown that firms manage their inventory stock in response to monetary policy shocks.[4] When firms face unexpectedly high demand, they reduce their inventory stock and, consequently, the sales-to-stock ratio rises. To reflect this dynamic response of firms, I allow firms to accumulate inventory inventories as a second ingredient of the model. Dynamic adjustments of inventory stock enable firms to smooth out the marginal cost of production.

The first contribution of this chapter to the recently burgeoning HANK literature is to develop a model that generates plausible responses of the real wage and profits to monetary policy shocks. Because of the two features explained above, the

[3]Introducing wage rigidity without an extensive margin adjustment does not necessarily lead to a substantially dampened response of marginal cost, especially when income effects on labor supply are absent. In this case, the aggregate labor supply also does not increase much when the wage responds little to shocks. When there is another factor of production, e.g., capital, firms then rely more intensively on the other factor. Thus, capital utilization increases inefficiently sharply, which results in a substantial increase in marginal cost.

[4]Jung and Yun (2005) show that a contractionary monetary policy shock leads to an increase in firms' finished goods inventory stock and a decrease in the ratio of sales to stock available for sale.

response of firms' marginal cost to monetary policy shocks is dampened. As a result, profits increase in response to an expansionary monetary policy shock, contrary to many standard New Keynesian models. In contrast, the response of the real wage is minimal due to the assumption of nominal wage rigidity. Instead, unemployment risk falls substantially as labor agencies post more vacancies. These responses are more in line with existing empirical evidence than the responses in standard New Keynesian models, which imply counter-factual real wage increases and declining profits after an expansionary monetary policy shock.

The second contribution of this paper is to complement the existing discussion on the transmission mechanisms of monetary policy shocks in a heterogeneous agent environment. To this end, I calibrate the model to match the distribution of assets and the heterogeneity in income and portfolio composition observed in the data. Importantly, in the model, substantial masses of households hold either no assets or only illiquid assets (wealthy hand-to-mouth households), as in the data. Because these subsets of households do not respond to changes in the real interest rate, the direct effect accounts for only a small fraction of the total aggregate consumption response to monetary policy shocks. By contrast, indirect effects, including the effects of changes in the real wage, profits, the job-finding rate, and fiscal instruments, explain most of the aggregate consumption response, as in Kaplan et al. (2018). However, the decomposition of the indirect effects is different from existing models. Since the real wage increases only a little following an expansionary monetary policy shock, its effects on aggregate consumption are muted. Instead, the increased job-finding rate increases the consumption of households that are relatively more vulnerable to unemployment risk.[5] Moreover, an increase in profits leads wealthy households to increase their consumption significantly, because they are the major claimants to profits in the model. Under my calibration, the effects of the job-finding rate and profits on aggregate consumption are larger than those of real wage changes.

The third contribution of my analysis is to provide a new perspective on the distributional consequences of monetary policy shocks. Existing work argues that expansionary monetary policy decreases inequality among households. However, I show that such an outcome is not guaranteed. Depending on model features, an expansionary monetary policy shock can be more beneficial for wealthy households. An unexpected increase in inflation causes losses to net nominal asset holders and redistributes wealth to net borrowers. Since wealthy households tend to hold a large amount of nominal assets and poor households are usually net borrowers, this *Fisher channel* contributes to a fall in wealth and consumption inequality after an expansionary monetary policy shock. However, wealthy households' portfolio mostly consists of illiquid assets. Thus, if the return on illiquid assets increases after an expansionary monetary policy shock, total asset income can be higher than before the shock despite the presence of the Fisher channel. In the model, the return on illiquid assets is proportional to profits. Hence, an increase in profits after an expansionary monetary policy shock translates into a higher return on illiquid assets. If the positive response of profits to expansionary monetary policy is large enough

[5] A rise in the job-finding rate leads both employed and unemployed households to increase their consumption. First, a larger fraction of unemployed households become employed and increase their consumption as their income rises significantly. Second, employed households increase their consumption by reducing their precautionary savings.

to offset the fall in the real interest rate and is much larger in magnitude than the changes in the real wage, an expansionary monetary policy shock can lead to a rise in inequality. This is because wealthy households hold the majority of claims to profits while other households rely almost exclusively on labor income. In the baseline model, an expansionary monetary policy shock leads to more pronounced consumption responses from wealthy households than from other households. In the long run, expansionary monetary policy also leads to a higher welfare gain for wealthy households, measured as consumption equivalents.[6]

Finally, I show that the distributional consequences of monetary policy depend importantly on the fiscal policy response. Monetary policy shocks result in changes in the real value of government debt. In response to these changes, the fiscal authority needs to adjust its fiscal instruments, including government purchases, lump-sum transfers, and tax rates, to meet the solvency condition or inter-temporal budget constraint. When the government increases purchases or reduces tax rates, an expansionary monetary policy shock has weak distributional consequences. An increase in government purchases implies a one-to-one increase in aggregate demand. Such an increase in demand leads to a larger increase in profits, which benefits wealthy households disproportionately. A proportional decrease in all tax rates is also regres-

[6] An expansionary monetary policy shock benefits households at the left end of wealth distribution by increasing the job-finding rate and reducing the real value of their debt. In contrast, the benefit for households at the middle of wealth distribution is limited. This is because they mostly rely on labor income, which does not respond much, and are on average net holders of government bonds, whose real return falls in response to a fall in the policy rate. Thus, inequality among these households tends to shrink after an expansionary monetary policy shock. However, households at the right end of the wealth distribution also enjoy a substantial gain because of the increase in profits or, equivalently, the return on the illiquid asset. Thus overall inequality can increase after an expansionary monetary policy shock.

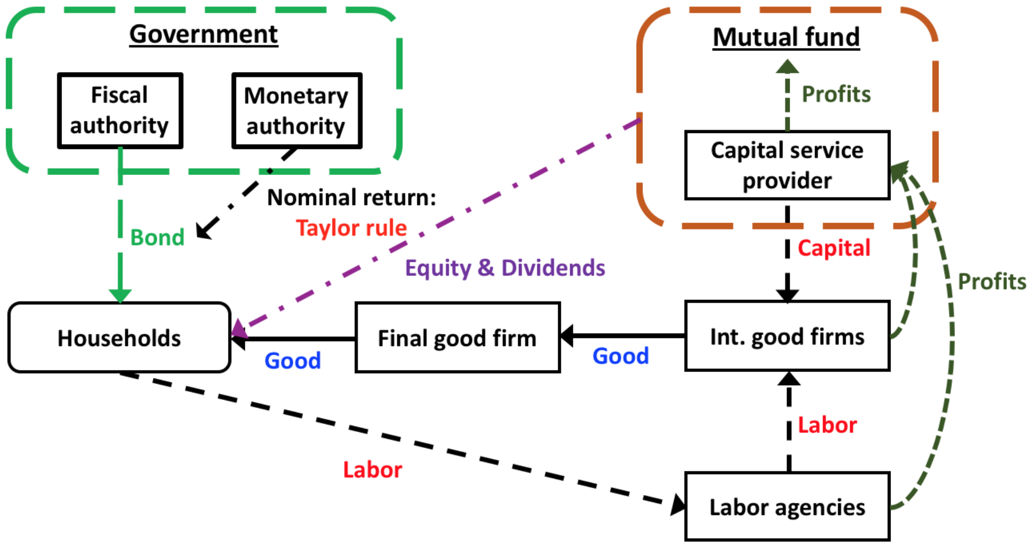
sive. In the model, the only case in which an expansionary monetary policy shock clearly reduces inequality is when the fiscal authority increases lump-sum transfers. The result that the distributional consequences of monetary policy heavily depend on the nature of fiscal responses casts doubt on the argument that an expansionary monetary policy shock reduces inequality without further conditioning on the fiscal response.

The rest of chapter is organized as follows. Section 2 develops the model. Section 3 discusses the calibration of the model. Section 4 analyzes the transmission mechanisms and the distributional consequences of monetary policy shocks in the baseline model. Section 5 compares the baseline model with a more standard model with a flexible wage, focusing on the responses of aggregate variables and their implications for the effects of monetary policy shocks on inequality. Section 6 examines the implications of alternative fiscal responses. Section 7 concludes.

1.2 Model

In this section, I develop a New Keynesian model with heterogeneous agents featuring two assets, labor market frictions combined with wage rigidity, and inventory holding. The model consists of households, firms, labor agencies, a mutual fund, and government authorities. Households provide labor, consume the final good, and invest in two types of assets. Firms produce goods by utilizing labor and capital services. A mutual fund owns all the firms in the model. It collects all profits, and distributes a part of them to households as dividends in proportion to their equity

Figure 1.1: Model overview



Notes: The figure shows how economic agents in the model interact with each other. The arrows represent the movements of goods, production factors, assets, and profits. The payments associated with factors, goods, and asset transactions are not shown.

holding. The remaining portion is given as compensation to a certain fraction of households that serve as entrepreneurs. The fiscal authority collects taxes, chooses government purchases and lump-sum transfers, and pays unemployment benefits. It also issues a one-period nominal government bond. The monetary authority sets the nominal rate of return on the government bond. Figure 1.1 shows an overview of the model. In the following subsections, I describe each agent's problem and their optimal choices in more detail.

1.2.1 Households

There is a unit mass of households, which are characterized by their idiosyncratic productivity, employment status, asset holding, and adjustment costs. Each period they earn income and choose the amount of consumption and savings. When they are employed, they also decide their labor supply. Finally, a certain fraction of households die randomly at the beginning of each period.[7] The Bellman equation for the household is given as follows.

$$V(a_t, b_t | s_t, e_t, \chi_t, X_t) = \max_{\{c_t, n_t, a_{t+1}, b_{t+1}\}} u(c_t, n_t) - \mathbb{1}_{\{a_{t+1} \neq a_t\}} \chi_t + \beta(1-\zeta)\mathbb{E}\Big[V(a_{t+1}, b_{t+1} | s_{t+1}, e_{t+1}, \chi_{t+1}, X_{t+1})\Big] \quad (1.1)$$

s.t.

$$c_t + q_t a_{t+1} + b_{t+1} = (1-\tau_t^y) y_t + (q_t + \tilde{r}_t^a) a_t + \frac{\tilde{R}_t}{\pi_t} b_t + T_t \quad (1.2)$$

$$a_{t+1} \geq 0 \ , \quad b_{t+1} \geq \underline{\mathrm{b}} \quad (1.3)$$

where a_t, b_t, c_t, and n_t are equity and bond holding, consumption, and labor supply respectively. $u(c_t, n_t)$ is instantaneous utility from consuming c_t units of the final good and supplying n_t units of labor. q_t and $\tilde{r}_t^a$ are the price and the dividend rate of equity respectively. $\tilde{R}_t$ is the gross nominal rate of return that is applied to a one-period government bond issued in period $t-1$. π_t is the gross inflation rate from period $t-1$ to period t. τ_t^y and T_t are the income tax rate and the lump-sum transfer from the government respectively. ζ is the probability of death. X_t denotes the aggregate state of the model economy after the employment status transition and before the production of goods. Specifically, X_t consists of the aggregate capital stock, aggregate shocks, the size of the aggregate labor force after the labor market transitions, and households' distribution over their idiosyncratic states.

[7] Upon death, households' assets are collected and distributed to surviving households as a return in proportion to their asset holding. To denote the rate of return that reflects such a hypothetical annuity arrangement, I use tilde in the notation.

The household's income y_t depends on the level of idiosyncratic productivity s_t and its employment status e_t. The former evolves according to a first-order Markov process. Also, the household's employment status varies over time. Specifically, the household can be employed, unemployed or work as an entrepreneur. While employed ($e_t = 1$), the household earns a wage in proportion to its idiosyncratic productivity and the amount of labor supply. If unemployed ($n_t = 2$), the household receives an unemployment benefit from the fiscal authority based on the steady state wage that it could earn if it were employed. Finally, if the household becomes an entrepreneur ($n_t = 3$), its income is equal to a fraction of the profits. Formally, y_t is given as follows.

$$y_t = \begin{cases} w_t s_t n_t & \text{for employed} \quad (e_t = 1) \\ \bar{w} \upsilon \min\{s_t, \bar{s}\} & \text{for unemployed} \ \ (e_t = 2) \\ \nu \Pi_t & \text{for entrepreneur} \ (e_t = 3) \end{cases} \tag{1.4}$$

where υ is the replacement ratio and ν is the share of the profits that is given to the entrepreneurs. w_t is the market wage per efficient unit of labor, and $\bar{w}$ is its steady state value. $\bar{s}$ and Π_t denote the average productivity and profits respectively.

Households' employment status transitions are governed by exogenous probabilities and endogenous labor market outcomes. First, the transitions in and out of the entrepreneurial state occur at the end of a period with given probabilities. Specifically, households that are entrepreneurs at the end of the period lose their job and become unemployed with the probability $\tilde{P}_e$. At the same time, a fraction P_e of other households are newly hired as entrepreneurs. Meanwhile, at the beginning of each period, employed households lose their job with exogenous probability λ. Once households become unemployed, they look for a new job by searching in the labor market.[8] The mass of job seekers and the number of vacancies jointly determine the job-finding rate, which I denote as f. Unemployed households become employed with probability f. Once the employment status in a given period is determined, the household provides labor or works as an entrepreneur and earns income according to its employment status.

Regarding savings, the household chooses not only the total amount but also the composition of its portfolio. The model has two types of assets, government bonds and equity. Equity is assumed to be illiquid, i.e., adjusting the amount of equity holding incurs an adjustment cost. Specifically, the household suffers a loss of utility of χ_t when $a_{t+1} \neq a_t$. As in Bayer et al. (2019), the utility adjustment cost χ_t is assumed to follow a logistic distribution, which is identically and independently distributed across households.[9]

$$\chi_t \ \sim \ F(\chi_t) = \frac{1}{1 + \exp\{-\frac{\chi_t - \mu_\chi}{\sigma_\chi}\}} \tag{1.5}$$

where $F(\cdot)$ denotes the CDF of the logistic distribution function. Based on the

[8] I assume that search intensity is exogenous, i.e., households do not decide whether to participate in the labor market or not. Instead, all unemployed households become job seekers.

[9] Bayer et al. (2019) show that introducing stochastic adjustment costs in the form of utility costs preserves the concavity of the household's value functions in a similar environment. Moreover, assuming a logistic distribution provides a closed-form solution for the expected adjustment cost, which facilitates the numerical computation further. Finally, Kaplan and Violante (2014) find that the choice of pecuniary or utility adjustment costs does not make any significant difference in the results for the portfolio composition problem.

realized value for χ_t, households decide whether to adjust their equity holding or not. If they choose not to adjust, they receive dividends and adjust bond holding.[10] In contrast, if households choose to adjust, they can buy and sell equity at price q_t, which varies each period based on the aggregate status of the economy, but experience a loss of utility of χ_t.

On the liability side, households can borrow up to an exogenous amount $\underline{\text{b}}$ in the form of the liquid asset. To prevent arbitrage and excessive borrowing in the model, I assume that households must pay a premium $\underline{\text{R}}$ if they borrow.[11] Consequently, the borrowing rate is higher than the saving rate, as follows.

$$\tilde{R}_{t+1} = \begin{cases} R_{t+1}/(1-\zeta) & \text{if } b_{t+1} \geq 0 \\ (R_{t+1} + \underline{\text{R}})/(1-\zeta) & \text{if } b_{t+1} < 0 \end{cases} \tag{1.6}$$

For the household's utility, I use the following form of Greenwood-Hercowitz-Huffman (GHH) preference (Greenwood et al. (1988)).

$$u(c_t, n_t) = \frac{\left[c_t - \psi s_t \frac{n_t^{1+\xi}}{1+\xi}\right]^{1-\sigma}}{1-\sigma} \tag{1.7}$$

where ξ and σ are the inverse of the Frisch elasticity of labor supply and the coefficient of relative risk aversion, respectively. By interacting the disutility of labor supply with the level of idiosyncratic productivity, I guarantee that all employed

[10] The dividends that households receive consist of a fraction of profits given by the mutual fund and annuity payments on equity holding.

[11] Following Kaplan et al. (2018), I assume that the borrowing premium is a wasteful intermediation cost. That is, the intermediation cost does not appear in the government's budget constraint or firms' cash flow.

households supply the same amount of labor based on the market wage. ψ is set to ensure a steady state labor supply of one.

Before deriving the household's optimality conditions, I first express the household's next period expected value, depending on its current period employment status. For brevity of notation, I drop households' idiosyncratic productivity, the utility cost of adjustment, and the aggregate states at the beginning of period $t+1$ in the following equations.

$$\mathbb{E}\Big[V(a_{t+1}, b_{t+1}|e_{t+1})|e_t = 1\Big] = \mathbb{E}\Big[P_e V(a_{t+1}, b_{t+1}|3) + (1 - P_e)$$
$$\times \big[\{1 - \lambda + \lambda f(\tilde{X}_{t+1})\}V(a_{t+1}, b_{t+1}|1) + (\lambda\{1 - f(\tilde{X}_{t+1})\})V(a_{t+1}, b_{t+1}|2)\big]\Big] \quad (1.8)$$
$$\mathbb{E}\Big[V(a_{t+1}, b_{t+1}|e_{t+1})|e_t = 2\Big] = \mathbb{E}\Big[P_e V(a_{t+1}, b_{t+1}|3)$$
$$+ (1 - P_e) \times \big[f(\tilde{X}_{t+1})V(a_{t+1}, b_{t+1}|1) + \{1 - f(\tilde{X}_{t+1})\}V(a_{t+1}, b_{t+1}|2)\big]\Big] \quad (1.9)$$
$$\mathbb{E}\Big[V(a_{t+1}, b_{t+1}|e_{t+1})|e_t = 3\Big] = \mathbb{E}\Big[(1 - \tilde{P}_e)V(a_{t+1}, b_{t+1}|3)$$
$$+ \tilde{P}_e \times \big[f(\tilde{X}_{t+1})V(a_{t+1}, b_{t+1}|1) + \{1 - f(\tilde{X}_{t+1})\}V(a_{t+1}, b_{t+1}|2, X_{t+1})\big]\Big] \quad (1.10)$$

where $\tilde{X}$ denotes the aggregate state before labor market transitions. Specifically, it consists of the aggregate capital stock, aggregate shocks, the size of the labor force and households' distribution over idiosyncratic states before labor market transitions. Equation (1.8) shows next period's expected value of households that are currently employed. Equations (1.9) and (1.10) show the expected values of currently unemployed households and households that currently work as entrepreneurs, respectively. The above expressions take into account both exogenous transition probabilities and the endogenous evolution of the job finding rate.

Next, I describe the first order conditions to the household's problem. First, employed households decide their labor supply according to the following intra-temporal optimality condition.

$$\psi n_t^{\xi} = w_t \tag{1.11}$$

If households are unemployed or work as entrepreneurs, $n_t = 0$, since their income does not depend on n_t. Next, I turn to the optimality conditions regarding asset holding. Let V_a and V_b denote the partial derivatives of the value function with respect to equity and bond holding, respectively. In addition, let u_c denote the derivative of the instantaneous utility function with respect to consumption. Then, the household's optimality conditions regarding asset holding can be expressed as follows. For simplicity of notation, I drop all exogenous states in the following.

$$q_t u_c(c_t, n_t) \geq \beta(1-\zeta)\mathbb{E}\Big[V_a(a_{t+1}, b_{t+1})\Big] \text{ with equality if } a_{t+1} > 0 \tag{1.12}$$

$$u_c(c_t, n_t) \geq \beta(1-\zeta)\mathbb{E}\Big[V_b(a_{t+1}, b_{t+1})\Big] \text{ with equality if } b_{t+1} > 0 \tag{1.13}$$

Note that equation (1.12) is a necessary condition only for households that adjust their asset holdings, $a_{t+1} \neq a_t$. The envelope conditions are given as follows.

$$V_a(a_t, b_t) = \begin{cases} (q_t + \tilde{r}_t^a) u_c(c_t^A, n_t) & \text{if } a_{t+1} \neq a_t \\ \tilde{r}_t^a u_c(c_t^N, n_t) + \beta(1-\zeta)\mathbb{E}\Big[V_a(a_t, b_{t+1})\Big] & \text{if } a_{t+1} = a_t \end{cases} \tag{1.14}$$

$$V_b(a_t, b_t) = \begin{cases} \frac{\tilde{R}_t}{\pi_t} u_c(c_t^A, n_t) & \text{if } a_{t+1} \neq a_t \\ \frac{\tilde{R}_t}{\pi_t} u_c(c_t^N, n_t) & \text{if } a_{t+1} = a_t \end{cases} \tag{1.15}$$

where c_t^A and c_t^N are the optimal consumption when the household chooses to adjust and not to adjust its equity holding, respectively. As shown in the above envelope conditions, the household's optimality conditions depend on its expectation on the adjustment of its equity holdings in the next period. The household adjusts its equity holding in period t if the following condition is satisfied.

$$V^A(a_t, b_t) - \chi_t \geq V^N(a_t, b_t) \tag{1.16}$$

where V^A and V^N denote the value of households when they adjust or do not adjust their equity holding respectively. Since the utility transaction cost is drawn from the logistic distribution, I can describe the probability of adjustment $P^*(a_t, b_t)$ as follows.

$$\begin{aligned} P^*(a_t, b_t) &= P\Big[\chi_t \leq V^A(a_t, b_t) - V^N(a_t, b_t)\Big] \\ &= F\Big[V^A(a_t, b_t) - V^N(a_t, b_t)\Big] \end{aligned} \tag{1.17}$$

Given the probability of adjustment, the household's Euler equation with respect to each asset can be described as follows.

$$q_t u_c(c_t, n_t) \geq \beta \mathbb{E}\Big[P^*(a_{t+1}, b_{t+1})\big\{q_{t+1} + r^a_{t+1}\big\} u_c(c^A_{t+1}, n_{t+1}) + \big\{1 - P^*(a_{t+1}, b_{t+1})\big\} \\ \times r^a_{t+1} u_c(c^N_{t+1}, n_{t+1}) + \big\{1 - P^*(a_{t+1}, b_{t+1})\big\}\mathbb{E}\big[V_a(a_{t+1}, b_{t+2})\big]\Big] \\ \text{with equality if } a_{t+1} \neq a_t \tag{1.18}$$

$$u_c(c_t, n_t) \geq \beta \mathbb{E}\Big[P^*(a_{t+1}, b_{t+1})\frac{R_{t+1}}{\pi_{t+1}} u_c(c^A_{t+1}, n_{t+1}) + \{1 - P^*(a_{t+1}, b_{t+1})\} \\ \times \frac{R_{t+1}}{\pi_{t+1}} u_c(c^N_{t+1}, n_{t+1})\Big] \text{ with equality if } b_{t+1} > 0 \tag{1.19}$$

1.2.2 Final good firm

The final good firm produces a final good using intermediate goods as inputs with a CES production function in a perfectly competitive environment, taking goods' prices as given. Note that the final good firm regards intermediate good producers' inventory stock and newly-produced goods as perfect substitutes when purchasing inputs to produce the final good.[12] The final good firm's problem is described as follows.

$$\max_{S_t, \{S_{jt}\}_{j\in(0,1)}} P_t S_t - \int_0^1 P_{jt} S_{jt} dj \quad \text{s.t. } S_t = \left\{ \int_0^1 \left(\frac{a_{jt}}{a_t}\right)^{\frac{\varphi}{\eta}} S_{jt}^{\frac{\eta-1}{\eta}} dj \right\}^{\frac{\eta}{\eta-1}} \tag{1.20}$$

[12] Intermediate good producers hold inventories and sell them to the final good firm.

where S_t and S_{jt} denote the final good sales and intermediate firm j's sales respectively. a_t and a_{jt} are the aggregate input stock and intermediate good firm j's stock available for sale respectively.[13] φ denotes the elasticity of intermediate good sale with respect to the available stock. The above aggregator implies that intermediate good firm j's sales are higher than other firms' sales if firm j has a larger amount of stock for sale than other firms. This feature provides an additional incentive for intermediate good firms to accumulate inventory stock. As discussed in Bils and Kahn (2000) and Jung and Yun (2005), such an incentive is necessary to make firms hold a meaningful amount of inventory stock at the steady state. η is the elasticity of substitution across differentiated intermediate goods. The first order conditions imply the following demand for intermediate goods and the aggregate price index.

$$S_{jt} = \left(\frac{a_{jt}}{a_t}\right)^{\varphi} \left(\frac{P_{jt}}{P_t}\right)^{-\eta} S_t \tag{1.22}$$

$$P_t = \left\{ \int_0^1 P_{jt}^{1-\eta} dj \right\}^{\frac{1}{1-\eta}} \tag{1.23}$$

1.2.3 Intermediate good firms

In the model, a unit mass of intermediate good firms indexed by $j \in (0, 1)$ produces differentiated goods using labor and capital rental services. Each intermediate good firm faces a downward-sloping demand curve and has monopoly power

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[13] a_t is given by

$$a_t = \int_0^1 a_{j,t} dj \tag{1.21}$$

to set a price for its product. Firms' price setting is subject to a nominal rigidity in the form of Rotemberg (1982) adjustment costs. In addition, a working capital constraint applies to intermediate good firms as in Christiano et al. (2005). Specifically, intermediate good firms have to pay the labor and capital rental rate in advance by borrowing at interest rate R_{t+1}. Let $J_I(i_{jt-1}, P_{jt-1}|X_t)$ denote the value of firm j with previous period price P_{jt-1} and inventory holding i_{jt-1} under aggregate state X_t. Importantly, intermediate good firms are allowed to hold inventory stock. Thus, intermediate good firms do not necessarily sell all the goods they produce in the current period. They decide not only how much to produce but also how much to sell. The intermediate good firm's problem can be described as follows.

$$\begin{aligned} J_I(i_{jt-1}, P_{jt-1}|X_t) = \max_{\{P_{jt}, L_{jt}, K_{jt}\}} \left(\frac{P_{jt}}{P_t}\right) S_{jt} - R_{t+1}(r_t^l L_{jt} + r_t^k K_{jt}) - \Xi \\ - \frac{\eta}{2\kappa}\left(\log \frac{P_{jt}/P_{jt-1}}{\bar{\pi}}\right)^2 S_t - \frac{\phi_x}{2}\left(\log \frac{x_{jt}}{\bar{x}}\right)^2 S_t + \mathbb{E}\big[\Lambda_{t,t+1} J_I(i_{jt}, P_{jt}|X_{t+1})\big] \end{aligned} \tag{1.24}$$

s.t.

$$S_{jt} = \left(\frac{a_{jt}}{a_t}\right)^{\varphi} \left(\frac{P_{jt}}{P_t}\right)^{-\eta} S_t \tag{1.25}$$

$$a_{jt} = Y_{jt} + (1-\delta_i) i_{jt-1} \tag{1.26}$$

$$Y_{jt} = Z_t L_{jt}^{1-\theta} K_{jt}^{\theta} \tag{1.27}$$

$$i_{jt} = a_{jt} - S_{jt} \tag{1.28}$$

$$x_{jt} = S_{jt}/a_{jt} \tag{1.29}$$

$$\log(Z_{t+1}) = \rho_Z \log(Z_t) + \epsilon_{Z,t} \quad , \quad \epsilon_{Z,t} \sim \text{ i.i.d } \; N(0, \sigma_Z^2) \tag{1.30}$$

where r_t^l and r_t^k are the labor and capital rental rates respectively. L_{jt} and K_{jt} denote the amount of labor and capital rental services used by firm j. R_{t+1} is the nominal interest rate that is applied to the working capital. Ξ is a fixed cost of operation.[14] κ determines the degree of nominal rigidity in the model jointly with the elasticity of substitution η. $\bar{\pi}$ is the steady state inflation rate.[15] If the individual firm's price growth is not equal to the steady state inflation rate, it has to pay adjustment costs in units of the final good. $\Lambda_{t,t+1}$ denotes the stochastic discount factor that each firm uses to discount future cash flows. θ is the capital share of the production. Total factor productivity Z_t follows an AR(1) process as shown in equation (1.30). ρ_Z and σ_Z denote the autocorrelation coefficient and the standard deviation of Z respectively.

Equation (1.25) describes the demand for firm j's good, which reflects a benefit of holding inventory stock, i.e., boosting sales. Equation (1.26) shows that the firm j's current period stock of goods for sale is the sum of the newly-produced good and the undepreciated inventory stock, where δ_i denotes the inventory depreciation rate. The inventory stock at the end of the current period is the remaining stock after sales as shown in equation (1.28). Equation (1.29) is the definition of the sales-to-stock ratio. I assume that, the firm pays a cost if its sales-to-stock ratio deviates from its steady state target, following Jung and Yun (2005). This feature prevents overly volatile inventory stock adjustments.

[14] In principle, an intermediate good firm should choose not to operate if the revenue that it can achieve in a given period is less than the fixed cost. I do not allow such a choice. However, under the parameterizations examined in this chapter, the firm's profit is always positive and they optimally continue to produce.

[15] The current expression for the adjustment cost implies that the firm can index its price to the steady state inflation rate without paying any costs.

The intermediate good firm's problem can be divided into two parts. First, the firm minimizes total costs by choosing the optimal amount of labor L_{jt} and capital services K_{jt} that can produce a given amount of the intermediate good Y_{jt}. Then, for the given marginal cost, the firm sets the price for its good and the amount of inventory holding that maximizes expected discounted cash flows. The former determines the amount of sales and the latter determines the amount of production. Note that the firm's sales may or may not equal production Y_{jt} if it accumulates inventory stock. The firm holds unsold goods as inventory stock. The optimality conditions and the symmetric equilibrium assumption lead to the following expressions for marginal cost and New Keynesian Phillips curve.[16]

$$\begin{aligned} MC_t &= R_{t+1}\frac{(r_t^k)^\theta (r_t^l)^{1-\theta}}{Z_t} \times \left(\frac{1}{\theta}\right)^\theta \times \left(\frac{1}{1-\theta}\right)^{1-\theta} \\ &= \varphi x_t + \phi_x(1-\varphi)\big(\log x_t/\bar{x}\big)x_t + (1-\varphi x_t)(1-\delta_i)\mathbb{E}\Big[\Lambda_{t,t+1}MC_{t+1}\Big] \qquad (1.31) \end{aligned}$$

$$\begin{aligned} \log \pi_t - \log \bar{\pi} &= \mathbb{E}\left[\Lambda_{t,t+1}\frac{S_{t+1}}{S_t}\Big\{\log \pi_{t+1} - \log \bar{\pi}\Big\}\right] \\ &\quad + \kappa\left\{(1-\delta_i)\mathbb{E}\Big[\Lambda_{t,t+1}MC_{t+1}\Big] - \frac{\eta-1}{\eta} + \phi_x\big(\log x_t - \log \bar{x}\big)\right\} \qquad (1.32) \end{aligned}$$

Equation (1.31) implies that marginal cost of production should equal marginal benefit of production, which consists of the benefit from increased sales and saving production cost next period net of sales-to-stock ratio adjustment costs. Also, Equation (1.32) shows that the current period inflation rate depends on the current cost of deviating from the target sales-to-stock ratio, the expected future marginal

[16] For a symmetric equilibrium, I assume that the initial inventory stock $i_{j,-1}$, and the initial price level $P_{j,-1}$ are equal across firms.

cost of production, and the expected future inflation rate.

1.2.4 Labor agencies

I model labor agencies as in Gornemann et al. (2016). Agencies post vacancies to hire labor from households. While a labor agency is matched to a household, it rents labor services to intermediate good firms at a competitive rental rate r_t^l.[17] A match between a household and a labor agency is terminated if the household dies; if the household survives but becomes an entrepreneur, which occurs with probability P_e; and if the household is separated from the match, which occurs with an exogenous probability λ. Given the description of the problem so far, the labor agency's value function can be expressed as follows.

$$J_L(s_t|\tilde{X}_t) = (r_t^l - w_t)s_t n_t + \mathbb{E}\Big[\Lambda_{t,t+1}(1-\zeta)(1-\lambda)(1-P_e)J_L(s_{t+1}|\tilde{X}_{t+1})\Big] \quad (1.33)$$

where w_t is the real wage that the labor agency pays to the household per labor efficiency unit and $s_t n_t$ is the household's supply of efficiency units. In principle, labor agencies and households should negotiate the wage that applies to their match.[18] However, for simplicity, I follow Gornemann et al. (2016) and assume that wage

[17] Labor agencies do not have monopoly power in supplying labor services. However, this does not necessarily imply that they cannot enjoy a mark-up. Because of the fixed cost of posting vacancies, a certain amount of mark-up is needed to motivate labor agencies to post vacancies, thereby supplying labor services.

[18] Since each household's outside option depends not only on their idiosyncratic productivity but also on their asset holding and the level of adjustment cost, the equilibrium wage can differ at each point in the idiosyncratic state space.

determination takes the following form.[19]

$$\frac{w_t}{\bar{w}} = \left(\frac{r_t^l}{\bar{r}^l}\right)^{1-\rho_w} \left\{\frac{w_{t-1}}{\bar{w}} \times \left(\frac{\bar{\pi}}{\pi_t}\right)^d\right\}^{\rho_w} \tag{1.34}$$

where variables with bars denote steady state values. Equation (1.34) implies nominal wage rigidity and can be interpreted as follows. First, a fraction $1 - \rho_w$ of the real wage is adjusted in proportion to changes in the labor rental price, which reflects the marginal productivity of labor. The nominal wage rigidity is applied to the remaining ρ_w fraction of the nominal wage, of which a fraction d is indexed to the steady state inflation rate, while the rest is indexed to the current period inflation rate. Due to nominal wage rigidity and insufficient indexation, an increase in the inflation rate leads to a fall in the real wage. Thus, in response to a demand shock, the real wage exhibits rigidity.[20] This wage formula resembles Calvo-type nominal wage rigidity.

For the matching function, I follow den Haan et al. (2000) and assume the following form.

$$M(\tilde{X}_t, V_t) = \frac{\left(U(\tilde{X}_t) + \lambda N(\tilde{X}_t)\right)V_t}{\left\{\left(U(\tilde{X}_t) + \lambda N(\tilde{X}_t)\right)^{\alpha} + V_t^{\alpha}\right\}^{\frac{1}{\alpha}}} \quad , \quad \alpha > 0 \tag{1.35}$$

[19]Note that, for the wage implied by the equation (1.34) to be an equilibrium wage, both labor agencies and households should not want to terminate the match between them. Under the parameterizations and the simulations examined in this chapter, wages implied by the wage equation always remain in the bargaining set. Thus maintaining a match is always beneficial for both labor agencies and households.

[20]By contrast, in response to supply shocks such as total factor productivity (TFP), the wage varies substantially, because higher TFP leads to an increase in the labor rental price and a fall in the inflation rate at the same time.

where V_t is the number of vacancies while $U(\cdot)$ and $N(\cdot)$ are the mass of unemployed and employed households.[21] The above matching function guarantees that the job finding rate and the vacancy filling rate are always between zero and one. Finally, the labor agencies are subject to the following free entry condition for vacancies.

$$\iota = \frac{M(\tilde{X}_t, V_t)}{V_t} \int J_L(s_t|\tilde{X}_t) d\mu_t \tag{1.36}$$

where μ_t is the distribution of households over their idiosyncratic states. The above condition implies that labor agencies keep posting vacancies until the vacancy posting cost ι (LHS) becomes equal to the expected value of matching (RHS).

1.2.5 Mutual fund

All firms in the model are owned by a mutual fund, which hires a fraction of households as entrepreneurs for the management of these firms. Also, the fund holds the capital stock, rents it to the firms at a competitive rate, and issues equity to finance investment. The dividend on equity is a fraction of the profits. The other fraction is given to entrepreneurs as compensation for their work. The collection of the profits, capital rental and investment are done by a capital service provider within the mutual fund. This provider determines the utilization rate of capital based on the market rental rate and converts the final good into new capital stock. Let $J_K(K_t|X_t)$ denote the value of the capital service provider with the capital stock

[21] I assume that the mass of the households is one in the model. However, since there is a mass of households that work as entrepreneurs U is not equal to $1 - N$.

K_t and under the aggregate state X_t. Then,

$$J_K(K_t|X_t) = \max_{\{v_t, K_{t+1}\}} (1-\tau_t^a)(1-\nu)\Pi_t - K_{t+1} + K_t - \Phi(K_{t+1}, K_t) + \mathbb{E}_t\Big[\Lambda_{t,t+1} J_K(K_{t+1}|X_{t+1})\Big] \tag{1.37}$$

where τ_t^a and v_t are the profit tax rate and the capital utilization rate respectively. Φ is the capital adjustment cost, whose functional form is given as follows.

$$\Phi(K_{t+1}, K_t) = \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{1.38}$$

where the coefficient $\phi > 0$ represents the extent of adjustment costs. Π_t is before-tax profits, given as follows.

$$\Pi_t = \Big\{r_t^k v_t - \delta(v_t) + \Psi_t\Big\} K_t \tag{1.39}$$

$$\Psi_t = \left[S_t\left\{1 - \frac{\eta}{2\kappa}\left(\log\frac{\pi_t}{\bar{\pi}}\right)^2 - \frac{\phi_x}{2}\left(\log\frac{x_t}{\bar{x}}\right)^2\right\} - MC_t Y_t - \Xi + (r_t^l - w_t)\bar{s}L_t - \iota V_t\right] / K_t \tag{1.40}$$

where Ψ_t is the sum of the profits from intermediate good firms and labor agencies per unit of capital stock. I assume that the capital service provider takes Ψ_t as given. $\delta(\cdot)$ and L_t are the variable depreciation rate and the aggregate hours of work . For the depreciation rate, I follow Greenwood et al. (1988).

$$\delta(v_t) = \delta_0 v_t^{\delta_1} \quad , \quad \delta_1 > 1 \tag{1.41}$$

where δ_0 is the depreciation rate under full utilization and δ_1 governs the degree of acceleration of depreciation with utilization. The optimal utilization rate satisfies the following condition.

$$r_t^k \geq \delta'(v_t) \quad \text{with equality if } \ 0 < v_t < 1 \tag{1.42}$$

Finally, the inter-temporal optimality condition of the capital service provider is given as follows.

$$1 = \mathbb{E}\left[\Lambda_{t,t+1}\left\{\frac{(1-\tau_{t+1}^a)(1-\nu)\Pi_{t+1} + 1 - \Phi_2(K_{t+2}, K_{t+1})}{1 + \Phi_1(K_{t+1}, K_t)}\right\}\right] \tag{1.43}$$

where Φ_1 and Φ_2 are the partial derivatives of the adjustment cost Φ with respect to the first and second arguments, respectively.

The capital service provider collects all profits and passes them onto the mutual fund. Then, the fund chooses the amount of equity issuance and the dividend rate. Importantly, I assume that the fund operates in a perfectly competitive environment. Thus, the fund cannot have retained earnings. Then, the following equation should hold.[22]

$$(1-\tau_t^a)(1-\nu)\Pi_t + K_t - K_{t+1} - \Phi(K_{t+1}, K_t) + q_t(K_{t+1} - K_t) = r_t^a K_t \tag{1.44}$$

Since the fund faces perfect competition, the price of equity, which is equivalent to

[22] Equation (1.44) reflects the market clearing condition for equity. That is, $A_t = K_t$ for all t, where A_t is the aggregate equity demand.

capital holding, should be equal to marginal cost of capital accumulation. That is, we have the following expression for the equity price.

$$q_t = 1 + \Phi_1(K_{t+1}, K_t) \tag{1.45}$$

By plugging (1.45) into the equation (1.44) and exploiting the homogeneity of degree zero of $\Phi(\cdot)$, we have the following expression for the dividend rate.

$$r_t^a = (1 - \tau_t^a)(1 - \nu)\Pi_t - \Phi_1(K_{t+1}, K_t) - \Phi_2(K_{t+1}, K_t) \tag{1.46}$$

Equation (1.45) and (1.46) always hold regardless of the stochastic discount factor. In this chapter, I assume the following form of the stochastic discount factor.

$$\Lambda_{t,t+1} = \frac{q_t}{q_{t+1} + r_{t+1}^a} \tag{1.47}$$

The above stochastic discount factor guarantees that the inter-temporal optimality condition regarding capital accumulation always holds. Thus, in the current setting, households collectively decide the next period aggregate capital stock, and the capital service provider simply meets the demand for new capital stock.[23]

1.2.6 Government

(Fiscal authority)

[23] I experimented with different stochastic factors, such as the household's discount factor, the inverse of the rate of return on both assets, and the marginal rate of substitution of aggregate consumption, to examine the robustness of the results. None of the results presented in this chapter were significantly affected by the choice of the stochastic discount factor.

The fiscal authority collects taxes and issues bonds to finance government purchases, lump-sum transfers and unemployment benefits. Following Bayer et al. (2019), I assume that the government issues bonds by the following rule to stabilize its debt level around its steady state.[24]

$$\frac{B_{t+1}}{\bar{B}} = \left(\frac{R_t/\pi_t \times B_t}{\bar{R}/\bar{\pi} \times \bar{B}}\right)^{\rho_B} \quad , \; 0 \leq \rho_B < 1 \tag{1.48}$$

where variables with bars denote steady state values. The parameter ρ_B governs how rapidly the fiscal authority stabilizes its debt. For instance, if $\rho_B = 0$, the fiscal authority reverts the level of government debt back to the steady state level immediately by adjusting available fiscal instruments, i.e. government purchases, lump-sum transfers, or tax rates. If ρ_B is close to one, the government debt level can deviate from its steady state level for a substantial amount of time.

Since agents in the model form rational expectations, the government must meet its solvency condition or inter-temporal budget constraint.

$$B_t = \sum_{l \geq t}^{\infty} \left\{ \prod_{i=t}^{l} \left(\frac{\pi_i}{R_i}\right) \right\} \left\{ \mathbb{T}_l - (G_l + T_l + D_l) \right\} \tag{1.49}$$

where $\mathbb{T}$, G, T, and D are tax revenue, government purchases, lump-sum transfers (or taxes) and unemployment benefits respectively. Equation (1.49) implies that, each period, the debt level should be equal to the present discounted value of all

[24] The households in the model are not Ricardian, in the sense that they value the liquidity provided by the bond, and thus are, not indifferent between tax financing and debt financing. Therefore such a rule is very important for the existence of a stable equilibrium. For detailed discussion, see Bayer et al. (2019).

future government surpluses. When the real value of government debt changes, at least one of the four instruments on the right-hand side must adjust to meet the solvency condition. In the baseline model, I assume that the government adjusts its goods purchases to meet the following budget constraint on a period-by-period basis.

$$\frac{R_t}{\pi_t}B_t - B_{t+1} = \Bigg[\tau_t^y\Big\{\int \mathbb{1}_{\{n_t=1\}} s_t d\mu_t + \int \mathbb{1}_{\{n_t=2\}} \min\{\upsilon s_t, \bar{s}\} d\mu_t + \nu\Pi_t\Big\} + \tau_t^a(1-\nu)\Pi_t\Bigg] - (G_t + T_t + D_t) \qquad (1.50)$$

(Monetary authority)

The monetary authority is standard. It sets the nominal rate of the return on the government bond, following a Taylor-type rule with interest rate smoothing as follows.

$$\frac{R_{t+1}}{\bar{R}} = \left(\frac{R_t}{\bar{R}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\Big\{\exp\left(u_t - \bar{u}\right)\Big\}^{\phi_u}\right]^{1-\rho_R} \exp(\epsilon_{R,t}) \ \ , \ \epsilon_{R,t} \ \sim \ \text{i.i.d } N(0, \sigma_R^2) \qquad (1.51)$$

where $0 < \rho_R < 1$ is the interest rate smoothing parameter, $\phi_\pi > 1$ and $\phi_u > 0$ are the responsiveness of the nominal rate to inflation and the unemployment rate gap, respectively, and $\epsilon_{R,t}$ denotes a monetary policy shock.[25]

The description of the model has ended. The market clearing conditoons, the definition of a recursive equilibrium, and the numerical method are explained in the

[25]In this paper, I do not consider the case in which the policy rate is constrained by the zero lower bound.

appendix.

1.3 Calibration

One of the primary goals of this chapter is to examine the transmission mechanisms of monetary policy shocks. On this subject, Kaplan et al. (2018) argue that indirect effects, such as the effects of changes in the real wage, far outweigh the direct effect of changes in the real interest rate in determining the response of the aggregate consumption to monetary policy shocks. They find that matching the empirical asset holding distribution is of great importance in generating their result.[26] Thus, to have a fair comparison of the results, I seek for a calibration for which the model can match similar targets used in Kaplan et al. (2018). Specifically, I calibrate the model to match moments of asset distribution such as the share of borrowers, the mass of households with zero assets, the mass of wealthy hand-to-mouth households (households with zero liquid assets but a significant amount of illiquid assets), and so on. Given that the model has many features and parameters, I use standard parameter values used in the literature for the most part and limit the number of parameters that are internally calibrated. One period in the model corresponds to one quarter in the data.

[26] Kaplan et al. (2018) emphasize that the mass of households with low liquid wealth is directly related to the overall sensitivity of consumption to interest rate changes, while the top end of the liquid asset distribution is critical for plausible redistributive effects of monetary policy shocks.

Table 1.1: Parametrization 1/3

Parameter	Value	Description	Target or reference
Households			
σ	1.5	Relative risk aversion	Standard value
β	0.9909	Household's discount factor	Bond to GDP ratio
ξ	3	Inverse Frisch elasticity	Chetty et al. (2011)
ψ	0.7164	Scale parameter for disutility of labor	Normalization
ζ	1/181	Probability of death	Average life-span
μ_χ	10.6229	Mean of χ dist	Average prob. of adjust
σ_χ	3.7543	Scale parameter for χ dist	Top 10% illiquid asset share
Labor market			
λ	0.1	Exogenous job separation rate	Standard value
ι	0.1	Vacancy posting cost	Gornemann et al. (2016)
$\bar{w}$	1.0235	SS wage	SS unemployment rate
α	1.7127	Matching elasticity	SS vacancy filling rate
ρ_w	0.75	Degree of staggered wage setting	Calvo sticky wage lit.
d	0.75	Indexation to SS inflation rate	Calvo sticky wage lit.

1.3.1 Data and parametrization

For the calibration, I intentionally use the same data, i.e., SCF 2004, and the same categorization of assets and liabilities as Kaplan et al. (2018). In the model, there are two kinds of assets, liquid and illiquid assets. In the data, I consider all deposits in financial institutions (checking, savings, call, and money market accounts), and bonds net of revolving consumer credit as liquid assets. I define illiquid assets as the sum of real estate wealth net of mortgage debt and equity in the corporate and non-corporate business sectors. Illiquid assets also include consumer durables, such as vehicles, net of non-revolving debt. Households with negative illiquid assets are excluded, as short positions in the illiquid asset are not allowed in the model.

Table 1.1 shows the set of parameters related to households and the labor market. For the coefficient of relative risk aversion, I use 1.5, which is within the range of values used in the literature. The household's subjective discount factor is closely related to the household's demand for liquid assets. Thus, it is internally calibrated at 0.9909 to match the bond to GDP ratio in the model.[27] For the inverse Frisch elasticity, I use the value reported in Chetty et al. (2011) for the intensive margin, which implies $\xi = 3$. The scaling parameter for the disutility of labor is set so that the steady state labor supply is equal to one. The probability of death used in the model implies an average working life-span of 45 years.

The distribution of the utility cost of adjustment mainly determines the frequency of adjustment and households' demand for the illiquid asset. The calibrated adjustment costs imply an average adjustment frequency at the steady state of 5.7% per quarter, which is slightly higher than the implied adjustment probability in Kaplan and Violante (2014).[28] Also, the utility cost distribution leads to an illiquid asset share of top 10% wealthy households of about 70%.

As for the labor market parameters, I set the probability of exogenous job separation at 10%, which is a standard value used in the literature. Following Christiano et al. (2016), I assume that the steady state unemployment rate and vacancy-filling rate are 5.5% and 70%, respectively. Given the value of 0.1 for the vacancy posting cost, which is slightly smaller the value used in Gornemann et al. (2016), these values imply state wage of 1.0235 and a matching elasticity with respect to the number of searchers of 1.7127. The two parameters governing wage rigidity in

[27] Since all aspects of the model are inter-connected, there is no clear one-to-one relation between a parameter and a moment. However, I explain as if there exist such one-to-one relations based on the impact of particular model parameters on particular moments.

[28] Kaplan and Violante (2014) compute a participation frequency of 4.5% given a fixed adjustment cost of $500.

Table 1.2: Parametrization 2/3

Parameter	Value	Description	Target or reference
Int. good			
η	3	Elasticity of substitution	Gornemann et al. (2016)
Ξ	0.1266	Fixed cost of operation	Capital income share
θ	0.27	Capital share of production	Labor income share
κ	0.08	Slope of the Phillips curve	Standard value
$\bar{\pi}$	1.005	steady state inflation rate	Standard value
φ	0.1967	Sales elasticity w.r.t. the stock	SS sales-to-stock ratio of 0.25
ϕ_x	0.01	Sales-to-stock adj. costs	Jung and Yun (2005)
δ_i	0.01	Inventory depreciation rate	Jung and Yun (2005)
Mutual fund			
δ_0	0.02	Depreciation rate under full utilization	SS depreciation rate
δ_1	1.69	Elasticity of depreciation rate	SS utilization rate
ϕ	10	Capital adjustment cost	Standard value
ν	0.2299	Ent. share of profits	Gini Networth

the model, ρ_w and d, are both set to 0.75, which are standard parameter values used in models with Calvo wage setting, such as Erceg et al. (2000) and Schmitt-Grohe and Uribe (2006).

I set the elasticity of substitution among intermediate inputs to 3, the value in Gornemann et al. (2016). The low value for this parameter has several important implications in the model. First, it results in a substantial amount of profits at the steady state. Thus, for a high enough aggregate capital stock, the dividend rate can be much higher than the rate of return on the liquid asset even after a fraction of profits is given to entrepreneur households. This facilitates matching the illiquid asset-to-GDP ratio in the data. Second, a lower elasticity of substitution leads to a higher degree of quantity response of the intermediate good firms. The elasticity of substitution represents the degree of competition in the intermediate goods market. If η is high, i.e., the competition in the market is severe, it is hard for firms to raise their prices following an increase in demand. Hence, the decrease in mark-ups is larger, and, thus, profits fall in response to an expansionary monetary policy shock. In contrast, if the elasticity of substitution is low, the fall in the mark-up is limited, and hence profits can increase in response to a decrease in the real interest rate.

I set the slope of Phillips curve to 0.08, a standard value used in the literature. The fixed cost of operation is set to imply a steady state capital income share of about 30%.[29] The steady state inflation rate is set to 2% per annum, which is consistent with the Fed's current inflation target. The capital share of the production is set to 0.27, which leads to a steady state labor income share of around 60%. For the parameters related to inventory holding, I follow Jung and Yun (2005). The elasticity of sales with respect to the stock of goods is calibrated at 0.1967, which implies a steady state sales-to-stock ratio of 25%. The coefficient of sales-to-stock adjustment costs and the inventory depreciation rate are set to the values in Jung and Yun (2005).

The first parameter regarding variable depreciation rate δ_0 is set to 0.02, which implies an annual depreciation rate of 8% under full utilization. The parameter that governs the degree of the acceleration of depreciation δ_1 is set to 1.69. This choice of parameter values regarding variable depreciation leads to a steady state capital utilization rate of about 65%. The coefficient of capital adjustment costs is set to 10, which is a standard value used in the literature. The entrepreneur share of profits is internally calibrated to match the Gini index of net worth, i.e., assets minus liabilities, since entrepreneur households drive overall wealth inequality by

[29] The current calibration for the elasticity of substitution implies a steady state mark-up of 50%. Thus, even though a fraction of profits is given to some households as a compensation for working as entrepreneurs, a high mark up can lead to too high of a capital income share if there is no fixed cost.

Table 1.3: Parametrization 3/3

Parameter	Value	Description	Target or reference
Government			
τ^w	0.3	SS Income tax rate	Data
τ^a	0.3	SS Profit tax rate	Data
υ	0.4	Replacement ratio	Data
ρ^B	0.85	Degree of debt stabilization	Bayer et al. (2019)
T	10.7%	SS lump-sum transfer to GDP ratio	Top 10% other income share
$\underline{\mathrm{R}}$	3.5%	Borrowing premium	Share of wealthy HtM HH
$\underline{\mathrm{b}}$	1.2705	Borrowing limit	Fraction of borrowers
Central bank			
$\bar{R}$	1.0080	SS policy rate	Share of HH with zero assets
$\bar{\pi}$	1.005	Target gross inflation rate	Fed's current target
ρ^R	0.85	Degree of interest rate smoothing	Standard value
ϕ_π	1.5	Response to inflation gap	Standard value
ϕ_u	0.1	Response to unemployment gap	Standard value
TFP shock			
ρ_Z	0.9	Auto-correlation of TFP shock	Standard value

holding a significantly larger amount of assets than other households in the model.

The parameters for the fiscal and monetary authorities and exogenous processes are mostly set to values that are widely used in the literature. I assume that both the income and profit tax rates are 30%.[30] I set the replacement ratio of unemployment benefits to 40% as in Shimer (2005). The degree of persistence in the bond issuance rule is set to 0.85, following Bayer et al. (2019). The lump-sum transfer is calibrated at 10.7% of GDP in the model. This implies a share of other income of top 10% wealthy households of 5% in the model.[31] The borrowing premium is set to 3.5% per quarter to enable the model to match the share of wealthy hand-to-mouth households in SCF 2004, which is 20%. The borrowing limit is also internally calibrated to match the fraction of households with debt, which is 15% in

[30] Before 2017, the U.S. profit tax rate was above 30%. However, it fell to 21% after the introduction of the 2017 Tax Cuts and Jobs Act. The income tax rate ranges from 10% up to around 40% based on the amount of income.

[31] Other income share of top 10% wealthy households in the model is slightly lower than the corresponding value in SCF 2004, which is 7%.

Table 1.4: Productivities

Symbol	Value
s_1	0.2488
s_2	0.9148
s_3	1.0000
s_4	1.0931
s_5	5.0287
Ent	-

Table 1.5: Transition matrix

		tomorrow					
		s_1	s_2	s_3	s_4	s_5	Ent
today	s_1	0.9308	0.0567	0.0020	0.0000	0.0100	0.0005
	s_2	0.0098	0.8194	0.1587	0.0016	0.0100	0.0005
	s_3	0.0020	0.1599	0.6677	0.1599	0.0100	0.0005
	s_4	0.0000	0.0016	0.1603	0.8276	0.0100	0.0005
	s_5	0.0389	0.0389	0.0389	0.0389	0.8440	0.0005
	Ent	0.0385	0.0385	0.0385	0.0385	0.0385	0.8076

the data.

The steady state quarterly nominal rate of return on the government bond is set to 1.008 to match the share of households with zero assets. Combined with the inflation target of 2% per annum in the model, the calibrated policy rate implies a steady-state real liquid asset return of 1.2% per annum. I set parameters related to the Taylor rule to standard values. The degree of interest rate smoothing is set to 0.85. The responsiveness of the nominal rate to the inflation gap and unemployment gap are set to 1.5 and 0.1 respectively. I set the auto-correlation of the TFP shock to 0.9, which is also a standard value.[32]

Finally, the income process, which is the ultimate source of the inequality in the model, is reverse-engineered to match asset holding and wealth inequality in the data.[33] First, I set the baseline income process for s_t as a standard AR(1) with three states using Tauchen's method for the discretization. I set the autocorrelation and standard deviation of quarterly income to 0.93 and 0.03 respectively, both of which are close to values typically used in the literature.[34] In addition to this baseline

[32]Since I rely on a linearization of the model to solve for the recursive equilibrium, the size of the shocks does not matter for the impulse response analysis.

[33]The income Gini index at the steady state is 0.39, which is somewhat lower than the Gini index computed from the SCF 2004, which is 0.56.

[34]For instance, McKay et al. (2016) uses 0.966 and 0.33 for their income AR(1) income process in a version of their HANK model.

Table 1.6: Model fit 1

	Data	Model
Capital to output ratio	**2.92**	**2.92**
Bond to output ratio	**0.26**	**0.26**
Gini index for net worth	**0.81**	**0.82**
Fraction with b < 0	**0.15**	**0.15**
Fraction with b = 0 and a > 0	**0.20**	**0.20**
Fraction with b = 0 and a = 0	**0.10**	**0.10**
Fraction with a = 0	0.21	0.25

Notes: The data moments are computed from SCF 2004. a: illiquid, b: liquid assets

process, I add two boundary states (super low-skilled and super high-skilled) to enable the model to generate realistic wealth inequality. I set the probability of becoming an entrepreneur P_e to 0.05%. Then, I calibrate the probability of leaving the entrepreneur state to match the liquid asset share of top 10% wealthy households. The resulting parameter value for $\tilde{P}_e$ is 19.24%.[35] These parameter values imply that the profit income share for top 10% wealthy households is 21%, which is close to the corresponding value of 23% in SCF 2004. Tables 1.4 and 1.5 show the values for idiosyncratic productivity and the transition matrix of skills and the entrepreneur state.[36]

1.3.2 The fit of the model

Tables 1.6 and 1.7 show the model's performance in matching the targets related to aggregate asset holding and the distribution of assets. As discussed in the previous section, I use the capital-to-output ratio, bond-to-output ratio, and Gini

[35] Since entrepreneur households earn a large amount of income and face very high unemployment risk, they accumulate a lot of wealth, especially in the form of the liquid asset.

[36] Table 1.5 does not take into account employment status transition probabilities, i.e., job-finding and separation rates.

Table 1.7: Model fit 2

	Liquid		Illiquid	
Moments	Data	Model	Data	Model
Top 0.1 percent share	17	18	12	4
Top 1 percent share	47	49	33	22
Top 10 percent share	**86**	**87**	**70**	**73**
Bottom 50 percent share	-4	0	3	1
Bottom 25 percent share	-5	-2	0.1	0
Gini Coefficient	0.98	0.94	0.81	0.84

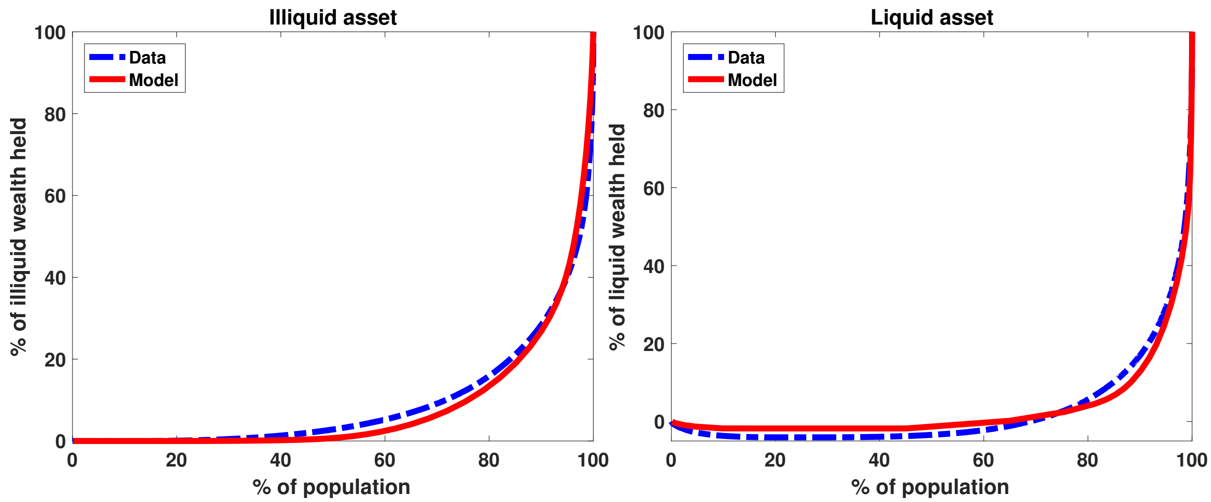
Notes: The data moments are computed from SCF 2004.

index for net worth as targets. In addition, I target the fraction of borrowers, the share of wealthy hand-to-mouth households, and the share of households with zero assets, because these moments determine the sensitivity of aggregate consumption to changes in the real interest rate and relevant distributional effects in the model. As shown in Tables 1.6, the model matches these targets up to two decimal points, except for the Gini index for net worth. Moreover, the model is reasonably good at matching non-targeted moments. In the model, the fraction of households with zero illiquid assets is 25% in the model, which is slightly higher than but close to the corresponding value of 21% in the data.

Table 1.7 looks more closely at the distributions of asset holdings in the model and in the data. Among the moments presented in the table, only the top 10% shares of liquid and illiquid asset holding are targeted.[37] Nevertheless, the model captures overall inequality for each type of asset very well. For the liquid asset, the model matches even the top 0.1% and top 1% asset shares very closely. The liquid asset shares of the top 0.1% and 1% wealthy households are 18% and 49%

[37] As in Kaplan et al. (2018), wealth percentiles are defined based on *each* asset distribution.

Figure 1.2: Lorenz curves in the data and the model

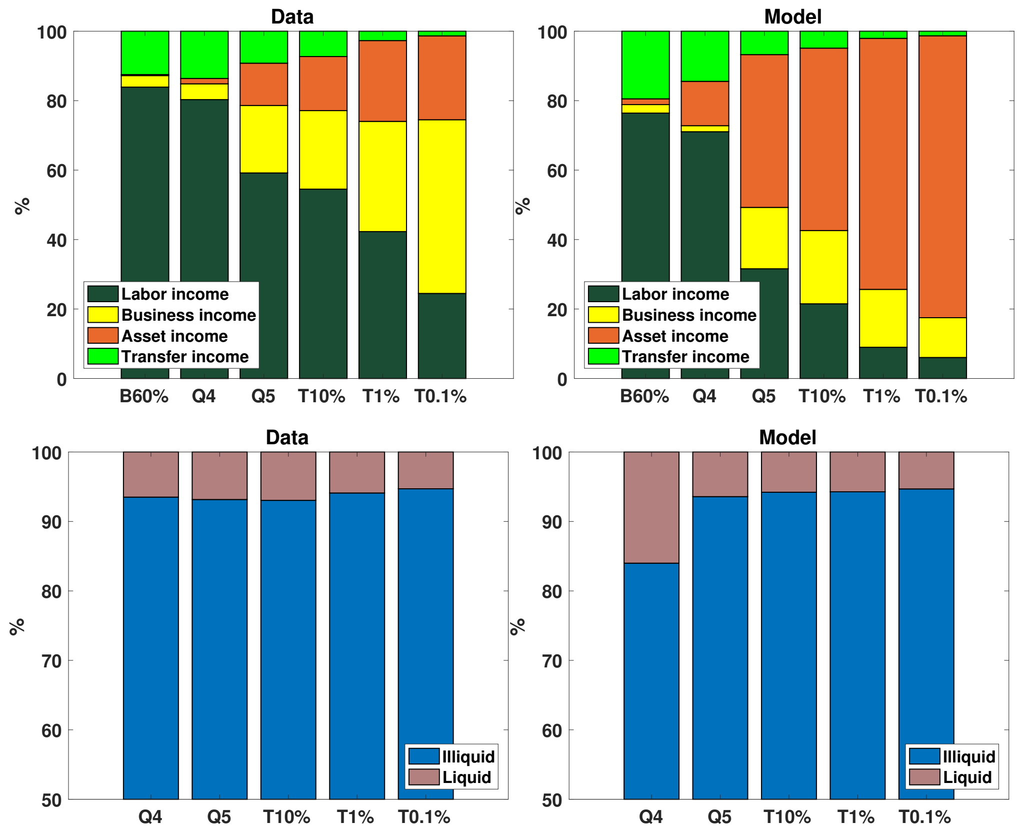


Notes: The figure shows asset holding inequality in the data and in the model using Lorenz curves. For the definition of liquid and illiquid asset in the data, see the main text.

in the model while they are 17% and 47% in the data. By contrast, households in the model are less indebted than households in the data, as shown by relatively poor households' liquid asset shares. The liquid asset share of households in the bottom 25% of wealth is -5% in the data but only -2% in the model. The model is less successful in matching the top right-end of the distribution of illiquid assets. The illiquid asset shares of top 0.1% and 1% wealthy households are 12% and 33% in the data, while they are only 4% and 22% in the model. However, given that matching the right tail of wealth distribution is known to be difficult, the model does a reasonable job in generating realistic asset holding inequality.

Figure 1.2 shows the degree of asset holding inequality using Lorenz curves. The Lorenz curves in the model and in the data look similar, which confirms the model's ability to capture the degree of asset holding inequality observed in the data. Illiquid asset holding is slightly more concentrated in the right-tail of the

Figure 1.3: Income composition in the data (SCF 2004) and in the model



Notes: The above and bottom panels show income and asset composition in the data and the model respectively. For the classification of income and asset types, see the main text.

distribution in the model than in the data, but the differences are not substantial.[38] The liquid asset Lorenz curve again confirms that households in the left tail of the distribution are less indebted in the model than in the data. However, the model captures well the concentration of liquid asset holding among households in the right tail of the distribution.

Regarding income composition, the model captures essential characteristics of heterogeneity depending on households' wealth.[39] Both in the data and the model,

[38]The illiquid asset holdings are more concentrated at the top 1% level in the data. However, at the top 10% level, illiquid asset holdings are more ceoncentrated in the model.

[39]In the data, labor income consists of income from wages and salaries. Business income is income from ownership of a business. Asset income includes interest, dividends, and capital gains or losses.

the main source of income for the bottom 80% of households is labor income, which accounts for around 80% of their total income. In contrast, top 10% households earn a substantial amount of income from assets and business activities. However, the combined share of business and asset income in the total income of wealthy households is larger in the model than in the data.

As for portfolio composition, more than 90% of wealthy households' portfolio is in illiquid assets both in the model and in the data. However, there are no noticeable differences across different wealth groups in terms of portfolio composition in the data. This is mainly because most of households in the data own houses, an important illiquid asset. If I consider only a fraction of residential houses as productive illiquid assets, as in Kaplan et al. (2018), the illiquid asset share in households' portfolio increases in household' wealth, as in the model.[40]

1.4 The effects of conventional monetary policy

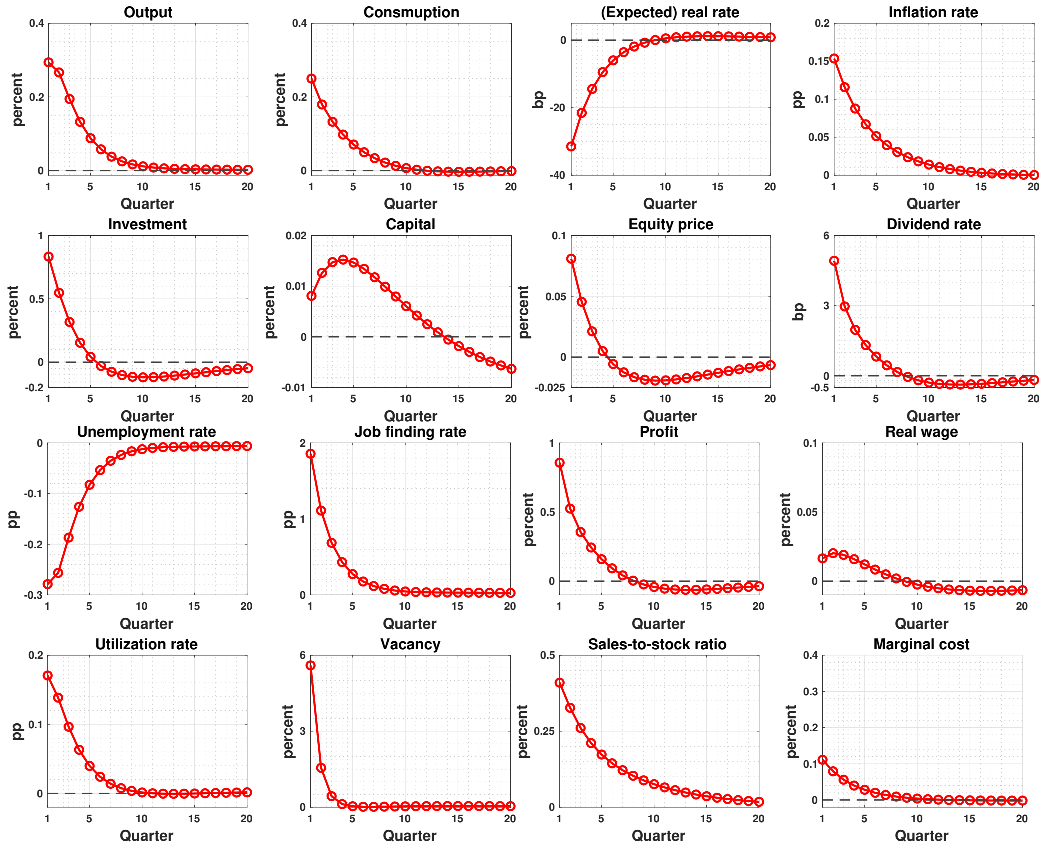
In this section, I present the main results from the model regarding the effects of conventional monetary policy shocks. Throughout the section, I focus on the effects of an expansionary monetary policy shock.[41] First, I show how aggregate variables such as output, consumption, investment, the inflation rate, wages and asset returns respond to a fall in the policy rate. Especially, I focus on how relevant

Transfer income includes social security benefits and unemployment benefits. In the model, labor income is income from wages. Entrepreneurs earn business income in the model. Asset income consists of the return on bonds and equity. Transfer income consists of the lump-sum transfer from the fiscal authority and unemployment benefits.

[40] In an earlier version of Kaplan et al. (2018), the authors regarded only 40% of the housing wealth as productive illiquid assets in the SCF data.

[41] Since the solution method relies on a linearization of the system around the steady state, the results from a contractionary monetary policy shock are symmetric.

Figure 1.4: IRFs to an expansionary MP shock



Notes: Impulses responses to annualized 25 bp expansionary monetary policy shock. Inflation rate, unemployment rate, job finding rate, and capital utilization rate are expressed in terms of percentage point deviations from steady state. Nominal and real rate, and dividend rate are shown as basis point deviations from steady state. Inflation rate, nominal and real rate, and dividend rate are annualized. All other variables are shown as percent deviations from steady state.

features of the model help generate an increase in profits when an expansionary monetary policy shock hits the economy. Then, I decompose the total effect of an expansionary monetary policy shock into direct and indirect effects to examine the transmission mechanisms. Finally, I study the implications of these transmission channels for their distributional consequences.

1.4.1 Impulse responses of aggregate variables

In this subsection, I show the impulse responses of aggregate variables to a minus 6.25 bp (minus 25 bp per annum) shock to the policy rate ($\epsilon_{R,0} = -0.000625$). After an expansionary monetary policy shock, the expected real interest rate falls and, by inter-temporal substitutions, households increase their demand for goods. In response to an increase in demand, the firms raise their prices. However, due to price adjustment costs (nominal rigidity) and the possible substitution across different goods (real rigidity), the increase in prices is limited. Instead, firms increase their production, which leads to a rise in factor demands. Such an increase in factor demands results in higher rental rates for both labor and capital. As the market wage exhibits rigidity, a higher rental rate for labor services implies a greater mark-up for labor agencies, which induces them to post more vacancies. Thus, the job-finding rate increases, and the unemployment rate falls. As more households become employed, the demand for goods further rises. Households whose employment status has not changed also increase their demand, since the precautionary motive for savings becomes weaker due to lower unemployment risk and a higher job-finding rate. Similarly, a higher rental rate for capital leads to a higher capital utilization rate.

In principle, higher demand for goods can contribute to an increase in profits. However, if all firms increase production, marginal cost can rise substantially. As a consequence, mark-ups may shrink significantly, and profits may fall. In the model, firms avoid such a result by increasing production only partially, and instead resorting to their inventory stock to meet demand. That is, firms reduce their inventory stock, and increase the sales-to-stock ratio. As a result, the increase in marginal cost is limited. This dynamic adjustment of inventory stock, jointly with wage rigidity, contributes to an increase in profits. Hence, the dividend rate also rises. As the return on the illiquid asset rises, households shift their portfolio towards the illiquid asset. Hence, investment and the aggregate capital stock increase.

Quantitatively, after a 25 bp (annualized) expansionary monetary policy shock, output increases by 0.3% on impact and gradually converges back to the initial steady state level.[42] Consumption increases on impact by 0.25%, which is slightly lower than the response of output. As is well-known, investment responds more strongly, increasing by more than 0.8% on impact. In order to meet the expansion in activity, the capital utilization rate increases by 0.17 percentage points. Importantly, the demand for labor services goes up, but due to the assumed wage rigidity, the real wage does not respond much, increasing only by less than 0.02% on impact. The weak response of the real wage is in line with Christiano et al. (2005), Christiano et al. (2016), and Gornemann et al. (2016) in terms of magnitude. Since the real wage does not respond much, the rise in the labor rental rate leads to a substantial increase in the number of vacancies. On impact, vacancies rise by almost 6%.[43] As

[42]Like many other HANK models in the literature, the model in this chapter cannot generate hump-shaped responses in output, consumption, and other aggregate variables to monetary policy shocks, because it lacks mechanisms such as habits in consumption and investment adjustment costs, which help generate a hump-shaped response of these variables in a representative agent model.

[43]The vacancy posting responds in a front-loaded manner. After an initial peak, it rapidly goes back to the steady state level. This is because the vacancy posting decision takes into account the expectation on all future values from a match. Though the response of vacancy posting is transient, the unemployment rate and the job-finding rate respond more persistently.

a result, the job-finding rate increases by 1.9%, and the unemployment rate falls by 0.28%.

Firms reduce their inventory stocks gradually over several periods in response to the increase in factor prices. On impact, firms reduce their inventory by 0.06%, and the magnitude of the decrease peaks at 0.12% after six quarters. The response of the sales-to-stock ratio is more immediate. It rises by 0.4% on impact and converges back to the steady state level monotonically. These responses of inventory stock and the sales-to-stock ratio are consistent with the findings of Jung and Yun (2005).[44]

Because of wage rigidity and inventory stock adjustments, profits increase substantially in response to an expansionary monetary policy shock, rising by 0.85% on impact. This response is in stark contrast to standard New Keynesian models, in which profits decrease after an expansionary monetary policy shock. As I will show momentarily, this difference in the response of profits (and the real wage) leads to different results on the transmission mechanisms as well as the distributional consequences of monetary policy shocks.

1.4.2 Transmission mechanisms of monetary policy in a HANK

To analyze transmission mechanisms, I decompose the effects of an expansionary monetary policy shock on consumption into direct and indirect effects, following Kaplan et al. (2018). Note that aggregate consumption in any given period t can be written as a function of both current and future (expected) values of the real

[44] As with other aggregate variables, the model cannot generate a hump-shaped response of the sales-to-stock ratio. The magnitude of the inventory stock response is somewhat smaller than in Jung and Yun (2005).

interest rate r, the equity price q, profits Π, the real wage w, the job finding rate f, and relevant fiscal instruments. That is, we have

$$C_t^* = C_t(\{r_\tau^*, q_\tau^*, \Pi_\tau^*, w_\tau^*, f_\tau^*\}_{\tau=t}^\infty) \tag{1.52}$$

in the baseline model, where I assume that the fiscal authority adjusts government purchases in response to changes in the real value of government debt. By totally differentiating (1.52), we have the following decomposition.[45]

$$dC_t^* = \underbrace{\sum_{\tau=t}^{\infty} \frac{\partial C_t}{\partial r_\tau} dr_\tau}_{\text{direct effect}} + \underbrace{\sum_{\tau=t}^{\infty} \left(\frac{\partial C_t}{\partial q_\tau} dq_\tau + \frac{\partial C_t}{\partial \Pi_\tau} d\Pi_\tau + \frac{\partial C_t}{\partial w_\tau} dw_\tau + \frac{\partial C_t}{\partial f_\tau} df_\tau \right)}_{\text{indirect effects}} \tag{1.53}$$

Figure 1.5 shows the direct and indirect effects of an expansionary monetary policy shock on aggregate consumption over the first twelve quarters. Table 1.8 shows the average effect over the first year. First, as in Kaplan et al. (2018), the direct effect of changes in the real interest rate only accounts for a small proportion of the overall consumption response. Under the current calibration, the direct effect explains only 24% of the consumption response over the first year. As discussed in Kaplan et al. (2018), this is because a substantial fraction of households do not hold

[45] I compute (1.53) numerically as follows. First, I compute the path of equilibrium prices and the job finding rate under an expansionary monetary policy shock using the solution that I get from a perturbation method. With that path, I solve the household's problem backwardly from T to 0 for a large T, where T is chosen to guarantee that the model economy has reverted back to the steady state in period T. A shock occurs in period 0. At each period, I compute the household's value function by taking into account the household's expectation on future prices and labor market conditions. With these value functions, I derive the household's optimal decisions and compute aggregate consumption by summing up individual consumption over the state space. I repeat this computation for different paths in which only one variable follows the equilibrium path and others remain at their steady state values. Then, I compute the difference between the computed consumption response and the steady state consumption level to measure the effect of each variable shown in the decomposition.

Figure 1.5: Consumption decomposition

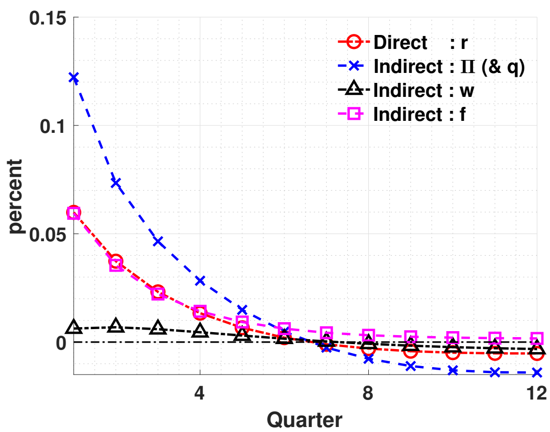


Table 1.8: Average response over the year

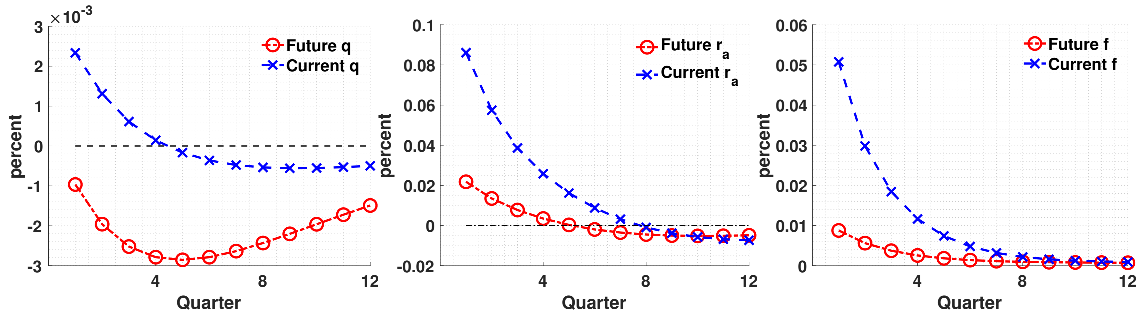
r change (pp annual)	-0.21
Elasticities $(\epsilon_X = \frac{\Delta X}{X}/\Delta r)$	
Output (Y)	-4.29
Investment (I)	-8.94
Consumption (C)	-3.19
Bond (B)	1.89
Equity (A)	-0.24
Decomposition of C (%)	
Direct effect (r)	**24**
Indirect effects	**76**
Profits (and equity price) (Π & q)	48
Wage (w)	4
Job finding rate (f)	24

Notes: The figure shows the percentage deviations of consumption from its steady state level. Elasticities are shown as semi-elasticities. The decomposition into the direct and indirect effects is approximately exhaustive. Both effects add up to 100%. The fiscal authority is assumed to adjust lump-sum transfer for given bond-issuance rule in response to monetary policy shocks. Profit is the profit given to entrepreneurs as compensations. The profits given to the equity holders are reflected in the equity return.

liquid assets at the steady state. Especially, wealthy hand-to-mouth households play a crucial role in driving this result. Since they do not hold liquid assets and face a borrowing premium, they do not respond to a small change in the real interest rate. Moreover, since they are relatively wealthy, their consumption response is important in determining the overall consumption response.

As in Kaplan et al. (2018), most of the aggregate consumption response is due to indirect effects, which account for 76% of the consumption response over the first year. However, unlike in Kaplan et al. (2018), the effect of changes in the real wage is limited, due to wage rigidity. Over the first year, real wage changes account for only 4% of the total response. Meanwhile, increases in profits and the job-finding rate contribute substantially to the increase in aggregate consumption than the real wage changes. Surprisingly, the increase in profits accounts for almost half of the total consumption response over the first year. This is because the

Figure 1.6: Effects of current and future variables



Notes: The left, middle and right panel show the effect of current and future equity price, dividend rate, and job finding rate respectively. When a current variable changes, future values of the variable are expected to take on its steady state value. When future variables are expected to take on new equilibrium values, the current value for the variable is held fixed at the steady state value. All panels shows the percentage deviations from the steady state.

magnitude of the increase in profits is much larger than that of the real wage. Even though the increased profits are mostly distributed to wealthy households, whose marginal propensity to consume is low, the response of aggregate consumption is still substantial, because the magnitude of the profit response is sizable. The increase in the job-finding rate also has a large impact, accounting for 24% of the aggregate consumption response over the first year.

In Figure 1.6, I further decompose the effect of the equity price, the dividend rate (profits), and the job-finding rate on consumption into the effects of current and future variables. Specifically, I compute the consumption response when only the current value of a variable changes, while future values are expected to remain at its steady state level. Also, I compute the consumption response when future values of a variable are expected to change, while the current value is held fixed at the steady state level. As shown in the left panel of the figure, positive consumption response to equity prices mostly comes from the change in the current price. The higher current price of the equity has a positive income effect for households that choose to sell their equity. In addition, a higher current price for a given expectation on future prices implies a lower expected capital gain. Thus, households save less in equity and consume more instead. Combining these two effects, consumption increases in response to an increase in the current equity price. In contrast, an increase in the future expected equity price implies a higher expected capital gain, and thus leads to a fall in consumption. Since the effects of current and future equity prices offset each other, the overall effect of equity prices on the aggregate consumption is not large.

Changes in the dividend rate induced by the increase in profits have more substantial effects than the equity price on aggregate consumption. Both changes in current and future dividend rates lead to an increase in current consumption. A higher current dividend rate contributes to higher consumption via positive income effects. What is surprising is the effect of the future dividend rate. In general, a higher return on saving leads to a fall in current consumption via inter-temporal substitution. In this model, however, higher future dividend rates also result in higher current period consumption because of the two asset structure of the model. When the expected return on equity rises, households re-optimize their portfolio towards equity. That is, households reduce bond holdings and, instead redirect their savings into equity. However, the withdrawals from bond holding are larger than saving into equity, since the return on equity is much higher than the bond return. Households can achieve the desired return on their portfolio with a smaller amount of savings. Thus, households increase both consumption and equity investment at the same time when future dividend rates increase. This is why the increased dividend rate has a strong effect on aggregate consumption. Both income and substitution effects lead to an increase in current aggregate consumption.

Finally, increases in both current and future job-finding rates increase aggregate consumption. A rise in the current job-finding rate allows more unemployed households to find a job. For such households, the income effect is substantial, since their income increases upon employment by more than 60% on average, given the replacement ratio of 40%. Thus, these households increase their consumption substantially. On the contrary, higher future job-finding rates do not have any income effect. No households earn more in the current period from higher future job-finding rates. However, households save less or borrow more in the face of lower unemployment risk. This effect applies to all households in the model regardless of their current employment status, unless they are constrained by the borrowing limit. Quantitatively, the effects of lower future unemployment risk are much smaller than the income effect of the current job-finding rate. Since a substantial fraction of households hold zero liquid assets and face a high borrowing premium, they cannot reduce their savings and do not borrow even though unemployment risk has fallen. The increase in the future job-finding rate only leads to a 0.01% increase in aggregate consumption on impact. In contrast, the higher current job-finding rate results in a roughly five times larger response of aggregate consumption on impact.

In sum, indirect effects are dominant in determining the aggregate consumption response, as in Kaplan et al. (2018). However, the main driver of the indirect effect is not changes in the real wage, but rather the increase in profits and a higher

Figure 1.7: Decomposition across wealth groups

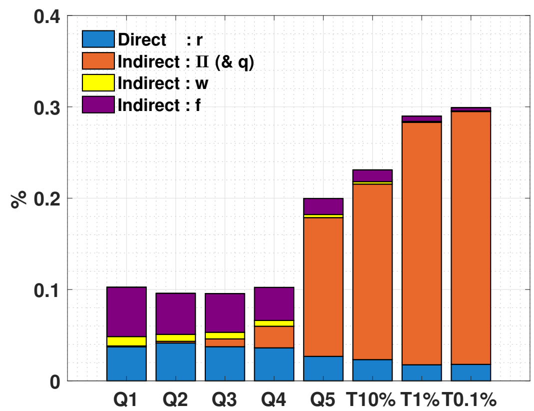


Table 1.9: Welfare gains

Q1	1.01
Q2	0.65
Q3	0.43
Q4	0.37
Q5	1.24
Top 10 %	1.64
Top 1 %	1.77
Top 0.1 %	1.87
Employed	0.61
Unemployed	2.26
Entrepreneur	11.9
Average	0.73

Notes: The figure shows the decomposition of each group's average consumption response over the first year in terms of the percentage deviations from the steady state. The table shows welfare gain of each group from an expansionary monetary policy shock in terms of the consumption equivalent. The unit for consumption equivalent is basis point.

job-finding rate.

1.4.3 Distributional consequences of conventional monetary policy

Now, I turn to the distributional implications of conventional monetary policy. To this end, I sort households into quintiles based on their net worth and examine the effects of an expansionary monetary policy shock on each wealth group.[46] In addition, I analyze the households with top 10%, 1% and 0.1% net worth for a better understanding of the effects of monetary policy on wealthy households, which is one of the focuses of this chapter.

As shown in Figure 1.7, the average consumption response to an expansionary

[46]The quintile analysis is based on the classification of households in the period of the shock. Over time, households' working status changes and, consequently, transitions of households across different wealth groups occur. I do not take into account such transitions in the analysis. Instead, I follow the same households over time. For instance, households in the third quintile in the third period after the shock are households in the third quintile on impact. Thus, the value of their net worth may be greater than the bottom 60% or less than the bottom 40% of households' net worth in the third period.

monetary policy shock is much larger for households with top 20% or larger wealth than for other households. Households in the lower four quintiles increase their consumption by around 0.1% over the first year. In contrast, households in the fifth quintile increase their consumption by 0.2%. The increase in consumption of top 0.1% wealthy households is almost 0.3%. The result that wealthier households increase their consumption the most is striking, given the heterogeneity in MPC across wealth groups. As shown in Table 1.10, households' MPC is decreasing in the amount of wealth.

The difference in consumption responses is due to the heterogeneity in households' portfolio and income composition as well as different magnitudes of the responses of aggregate variables to monetary policy shocks. Figure 1.7 shows that not only the magnitude but also the decomposition of the response are different across wealth groups. For households in the bottom 80% wealth groups, labor income is the most important component of their total income. Since the real wage does not respond much to an expansionary monetary policy shock, the benefit from changes in the real wage is limited for these households. They instead benefit from a higher job-finding rate and a lower real debt burden. By contrast, households in higher wealth groups do not rely much on labor income, so changes in the real wage or the job-finding rate do not affect their consumption much. Instead, most of their consumption response comes from the rise in profits, because these households hold a large amount of the illiquid asset, which is a claim on profits. Since the increase in profits is much larger than changes in the real wage or the job-finding rate, consumption responses over the first year are much larger for wealthy households than

Figure 1.8: MPC distribution

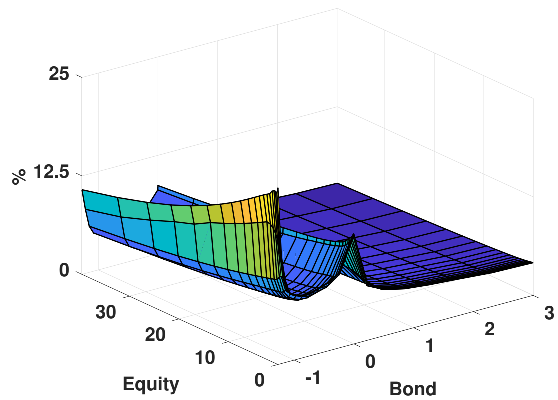


Table 1.10: Average MPCs

Wealth group	MPC
Q1	21.0
Q2	12.8
Q3	8.3
Q4	6.4
Q5	5.7
Top 10 %	5.6
Top 1 %	2.7
Top 0.1 %	1.6
Average	10.6

Notes: Marginal propensity to consume is computed as the ratio of the difference in consumption to an unexpected lump-sum transfer of 0.05. The figure shows mean MPC over idiosyncratic productivity and working status. Average MPC for each wealth group is shown in the table. The unit for average MPC is percentage.

for other households.

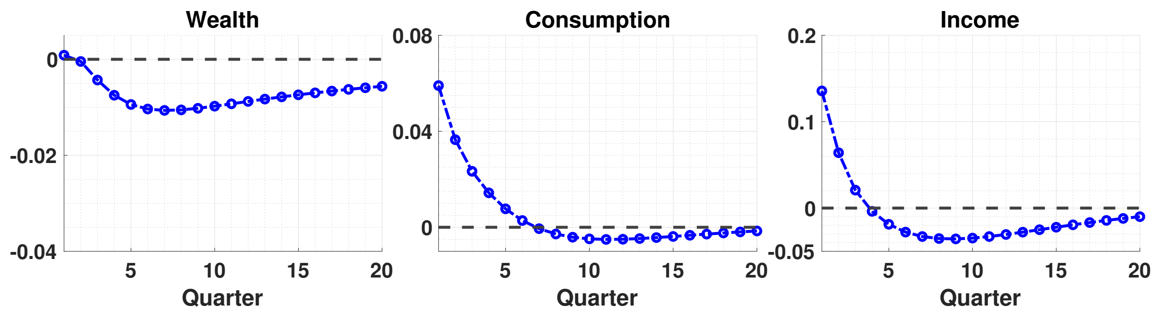
So far, I have focused on short-term responses. To evaluate long-term distributional consequences of conventional monetary policy shocks, I compare welfare gains or losses in terms of lifetime consumption equivalents. Specifically, I compute the ratio of steady state consumption that households would require each period as compensation for forgoing a 25 bp expansionary monetary policy shock (per annum). For instance, the entry of "1.01" in row "Q1" indicates that the households in the first wealth quintile on average would feel indifferent between the steady state and experiencing a 25 bp expansionary monetary policy shock if they received 1.01 basis points of extra steady state consumption each period. As shown in Table 1.8, this welfare gain is relatively small, because the size of the shock is small and a substantial amount of the benefits from an expansion is wasted on higher government purchases. The average consumption equivalent across all households is only 0.73 basis points.

More interestingly, the magnitude of welfare gain is not monotonically decreasing in wealth. In Table 1.9, the welfare gain is actually largest for the wealthiest households and smallest for households in the fourth quintile. This result is in stark contrast with previous studies, e.g., Gornemann et al. (2016), which found that the welfare gain is decreasing in wealth. In the first quintile, relatively more households are unemployed and in debt.[47] Thus, these households benefit from an increase in the job-finding rate and a fall in the real interest rate. However, households in the third and fourth quintile are less likely to be unemployed and have a positive amount of liquid assets on average. Thus, the benefits from a monetary expansion is limited for these groups unless the real wage increases substantially. In contrast, wealthier households enjoy a significantly larger welfare gain, because they are major claimants to profits and the response of profits is much larger than those of the real wage and the job-finding rate. Grouping households by working status, employed households enjoy the smallest gain while entrepreneurs experience the biggest gain, which is about 20 times larger than the gain of employed households. This is because most of entrepreneurs' income comes from profits.

As another way of measuring the distributional consequences of monetary policy, I compute the impulse responses of Gini indices for wealth, consumption, and total income. The vertical axis of the figure indicates absolute differences. For instance, if the index rises from 65.0 to 65.2, then this will appear as 0.2 on the vetical axis. As shown in Figure 1.9, the impulse responses of Gini indices are con-

[47] The unemployment rate among households in the first quintile is 6.5%, which is 1% larger than the average in the model economy. Also, average liquid asset holding is negative for these households.

Figure 1.9: Responses of Gini indices



Notes: Impulse responses of Gini indices are shown as the absolute differences in indices. Wealth consists of bond and equity holdings. Income includes all types of income, including labor earning, asset income such as interest, dividends, and capital gain, unemployment benefit and government transfer.

sistent with the results so far. After the shock, both consumption and income Ginis increase from their steady state value, indicating higher consumption and income inequality. As before, this is mainly due to the substantial increase in profits after an expansionary monetary policy shock. Since the right tail of the wealth distribution mostly consists of entrepreneurs and households that hold a large amount of equity, a disproportionately large increase in profits leads to higher inequality in income and consumption. Meanwhile, the wealth Gini decreases after an expansionary monetary policy shock. However, this does not imply that absolute differences in wealth across households have decreased. The increase in wealth levels is actually larger for wealthier households. However, since the steady state level of wealth is much smaller for poor households, the increase in wealth following expansionary monetary policy is proportionally much larger for poor households. Indeed, the fall in wealth Gini is mostly driven by the responses of households in the first and second quintiles, whose absolute level of wealth is very small.

So far I have studied the distributional consequences of monetary policy shocks using various measures. From the results presented in this section, it is hard to argue that an expansionary monetary policy shock reduces inequality. In my baseline model, an expansionary monetary policy shock increases income and consumption inequality and provides a larger welfare gain to households in the right tail of the wealth distribution.

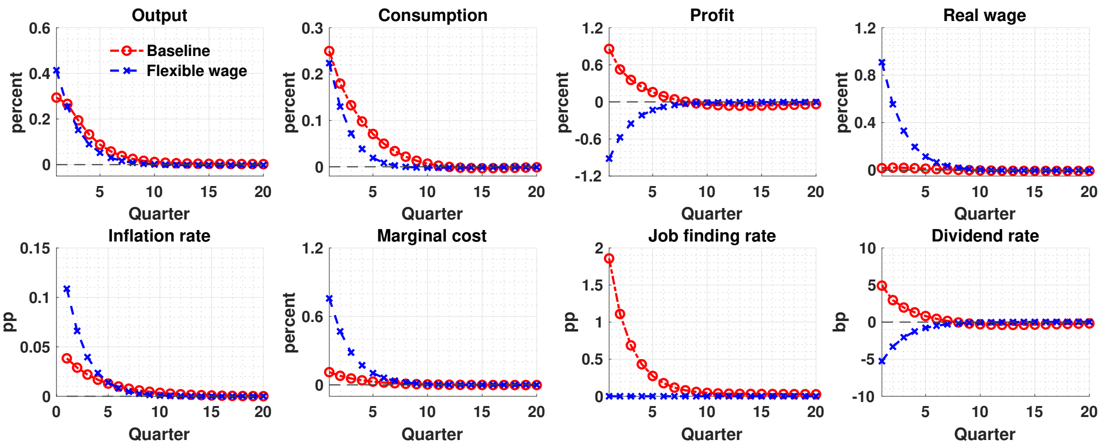
1.5 A model with flexible wages

Wage rigidity is one of the key features in this model that contributes to the pro-cyclical response of profits to monetary policy shocks. Then, a natural question that arises is how the results would change if I instead assumed flexible wage setting, as in standard New Keynesian models. In this section, I solve the model with flexible wages and compare the results with those presented in the previous section. For comparison, I maintain the structure of the previous model, but set ρ_w to zero so that the real wage responds one-to-one with the labor service rental rate, which in turn closely tracks the marginal productivity of labor. Moreover, I do not allow changes in inventory stock or the number of vacancies in the alternative model.[48] Not surprisingly, profits fall and the real wage increases substantially after an expansionary monetary policy shock in the alternative model, as in many standard New Keynesian models.[49]

[48] As sales change while the inventory stock remains fixed, the sales-to-stock ratio varies. To eliminate any losses associated with changes in the sales-to-stock ratio, I additionally assume that the sales-to-stock ratio adjustment cost is zero in the alternative model.

[49] Having other features such as inventory stock and labor market frictions help dampen the magnitude of countercyclical profit response, but those features cannot change the sign of the profit response.

Figure 1.10: IRFs to an expansionary MP shock: rigid vs flexible wage



Notes: Impulse responses after 25 bp (annualized) expansionary monetary policy shock. For the flexible wage case, ρ_w and ϵ_w are set to 0 and 1, respectively. Output, consumption, profit, marginal cost and wage are shown as the percent deviations from the steady state. Real interest rate and job finding rate are expressed in terms of the percentage point deviations from the steady state. Dividend rate is shown as the basis point deviations. Real interest rate and dividend rate are annualized rate.

1.5.1 Impulse responses and transmission mechanisms

Figure 1.10 compares the impulse responses of the two models. The key differences are in the responses of profits and the real wage. As shown in the figure, the real wage increases substantially when wage-setting is flexible. Since firms are not allowed to adjust their inventory stock in the flexible wage model, they must expand their production more in response to an increase in aggregate demand. This increase in production drives up the real wage by raising the demand for labor. Even though extensive margin adjustment of labor supply is not allowed, employed households supply more labor as the real wage rises. Thus, output rises more on impact in the flexible wage model, by 0.5% compared to 0.3% in the baseline model. However, as the marginal cost of production increases substantially due to the higher real wage, profits fall despite the increase in sales. Profits fall by 1% in the flexible wage

Figure 1.11: Consumption decomposition in flexible wage case

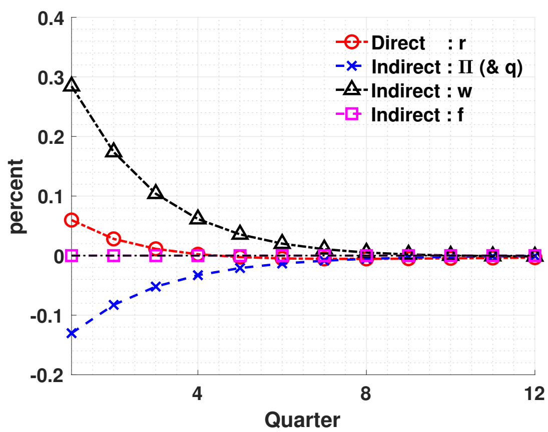


Table 1.11: Average responses

Wage setting	Rigid	Flexible
r change	-0.21	-0.30
Elasticities ($\epsilon_X = \frac{\Delta X}{X} / \Delta r$)		
Output (Y)	-4.21	-3.03
Investment (I)	-8.94	-6.56
Consumption (C)	-3.19	-1.55
Bond (B)	1.89	2.12
Capital (K)	-0.24	-0.07
Decomposition of C (%)		
Direct effect (r)	**24**	**24**
Indirect effect	**76**	**76**
Profit and equity price (Π & q)	48	-68
Wage and profit (w)	4	144
Job finding rate (f)	24	0

Notes: The figure shows the percentage deviations of consumption from its steady state level. Elasticities are shown as semi-elasticities. The decomposition into the direct and indirect effects is approximately exhaustive. Both effects add up to 100%. The fiscal authority is assumed to adjust lump-sum transfer for given bond-issuance rule in response to monetary policy shocks. Profit is the profit given to entrepreneurs as compensations. The profits given to the equity holders are reflected in the equity return.

model, while they increase by 1% in the baseline model. Consequently, the dividend rate falls by 6 basis points per annum on impact in the flexible wage model, while increasing by roughly the amount in the baseline model.

Because of the differences in the response of aggregate variables, the transmission mechanisms of monetary policy shocks are different in the two models. In both models, the indirect effects are larger than the direct effect. However, unlike in the baseline model, the largest consumption responses come from the increase in the real wage. On impact, aggregate consumption increases by about 0.3% in the flexible wage model. Over the first year, changes in the real wage account for more than 100% of the overall response. In contrast, the aggregate consumption response from changes in profits is negative in the flexible wage model, since profits fall substantially after the shock. The significant fall in the dividend rate induced by the decrease in profits causes aggregate consumption to fall by more than 0.1%

Figure 1.12: Decomposition across wealth groups

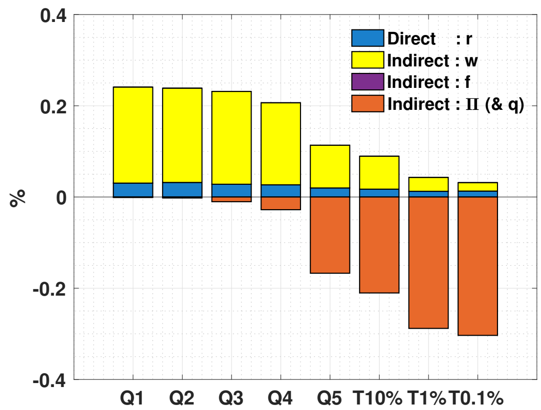


Table 1.12: Welfare gains

Wage setting	Rigid	Flexible
Q1	1.01	3.53
Q2	0.65	3.11
Q3	0.43	2.31
Q4	0.37	0.87
Q5	1.24	-1.68
Top 10 %	1.64	-2.85
Top 1 %	1.77	-4.70
Top 0.1 %	1.87	-4.63
Employed	0.61	1.71
Unemployed	2.26	1.33
Entrepreneur	11.9	-30.3
Avg.	0.73	1.61

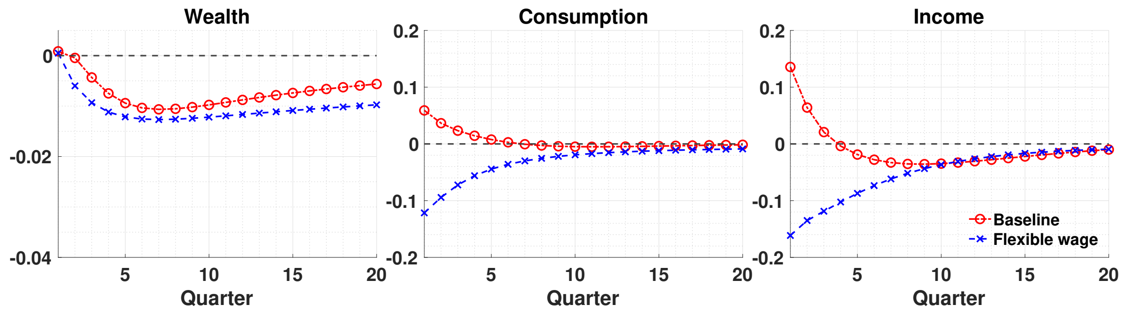
Notes: The figure shows the decomposition of each group's average consumption response over the first year in terms of the percentage deviations from the steady state. The table shows welfare gain of each group from an expansionary monetary policy shock in terms of the consumption equivalent. The unit for consumption equivalent is basis point.

in the flexible wage model. Over the first year, around 68% of the consumption increase due to changes in the real wage is offset by the fall in the dividend rate. These results are consistent with Kaplan et al. (2018): the indirect effects are much larger than the direct effect, and most of the indirect effects are due to changes in the real wage.

1.5.2 Distributional consequences of monetary policy

The two models are in stark contrast regarding the distributional consequences of monetary policy shocks. Unlike in the baseline model, there are clear winners and losers from an expansionary monetary policy shock in the flexible wage model. As shown in Figure 1.12, the bottom 80% of households increase their consumption while the top 20% experience a fall in consumption in the first year after the shock. The decomposition shows that most of the increase in consumption for the bottom

Figure 1.13: Responses of Gini indices



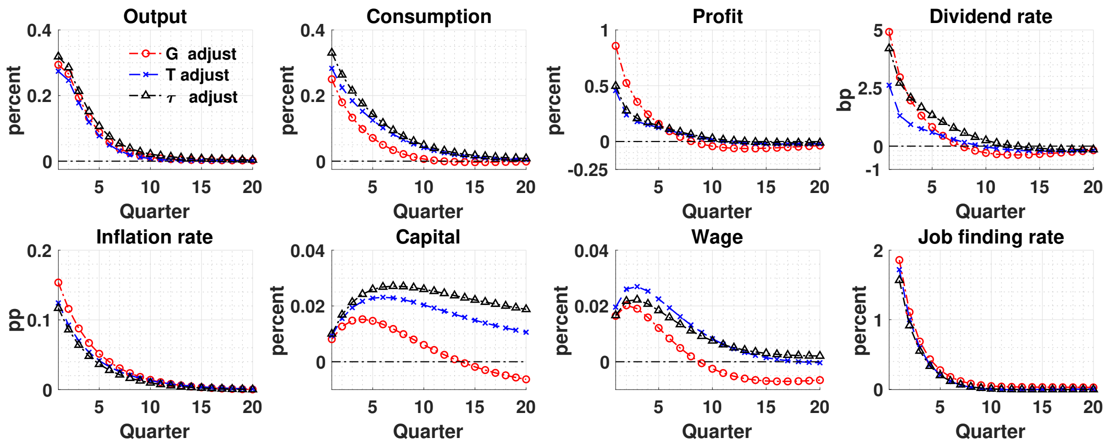
Notes: Impulse responses of Gini indices are shown as the absolute differences in indices. Wealth consists of bond and equity holdings. Income includes all types of income, including labor earning, asset income, unemployment benefit and government transfer.

80% is due to an increase in the real wage, while the fall in profits explains the fall in consumption for the top 20%. Contrary to the baseline model, the consumption equivalent welfare gain is decreasing in the amount of wealth in the flexible wage model. Indeed, the top 20% wealth group experiences welfare losses. Households in the first quintile would need to receive 3.53 basis points of their steady state consumption to forgo an expansionary monetary policy shock. In sharp contrast, top 0.1% households are willing to give up 4.63 basis points of their steady state consumption to avoid a decrease in the policy rate. Figuratively, an expansionary monetary policy shock is doves for the poor but hawks for the rich when the real wage is flexible.

The impulse responses of the Gini indices also confirm the strong distributional consequences of monetary policy shocks in the flexible wage model. Figure 1.13 shows the responses of wealth, consumption, and income Gini indices both in the baseline and the flexible wage model. As shown in the figure, an expansionary monetary policy shock reduces wealth, consumption, and income inequality in the flexible wage model, with the largest change for income inequality. This is due to drastic differences in the responses of profits and wages to monetary policy shocks in the flexible wage model. Since the income of poor households rises while wealthy households' income falls, income inequality substantially falls after an expansionary monetary policy shock. Only part of the lower income inequality translates into consumption inequality, since the MPC is less than one. Finally, in both models, wealth inequality falls after an expansionary monetary policy shock. The magnitude of the fall is only slightly larger in the flexible wage model, however, because poor households have a much smaller marginal propensity to save than wealthy households.

The results presented in this section are consistent with existing results showing that an expansionary monetary policy shock reduces inequality. However, in this chapter, I show that this result crucially depends on the responses of profits and the real wage in the model. Based on my results, I argue that expansionary monetary policy shocks lead to a strong reduction in inequality when the real wage is flexible and profits fall after an expansionary monetary policy shock. However, if profits increase substantially, as in the data, an expansionary monetary policy shock can increase inequality. In other words, the distributional consequences of monetary policy shocks depend on model features, and are not a robust feature of New Keynesian models.

Figure 1.14: IRFs with different fiscal policy responses



Notes: Impulses responses to annualized 25 bp expansionary monetary policy shock. G, T, and τ denote government purchase, lump-sum transfer, and tax rates respectively. Inflation rate, unemployment rate, job finding rate, and capital utilization rate are expressed in terms of percentage point deviations from steady state. Nominal and real rate, and dividend rate are shown as basis point deviations from steady state. Inflation rate, nominal and real rate, and dividend rate are annualized. All other variables are shows as percent deviations from steady state.

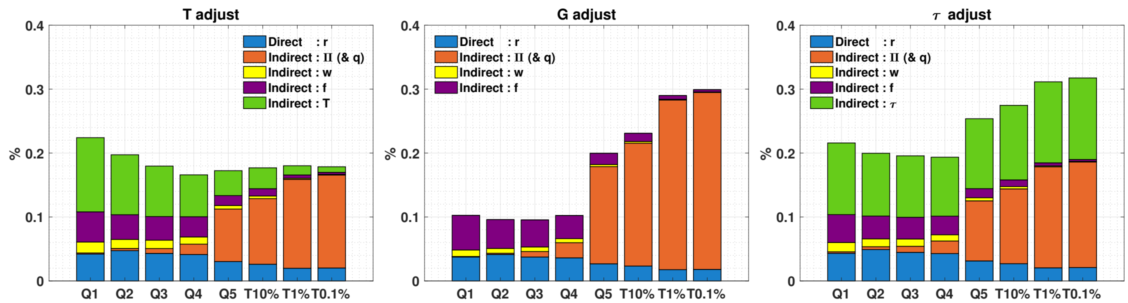
1.6 The role of fiscal policy

So far I have assumed that the fiscal authority increases government purchases when the real value of government debt declines due to expansionary monetary policy shocks. Alternatively, the fiscal authority could increase lump-sum transfers or reduce tax rates to meet the inter-temporal budget constraint. In this section, I examine the implications of alternative assumptions on fiscal policy and compare the results with the baseline model.[50]

Figure 1.14 shows impulse responses of aggregate variables under different assumptions on how fiscal policy is conducted. Overall, the responses are similar across models with different assumptions. If the fiscal authority chooses to adjust government purchases as in the baseline case, this leads to a one-for-one increase in

[50] Regarding tax rate adjustment, I adjust both income and profit tax rates by the same amount.

Figure 1.15: Consumption decomposition over wealth groups



Notes: The figure shows the decomposition of each group's average consumption response over the first year in terms of the percentage deviations from the steady state under different assumptions on fiscal responses.

the aggregate demand for goods. Thus, the initial increase in the aggregate demand is largest when the fiscal authority adjusts government purchases in response to expansionary monetary policy shocks. Consequently, the increase in profits is largest in the baseline model. However, since higher demand leads to a higher inflation rate, the increase in the real wage is smallest in the baseline model, as the wage is subject to nominal rigidity. Therefore, the consumption response is smallest in the baseline model as well.[51] In addition, the increase in public consumption crowds out private investment and results in a faster depreciation of the aggregate capital stock. Overall, tax cuts lead to the largest responses of output, consumption, and capital accumulation, though the differences are not so substantial.

Turning to transmission mechanisms, Table 1.13 shows that the relative importance of direct and indirect effects is similar across different fiscal responses;

[51] This result is in sharp contrast to the result in Kaplan et al. (2018), where higher government purchases lead to larger responses not only of output but also of consumption. This is due to their assumption of wage flexibility. When the wage is flexible, the benefit of higher output is distributed to the households in the form of an increase in the wage. Thus, the expansion triggered by government purchases also leads to a larger response in consumption and investment. However, if the wage is assumed to be rigid, the benefit of higher output mostly accrues to equity holders and entrepreneurs. Since they are households with a low MPC, further stimulus in aggregate consumption is smaller. Consequently, aggregate consumption and investment increase by less when government purchases are adjusted.

Table 1.13: Transmission mechanisms under different fiscal policies

	T	G	τ
r change (pp annual)	-0.20	-0.21	-0.19
Elasticities ϵ_X			
Output (Y)	-4.18	-4.29	-5.03
Investment (I)	-11.48	-8.94	-12.98
Consumption (C)	-4.31	-3.19	-5.10
Bond B	1.85	1.89	1.84
Equity A	-0.34	-0.24	-0.38
Decomposition of C			
Direct effect (r)	21	24	18
Substitution effect	24	28	21
Income effect	-3	-4	-3
Indirect effect	79	76	82
Profits and equity price (Π & q)	21	48	20
Wage (w)	6	4	4
Job finding rate (f)	16	24	12
Lump-sum transfer (T) of tax rates (τ)	36	0	46

the direct effect accounts for around 20% of the consumption response in the first year while the remaining 80% is accounted for by indirect effects. Depending on the fiscal policy, the decomposition of the indirect effects is quantitatively slightly different across models. The effect of the increase in profits is largest in the baseline model. When the fiscal authority boosts aggregate demand indirectly by increasing lump-sum transfers or reducing tax rates, the response of profits is smaller than in the baseline model, and therefore, the contribution of profits to the increase in aggregate consumption is smaller. In the baseline model, about 50% of the consumption response is due to profits, while this contribution is about 20% in the alternative models. Similarly, the effect of changes in the job-finding rate is smaller when the fiscal authority adjusts lump-sum transfers or tax rates instead of government purchases. Reducing tax rates leads to a larger response in consumption than increasing transfers, because lower taxes increase household labor supply and the

Figure 1.16: Responses of Gini indices

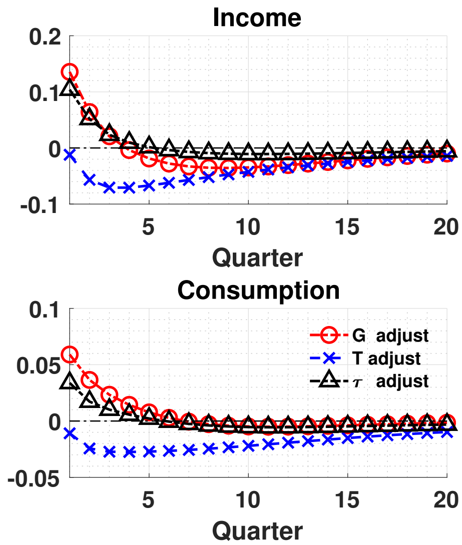


Table 1.14: Welfare gains

	T	G	τ
Q1	5.30	1.01	3.03
Q2	4.07	0.65	2.62
Q3	3.25	0.43	2.23
Q4	2.29	0.37	1.86
Q5	1.52	1.24	2.22
Top 10 %	1.21	1.64	2.39
Top 1 %	0.28	1.77	2.12
Top 0.1 %	0.25	1.87	2.18
Employed	3.16	0.61	2.28
Unemployed	4.77	2.26	3.59
Entrepreneur	2.30	11.9	9.77
Avg.	3.25	0.73	2.38

Notes: G, T, and τ denote government purchase, lump-sum transfer and tax rates respectively. The left figure shows impulse responses of Gini indices for income and consumption as the absolute difference from the steady state level. The right table shows welfare gain of each group from an expansionary monetary policy shock in terms of the consumption equivalent. The unit for consumption equivalent is basis point.

dividend rate.[52]

The differences in households' consumption responses across wealth groups are more evident. As shown in Figure 1.15, the consumption response of bottom 80% households is much larger when the fiscal authority adjusts lump-sum transfers or tax rates, while the consumption response from wealthy households is much smaller when the government adjusts transfers than in the baseline model. These results show that the short-term distributional consequences from a monetary expansion are sensitive to the fiscal responses.

The responses of Gini indices, as well as consumption equivalents, confirm that the distributional consequences of monetary policy shocks depend on fiscal

[52]Note that, in the model, the dividend rate is proportional to after-tax profits.

policy. An expansionary monetary policy shock has linear welfare effects across the wealth distribution only when the fiscal authority responds to the shock by adjusting lump-sum transfers. Both income and consumption Gini indices decrease under the lump-sum transfer adjustment. By contrast, in the other two cases, both income and consumption inequality increase. Likewise, welfare gains measured as consumption equivalents are monotonically decreasing in wealth only when lump-sum transfers are adjusted. If the government adjusts government purchases or tax rates instead, welfare gains tend to be higher for wealthier households than for the middle class. If the government adjusts its purchases in response to an expansionary monetary policy shock, the welfare gains are largest for the top 0.1% households.

The results in this section show that, in a HANK model with realistic features, the aggregate and distributional effects of monetary policy shocks depend on fiscal policy responses.

1.7 Conclusion

In this chapter, I build a heterogeneous agents New Keynesian model with two assets, wage rigidity, job market frictions, and inventory holding to study the effect of monetary policy on consumption and inequality. The most important innovation of this chapter is that the model generates pro-cyclical responses of profits to monetary policy shocks, unlike many standard New Keynesian models. Given that wages are the biggest component of income for most households, while wealthy households are the major claimants to profits in the economy, the model developed in this chapter provides a proper setting for studying the transmission mechanisms and the distributional consequences of conventional monetary policy shocks.

Regarding the transmission mechanism of monetary policy shocks, indirect effects through changes in variables other than the real interest rate are stronger than the direct effect of the real interest rate through intertemporal substitution as in Kaplan et al. (2018). However, the decomposition of the indirect effects provides a more detailed understanding of the mechanisms. First, the real wage has little effect on aggregate consumption, since it responds little to monetary policy shocks, while the job-finding rate has a much stronger impact. More importantly, the increase in profits results in a substantial increase in consumption, particularly of wealthy households.

Consequently, the distributional consequences of monetary policy are much weaker or even work in the opposite direction compared to what is found in existing work. An expansionary monetary policy shock reduces wealth inequality but increases consumption and income inequality in the baseline model. Welfare gains measured as consumption equivalents are also larger for wealthier households. This is because the increase in profits is substantial, while the real wage responds little to an expansionary monetary policy shock. This result is in stark contrast with existing work suggesting that an expansionary monetary policy shock lowers inequality.

Finally, I also show that the distributional consequences of monetary policy shocks crucially depend on fiscal policy. An expansionary monetary policy shock reduces inequality only when the government increases lump-sum transfers in response to a fall in the real value of government debt. In alternative cases where the fiscal authority adjusts government purchases or tax rates, welfare gains are larger for wealthier households than for the middle class. Given that lump-transfers are progressive and tax cuts are regressive, it is not surprising that fiscal responses affect the distributional consequences of monetary policy.

In this chapter, I provide alternative results on the distributional consequences of conventional monetary policy shocks, and emphasize the importance of the interactions between monetary and fiscal policy in shaping not only the aggregate but also the distributional consequences of conventional monetary policy. I leave the implications of these results for the conduct of optimal monetary and/or fiscal policy and the effects of unconventional monetary policy to future study.

Chapter 2: Quantitative Easing and Inequality

2.1 Introduction

In recent decades, income and wealth inequality have been increasing in the United States, motivating the use of heterogeneous-agent macroeconomic models, in which the propagation of aggregate fluctuations and the effectiveness of policy interventions are evaluated within frameworks that capture the large degree of household inequality present in the data. A particularly lively debate has centered on the distributional consequences of monetary policy.[1] While much of the literature focuses on conventional monetary policy, much less has been established about how quantitative easing (QE) affects welfare across the wealth distribution. Though effective at stimulating aggregate economic activity, the quantitative easing policies launched by the Federal Reserve in the aftermath of the Great Recession have often been criticized for exacerbating already wide disparities in income and wealth among U.S. households.[2] At the same time, the persistent decline in the natural interest rate has increased the likelihood that the economy will often find itself at the effective lower bound (ELB) on the policy rate, and that monetary authorities will need to turn to

[1] For trends in inequality, see Heathcote et al. (2010), Saez and Zucman (2016), and Gould (2019). For the discussion on inequality and monetary policy, see Yellen (2014), Bernanke (2015) and Draghi (2016).

[2] See, for instance, Schwartz (2013) and Cohan (2014).

QE as a stabilization tool.[3] Yet, whether or how QE raises inequality remains an open question and a topic of heated debate.

Gauging QE's distributional effects is a challenging task, as various forces compete in determining the net effects of QE on inequality. First, QE can exacerbate income and wealth inequality by raising profits and asset prices. Since stocks and equity, i.e., claims for profits, are mainly held by the top of the wealth distribution, QE might disproportionately benefit that part of the distribution. Conversely, QE can reduce inequality by lowering the unemployment rate, which mainly benefits the bottom of the wealth and income distributions, or by stimulating wage growth, boosting income shares in the middle of the distribution.[4] Finally, higher inflation induced by QE re-distributes wealth from savers to debtors by lowering real rates. A proper evaluation of the net effect of QE on inequality needs to take into account these channels comprehensively.[5]

This paper provides a structural evaluation of the aggregate and distributional consequences of the Federal Reserve's QE program using a medium-scale HANK model that can capture and quantify the dynamics of the channels mentioned above. Two key requirements are necessary for a model to fulfill this task. First, a model should match households' wealth and income composition in the data. Second, it should generate empirically plausible responses of variables that affect households'

[3] See, among others, Laubach and Williams (2016) and Holston et al. (2017).

[4] Heathcote et al. (2010) show that earnings at the bottom of the income distribution are mainly affected by changes in the unemployment rate and hours worked while earnings at the top of the income distribution are mainly affected by changes in hourly wage. Thus, as long as QE has stronger effects on unemployment rates than on real wages, it can reduce income inequality.

[5] For a more detailed discussion on the relevant channels, see Coibion et al. (2017) and Amaral (2017).

wealth and income, such as profits, asset prices, wages, and the unemployment rate. The interaction of these two factors, i.e., wealth/income components and their responses to QE, will determine the winners and losers from the QE policy.

To meet the first requirement, the model features portfolio choice and endogenous unemployment. Households can hold two types of assets (deposits/equity), and their working status (employed/unemployed) varies endogenously over time. As in Bayer et al. (2019), I introduce an additional working status under which households receive a fraction of profits as income without supplying labor. Because of these features, households in the model have a heterogeneous composition of wealth (deposits/equity) and income (labor, assets, and business income) and heterogeneous exposures to unemployment risk. In steady state, the top 10% wealthy households hold about 70% of total wealth, mostly in the form of equity, and the sum of business and asset income accounts for about 50% of their total income, consistent with U.S. data. In contrast, households in the lower 80% of the wealth distribution rely mostly on labor income, and a larger share of the households at the bottom of the distribution are unemployed, and thus, more vulnerable to unemployment risk.[6]

Regarding the second requirement, I first address a well-known problem of New Keynesian models, namely the counter-cyclical response of profits to monetary policy shocks, which is inconsistent with the empirical evidence.[7] Fixing this problem is

[6]According to the 2007 SCF data, labor income, i.e., wage and salary, accounts for about 80% of the total income of the bottom 80% of the wealth distribution, while most of the remaining income consists of transfer income. In stark contrast, for the top 0.1% wealthy households, the share of labor income is only 16%, and transfer income accounts for less than 1% of their total income. The remaining 85% consists mostly of income sources related to profits, i.e., business income and dividends. For the top 10% households, the ratio is about 50%.

[7]In the appendix, I provide empirical evidence on the responses of profits, wages, and unempoyment rates to monetary policy shocks, using a structural VAR model. See also Christiano et al.

crucial for studying QE's distributional effects, given the importance of profits for wealthy households. For this purpose, I assume a substantial share of fixed cost in production, inspired by Anderson et al. (2018), in addition to wage rigidity and an extensive margin of labor supply.[8] These features help the model generate a procyclical profit response to monetary policy shocks.

I estimate the model using Bayesian methods to estimate the shock processes that pushed the economy to the ELB and discipline the parameter values that determine the model's dynamic responses to monetary policy. Importantly, I explicitly take into account the binding ELB constraint and Fed's QE operations between 2009 and 2015. Because of highly volatile profits and relatively stable and sluggish wage dynamics in the data, the estimated parameter values suggest a high degree of wage rigidity and a relatively flexible extensive margin adjustment of labor supply. Thus, the estimated model generates a strongly procyclical response of profits and employment to conventional monetary policy shocks while the real wage hardly changes, consistent with the existing empirical evidence.

In the model, QE takes the form of the central bank's direct private asset purchase as in Gertler and Karadi (2011). By transforming the demand for a non-productive asset (government bonds or deposits) into a productive asset (capital), QE increases aggregate demand and undoes the contractionary effects of the binding ELB constraint, which are equivalent to those of a series of contractionary interest

(2005), Coibion et al. (2017), and Lenza and Slacalek (2018) for further evidence.

[8]Anderson et al. (2018) shows, using microdata on the retail sector, that net operating profit margins are strongly procyclical while gross margins, which are proportional to the inverse of the real marginal cost, are mildly procyclical or acyclical over the business cycle. They interpret that their results suggest the presence of sizeable fixed costs.

rate shocks. The model suggests that, between 2009 and 2015, QE on average generated 3.3% and 0.9% higher profits and equity prices, a 1.5% lower unemployment rate, but only 0.1% higher real wages, compared to the counterfactual of no QE intervention.

Together with heterogeneity in households' wealth/income composition and exposure to unemployment risk, these aggregate effects generated non-linear distributional effects. The top decile's income and consumption shares increased by 0.17 and 0.06 percentage points during the ELB episode, mainly because of higher profits and equity prices. However, at the same time, QE reduced the wealth and income Gini indices by 0.05 and 0.04 percentage points on average by lowering the unemployment rate. As to welfare gains, stimulative effects of QE improved welfare for all households, and the average welfare gain was equivalent to 0.27 percent of lifetime consumption. However, welfare gains were U-shaped. QE benefited households at both ends of the wealth distribution more than the middle class. The bottom and the top decile (1%) enjoyed gains of about 0.3% (0.33%). Conversely, the welfare gain of the middle 60% was about 0.26% in terms of consumption equivalents.

Interestingly, the simulation results suggest that QE had *less* adverse effects on inequality than conventional monetary policy. If the Federal Reserve had been able to conduct conventional monetary policy according to an estimated Taylor rule, only the bottom 1% and the top 10% would have enjoyed higher welfare gains. The difference between QE and conventional monetary policy reflects the dynamics of real rates. Lower real rates disproportionately benefit households at the bottom of the wealth distribution, who are mostly debtors, while hurting the remaining savers.

However, the financial sector also substantially benefits from the lower financing costs.[9] The benefits of the financial sector are transferred mostly to the top of the wealth distribution as a part of aggregate profits. Hence, households in the top decile of the wealth distribution end up experiencing higher welfare gains under conventional monetary policy despite the direct welfare loss from lower real rates.

Finally, I find that about half of QE's aggregate effects were due to the expansionary forward guidance and the prolonged periods of zero policy rates. In the model, forward guidance takes the form of exogenously imposing an expected ELB duration, i.e., the number of periods during which the central bank commits to maintaining the policy rate at zero or the effective lower bound, as in Jones (2017). If the expected ELB duration is longer than the duration determined by the policy rule and the expected evolution of the economy, the effects of forward guidance are equivalent to those of anticipated future expansionary interest rate shocks. I estimate an expected ELB duration that is longer than the expected duration implied by fundamentals alone for the entire period between 2009 and 2015. Moreover, I estimate additional stimulus, in the form of rates that were lower for longer: the Fed could have set a positive rate as early as 2014 Q3, but maintained the rate at zero until the end of 2015. Consequently, the economy experienced additional stimulus that accounts for about 55% of the total stimulus effects of unconventional monetary policies. However, forward guidance and the extended periods of zero policy rates also amplified the adverse effects of QE on inequality, further increasing the

[9] QE boosts banks' net worth by increasing equity prices. However, simultaneously, it decreases the expected gross rate of return on their assets, i.e., profitability. In contrast, conventional monetary policy does not directly increase banks' net worth, but in the absence of general equilibrium feedbacks, it increases the profitability of banks by lowering their financing costs.

top 10%'s income share by 0.09 percentage points on average.

Related Literature

(Empirical literature)

My work contributes to a growing literature studying the distributional consequences of conventional and unconventional monetary policy measures. Regarding conventional monetary policy, Coibion et al. (2017) show, using the Consumer Expenditure Survey (CEX), that a conventional monetary tightening raises the measures of income and consumption inequality, including the Gini index and the inter-decile ratio. Mumtaz and Theophilopoulou (2017) and Furceri et al. (2018) find similar results for the U.K. and for a panel of advanced and emerging countries, respectively. In contrast, Inui et al. (2017) show, using the microdata on households' income in Japan, that an expansionary interest rate shock does not have a significant impact on income inequality. Using structural VAR models, Davtyan (2017) and Hafemann et al. (2018) find that a contractionary (expansionary) interest rate shock reduces (increases) income inequality, as measured by the Gini index. My results underscore the sensitivity of conclusions to the inclusion of the wealthiest households: to the extent that the CEX under-represents this group, it reflects what the model predicts for the lower 90% of the wealth distribution, for whom it is indeed the case that tightening raises inequality.

In the case of unconventional monetary policies, the empirical literature is relatively limited, and the results are even more mixed. In the U.S., Bivens (2015) argues that QE significantly reduced income inequality by helping the economy achieve full employment, but had negligible effects on wealth inequality. Conversely, Montecino and Epstein (2015) examine the contribution of QE to the observed changes in households' income in 2010 and 2013 SCF data, and they conclude that QE increased income inequality mainly via its strongly positive impact on equity returns.

There are also cross-region differences in the results. Saiki and Frost (2014) show that an increase in assets held by the Bank of Japan (BOJ) raised income inequality in Japan, as measured by the top to the bottom quintile income ratio. Similarly, Taghizadeh-Hesary et al. (2020) argue that BOJ's unconventional monetary policies, including zero and negative interest rates, raised the top 10% to the bottom 10% income ratio. The existing work on European countries suggests that QE reduces income inequality mainly via its positive impact on labor markets. Casiraghi et al. (2018) estimate the aggregate effects of QE using the semi-structural model of the Italian economy and apply them to households' distribution in the data. They conclude that QE mostly benefits households at the bottom of the wealth and income distribution because their earnings respond more strongly and positively to QE operations. Using a similar methodology, Lenza and Slacalek (2018) also find that the ECB's QE programs reduce the income Gini index in European countries mainly by lowering unemployment rates, while leaving the wealth Gini largely unchanged. In contrast, Bank of England (2012) and Domanski et al. (2016) find that QE increases wealth inequality mainly by increasing equity prices.

Competing results in the empirical literature show the importance of a comprehensive examination of competing channels as well as the consideration of country-specific characteristics, such as the preexisting degree of inequality, the flexibility of labor markets, and the structure of equity markets and the financial sector. I therefore adopt a dynamic structural general equilibrium (DSGE) analysis approach to examine the impact of QE on inequality. In terms of the methodology, my work is closely related to Casiraghi et al. (2018) and Lenza and Slacalek (2018) since I estimate the aggregate effects of QE and apply them to the household distribution to evaluate the contribution of each channel as well as the overall distributional consequences. However, while they estimate the aggregate and distributional consequences of QE separately with an auxiliary assumption on how the aggregate effects are distributed across the distribution of households, I evaluate both aggregate and distributional effects at the same time with a unified framework. Moreover, I compare the effects of QE with those of conventional monetary policy and the other type of unconventional monetary policy, i.e., forward guidance.

(Theoretical literature)

Regarding my theoretical contribution, this paper lies at the intersection of three key literatures: 1) the literature studying macroeconomic fluctuations using a HANK framework, 2) the literature studying unconventional monetary policies, and 3) the literature on Bayesian estimation of DSGE models.

In the HANK literature, my model builds on existing work that has focused on the transmission mechanisms or distributional consequences of conventional monetary policy shocks. The seminal paper of Kaplan et al. (2018) shows that the presence of wealthy hand-to-mouth households drastically changes transmission mechanisms of monetary policy shocks in a HANK model compared to those in a representative agent New Keynesian (RANK) model. Similarly, Luetticke (2020) and Auclert et al. (2020b) examine the transmission mechanisms of conventional monetary policy shocks and emphasize the importance of investment in the transmission of these shocks. Unlike these papers, Gornemann et al. (2016) examine the distributional consequences of interest rate shocks and find that a contractionary monetary policy shock increases inequality by reducing employment while increasing the return on assets.

My work is also closely related to the literature studying unconventional monetary policies. The model in this paper draws on Gertler and Karadi (2011), who propose a framework for the analysis of a central bank's large scale asset purchase program, which features frictional financial intermediation to break the irrelevance result of Wallace (1981). Also, as in Del Negro et al. (2017) and Chen et al. (2012), I evaluate the effects of the Federal Reserve's large-scale asset purchase programs during the ELB episode, but the focus is on distributional consequences rather than the programs' aggregate effects. Jones (2017) studies the effects of forward guidance in the form of exogenous ELB durations, and I adopt the same approach in simulating the ELB episode.

In addition, Hohberger et al. (2020) study the distributional consequences of unconventional monetary policy, using a small open economy two agents (saver-spender) New Keynesian model (TANK). They conclude that an expansionary QE shock reduces income inequality, measured by the income shares of the two agents, in the medium-run. Cui and Sterk (2018) also examine the effects of QE but focus on the efficacy and the stability of the policy as a regular monetary policy tool rather than on its distributional effects.

Finally, my work also contributes to the literature on estimating HANK models. Bayer et al. (2020) extend the work of Smets and Wouters (2007) and Justiniano et al. (2011) and estimate a HANK model using Bayesian techniques to study the drivers of inequality in the U.S. during the post-war period. Auclert et al. (2020b) also estimate their HANK model to discipline the parameter governing the degree of sticky expectation in their model using the aggregate data of the U.S. economy. I contribute to this literature by estimating a HANK model with an occasionally binding constraint and unconventional monetary policies. To the best of my knowledge, this paper is the first to estimate a HANK model with an occasionally binding ELB constraint.

The remainder of the paper proceeds as follows. Section 2 describes the model. Section 3 explains the parametrization and estimation strategy and presents the estimation results. Section 4 conducts counterfactuals to examine the aggregate and distributional effects of QE during the ELB episode. Section 5 compares the effects of QE with the effect of conventional monetary policy. Section 6 examines the aggregate and distributional effects of forward guidance during the ELB episode. Section 7 checks the robustness of the results. Section 8 concludes.

2.2 Model

The model introduces financial intermediaries, the ELB, and QE in the form of central bank asset purchases, into a medium-scale DSGE model with heterogeneous households, uninsurable income risk, aggregate uncertainty, and a two-asset structure. The household block mostly follows the HANK models of Kaplan et al. (2018) and Bayer et al. (2019), while the modeling of financial intermediaries and QE draws on the work of Gertler and Karadi (2011). On the supply side, I incorporate frictional labor markets, a fixed cost of production, and wage and price rigidities. These features help the model generate empirically plausible dynamics for key aggregate variables such as wages, unemployment, equity prices, and profits. They also shape the relative contributions of different transmission channels of unconventional monetary policy and determine its distributional consequences.

2.2.1 Household

There is a unit mass of households, who are *ex-ante* identical but *ex-post* heterogeneous due to the evolution of their idiosyncratic productivity s, holdings of illiquid and liquid assets, a and b, and employment status e. In each period, households are employed, unemployed, or business owners. There are exogenous and endogenous transitions between working status, as I will explain below.

Households derive utility from consumption and disutility from labor, and die with exogenous probability ζ each period.[10] Conditional on surviving, they discount

[10] As in Kaplan et al. (2018), I assume that upon death, a household is replaced by a new

the future at rate $\beta \in (0,1)$, solving

$$\max_{\{a_{it+1}, b_{it+1}, c_{it}, n_{it}\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t (1-\zeta)^t \left\{ u(c_{it}, n_{it} | s_{it}, e_{it}) - \chi_{it} \mathbb{1}_{\{a_{it+1} \neq a_{it}\}} \right\} \right] \quad , \tag{2.1}$$

where c_{it} is consumption, n_{it} is labor supply, a_{it+1} is illiquid asset holding, and b_{it+1} is liquid asset holding; s_{it} and e_{it} are idiosyncratic productivity and employment status, respectively; and χ_{it} is the stochastic disutility incurred when the household adjusts its portfolio of illiquid holdings.[11] $\mathbb{1}_{\{a_{it+1} \neq a_{it}\}}$ is an indicator function equal to 1 in periods in which the household changes its holdings a_{it} of illiquid assets. The period utility function has the specification of Greenwood et al. (1988),

$$u(c_{it}, n_{it}) = \frac{\left[c_{it} - \psi s_{it} \frac{n_{it}^{1+\xi}}{1+\xi} \right]^{1-\sigma}}{1-\sigma} \quad , \tag{2.2}$$

where σ is the inverse of the elasticity of inter-temporal substitution (IES), ξ is the inverse of the Frisch elasticity of labor supply, and ψ is a parameter that scales the steady state hours worked to 1. As in Bayer et al. (2019), I assume that the disutility from supplying labor is proportional to the productivity level. Under this preference specification, all employed households choose the same amount of hours worked, as a function solely of the real wage.[12] This feature significantly facilitates

household with zero wealth and the wealth of the deceased is redistributed to surviving households in proportion to their asset holdings. Stochastic death helps the model generate a substantial mass of households with zero assets at the steady state.

[11] Without adjustment costs, both assets become perfect substitutes.

[12] Auclert et al. (2020a) point out that the consumption-labor complementarity embedded in GHH preferences leads to unreasonably high fiscal and monetary policy multipliers in a model with frictionless labor markets and flexible wages. In contrast, Broer et al. (2019) show that, under the separable preferences, a countercyclical response of profits to accommodative monetary policy can lead to an undesirable amplification because of its negative income effect on labor supply. Since

the computation.

Households optimally choose consumption, hours worked, and portfolio composition subject to the following budget constraint and borrowing limits:

$$c_{it} + q_t a_{it+1} + b_{it+1} = (1-\tau) y_{it} + (q_t + r_t^a) a_{it} + (1 + r_t^b) b_{it} + T_t \quad , \tag{2.3}$$

$$a_{it+1} \geq 0 \ , \quad b_{it+1} \geq \underline{\mathrm{b}} \quad , \tag{2.4}$$

where q_t is the price of illiquid assets, r_t^a is its dividend rate, and r_t^b is the net real rate of return on liquid assets. T_t is the lump-sum transfer from the government and the money market mutual fund. The tax rate on households' income is denoted by τ.[13] The period income y_{it} depends on the household's working status,

$$y_{it} = \begin{cases} w_t s_{it} n_{it} & \text{for employed} \qquad (e_{it} = 1) \\ wv \min\{s_{it}, s\} & \text{for unemployed} \qquad (e_{it} = 2) \\ \nu \Pi_t & \text{for business owners} \qquad (e_{it} = 3) \end{cases} \tag{2.5}$$

where w_t is the real wage per efficiency unit, and w its steady state value. Employed households earn wage income that is proportional to their productivity. Idiosyncratic productivity evolves according to a first-order Markov process. If unemployed, households receive unemployment benefits equal to a fraction of their steady state labor income, based on their current productivity but capped by the average

my model features frictional labor markets and sticky wages and generates procyclical responses of profits to demand shocks, it is free from these problems.

[13] In the model, taxes are levied on households' income and firms' profits. For simplicity, I assume that tax rates are the same for both types of tax bases. Thus, I use the same notation for both types of taxes.

productivity s, with the replacement ratio υ. If households become business owners, they receive a fraction of profits as income.

Working status evolves as follows: At the beginning of the period, an employed household becomes unemployed with an exogenous separation rate λ, while business owners lose their ownership state with an exogenous probability $\tilde{P}^e$ and also become unemployed. The newly unemployed households search for jobs along with previously unemployed households. The job finding rate f is determined endogenously, based on the aggregate state of the economy. At the end of the period, a fraction P_e of non-business owners become business owners.[14]

Households transfer wealth inter-temporally via two assets. Liquid assets b are subject to an exogenous borrowing limit $\underline{b}$, and pay a real rate that depends on whether the household borrows or saves:

$$r_t^b = \begin{cases} \frac{1+i_t}{\pi_t} & \text{if } b_{it} \geq 0 \\ \frac{1+i_t+i}{\pi_t} & \text{if } b_{it} < 0 \end{cases}, \tag{2.6}$$

where i_t is the nominal interest rate, π_t is the gross inflation rate, and i is the nominal borrowing premium. Illiquid assets a_t, earn return r_t^a.[15] Adjusting illiquid asset

[14]The introduction of business owners helps the model match the overall wealth inequality in the data as they are the highest income groups in the model.

[15]In Kaplan et al. (2018), households do not earn any income from their illiquid assets if they do not make withdrawals. Instead, the dividends are automatically re-invested in the illiquid asset. This difference in the treatment of illiquid asset returns is mainly due to the difference in our perspective on illiquid asset. Kaplan et al. (2018) view housing as the major type of illiquid assets. Since most housing is for residential purposes, it is natural to assume that households receive no pecuniary income from their own house. However, in this paper, I focus on claims on profits, such as stocks and proprietorship of a business, as the main type of illiquid asset. In the case of housing, I mainly focus on property that yields rental income, which is disproportionately held by wealthier households in the data. Therefore, it is natural to assume that households enjoy a liquid stream of dividends by holding illiquid assets in my model.

holding incurs a utility cost, which, for tractability, I assume is stochastic.[16] Following Bayer et al. (2019), the independently and identically distributed adjustment costs are drawn from the logistic distribution, with cumulative probability

$$F(\chi_t) = \frac{1}{1 + \exp\left\{ - \frac{\chi_t - \mu_\chi}{\sigma_\chi} \right\}} \quad , \tag{2.7}$$

where μ_x and σ_x are the location and the scale parameter of the logistic distribution, respectively.

2.2.2 Final good firm

The final good is a standard CES aggregator,

$$Y_t = \left[\int Y_{jt}^{\frac{\eta_t - 1}{\eta_t}} dj \right]^{\frac{\eta_t}{\eta_t - 1}} \quad , \tag{2.8}$$

where Y_{jt} is firm j's intermediate good, and η_t is the time-varying elasticity of substitution. Profit maximization yields individual demand and the associated aggregate price index,

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\eta_t} Y_t \tag{2.9}$$

$$P_t = \int P_{jt}^{1-\eta_t} dj \tag{2.10}$$

where P_{jt} is good j's price.

[16] A stochastic adjustment cost preserves the global concavity of the household's value function, facilitating the computation of households' optimal policies.

2.2.3 Intermediate goods firms

There is a continuum of intermediate good firms that produce differentiated products using labor and capital rental services in a monopolistically competitive environment. The production technology is characterized by a standard Cobb-Douglas production function,

$$Y_{j,t} = Z_t K_{j,t}^{\theta} L_{j,t}^{1-\theta} \quad , \tag{2.11}$$

where $K_{j,t}$ and $L_{j,t}$ are capital and labor rental services, respectively, Z_t is total factor productivity, and θ is the share of capital in production.

Each firm maximizes the following expected present discounted value of future profits subject to its demand (2.9) and the production function (2.11).

$$\max_{\{P_{jt},L_{jt},K_{jt}\}} \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\Lambda_{0,t} \Pi_t^I \right] \quad , \quad \Pi_{jt}^I = P_{jt} Y_{jt} / P_t - r_t^l L_{jt} - r_t^k K_{jt} - \Phi^P(P_{jt}, P_{jt-1}) - \Psi_t^F Y \quad , \tag{2.12}$$

where r_t^l is the labor rental rate, r_t^k is the capital rental rate, and $\Psi_t^F Y$ is the fixed cost of operation. As I will discuss later, the fixed cost of operation plays an important role in generating procyclical responses of profits to demand shocks in the model. The fixed cost is a random proportion of the steady state output and follows an AR(1) process.

Firms use business owners' average marginal rate of substitution, which is denoted by $\Lambda_{t,t+1}$, to discount future cash flows. Price adjustment costs a lá Rotemberg

(1982) are given by

$$\Phi^P(P_{jt}, P_{jt-1}) = \frac{\eta_t}{2\kappa}\left(\log\frac{P_{jt}}{P_{jt-1}} - \log \pi_{t-1}^{\iota_p}\pi^{1-\iota_p}\right)^2 Y_t \quad , \tag{2.13}$$

where κ is the slope of the Phillips curve and ι_p is the degree of backward looking price-setting behavior in an equivalent Calvo price-setting setup.[17]

The first order conditions for labor and capital rental services are standard and given by

$$r_t^k = \theta \mathrm{MC}_t\left(\frac{Y_{jt}}{K_{jt}}\right) \ , \quad r_t^l = (1-\theta)\mathrm{MC}_t\left(\frac{Y_{jt}}{L_{jt}}\right) \quad , \tag{2.14}$$

where MC_t is the Lagrange multiplier on the production constraint of the cost minimization problem, which represents the real marginal cost,

$$\mathrm{MC}_t = \frac{(r_t^k)^\theta (r_t^l)^{1-\theta}}{Z_t}\left(\frac{1}{\theta}\right)^\theta\left(\frac{1}{1-\theta}\right)^{1-\theta} \ , \tag{2.15}$$

The optimality conditions regarding firms' price setting under the symmetric equilibrium assumption yield the following Phillips curve.

$$\log \pi_t - \log \pi_{t-1}^{\iota_p}\pi^{1-\iota_p} = \mathbb{E}_t\Big[\Lambda_{t,t+1}\big(\log \pi_{t+1} - \log \pi_t^{\iota_p}\pi^{1-\iota_p}\big)\Big] + \kappa\left(\mathrm{MC}_t - \frac{1}{\Psi_t^p}\right) \ , \tag{2.16}$$

where $\Psi_t^p = \frac{\eta_t}{\eta_t - 1}$ is the price mark-up shock.

[17] In the equivalent Calvo pricing model, ι_p denotes the degree of indexation to the previous inflation rate when firms are not allowed to adjust their price.

2.2.4 Labor agencies

Labor agencies work as an intermediary between households and intermediate good firms. They post vacancies to hire households and provide labor services to firms. A household can supply labor only via a labor agency.

A labor agency that is matched to a household i earns the margin between the labor rental rate that the intermediate good firms pay, and the wage paid to the household.

$$(r_t^l - w_t - \Xi^L)s_{it}n_{it} \tag{2.17}$$

where Ξ^L is the cost for maintaining a match.[18]

For the determination of the real wage w_t, I follow Gornemann et al. (2016) and assume a function of the form.[19]

$$\frac{w_t}{w} = \left\{\epsilon_{w,t}\left(\frac{r_t^l}{r^l}\right)\right\}^{(1-\rho_w)} \times \left\{\frac{w_{t-1}}{w} \times \left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota_w} \times \left(\frac{\pi}{\pi_t}\right)^{1-\iota_w}\right\}^{\rho_w} \quad , \quad 0 < \rho_w \ , \ \iota_w < 1 \tag{2.18}$$

[18]The cost Ξ^L is introduced only to enable the estimation of the vacancy posting cost. I adjust Ξ^L to make sure that the expected value of a matching equals the vacancy posting cost in the estimatil.

[19]In principle, one would need to solve a bargaining problem to find the equilibrium wage that applies to a match between an agency and a household. However, since each household's outside option depends not only on their idiosyncratic productivity but also on their asset holding and the level of adjustment costs, the equilibrium wage can differ at each point in the idiosyncratic state space. This feature of the model makes computing wages as a solution to a bargaining problem infeasible. However, there exists a set of wages that support an equilibrium, and a given wage function can support an equilibrium as long as the wage given by the function belongs to such a set. Under the parameterizations and the simulations examined in this paper, the wages implied by the wage function always remain in the bargaining set. Thus, maintaining a match is always beneficial for both labor agencies and households.

Equation (2.18) implies a wage determination mechanism that is similar to Calvo wage setting. First, a fraction ρ_w of the wage is subject to nominal wage rigidity.[20] Specifically, the fraction ι_w of this part of the wage adjusts based on the previous inflation rate π_{t-1} while the fraction $1-\iota_w$ adjusts based on the steady state inflation rate.[21] The remaining fraction $1-\rho_w$ varies with the labor rental rate r_t^l. The responsiveness of the real wage to its rental rate can change due to an exogenous shock $\epsilon_{w,t}$ that follows an i.i.d. process.[22]

In a given period, a match between a household and a labor agency ends in the following three cases: (i) if a matched household dies (probability ζ), (ii) if the match is exogenously dissolved (probability λ) or (iii) if a matched household becomes a business owner (probability P^e).

Given the termination probability, a labor agency's value is given by

$$J^L(s_{it}) = (r_t^l - w_t - \Xi^L)s_{it}n_{it} + \mathbb{E}\Big[\Lambda_{t,t+1}(1-\zeta)(1-\lambda)(1-P_e)J^L(s_{it+1})\Big] \quad , \tag{2.19}$$

where $\Lambda_{t,t+1}$ is the same discount factor used by intermediate goods firms, i.e., the average MRS of business owners.

[20] A difference is that in Calvo setting a wage setter expects the possibility that the wage cannot be adjusted in the future. That is, Calvo wage setting is forward-looking. The wage function used in this paper does not feature forward looking behavior.

[21] One can interpret ι_w as the degree of indexation to the previous inflation rate in a Calvo sticky wage model.

[22] Assuming $\epsilon_{w,t}$ as an autoregressive process does not affect the estimation results. Thus, to reduce the number of parameters to be estimated, I assume that the wage shock is i.i.d. Note that even though the shock is i.i.d, it propagates as the wage exhibits inertia.

The total number of vacancies is determined by the free-entry condition,

$$\iota = \frac{M_t}{V_t} \int J^L(s_t) d\mu_t(s_{it}) \quad , \tag{2.20}$$

where ι is the vacancy posting cost, V_t is the total number of vacancies, and $\mu_t(s_{it})$ is the household distribution over idiosyncratic productivity.

Finally, to determine the number of matches, I follow den Haan et al. (2000) and use the following matching function

$$M_t = \frac{\Big(U_t + \lambda N_t\Big) V_t}{\Big\{\Big(U_t + \lambda N_t\Big)^{\alpha} + V_t^{\alpha}\Big\}^{\frac{1}{\alpha}}} \quad , \quad \alpha > 0 \quad , \tag{2.21}$$

where U_t is the mass of unemployed households at the beginning of period t, N_t is the mass of employed households at the beginning of period t, and V_t is the total number of vacancies.[23] The parameter α determines the efficiency of matching process in the model. The job-finding rate is determined by $\frac{M_t}{U_t + \lambda N_t}$.

2.2.5 Capital firm

A representative capital firm determines the capital utilization rate and accumulates capital as demanded by investors, i.e., households and banks.[24] For a given

[23]Note that, since a certain fraction of households belong to the business owners group, the sum of the masses of unemployed and employed household is not equal to 1.

[24]To simplify price determination, I assume that the capital accumulation is determined entirely by the demand side. This assumption implies that the capital firm does not solve the dynamic problem associated with capital accumulation.

capital stock K_t, the capital firm earns the following profit

$$r_t^k v_t K_t - \delta(v_t) K_t \quad , \tag{2.22}$$

where v_t and $\delta(\cdot)$ are the variable utilization rate and the variable depreciation rate. The first-order condition associated with capital utilization implies that the capital rental rate is equal to the marginal increase in the variable depreciation rate. That is,

$$r_t^k = \delta'(v_t) \quad , \tag{2.23}$$

For variable depreciation, I use a standard functional form used in Greenwood et al. (1988),

$$\delta(v_t) = \delta_0 v_t^{\delta_1} \quad , \quad \delta_1 > 1 \ , \tag{2.24}$$

where δ_0 is the depreciation rate under full utilization and δ_1 governs the degree of acceleration of depreciation.[25]

Regarding capital accumulation, I assume that the capital firm purchases new capital from its investment department on behalf of investors.[26] The investment department has a technology that can convert a unit of the final good to a unit of new capital subject to capital adjustment costs. Specifically, it makes profits as

[25] As I follow standard practice and set the steady state utilization rate to 1, δ_0 is equal to the steady state depreciation rate.

[26] For simplicity, I assume that these two entities operate independently from each other. Thus, one does not take into account the effects of its own decision on the other.

follows.

$$q_t K_{t+1} - \Psi_t^k \left\{ K_{t+1} + \frac{\phi}{2} \left(\log \frac{K_{t+1}}{K_t} \right)^2 K_{t+1} \right\}, \tag{2.25}$$

where Ψ_t^k is a shock to the efficiency of the capital production. In the sense that the shock affects the price of capital and the efficiency of capital transformation technology, it resembles an investment specific technology shock or the marginal efficiency of investment (MEI) shock in Justiniano et al. (2011).

With the assumption that the investment department discounts the future profits with the average MRS of business owners, one can derive the price of new capital as follows.

$$q_t = \Psi_t^k \left\{ 1 + \phi \log \frac{K_{t+1}}{K_t} + \frac{\phi}{2} \left(\log \frac{K_{t+1}}{K_t} \right)^2 \right\} - \mathbb{E}_t \left[\Lambda_{t,t+1} \Psi_{t+1}^k \phi \left(\log \frac{K_{t+2}}{K_{t+1}} \right) \frac{K_{t+2}}{K_{t+1}} \right] \tag{2.26}$$

Finally, investment expenditure is defined as

$$\tilde{I}_t = \Psi_t^k K_{t+1} \left\{ 1 + \frac{\phi}{2} \left(\log \frac{K_{t+1}}{K_t} \right)^2 \right\} - \{1 - \delta(v_t)\} K_t \tag{2.27}$$

2.2.6 Equity mutual fund

There exists a hypothetical mutual fund that owns all firms in the model. To distinguish it from the other type of mutual fund that I will introduce below, I call it the equity mutual fund. The roles of the equity mutual fund include collecting profits from firms, paying out dividends to shareholders, and issuing new equity for capital accumulation. I assume that the fund operates in a perfectly competitive environment. Thus, there are no retained earnings, and the fund pays out all profits as dividends. The funds acquired by issuing equity are transferred to the capital firm for the purchase of capital. The period cash-flow constraint of the equity mutual fund is as follows:

$$(1-\tau)(1-\nu)\Pi_t - q_t(K_{t+1}-K_t) + q_t(A_{t+1}-A_t) = r_t^a A_t \ , \tag{2.28}$$

where Π_t is the sum of all firms' profits and ν is the share of profits that is given to business owners.[27] The tax rate on firms' profits is denoted by τ. Given that the amount of aggregate capital is equal to the amount of equity in the model, the price of equity is equal to the price of new capital, and the dividend rate is

$$r_t^a = (1-\tau)(1-\nu)\Pi_t/K_t \ , \tag{2.29}$$

namely, the dividend rate is profits net of tax payments and net of the amount given to business owners, divided by total equity.

2.2.7 Banks

In practice, central banks' unconventional liquidity provision takes the form of purchases of assets held by financial institutions. To introduce such a policy into

[27] I assume that the fund itself is owned by business owners, and thus a fraction of profits is distributed to them regardless of their equity holding.

the model, I model banks as in Gertler and Karadi (2011).

There is a continuum of banks indexed by $j \in (0,1)$. Each bank takes deposits from savers and purchase equity. Bank j's balance sheet is given by

$$q_t A^b_{jt+1} = N_{jt} + B^b_{jt+1} \quad , \tag{2.30}$$

where A^b_{jt+1} and B^b_{jt+1} are bank j's equity holding and deposits at the end of period t, respectively. The bank's net worth at the beginning of period t is denoted by N_{jt}, which evolves as follows.

$$N_{jt+1} = R^a_{t+1} q_t A^b_{jt+1} - R_{t+1} B^b_{jt+1} \quad , \tag{2.31}$$

where $R^a_{t+1} = (q_{t+1} + r^a_{t+1})/q_t$ and $R_{t+1} = 1 + r^b_{t+1}$ are the gross real rate of return on illiquid and liquid assets, respectively.

As in Gertler and Karadi (2011), each period, only a θ_b fraction of banks continue to operate, while the remaining $1-\theta_b$ fraction exit the market. Let $J^b(N_{jt})$ denote the value of a surviving bank j. Under the environment described so far, the value of bank j is given by

$$J^b(N_{jt}) = \max_{\{A^b_{jt+1}, B^b_{jt+1}, N_{jt+1}\}} \mathbb{E}_t \Big[\Psi^b_t \Lambda_{t,t+1} \big\{ (1-\theta_b) N_{jt+1} + \theta_b J^b(N_{jt+1}) \big\} \Big] \tag{2.32}$$

s.t.

$$q_t A^b_{jt+1} = N_{jt} + B^b_{jt+1} \quad , \quad N_{jt+1} = R^a_{t+1} q_t A^b_{jt+1} - R_{t+1} B^b_{jt+1} \tag{2.33}$$

$$J^b(N_{jt}) \geq \Delta q_t A^b_{jt+1} \tag{2.34}$$

where Ψ_t^b denotes the aggregate risk premium shock, which follows an AR(1) process.[28] Like other firms in the model, banks are owned by the equity mutual fund and discount future cash flows, using the average MRS of business owners $\Lambda_{t,t+1}$. Howevevr, as shown in the above equation, banks' discount factor is perturbed by an exogenous risk premium shock Ψ_t^b. When a positive risk premium shock occurs, banks value future returns more and thus, demand more assets for a given equity premium.

Equitation (2.34) is the incentive compatibility constraint, which reflects a moral hazard problem assumed in Gertler and Karadi (2011). Specifically, banks would purchase assets to the point that the equation (2.34) holds with equality.[29] With a guess and verify approach, one can show that a bank j's value has the following expression.

$$J^b(N_{jt}) = \vartheta_t^a q_t A_{jt+1}^b + \vartheta_t^n N_{jt} \tag{2.35}$$

where ϑ_t^a and ϑ_t^b are the expected value of assets and net-worth, respectively.[30] Given that the incentive constraint always binds, as in Gertler and Karadi (2011), I have the following relationship between the amount of asset purchases and a bank's net-worth.

$$q_t A_{jt+1}^b = \frac{\vartheta_t^n}{\Delta - \vartheta_t^a} N_{jt} = \Theta_t N_{jt} \tag{2.36}$$

[28]The shock plays a similar role to the role of risk premium shocks in Smets and Wouters (2007) and the same shock applies to all banks.

[29]For a more detailed description of the incentive problem, see the appendix.

[30]For detailed description of these variables, see the appendix.

where Θ_t is the leverage ratio of banks. Since the leverage ratio does not depend on bank-specific variables, the above relation can be aggregated. That is, I have

$$q_t A_{t+1}^b = \Theta_t N_t \tag{2.37}$$

where A_{t+1}^b and N_t are the financial sector's equity holding and net-worth, respectively.

Given the law of motion for individual bank's net-worth, exogenous survival rate, and the assumption that existing banks are replaced with new banks with a seed fund given by the equity mutual fund, the law of motion for the aggregate net-worth of banks can be described as follows.

$$N_t = \theta_b \big\{ (R_t^a - R_t)\Theta_{t-1} + R_t \big\} N_{t-1} + \omega q_{t-1} A_t^b \tag{2.38}$$

where the last term is the seed fund for new banks, which is a ω fraction of the existing banks' asset holdings.

Finally, profits from the financial sector are the sum of the net-worth of existing banks, net of the seed fund given to new banks. For a more detailed description of the banking sector, see the appendix.

$$\Pi_t^b = (1 - \theta_b) \big\{ (R_t^a - R_t)\Theta_{t-1} + R_t \big\} N_{t-1} - \omega q_{t-1} A_t^b \ \ , \tag{2.39}$$

2.2.8 Money market mutual fund

The model features another type of hypothetical mutual fund, which I call the money market mutual fund. Its main role is to provide liquidity to the financial sector.[31] It receives contributions from the government and invests in liquid assets. With these contributions and the proceeds from its assets, the fund makes lump-sum transfers to households. Specifically, I assume that the fund smoothes out the flow of lump-sum transfers with the following objective.

$$\max_{\{T_t^m, B_{t+1}^m\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \Psi_t^l \beta_m^t \frac{(T_t^m)^{1-\sigma}}{1-\sigma} \right] \tag{2.40}$$

subject to

$$T_t^m + B_{t+1}^m = C_t^g + (1 + r_t^b) B_t^m \ , \tag{2.41}$$

where T_t^m and B_{t+1}^m are the fund's lump-sum transfer and liquid asset holding, respectively. The MMMF's IES is denoted by σ.[32] The contribution that the fund receives from the government is denoted by C_t^g. Unlike any other entities in the

[31] Since banks are levered investors, there should be an equivalent amount of liquid assets that correspond to banks' illiquid asset holdings in the model. If I assume that households are the sole entities that provide funds to banks, the share of liquid assets in households' portfolio should be high. However, the SCF data shows that households hold only about 10% of their total assets as liquid assets. Moreover, according to the Financial Account data (previously known as the Flow of Funds), the share of household liquid assets, e.g. checkable and time deposits and corporate bonds, in the domestic financial sector's liabilities, which includes deposits, bonds, open market paper, loans, and other liabilities, has been about 25% since 2000. Based on these facts, I assume that there is a significant non-household liquidity provider, in the form of the money market mutual fund.

[32] In principle, the MMMF's IES does not need to equal to the household's IES. However, to save the notation, I assume that MMMF and households have the same IES.

model, I assume that the fund discounts future lump-sum transfer flows with its own discount factor β_m.[33] Finally, the MMMF is subject to an AR(1) liquidity preference shock Ψ_t^l.[34]

2.2.9 Monetary authority

The monetary authority sets its policy rate according to a Taylor rule with interest rate smoothing.

$$i_{t+1} = \min\{0, \hat{i}_{t+1}\}$$
$$\frac{1+\hat{i}_{t+1}}{1+\hat{i}} = \left(\frac{1+\hat{i}_t}{1+\hat{i}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \exp\{-\phi_u(u_t - u)\}\right]^{1-\rho_R} \exp(\epsilon_{R,t}) \; , \qquad (2.42)$$

where $\epsilon_{R,t} \sim N(0, \sigma_R^2)$ is a monetary policy shock, $0 < \rho_R < 1$ is the degree of interest rate smoothing, and $\hat{i}_{t+1}$ and i_{t+1} are the shadow and actual policy rates, respectively. The responsiveness of the nominal rate to inflation and the unemployment gap are denoted by ϕ_π and ϕ_u. Note that the actual policy rate is constrained by the effective lower bound. Thus, if the shadow rate becomes negative, the policy rate can no longer respond to inflation and the unemployment gap.[35] However, in the model, the central bank can affect the economy even when the ELB binds, by adjusting its holdings of illiquid assets.

[33] The steady state optimality condition of the MMMF requires the MMMF's discount factor to be the inverse of the steady state real interest rate. However, because of idosyncratic income risks, the average of business owners' MRS is not equal to the inverse of the real rate at the steady state.

[34] If a positive liquidity preference shock occurs, the MMMF increases liquid asset investment and reduced transfers. Thus, consumption and the inflation rate fall.

[35] The effective lower bound for the policy rate does not need to be zero. In practice, several countries' central banks set negative policy rates. However, in the U.S., the Federal Reserve never set negative policy rates. In this paper, I assume that the effective lower bound is zero.

Central bank asset purchases (QE) have been modeled in different ways in the literature. For instance, Gertler and Karadi (2011) modeled QE as the central bank's direct purchase of private assets, i.e., capital, following a rule based on the equity premium. In contrast, Chen et al. (2012) modeled QE as an exchange between long-term and short-term government debt and did not propose any rule.

In this paper, I follow Gertler and Karadi (2011) and assume that the central bank directly purchases illiquid assets from the private sector. However, unlike Gertler and Karadi (2011), I model QE as a purely discretionary policy,

$$A_{t+1}^{\mathrm{CB}} = \Psi_t^{\mathrm{QE}} A^{\mathrm{CB}} \quad , \quad B_{t+1}^{\mathrm{CB}} = q_t A_{t+1}^{\mathrm{CB}} \ , \tag{2.43}$$

where A_{t+1}^{CB} is the central bank's illiquid asset holding at the end of period t and Ψ_t^{QE} is an AR(1) QE shock that determines the amount of asset purchases as a fraction of the central bank's steady state asset holding. As in Gertler and Karadi (2011), the central bank issues government bonds to finance its asset purchases.[36] B_{t+1}^{CB} denotes bonds issued by monetary authority in period t. From its asset holdings, the central bank earns cash flows, i.e., dividend income net of interest payments. The central bank remits all of its proceedings to the fiscal authority.

In addition to QE, the central bank can implement forward guidance in the

[36] As the formulation of QE policy implies, the central bank transforms demand for non-productive liquid assets (bonds) into demand for productive illiquid assets (capital/equity). Thus, QE policy directly increases investment. By increasing investment, QE policy also increases equity prices, which inflates banks' net worth. As banks' investment is proportional to their net worth, an increase in net worth can increase banks' equity purchases. However, a rise in equity prices also implies a fall in the expected gross rate of return on equity, which discourages equity investment. Similarly, an increase in equity prices can increase consumption via wealth effects. But, a fall in expected returns can reduce households' equity investment. The general equilibrium effects of QE depend on the relative magnitudes of these forces.

form of exogenous expected ELB durations if the economy is at the ELB.[37] That is, by assumption, the central bank can determine households' and firms' expectations regarding the number of periods during which the central bank would maintain the policy rate at zero. If the exogenous expected ELB duration is longer than the endogenous ELB duration (the number of periods during which the ELB constraint is expected to bind based on the central bank's policy rule), it is equivalent to agents expecting future negative (expansionary) interest shocks. Hence, via inter-temporal substitution, forward guidance also can stimulate economic activity.

2.2.10 Fiscal authority

The fiscal authority collects taxes and issues bonds to finance government purchases, unemployment benefits, lump-sum transfers, and contributions to the money market mutual fund. To ensure price level determinacy, I assume that the fiscal authority controls its debt according to the following simple autoregressive rule, as in Woodford (1995).

$$\frac{B^g_{t+1}}{B^g} = \left(\frac{R_t/\pi_t \times B^g_t}{R/\pi \times B^g}\right)^{\rho_B} \quad , \; 0 \leq \rho_B < 1 \quad , \tag{2.44}$$

where $\rho_B \in (0,1)$ is the pace of debt adjustment.

Since economic agents in the model form rational expectations, the government

[37]The model's solution at the ELB is computed backward using the method of Kulish et al. (2014) and Jones (2017), which requires an expected duration of the binding ELB constraint as a part of the solution. The methodology is similar to the OccBin method of Guerrieri and Iacoviello (2015), but allows the duration of the temporary regime to be exogenous.

should meet the following inter-temporal budget constraint.

$$B_t^g = \sum_{l \geq t}^{\infty} \left\{ \prod_{i=t}^{l} \left(\frac{\pi_i}{R_i} \right) \right\} \left\{ \mathbb{T}_l - (G_l + T_l^g + D_l + T_l^{CB} + C_l^g) \right\} \quad , \tag{2.45}$$

where $\mathbb{T}$, G, T^g, D, and C^g are tax revenues, government purchases, lump-sum transfers (or taxes) to households, unemployment benefits, and contributions to the MMMF, respectively. $T_t^{\mathrm{CB}} = q_t A_{t+1}^{\mathrm{CB}} - (q_t + r_t^a) A_t^{\mathrm{CB}} + R_t B_t^{\mathrm{CB}} - B_{t+1}^{\mathrm{CB}}$ is the transfer from (or to) the monetary authority.

Equation (2.45) implies that in each period, the debt level must be equal to the present discounted value of all future government surpluses. When the real value of government debt changes, at least one fiscal instrument must adjust to meet the solvency condition. In this paper, I assume that the fiscal authority adjusts its contribution to the MMMF to balance the budget, while government purchases are fixed and lump-sum transfer to households varies according to the following stochastic process.[38]

$$T_t^g = \left(1 - \frac{1}{\Psi_t^g} \right) Y \ , \tag{2.46}$$

where Ψ_t^g is a lump-sum transfer shock and Y is the steady state output.

[38] Because markets are incomplete and households value liquidity, the model is non-Ricardian. Thus, the fiscal responses matter, especially for the distributional effects of monetary policy. Given that there is only short-term government debt, the effects of these fiscal responses can be particularly strong, as shown in Lee (2019). However, the assumption that I adopt in this paper dampens the effect of the fiscal response. An increase in contributions to the MMMF will increase lump-sum transfers from it, but the responses are modest since I assume that the MMMF smoothes out lump-sum transfer flows.

2.2.11 Market clearing conditions

To close the model, I state the market clearing conditions for each market. The equity market clearing condition is

$$\underbrace{\underbrace{A^h_{t+1}}_{\text{households}} + \underbrace{A^b_{t+1}}_{\text{banks}} + \underbrace{A^{CB}_{t+1}}_{\text{central bank}}}_{\text{equity demand}} = \underbrace{K_{t+1}}_{\text{equity supply}}, \tag{2.47}$$

where $A^h_{t+1} = \int a_{t+1} d\mu_t$ is the aggregate equity demand of households. As shown above, three entities invest in equity: households, banks, and the central bank. The sum of their asset demands should equal the total equity supply, i.e., aggregate capital.

The market-clearing condition for liquid assets, i.e., bonds and deposits, is given as follows.[39]

$$\underbrace{\underbrace{B^h_{t+1}}_{\text{households}} + \underbrace{B^m_{t+1}}_{MMMF}}_{\text{liquid asset demand}} = \underbrace{\underbrace{B^b_{t+1}}_{\text{bank deposits}} + \underbrace{B^g_{t+1}}_{\text{gov. bond}} + \underbrace{B^{QE}_{t+1}}_{\text{bond for QE}}}_{\text{liquid asset supply}}, \tag{2.48}$$

where $B^h_{t+1} = \int b_{t+1} d\mu_t$ is the aggregate liquid asset demand of households. Note that, as households and the money market mutual fund do not distinguish between bonds and deposits, the composition of bank deposits and government bonds in the liquid asset market is determined by the supply side.

Market clearing for capital services implies that the capital stock utilized in

[39] In the model, agents do not distinguish between bonds and deposits.

the current period must equal the capital services demanded by the intermediate goods producers:

$$v_t K_t = \int_0^1 K_{j,t} dj = K^I \tag{2.49}$$

Similarly, the labor supplied by households (via labor agencies) must equal the labor services demanded by the intermediate good firms,

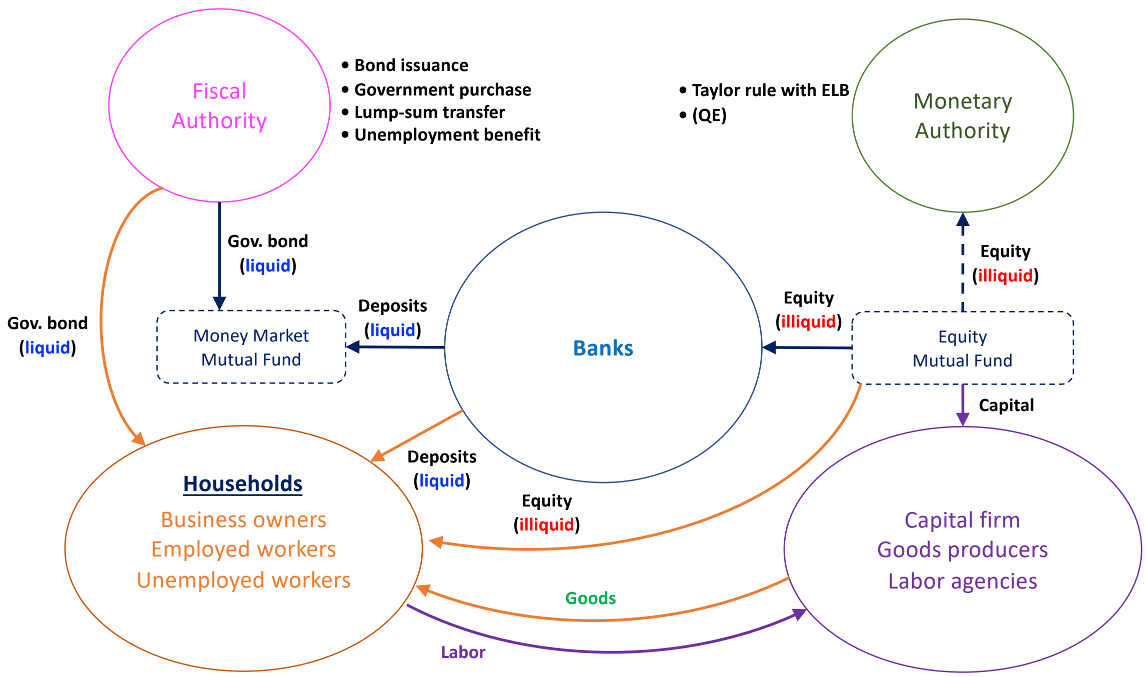
$$\int \mathbb{1}_{\{e_t=1\}} s_t n_t d\mu_t = \int_0^1 L_{j,t} dj \ . \tag{2.50}$$

If the above-mentioned markets clear, by Walras' law, the goods market also clears. Figure 2.1 summarizes the model.

2.2.12 Solution method

I solve the model using a perturbation method developed by Reiter (2009) and extended by Winberry (2018) and Bayer and Luetticke (2020). First, I solve the steady state of the model using an endogenous grid method developed by Carroll (2006). Then, I linearize the model around the steady state and apply a perturbation method. However, since the model features many idiosyncratic states, i.e., liquid and illiquid asset holdings, skill level, and working status, the linearized system's dimension is too large. Thus, a state-space reduction is required. For the state-space reduction, I adopt the method used by Bayer et al. (2019) and Bayer and Luetticke (2020). For the value function, I use Chebyshev polynomials with sparse grids to

Figure 2.1: The model overview



Notes: The figure shows economic agents in the model and the goods and asset flows among them. Arrows start from the entity that sells a certain good (good or labor) or an asset (liquid or illiquid). The dashed line indicates transactions associated with QE operations.

approximate deviations from the steady state. For the idiosyncratic distribution, I use a fixed copula, as suggested by Bayer and Luetticke (2020). I assume a time-invariant functional relationship between the joint and marginal distributions and use it to approximate the evolution of the distribution.

Also, to handle the occasionally binding constraint on the policy rate, I adopt the methodology of Kulish et al. (2014) and Jones (2017). Specifically, I treat the model with a binding ELB as a temporary alternative regime, while treating the model with a positive policy rate as a reference regime, and assume exogenous durations of the alternative regime during the estimation. For comparison with the results from the model with endogenous ELB durations, I adopt the method of Guerrieri and Iacoviello (2015). Further details on the numerical method and its application during the estimation procedure can be found in the appendix.

2.3 Calibration and Estimation

I adopt a two-stage approach to parametrize the model. First, I set a subset of parameters so that the model's steady state matches moments of households' wealth distribution and income composition in the micro-data. I then estimate the remaining parameters with full information Bayesian methods, using time-series data on aggregate macro variables. Importantly, I explicitly take into account the incidence of the binding ELB constraint and QE operations during the estimation.

2.3.1 Data for calibration

For the calibration, I mainly use data from the Survey of Consumer Finances (SCF), since it has detailed information on households' wealth and income composition.[40] I use the 2007 SCF, as it is the last survey before the implementation of QE.[41]

To map the model to the data, I first define liquid assets in the data as the sum of checking, savings and money market deposits, call accounts, certificates of deposit, bonds net of credit card balances, and other lines of credit. Illiquid assets are defined as the sum of all financial assets other than liquid assets, plus net housing wealth, business interest in corporate and non-corporate businesses,

[40] The SCF is well-suited for the study of inequality as it over-samples wealthy households. Specifically, two-thirds of respondents comprise a representative sample of U.S. households, while the remainder of respondents are over-sampled from wealthy households.

[41] Using a more recent survey for computing target moments would not lead to significant differences in parametrization as different surveys produce similar target moments.

minus installment loans. I include only 40% of net housing wealth in illiquid assets, following Kaplan et al. (2018), to take into account the presence of assets that are owned for purely residential purposes. Consumer durables, such as vehicles net of non-revolving debt, are also excluded from illiquid assets. Households with negative illiquid assets are excluded, as short positions in the illiquid asset are not allowed in the model.[42]

I decompose income into three categories; labor income, capital income, and transfer income. In the data, wages and salaries constitute labor income. I define capital income as the sum of business income and asset income.[43] Business income consists of profits from running businesses or farms. Asset income includes fixed interest on financial assets, dividends, and capital gains. Transfer income consists of miscellaneous transfer income, social security benefits, and pension income.

As Kaplan et al. (2018) have shown, the household wealth distribution matters for the responsiveness of a HANK model to monetary policy shocks. Hence, I target moments related to the household wealth distribution, such as the shares of borrowers, wealthy hand-to-mouth households, and households with zero assets.[44] However, gains and losses from monetary policy ultimately depend on households' income composition and the relative response of each income component to monetary policy.

Tables 2.1 and 2.2 show moments in the data and their model counterparts.

[42]Including such households does not make significant differences in the target moments.

[43]In the model, firms' profits constitute both business owners' income (business income) and income from equity holding (asset income). Thus, there is no clear distinction between business income and asset income in the model.

[44]As in Kaplan et al. (2018), wealthy hand-to-mouth households are defined as households with zero liquid but a positive amount of illiquid assets.

Table 2.1: Targeted moments and model fit 1

	Data	Model
Capital to output ratio	**3.03**	**3.02**
Liquid to illiquid asset ratio	**0.10**	**0.10**
Gini net worth	**0.82**	**0.83**
Fraction with b < 0	**0.14**	**0.15**
Fraction with b = 0 and a > 0	**0.20**	**0.20**
Fraction with b = 0 and a =0	**0.11**	**0.10**
Fraction with b = 0	0.31	0.30
Fraction with a = 0	0.14	0.26

Data : SCF 2007, NIPA

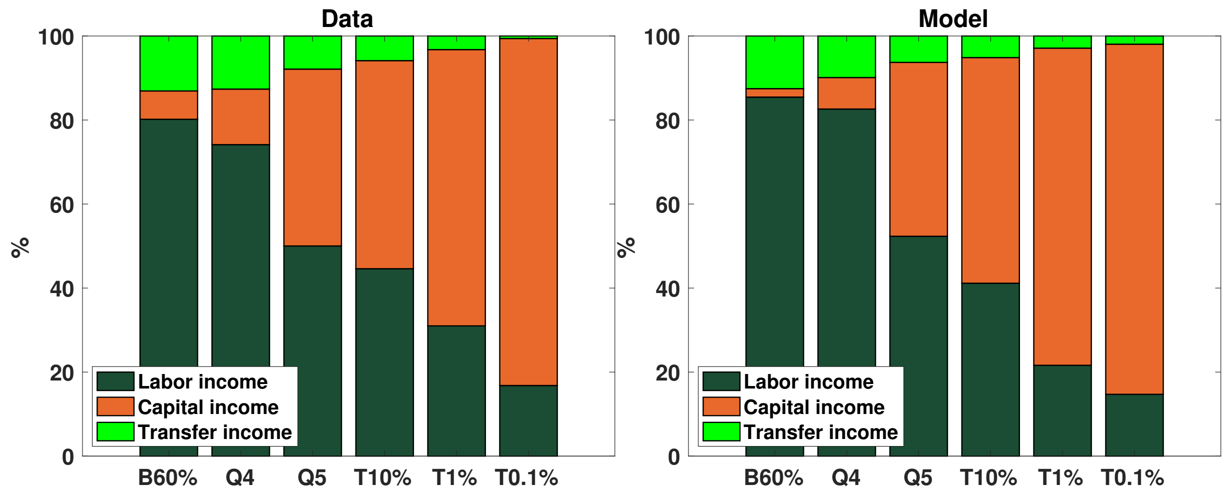
Table 2.2: Targeted moments and model fit 2

	Liquid Assets		***Illiquid Assets***	
Moments	Data	Model	Data	Model
Top 0.1 percent share	19	10	15	3
Top 1 percent share	45	39	38	19
Top 10 percent share	**84**	**84**	**74**	**73**
Bottom 50 percent share	-4	-3	3	1
Bottom 25 percent share	-5	-3	0.2	0
Gini Coefficient	0.97	0.95	0.82	0.85

Data : SCF 2007, *Notes* : The blue color indicates targeted moments.,

The targeted moments are shown in blue and bold text. As shown in the table, the model matches key moments in the data successfully. Specifically, the model is capable of matching the mass of households with zero liquid wealth in the data, which determines the overall responsiveness of consumption to changes in the interest rate. The overall amount of saving, the portfolio composition, wealth inequality, and indebtedness, as reflected in the capital to output ratio, aggregate liquid to illiquid asset ratio, the net worth Gini, and the share of households with debt, are also close between the model and the data. However, the model produces a larger mass of households with zero illiquid assets than in the data, because of the lack of housing in the model.

Figure 2.2: Income composition in the data and the model



Notes: Labor income in the data is the sum of wage and salary. Capital income is the sum of business income and asset income, which is the sum of interest and dividend income and capital gains. Transfer income includes unemployment benefits, social benefits, e.g., food stamps, and other miscellaneous transfers.

In Table 2.2, only the top 10% shares of liquid and illiquid wealth are targeted. Given that it is notoriously difficult to match the top end of the wealth distributions, the model does a reasonably good job generating an asset distribution close to the data.[45] As the table shows, more than 70% of each type of asset is held by the top 10% households.

Next, I discuss the income composition of households in different wealth groups. Inequality in households' wealth translates into heterogeneous household income composition in the model. As Figure 2.2 shows, the model closely matches households' income composition in the data. Both in the data and the model, the share of capital income, i.e., the sum of business and asset income, increases in households' wealth.[46] In contrast, the share of labor income, i.e., wages and salaries, decreases

[45]These shares are defined, in the model and in the data, relative to households' total asset holding, not to aggregate asset holding, since the moments are computed solely from the SCF, which only contains data on households.

[46]For more detailed income composition in the data, see Figure B.1 in the appendix.

in households' wealth. For households in the bottom 60% of the wealth distribution, labor income accounts for about 80% of total income in the data and the model. For households with top 0.1% wealth, which I targeted, the labor income share is 16% both in the data and the model. Their capital income share is 81% in the model and 83% in the data. Though the model is less successful in matching the wealthiest households' relative asset holdings, it successfully matches their income composition. In the model, the top 10% wealthy households' labor and capital income shares are 43% and 49%, respectively. In the data, the corresponding values are 42% and 53%.

2.3.2 Calibration

Table 2.3 shows the calibrated parameters. The model is a quarterly model. I set the inverse of the elasticity of inter-temporal substitution to 1.5, one of the standard values used in the literature. The discount factor is internally calibrated to match the mass of wealthy hand-to-mouth households, i.e., households with positive illiquid but zero liquid assets, which is 20% in the data.[47] The inverse of the Frisch elasticity of labor supply is set to 3, based on Chetty et al. (2011). The disutility of labor is set to ensure that employed households supply one unit of labor at the steady state. The probability of death implies an average working lifespan of 45 years as in Kaplan et al. (2018).

The distribution of illiquid asset adjustment costs affects the average adjustment frequency and inequality of illiquid asset holding in the model. The calibrated

[47] In the data, I define the zero assets as the assets whose value is less than 2,000 dollars.

Table 2.3: Calibrated parameters

Parameter	Value	Description	Reference or targets
Households			
σ	1.5	Relative risk aversion	Standard value
β	0.9932	HH discount factor	Mass of wealthy HtM HHs
ξ	3	Inverse Frisch elasticity	Chetty et al. (2011)
ψ	0.8476	Disutility of labor	SS labor supply of 1
ζ	1/180	Probability of death	Average life span of 45 years
μ_χ	9.0490	Mean of χ dist	SS adj. prob. of 6.5%
σ_χ	3.4205	Scale parameter for χ dist	Top 10% illiquid asset share
P_e	0.05%	Prob. of becoming business owner	Bayer et al. (2019)
$\tilde{P}_e$	20.6%	Prob. of losing business	Top 10% liquid asset share
Labor Market			
λ	0.1	Job separation rate	den Haan et al. (2000)
$\bar{w}$	1.2112	SS real wage	SS labor share/output ratio of 60%
α	1.7127	Matching efficiency	SS vacancy filling rate of 70%
Ξ^L	0.0076	Cost of maintaining a match	SS unemployment rate of 5.5%
Goods producers			
η	3	Elasticity of substitution	Gornemann et al. (2016)
θ	0.27	Share of capital in the prod fn	SS capital share/output ratio of 40%
Ξ/Y	0.2012	Ratio of the fixed cost/output	Capital/output ratio of 3.03
Capital firm			
δ_0	0.0150	SS depreciation rate	SS depreciation rate 6% (annual)
δ_1	1.0025	Elasticity of dep w.r.t. utilization	SS utilization rate of 1
Financial sector			
$\tilde{\Lambda}$	$(1+i)/\bar{\pi}$	MMMF's discount factor	SS optimality condition
τ_m	0.0533	MMMF contribution/tax	SS lump-sum transfer/output ratio 0.1
Δ	0.3410	Degree of limited enforcement	SS leverage ratio of 3
θ_b	0.97	Bank's survival rate	Gertler and Karadi (2011)
ω	0.0076	Initial net worth of new banks	Banks' equity share of 55%
ν	0.2380	Business owners' profit share	Gini Net worth
Government			
τ	0.30	Tax rate	Data
υ	0.4	Replacement ratio	Standard value
$\underline{\mathrm{i}}$	0.0253	Borrowing premium	Mass of HHs with zero assets
$\underline{\mathrm{b}}$	1.3006	Borrowing limit	Mass of HHs with debt
Central bank			
π	1.0050	Inflation target	Fed's target
$1+i$	1.0100	SS nominal rate	HHs' liquid/illiquid asset ratio
A^{CB}/Y	0.05	SS CB's assets/output ratio	Data
ρ^{QE}	0.99	Autocorrelation of QE shocks	See the main text

Table 2.4: Productivities

Symbol	Value
s_1	0.1812
s_2	0.8962
s_3	1.0000
s_4	1.1159
s_5	5.4425
Owner	-

Table 2.5: Transition matrix

		tomorrow					
		s_1	s_2	s_3	s_4	s_5	Owner
today	s_1	0.9054	0.0913	0.0020	0.0000	0.0050	0.0005
	s_2	0.0098	0.8988	0.0858	0.0000	0.0050	0.0005
	s_3	0.0020	0.0865	0.8195	0.0865	0.0050	0.0005
	s_4	0.0000	0.0000	0.0867	0.9078	0.0050	0.0005
	s_5	0.0395	0.0396	0.0395	0.0395	0.8415	0.0005
	Owner	0.0412	0.0412	0.0412	0.0412	0.0412	0.7938

adjustment costs imply an average adjustment frequency of 6.7% per quarter at the steady state, which is close to 6.5%, the value used in Bayer et al. (2020). Also, with the calibrated adjustment costs, the top 10% wealthy households hold 73% of total illiquid assets in the model, compared to 74% in the data.

The income process, which is the ultimate source of inequality in the model, is reverse-engineered to match asset holding and wealth inequality in the data. First, I set the income process for s_t as a standard AR(1) process with three states, using the Tauchen (1986) method for discretization. I set the autocorrelation and standard deviation of the quarterly income process to 0.98 and 0.02, based on Storesletten et al. (2004). In addition to this standard part, I add two boundary states (super low-skilled and super high-skilled) to match the wealth inequality in the data. I fix the probability of becoming a business owner P_e to 0.05%, which is similar to the value used in Bayer et al. (2019). Then, I calibrate the probability of leaving the business owner state, which represents top-income earners' income risk, to match the top 10% wealthy households' share of liquid asset. The resulting value for $\tilde{P}_e$ is 20.6%. Tables 2.4 and 2.5 show the values for idiosyncratic productivity and the state transition matrix for workers and business owners.[48]

I set exogenous job separation rate at 10%, following den Haan et al. (2000). Also, the steady state real wage is set to 1.2112 to have a ratio of labor income to output, net of fixed costs, of 60% at the steady state. I target a vacancy filling rate of 70%, based on den Haan et al. (2000), Ravenna and Walsh (2008), and Christiano et al. (2016). The target for the steady state unemployment rate is set to 5.5%, which is the average unemployment rate before the Great Recession in my sample. Matching these targets, for the given job separation rate, the steady state real wage, and the vacancy posting cost implies a matching efficiency of 1.7127 and the matching maintenance cost of 0.0076.[49]

For goods producers, I set the steady state elasticity of substitution to 3, following Gornemann et al. (2016). A relatively low elasticity of substitution implies a high steady state markup, which allows for a substantial share of the fixed cost in production. For the given value of labor agencies and other firms' profits, I set the fixed cost to match the capital to output ratio of 3.03 in the data.[50][51] The exponent of capital in the production function is set to 0.27, which implies the capital share, i.e., the sum of profits of intermediate good firms and capital rental payment, to

[48]Employment status transition probabilities, i.e., job-finding and separation rates, are discussed in the text.

[49]I estimate the vacancy posting cost and adjust the cost Ξ^L to ensure that the free entry condition is satisfied for given labor market parameter values. The value presented in Table 2.3 for Ξ^L corresponds to the value of the vacancy posting cost at the posterior mode.

[50]I measure aggregate capital as the current-cost net stock of private fixed assets from the Bureau of Economic Analysis. Consumer durables are not included.

[51]In the estimation, the vacancy posting cost varies. To ensure that the free entry condition holds, I adjust Ξ^L. However, adjusting Ξ^L changes the value of labor agencies at the steady state, which also affects the level of aggregate profits and the dividend rate. Thus, to maintain the steady state dividend rate, I also adjust the fixed cost of production for intermediate good firms, along with Ξ^L. The value presented in Table 2.3 is the level of the fixed cost that corresponds to the posterior mode of the vacancy posting cost.

output, net of fixed costs, of 40%.

The parameters associated with variable capital utilization are calibrated to match two targets; the steady state utilization rate and the depreciation rate. As is standard, I set the steady state utilization rate to 1. Then, I target a steady state depreciation rate of 6% (annualized), a standard value used in the literature. Matching these two targets results in $\delta_0 = 0.015$ and $\delta_1 = 1.0025$.

For the financial sector parametrization, I mainly follow Gertler and Karadi (2011). I target a steady state leverage ratio of 3, which implies $\Delta = 0.3304$. The survival rate of banks is 0.97, and ω is set to 0.0076 to match the banks' equtiy share of 55%. The money market mutual fund's discount factor is set to ensure that the steady state inter-temporal optimality condition holds for a given real rate of return on liquid assets.[52] The fraction of tax revenues that is given to the fund is set to 5.33% to ensure a tax rate of 30%, while matching the share of lump-sum transfers in the income of bottom 80%. Finally, the fraction of firms' profits that is given to business owners is set to 23.89%, which, together with the probability of becoming a business owner, contributes to the overall wealth inequality in the model.

For the government sector, I mostly use standard values. The replacement ratio is set to 40%, which is a standard value used in the literature. The tax rate is 30%. The levels of government purchases and lump-sum transfers are set to match the share of transfer income in the bottom 80% households' income and the tax rates of 30%. The borrowing premium of 2.53% is chosen to help match the mass of households with zero assets. Also, the borrowing limit is set to match the fraction

[52] At the steady state, $1 = \beta_m R$ should hold, where R is the steady state gross real interst rate.

of households with debt in the data.

The central bank's inflation target is set to 1.005, which is the current quarterly inflation target of the Federal Reserve. The steady state policy rate is calibrated to match households' liquid to illiquid asset ratio in the data. Also, I assume that the central bank's assets are equal to 5% of output at the steady state, based on the historical average before the implementation of QE. Finally, the auto-correlation of the central bank's assets is set to 0.99.

2.3.3 Data for estimation

For quantitative evaluation of the effects of QE, I estimate the remaining model parameters with Bayesian methods, using the following set of ten observables.[53]

$$\left[\Delta \log Y_t, \Delta \log C_t, \Delta \log \tilde{I}_t, \log \pi_t, \log(1+i_t), \log u_t, \Delta \log w_t, \log T_t^g, \log \Pi_t, \log A_t^{\mathrm{CB}}\right] \tag{2.51}$$

where Y_t, C_t, $\tilde{I}_t$, π_t, $1+i_t$, u_t, w_t, T_t^g, Π_t, and A_t^{CB} are 1) output, 2) consumption, 3) investment, 4) the inflation rate, 5) the nominal interest rate, 6), the real wage, 7) the unemployment rate, 8) lump-sum transfers, 9) corporate profits, and 10) the central bank's assets, respectively.

I measure output as real GDP and consumption as real personal consumption expenditure on non-durable goods and services. I define investment as the sum of private fixed investment on all types of fixed assets and personal consumption ex-

[53]For a more detailed description of observables, see the appendix.

penditure on durable goods. The inflation rate is defined as the quarterly percentage change of the GDP deflator. For the nominal interest rate, I use the effective Federal funds rate. The real wage in the model corresponds to the average hourly wage of production and non-supervisory employees in total private sector. The unemployment rate is the headline U-3 rate computed by the BLS. I measure lump-sum transfers as the sum of government's net current transfer payment and net capital transfer payment. For profits, I use after-tax corporate profits with inventory value adjustment and capital consumption adjustment. Lastly, I use all Federal Reserve bank assets to measure the central bank's asset in the model. The time period is from 1992 Q1 to 2018 Q4.[54]

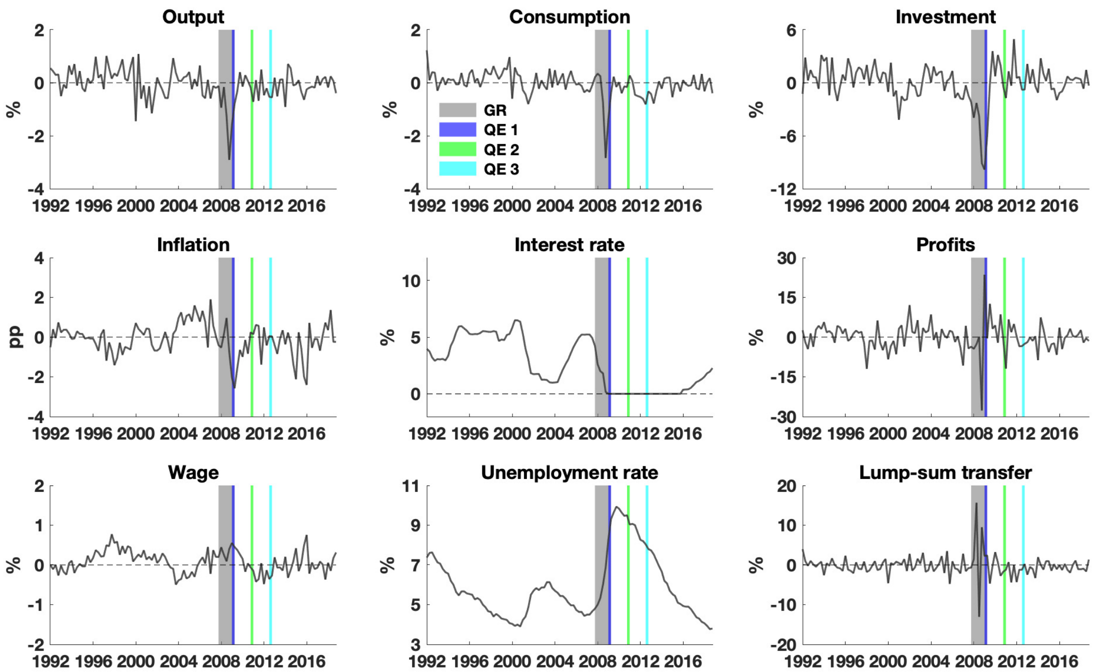
I assume the following shock processes: 1) the MMMF's liquidity preference shock Ψ_t^l, 2) the total factor productivity shock Z_t, 3) the price-mark up shock Ψ_t^p, 4) the wage shock $\epsilon_{w,t}$, 5) the investment technology shock Ψ_t^k, 6) the banks' risk premium shock Ψ_t^b, 7) lump-sum transfer shock Ψ_t^g, 8) monetary policy shock $\epsilon_{R,t}$ 9) the fixed cost shock Ψ_t^F, and 10) the QE shock Ψ_t^{QE}, i.e., the shock to the central bank's asset holding.[55]

Figure 2.3 shows the dynamics of observables during the sample period. As shown in the figure, output, consumption, and investment experienced the biggest

[54]I use relatively a short-sample period to avoid the periods with high interest rates, since households' optimal behavior is not consistent with the case in which the liquid asset return is higher than the return of illiquid assets. Thus, the dynamics of observables during the Great Recession and the ensuing ELB period are likely to affect the parameter values more significantly compared to when a longer sample is used. However, since the focus of this paper is on the dynamics of the economy during those periods, such an influence of the Great Recession is not a weakness in this paper.

[55]For the detailed description of the data, including mnemonic, and the summary of the shock processes, see the appendix.

Figure 2.3: Observables



Notes: The figure shows de-meaned quarterly growth rates of output, consumption, investment, real wages, lump-sum transfers, and corporate profits. The inflation rate is shown as the percentage point deviation from its target of 2%. The nominal interest rate (annualized) and unemployment rate are shown as levels (percentage points). Green, blue, green, and sky blue areas depict the Great Recession period, and the periods in which QE 1, 2, and 3 are implemented, respectively.

drops in their growth rates during 2009. Following the implementation of QE 1, investment recovered, showing consecutive positive growth rates during the ELB episode. However, output and consumption still showed very low or negative growth rates even after the end of the Great Recession.

The inflation rate did not change much at the beginning of the crisis. However, it fell significantly during 2009, and then started to recover after QE 1 was implemented. Likewise, the unemployment rate soared to 10%, and then started to decrease gradually after 2009. Profits exhibit their most volatile dynamics during the Great Recession. The quarterly growth rate of profits fell to almost minus 30%

in 2009, but recovered very quickly and remained mostly positive afterwards. In contrast, the real wage exhibits stable dynamics. During the Great Recession, real wages rose slightly, possibly because of a siginificant drop in the inflation rate.

As the contraction of the economy became severe and the inflation rate fell, conventional monetary policy reached its limit and the Fed embarked on so-called unconventional monetary policy, i.e., QE. As shown in the figure presented in the appendix, the central bank's balance sheet started to expand significantly starting in 2009. Compared to thieir pre-crisis level, the Federal Reserve's assets more than doubled during 2009 and continued to expand until the end of 2015, when the policy rate returned to a positive level for the first time since the Great Recession.

2.3.4 Estimation procedure

The biggest challenge associated with estimating a HANK model is to update the solution quickly for a new set of parameters. Even when using a perturbation method with a state-space reduction, it can take several minutes to find a new solution as the size of the equilibrium system is usually still large. Given that the estimation requires several hundreds of thousands of evaluations, estimation is infeasible with that amount of computation time. If one solves the model globally, then it takes immensely longer to solve a model, and thus calibration is the only viable option.

However, as Bayer et al. (2020) have shown, one can quickly update the solution if one restricts the set of parameters to be estimated to those that do not affect the steady state and uses auxiliary variables that summarize the effects of the household distribution over the idiosyncratic states on the aggregate variables.[56] The most time consuming part of the computation is the linearization of the model, which requires computing the Jacobian of the system. However, most of the equations are associated with the household's value function and the evolution of the distribution. Thus, if parameters do not directly affect the households' problem or the evolution of the distribution, these parts of the Jacobian do not need to be updated.[57] Thus, the number of elements in the Jacobian that need to be re-evaluated drastically decreases. Exploiting this idea, as proposed by Bayer and Luetticke (2020), I can update the solution very quickly in only several seconds, and thus the estimation is feasible.

When the model features an occasionally binding constraint, an evaluation of the likelihood requires additional steps as the solution depends on the expected ELB duration while the model is at the ELB. In this paper, I adopt the approach of Kulish et al. (2014), Jones (2017) and Jones et al. (2018) and assume a sequence of expected ELB durations during the estimation.[58] I estimate these durations along with other structural parameters of the model. Specifically, I apply the randomized blocking scheme developed by Chib and Ramamurthy (2010) and used in Kulish

[56] To be precise, if the households' optimal policies at the steady state do not change, the solution can be obtained quickly for a given set of parameters.

[57] For instance, the discount factor or the household's relative risk aversion affect the way that households respond to a given set of price variables, i.e., wages and asset returns. Thus, if one wants to estimate these parameters, the Jacobian related to the household's problem also needs to be updated.

[58] Alternatively, one can find the endogenous expected ELB duration in each period during the ELB episode in the estimation as in Guerrieri and Iacoviello (2017), Atkinson et al. (2019), and Cuba-Borda et al. (2019). However, this method is computationally burdensome, as it requires a repeated computation of the inverse of very large matrices.

et al. (2014).[59] For the likelihood evaluation, I use the inversion filter instead of the Kalman filter to speed up the estimation process.[60] For the estimation, 1,000,000 draws were evaluated. The first 200,000 draws were discarded as burn-in, and the remaining 800,000 draws were used to construct the posterior distributions of the structural parameters and the expected ELB durations.

[59] During the estimation, I make a draw for two blocks, a structural parameters block and an expected ELB duration block, in isolation. When making draws for the structural parameters, the expected ELB durations are fixed at the previously accepted values and vice versa. For the expected ELB duration draws, I first randomly sample the number of quarters to update from the discrete uniform distribution. Then, for the selected quarters, I draw new expected ELB durations from a discrete uniform proposal density and evaluate the likelihood. In this paper, I use a multinominal distribution with eight points adjacent to the existing expected ELB duration. That is, at each draw, I increase or decrease a subset of expected ELB durations bu up to four quarters. Based on the ratio of the likelihoods, the acceptance is determined. For the other block with structural parameters, a standard Metropolis-Hastings algorithm is used.

[60] When using the Kalman filter, I need to keep updating the state transition matrix, which takes a considerable amount of time given a large size of the equilibrium system.

Table 2.6: Prior and posterior distributions of structural parameters

Symbol	Description	Prior			Posterior		
		Prior Density	Mean	Std	Mode	10%	90%
		Frictions					
κ	Slope of Phillips curve	Gamma	0.10	0.02	0.0525	0.0340	0.0765
ι_p	Price indexation	Gamma	0.50	0.15	0.1219	0.0670	0.2069
ρ_w	Wage autocorrelation	Beta	0.50	0.20	0.7982	0.7065	0.8654
ι_w	Wage indexation	Beta	0.50	0.15	0.1835	0.1132	0.2639
ϕ	Capital adjustment cost	Normal	30.00	5.00	50.017	49.193	51.184
ι	Vacancy posting cost	Gamma	0.05	0.02	0.0317	0.0189	0.0495
		Government policy					
ρ_B	Bond issuance rule	Beta	0.50	0.20	0.5058	0.3998	0.6047
ρ_g	Lump-sum transfer shock AR	Beta	0.50	0.20	0.9986	0.9967	0.9995
σ_G	Lump-sum transfer shock std dev	Inverse-Gamma	0.10	2.00	0.1991	0.1815	0.2172
ρ_R	Interest rate smoothing	Beta	0.50	0.20	0.7927	0.7567	0.8271
σ_R	Interest rate shock std dev	Inverse-Gamma	0.10	2.00	0.1693	0.1481	0.1953
ϕ_π	Taylor rule inflation gap response	Normal	1.70	0.30	1.3101	1.1551	1.5231
ϕ_y	Taylor rule unemployment gap response	Gamma	0.10	0.05	0.3748	0.3307	0.4276
		Structural Shocks					
ρ_l	Liquidity preference shock AR	Beta	0.50	0.20	0.9997	0.9993	0.9999
σ_l	Liquidity preferences shock stt dev	Inverse-Gamma	0.10	2.00	0.0483	0.0431	0.0551
ρ_z	TFP shock AR	Beta	0.50	0.20	0.9952	0.9933	0.9965
σ_z	TFP shock std dev	Inverse-Gamma	0.10	2.00	0.5782	0.5320	0.6333
ρ_p	Price mark-up shock AR	Beta	0.50	0.20	0.9608	0.9457	0.9720
σ_p	Price mark-up shock std dev	Inverse-Gamma	0.10	2.00	1.6344	1.3283	2.1629
ρ_k	Investment shock AR	Beta	0.50	0.20	0.9784	0.9645	0.9900
σ_k	Investment shock std dev	Inverse-Gamma	0.10	2.00	0.0714	0.0658	0.0778
ρ_b	Risk premium shock AR	Beta	0.50	0.20	0.9887	0.9815	0.9941
σ_b	Risk premium shock std dev	Inverse-Gamma	0.10	2.00	0.1601	0.1436	0.1796
ρ_Ξ	Fixed cost shock AR	Beta	0.50	0.20	0.9505	0.9355	0.9643
σ_Ξ	Fixed cost shock std dev	Inverse-Gamma	0.10	2.00	0.9203	0.8380	1.0163
σ_w	Wage shock std dev	Inverse-Gamma	0.10	2.00	0.8324	0.5000	1.3296

Notes: The values for the standard deviations and the measurement error are multiplied by 100.

2.3.5 Prior and posterior distributions

For structural parameters, I mostly follow the literature and use standard priors. For the slope of the Phillips curve, I assume a gamma prior distribution with mean 0.1 and standard deviation 0.02, which is equivalent to a Calvo price contract with an average price duration of about one year. For the degree of indexation to previous inflation in price and wage setting, I use gamma priors with means and standard deviations of 0.50 and 0.15, respectively, following Smets and Wouters (2007) and Justiniano et al. (2011). Since the existing literature mostly uses investment adjustment costs, the prior distribution for capital adjustment costs is chosen in a heuristic way. Specifically, I assume a normal distribution with mean 30 and standard deviation 5 for the degree of capital adjustment frictions in the model. The prior for the autocorrelation of the real wage is set to a beta distribution with mean 0.5 and standard deviation 0.2.

For the policy parameters, I also use fairly standard distributions as priors. For the inflation gap response in the Taylor rule, I assume a normal prior distribution with mean 1.7 and standard deviation 0.2. For the unemployment gap response, I use a gamma prior with mean 0.1 and standard deviation 0.05. The priors for the bond autocorrelation, lump-sum transfer autocorrelation, and interest rate smoothing are set to beta distributions with mean 0.5 and standard deviation 0.2, which is a standard prior used in the literature.

For the shock processes, I use a beta distribution with mean 0.5 and standard deviation 0.2 for the autocorrelations and an inverse-gamma distribution with mean

0.001 and standard deviation 0.02 for the standard deviation of the shock, following Smets and Wouters (2007).

The estimated structural parameters imply a high degree of wage and price rigidity, a relatively low vacancy posting cost, and significant capital adjustment frictions.[61] The posterior distribution for the slope of the Phillips curve is centered around a relatively low value, implying a high degree of price rigidity in the model. The value of this parameter at the mode corresponds to an average price duration of 6 quarters in an equivalent Calvo price setting. Similarly, wage rigidity is also estimated to be very high, implying that only a fifth of the real wage adjusts in proportion to changes in labor productivity, as proxied by the labor rental rate. In contrast, the estimated vacancy posting cost is low.

The estimated parameter value for the capital adjustment cost is particularly high. This is because of the presence of banks in the model.[62] As I show in the appendix, these parameters imply that the model generate a strong and procyclical response of profits, equity prices, unemployment rates, and an almost acyclical real wage response to monetary policy shocks, consistent with the empirical evidence.[63]

The estimated policy parameters are fairly standard except for the autocorre-

[61] This estimation result is due to the relative dynamics of profits, wages, and unemployment rates in the data. As I showed, the real wage is very stable in the data. Moreover, it does not strongly comove with output. In contrast, profits are volatile and positively comove with output. The correlation between the growth rates of profits and output is about 0.3, while the correlation of output with the real wage is -0.1. Thus, in fitting the data, the model favors a high degree of wage rigidity. Also, the model requires a low vacancy posting cost to generate strongly procyclical profits in response to demand shocks.

[62] As in Gertler and Karadi (2011), a financial accelerator channel applies to banks in the model. Thus, bank assets respond strongly to changes in the equity price and the expected return, leading to high volatility of investment. Thus, a high degree of capital adjustment frictions is required to generate volatility of investment that is consistent with the data.

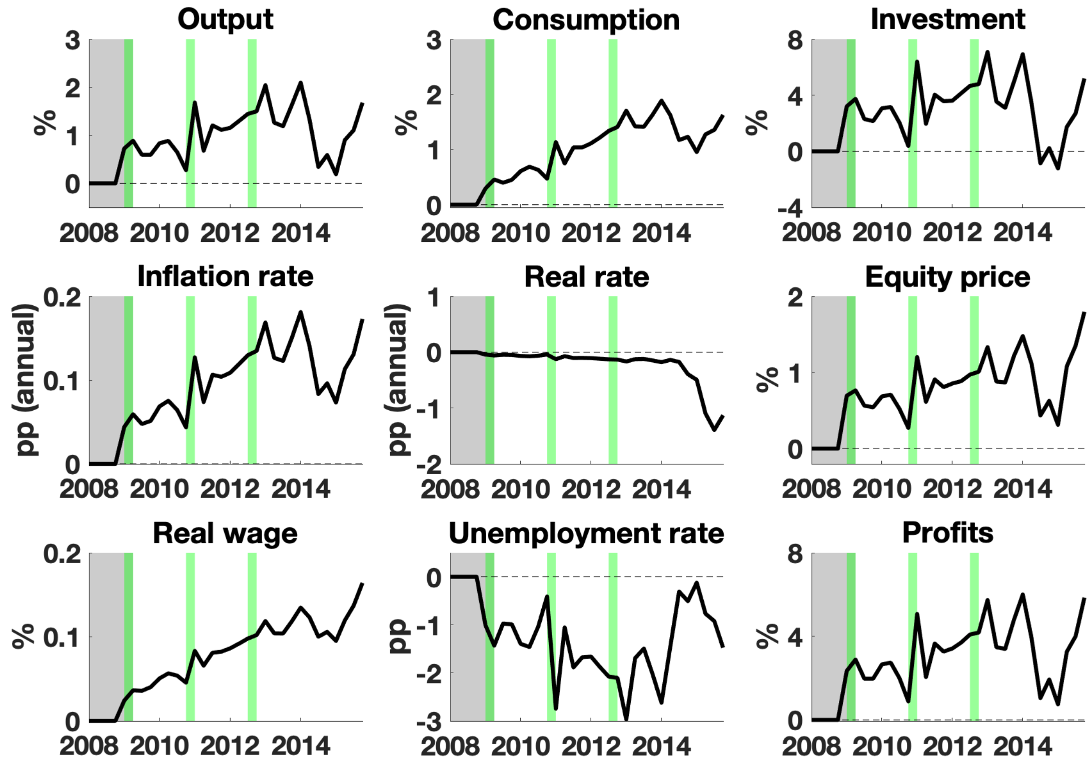
[63] Impulse response functions are reported in the appendix.

lation of the government bond issuance, which is low, and close to its prior mean, which is relatively low. However, such a low value of this parameter does not imply strong fiscal responses to exogenous shocks because of the assumption that the fiscal authority adjusts contributions to the MMMF. Since the MMMF smooths its transfer flows to households by assumption, its liquid saving fluctuates instead of government purchases or the government lump-sum transfer.

2.4 Quantitative Easing during the ELB episode

Now, I answer the central question of this paper: did QE raise inequality in the U.S. during the ELB episode? To this end, I conduct a counterfactual analysis that compares the economy's actual outcomes (as the baseline) to an alternative without QE. Since I use the inversion filter to extract structural shocks, the aggregate variables that correspond to the observables exactly follow the data counterparts in the baseline case. In the counterfactual case, the economy still experiences the same shock realizations. However, the central bank does not conduct unconventional monetary policies during the ELB episode. Instead, it maintains its asset holdings at their pre-crisis level, and it gives no forward guidance so that the expected ELB duration in each period is endogenously determined solely as a function of the aggregate state. Moreover, the central bank adheres to its interest rate rule as soon as fundamentals warrant nominal interest rate liftoff. By comparing these two cases, I gauge the effects of unconventional monetary policy relative to the scenario of a

Figure 2.4: Aggregate effects of QE



Notes: Except for inflation, the real rate, and the unemployment rate, variables are shown as percentage differences from the corresponding values in the alternative case with no policy interventions. The inflation rate, the real rate, and the unemployment rate are shown as the percentage point differences from their corresponding value in the alternative case.

passive central bank.[64]

2.4.1 Aggregate effects of QE

Figure 2.4 shows that, in the baseline case with QE output and consumption are about 1% higher on average than in the counterfactual case with no policy interventions. Likewise, investment is about 3% higher on average in the baseline case. The effects of QE on profits and the unemployment rate are particularly strong. On

[64]For this section, I use the term 'QE' as a shorthand to refer to both asset purchases and forward guidance.

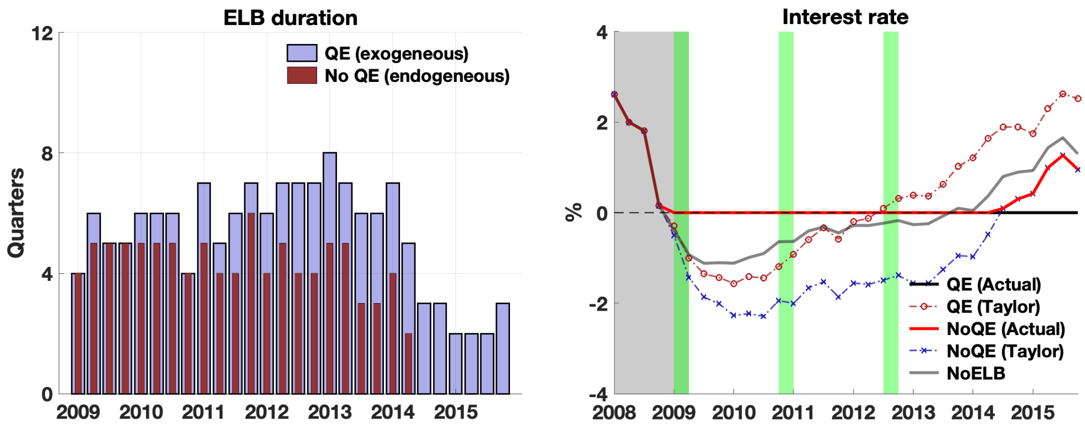
average, profits are about 3.3% higher during the ELB episode than in the case without QE. Similarly, on average, the unemployment rate is about 1.4% lower relative to what would have happened in the counterfactual case without QE. The effect of QE on equity prices is also quite significant. On average, the equity price is about 1% higher because of QE relative to the level when there were no unconventional policy interventions.[65] In contrast, the real wage differences are relatively small, with an average difference of 0.1% between the two cases. Importantly, these relative magnitudes of aggregate effects are similar to those of the effects of conventional monetary policy shocks, which implies that QE has the aggregate effects similar to those of expansionary interest rate shocks, and thus undoes the adverse effects of the binding ELB constraint.[66]

Notably, the aggregate effects of QE increase over time. This is because the effects of the central bank asset purchases accumulate and interact with the effects of forward guidance, i.e., longer exogenous expected ELB durations and policy rates staying at zero in later periods of the ELB episode. In Figure 2.5, the gap between the actual rate (solid red and black) and the policy rate prescribed by Taylor rule (red circle and blue cross) shows how severely the economy is constrained by the

[65]The magnitudes of the effects of unconventional monetary policies in the model fall into the ballpark of the existing estimates found in the literature. For instance, using a set of existing empirical and DSGE models, Chung et al. (2012) finds that the unemployment rate would be lower by 1.5% points compared to what would have happened absent policy interventions. Similarly, using the FRB/US model, Engen et al. (2015) report estimated effects of QE on unemployment rates ranging from a 0.8% point to 1.5% point decrease. Regarding the effects of QE on asset prices, Kiley (2014) finds that a policy-induced 100 bp decline in 10 year Treasury yields is associated with 1.5-3% increase in the equity price. Also, Rosa (2012) finds that an unanticipated expansionary QE announcement is associated with a stock price increase of 0.9%.

[66]The model's impulse responses to an expansionary interest rate shock are shown in Figure B.6 in the appendix.

Figure 2.5: ELB durations and interest rates



Notes: The left panel shows the expected ELB durations in the baseline case with QE and the counterfactual case with no policy interventions. In the counterfactual case, the endogenous ELB durations are obtained by applying the OccBin method of Guerrieri and Iacoviello (2015) for given shocks. The dark red bars show the endogenous ELB durations while the light purple bars show the exogenous durations. The right panel shows the dynamics of the actual policy rate and the policy rate prescribed by the estimated policy rule (Taylor rule) in the baseline and the counterfactual case. The thick black line and the red dotted line with circles show the actual policy rate and the Taylor rule prescription in the baseline case, respectively. The thick red line and the blue dotted line with crosses show the actual and the Taylor rule prescription in the counterfactual case. The light gray line shows the policy rate in the case in which the ELB does not bind.

ELB if the actual rate is higher than the rate prescribed by the rule.[67] Conversely, if the actual rate is lower than the prescribed rate, the gap reflects the degree of expansionary monetary policy. As shown in Figure 2.5, the estimated monetary policy rule prescribes higher policy rates in the baseline case than in the counterfactual case at the beginning of the ELB episode, which implies that the economy is less severely affected by the binding ELB constraint because of the central bank's asset purchases. Besides, the central bank maintains the policy rate at zero until the end of 2015, even though the Taylor rule implies positive rates starting in 2012. The central bank also maintains an expected ELB duration of two to three quarters until the end of the ELB episode in the baseline case, while it sets positive rates from 2014 Q4 in the counterfactual case. Because of this combination of unconventional

[67]The Taylor rule prescribed policy rates reflect the aggregate economic state, which is affected by unconventional monetary policies.

monetary policies, the economy experiences further stimulus effects, especially during the later periods of the ELB episode. In a later section of this paper, I isolate the effects of the central bank's asset purchases from the effects of other types of unconventional policies. But, in this section, I do not distinguish them and consider these various types of unconventional policies as a whole.

2.4.2 Distributional effects of QE

In this section, I evaluate the effects of QE on inequality in detail, using the Gini index, the top 10% share, and welfare gains, as measured by consumption equivalents. Iin addition to examining the overall effects, I compute the contribution of each variable that affects households' wealth and income, including the job finding rate to understand the underlying mechanisms.[68]

Given the aggregate effects of QE examined in the previous section, increases in profits and equity prices due to QE are much higher than those of the real wage, which pushes towards higher inequality. In contrast, the lower unemployment rate is likely to reduce inequality as it benefits households at the bottom of the wealth distribution.[69] Given that changes in the real interest rate are not so large because

[68]The decomposition method is similar to the microsimulation used in Casiraghi et al. (2018) and Lenza and Slacalek (2018). Specifically, I compute the evolution of inequality by feeding the expected paths of profits, real interest rates, wages, job-finding rates, lump-sum transfers, and equity prices in isolation. The household's optimal responses are computed based on each expected path. For more detail, see the appendix.

[69]Since unemployed households make only a small amount of labor income, they mostly belong at the bottom of the wealth distribution. In the model, at the beginning of 2009 Q1, the share of unemployed households at the bottom 10% of the wealth distribution is 8.75%, while the share is only 6.54% in the middle quintile. Accordingly, even though the job-finding rate increases uniformly across the household distribution, a larger number of households become employed at the bottom of the wealth distribution than in other groups. Thus, an increase in average income is larger than in other groups. Moreover, poor households are mostly hand-to-mouth. An increase in income leads to a larger increase in consumption and welfare than other groups.

of the binding ELB constraint, redistribution through real interest rate is unlikely to be large. Thus, the relative magnitudes of the first two channels are likely to determine the net effects of QE on inequality during the ELB episode.

Overall, QE had non-linear distributional effects during the ELB episode. QE benefited the top 10% of the wealth distribution substantially, by boosting profits and equity prices. At the same time, it also benefited the bottom 10% of the wealth distribution significantly by reducing unemployment rates. In contrast, the gains for the middle class were relatively small because of the small changes in the real wage. Because of this non-linear effect, QE can be seen as either increasing or decreasing inequality, depending on the measure of inequality used.

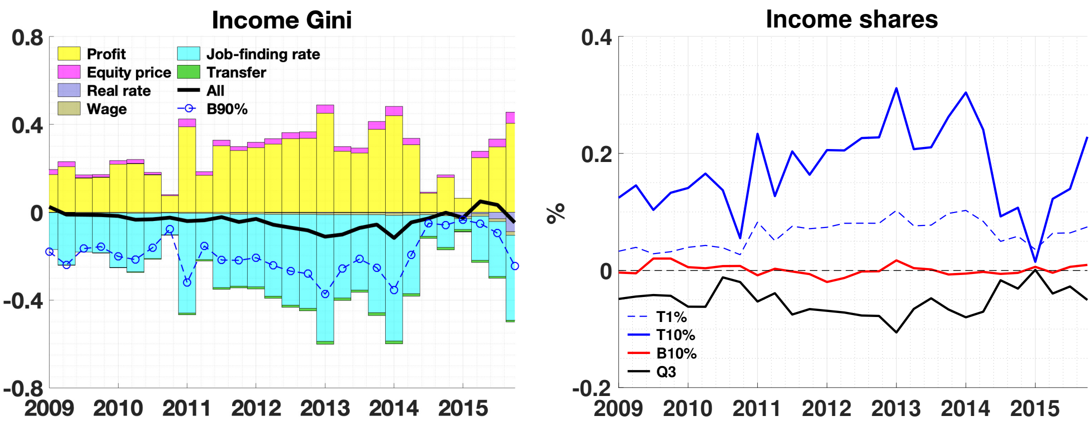
2.4.2.1 Income inequality

Figure 2.6 shows the evolution of the income Gini index and of income shares during the ELB episode. As shown in the left panel, QE modestly reduced the income Gini, relative to the counterfactual in which the Fed became passive once nominal rates reached the ELB.[70] Lower unemployment rates reduced the income Gini by up to 0.6 percentage points, consistent with policymakers' arguments that emphasized the positive impact of QE on the labor market.[71] Higher profits and equity prices offset about 80% of this effect, while transfers and wage growth had

[70] During the ELB episode, the income Gini was higher compared to its level at the beginning of the Great Recession, as shown in Figure B.14 in the appendix. QE accounts for only -2.5% of total changes in the income Gini index's, relative to the level at the beginning of the Great Recession, during the ELB episode. A small net effect is due to the result that the effects of higher profits and equity prices mostly offset the effects of lower unemployment rates.

[71] See, for instance, Bernanke (2015) and Draghi (2016).

Figure 2.6: Distributional effects of QE: Income inequality



Notes: The left panel shows the differences in the model-implied income Gini index (0 to 100) between the baseline and the counterfactual case. The thick black line shows the overall effect of QE, while each bar shows the contribution of each variable to the overall effect. The blue dotted line with circles shows the Gini index computed from households in the bottom 90% of the wealth distribution. The right panel shows the income share of the top 1% (dashed blue), 10% (blue), the middle quintile (black), and the bottom 10% (red) households, as the percentage point difference compared to the corresponding levels in the counterfactual case. Income is the sum of labor, capital, and transfer income. Capital gain is not included.

negligible effects.[72]

An interesting result is that the decrease in income inequality is larger when the income Gini index is computed only using households in the bottom 90% of the wealth distribution. The dynamics of the Gini index among the bottom 90% households closely follow the effects of QE on the job-finding rate. This is because the bottom 90% households have similar income composition and mostly rely on labor income.

The right panel of Figure 2.6 shows that QE widened the income gap between

[72]Note that capital gains are not included in the definition of income in the model because it is hard to keep track of the purchasing price of illiquid assets for each household. Though capital gain is not included, higher equity prices contribute to an increase in income for two reasons. First, when households sell their equity, they sell a lesser amount when the price is higher. Thus, after selling, households hold a larger amount of equity compared to when equity prices were lower. Besides, since households receive deceased households' equity holdings as a part of annuity arrangement, higher equity price increases equity holders' income from the annuity arrangement. If the capital gain is defined as the value of equity sold minus the steady state equity price, the income Gini is higher in 2015 in the baseline case than in the counterfactual case. However, for other years, income Gini is still lower in the baseline case.

the top 10% and the rest of households during the ELB episode. As shown in the figure, QE increased the top 1% and 10% income shares in aggregate income. Changes in the bottom 10% income share fluctuate around zero, while the income share of the middle quintile fell. This result underscores the failure of the income Gini to capture non-linear distributional effects of QE.[73] The income Gini falls mainly due to the reduced income gap between the bottom 10% and the middle class, which is large enough to offset the higher income gains for the top 10% in the calculation of the index.

2.4.2.2 Wealth inequality

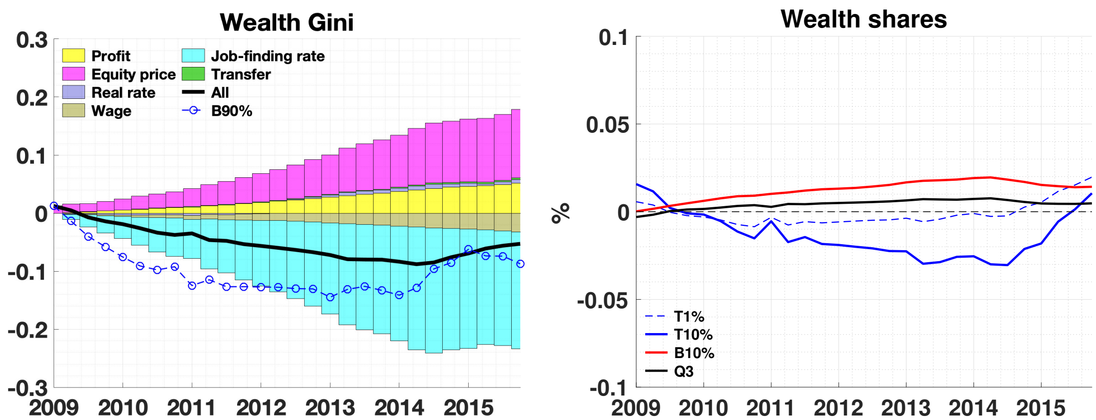
Despite its positive impact on equity prices, QE reduces the wealth Gini index slightly, and the main driver is also lower unemployment rates.[74] The literature has found similar results, but the existing work emphasizes competing effects of equity prices versus the savings redistribution or the role of house prices.[75] In the model, a redistribution from real interest rate changes is weak due to limited changes in the real rate, and there is no housing. Instead, households dynamically adjust their liquid savings to smooth consumption, and such behavior leads to a fall in wealth

[73]It is well known that many existing inequality measures, including Gini indices, do not guarantee subgroup consistency. Such a problem is particularly pronounced in the case of the consumption Gini in the model, which is presnted in the appendix. For a detailed discusion on the properties of the Gini index, see, for instance, Jurkatis and Strehl (2014).

[74]As shown in Figure B.14 in the appendix, the wealth Gini index increased by about one percentage point during the ELB episode, compared to its level at the beginning of the Great Recession. Changes in the wealth Gini index induced by QE accounts for about -5% of the total change during the ELB episode.

[75]Casiraghi et al. (2018) and Inui et al. (2017) both find that QE has only negligible effects on wealth inequality and argue that the effects of *portfolio composition channel* and *savings redistribution channel* cancel out each other. Lenza and Slacalek (2018) also find a similar result, but they emphasize the role of housing prices in offsetting the adverse effects of equity price on inequality.

Figure 2.7: Distributional effects of QE: Wealth inequality



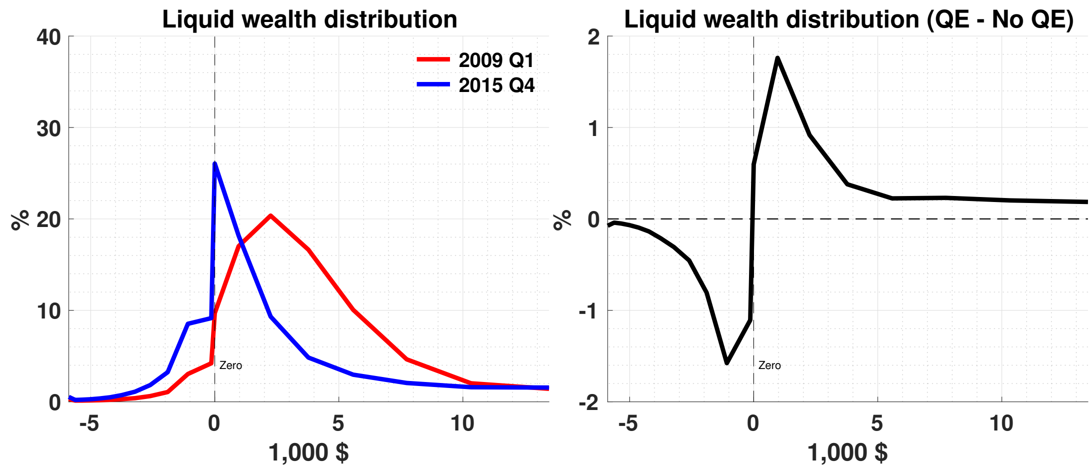
Notes: The left panel shows the differences in the model-implied wealth Gini index (0 to 100) between the baseline and the counterfactual case. The thick black line shows the overall effect of QE, while each bar shows the contribution of each variable to the overall effect. The blue dotted line with circles shows the Gini index computed from households in the bottom 90% of the wealth distribution. The right panel shows the model-implied income shares of the top 1% (dashed blue), 10% (blue), the middle quintile (black), and the bottom 10% (red) households, as the percentage point difference compared to the corresponding levels in the counterfactual case. Income is the sum of labor, capital, and transfer income. Capital gain is not included.

inequality. As shown in the left panel of Figure 2.8, during the ELB episode, a substantial share of households exhaust their liquid savings, and some of them become indebted mostly because of the higher unemployment rate.[76] By reducing the unemployment rate, QE helped households maintain their liquid wealth or prevented them from accumulating debt. The right panel of Figure 2.8 shows that QE increases the mass of households with a positive amount of liquid wealth and reduces the mass of indebted households at the end of the ELB episode. This result shows the importance of taking into account the dynamic responses of households' balance sheets, including debt, in evaluating changes in household wealth inequality.[77]

[76]Figure B.16 shows that, during the ELB episode, households in the first quintile of the wealth distribution experienced dramatic declines in wealth during the ELB episode, and they are the most vulnerable households to unemployment risks.

[77]Domanski et al. (2016) highlights the effects of QE on equity prices and argues that QE increases wealth inequality in European countries. However, Domanski et al. (2016) assume that household portfolios is fixed during the simulation.

Figure 2.8: Effects of QE on the liquid wealth distribution



Notes: The left panel shows model-implied households' liquid wealth distribution at the beginning and the end of the ELB episode. The red line shows the distribution in 2009 Q1. The blue line shows the distribution in 2015 Q4 in the baseline case. The black line in the right panel shows the differences in the liquid wealth distribution in 2015 Q4 between the baseline and the counterfactual case. A positve value implies that there is a larger mass in the baseline case than in the counterfactual case. Units are converted to dollar values, assuming that the steady state output is equal to real GDP per capita in 2009. Negative liquid wealth implies debt.

The results for wealth shares are consistent with the wealth Gini dynamics, unlike in the case of income inequality. That is, the top 1 and 10% wealth shares fall during the ELB episode in the baseline case, relative to their levels in the counterfactual case. At first, this result seems at odds with the effects of QE on income inequality. However, further analysis reveals that higher income of the top 10% induced by QE translates into higher inequality in illiquid asset holding, while benefits of the lower unemployment rate translate into lower inequality in liquid asset holding. As shown in the appendix, the latter effect mainly determines the dynamics of the overall wealth inequality. The top 1% and the top 10% equity holding share increases due to higher equity prices during the ELB episode, but these effects are swamped by reduced inequality in liquid asset holdings.[78]

[78] Though the overall wealth inequality has fallen, a rise in equity holding inequality, together with higher profits, leads to higher income inequality in later periods of the ELB episode.

2.4.2.3 Welfare effects

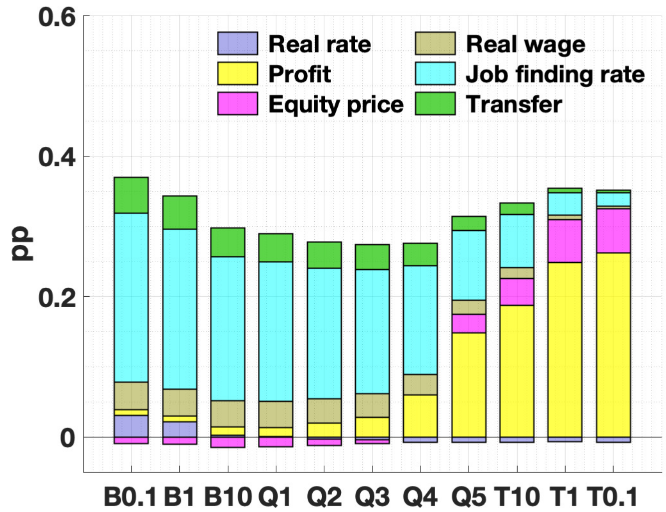
In this section, I examine who gains most in terms of welfare from QE during the ELB episode. To this end, I compute the consumption equivalents across different wealth groups. Specifically, I define wealth groups based on the distribution of wealth in 2009 Q1 and I keep track of these groups during the ELB episode, computing their consumption equivalents.[79] By comparing these groups' consumption in different cases, I compute different groups' consumption equivalents, defined as the fraction of lifetime consumption in the counterfactual case agents would be willing to give up to benefit from QE.

Figure 2.9 shows that QE benefits wealthy and poor households the most while leaving the smallest welfare gains for the middle quintile.[80] The average welfare gain from QE is equivalent to 0.27 percent of lifetime consumption. However, households at both ends of the wealth distribution enjoy higher than average welfare gains from QE while the middle class enjoys the least benefits: the consumption equivalent for the bottom and the top 1% households is about 0.06 percentage points higher than that of the middle 60%. The differences in welfare gains are due to different shares of the unemployed in each group and the groups' income and wealth composition. In 2009 Q1, the aggregate unemployment rate is 8.27%, but the share of unemployed

[79] Households' wealth distribution in 2009 Q1 is determined in 2008 Q4, and thus, is not affected by QE. Note also that, since households' wealth and working status vary over time, the composition of wealth groups also changes. Thus, for instance, households in the fifth quintile in 2009 Q1 do not necessarily belong to the fifth quintile in 2013 Q4. In computing the consumption equivalents, I need to follow the same households, and thus fix wealth groups. Also, as the sample ends in 2018 Q4, I assume that there are no shocks beyond that period.

[80] Across the working status, business owners enjoy the highest welfare benefit equivalent to 0.82% of lifetime consumption, followed by the unemployed with 0.35% of lifetime consumption. The welfare gain for the employed is equivalent to 0.27% of lifetime consumption.

Figure 2.9: Welfare effects of QE: Consumption Equivalents



Notes: The figure shows the welfare gains from QE in terms of consumption equivalents. Consumption equivalents are computed for differet groups of household wealth distribution as of the beginning of the ELB episode, assuming that there are no shocks after the end of the sample period. Bars in the positive region indicate welfare gains, while bars in the negative region reflect welfare losses. The sum of the height of the bars in the positive and negative regions show the net welfare gains. Boxes with different colors show the contribution of each variable. Units are percetage points. B0.1 (T0.1), B1 (T1), and B10 (T10) refer to the bottom (top) 0.1%, 1%, and 10% of the wealth distribution, respectively. Q1 to Q5 refer to the first to the fifth quintile.

households at the bottom 10% of the wealth distribution is 8.75%. In contrast, the share of unemployed households in the middle quintile is 6.54%. As a higher share of households is unemployed at the bottom of the wealth distribution, welfare gains from the higher job-finding rate are larger for them. In contrast, as profits and equity account for a significant proportion of the top 10% households' income and wealth, higher profits and equity prices lead to higher than average welfare gains for those households.

A noteworthy result is that the differences in welfare gains for the top 10% relative to others are smaller than the differences in income gains. The consumption equivalents for the bottom and the top 10% are similar, and the welfare gain is largest for the bottom 0.1%. This result is due to the expected effects of tapering in periods beyond the sample. During the ELB episode, wealthier households enjoy higher consumption gains that mirror higher income gains. However, as the economy enters into the tapering phase, households expect lower equity prices and profits.[81] Lower profits reflect the adverse effects of tapering on banks' net worth, but are not accompanied by equivalently higher unemployment rates. Moreover, as tapering generates downward pressures on the inflation rate, real wages are expected to be higher in the future. Accordingly, welfare gaps between the top 10% and the bottom 90% are smaller than the gaps in the income response during the ELB episode.

To recapitulate, I find that the wealth and income Gini indices are slightly lower during the ELB episode, mainly because QE's positive effects on employment are strong enough to offset its positive effects on profits. However, the Gini index fails to capture the strong income gains for the top 10% households whose income share rises. In terms of welfare gains, all households benefitted from QE, but both ends of the wealth distribution enjoyed higher gains relative to the middle. Overall, the welfare gaps across households are small relative to income gaps, because of the transient effects of QE on future profits and equity prices. I conclude that concerns about QE widening inequality are not supported by the experience of the Great Recession.

[81] The evolution of key variables during the tapering phase is shown in Figure B.18 in the appendix.

2.5 QE and Conventional Monetary Policy

The persistent decline in the natural interest rate in recent decades has spurred concerns about increasing incidence of ELB episodes going forward. As a result, the literature has started to discuss increasing the inflation target and thus the steady-state nominal policy rate, thereby securing more room for the operation of conventional monetary policy (CMP).[82] In this section, I compare QE and conventional monetary policy in terms of both aggregate and distributional consequences, to provide a reference for the benefit of avoiding the binding ELB constraint. Specifically, I ask what would have happened if policymakers had been able to lower the policy rate further, instead of relying on a package of unconventional policies. To model CMP, I assume that the central bank sets the policy rate according to the Taylor rule, ignoring the ELB constraint, but does not conduct any QE.[83] Specifically, the policy rate follows the gray line in Figure 2.5.

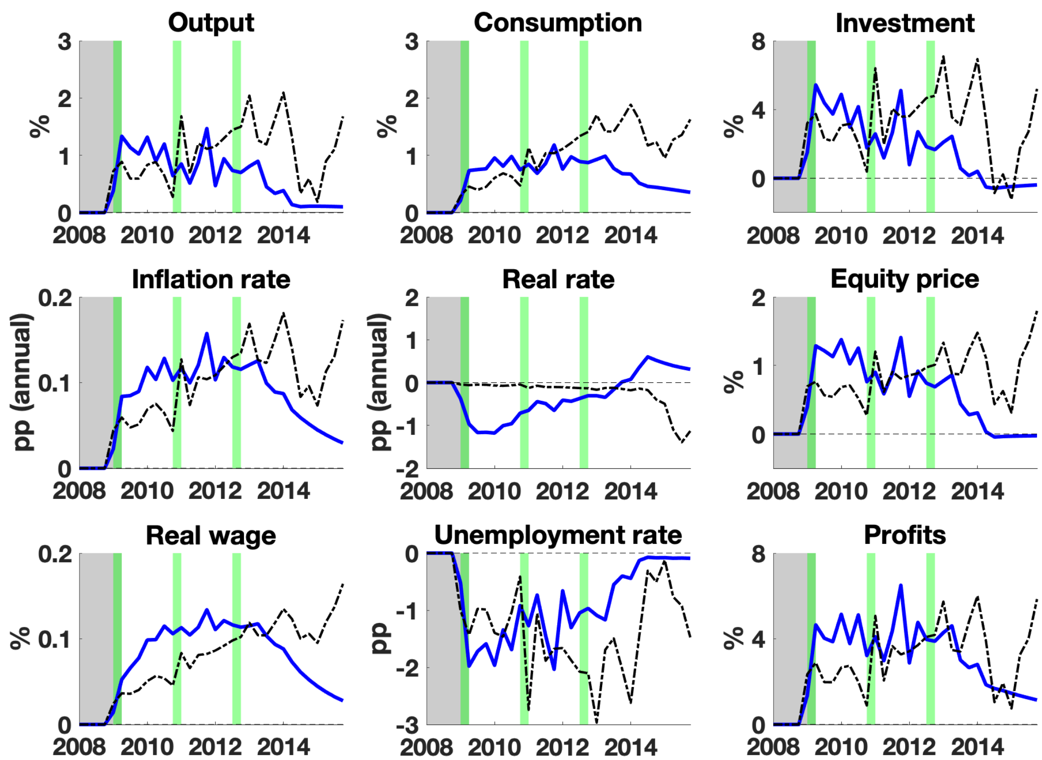
2.5.1 Aggregate effects

Figure 2.10 shows that CMP initially has stronger stimulus effects than QE, but in later periods, the effects are smaller than those of QE. As shown in Figure 2.5, the central bank in the QE case imposes longer expected ELB durations and

[82] See, for instance, Ball (2014),Blanchard et al. (2010), and Williams (2016).

[83] In the simulation, the nominal policy rate goes below zero. However, I do not interpret the results presented in this section as the effects of negative interest rates because saving in assets whose nominal rate is negative can be irrational in practice. Instead, I interpret the results as the effects of CMP when the nominal policy rate and the inflation rate are higher by the same amount. In this case, real interest rates are the same as in the baseline case, but the central bank has more room for lowering the nominal policy rate.

Figure 2.10: Aggregate effects: QE vs CMP



Notes: The solid blue and dashed blak lines show the effects of QE and CMP. Except for the inflation rate, the real interest rate, and the unemployment rate, variables are shown as percentage differences from the corresponding values in the alternative case with no policy interventions. The inflation rate, the real interest rate, and the unemployment rate are shown as the percentage point differences from their corresponding values in the alternative case.

maintains the policy rate at zero even after the Taylor rule prescribes positive rates. In the case of CMP, these effects are absent as the economy does not stay at the ELB. Thus, CMP (as prescribed by the Taylor rule) would have had weaker stimulus effects than QE in the later periods of the ELB episode.

What is somewhat surprising is the stronger initial effects of CMP than those of QE, especially given the large amount of central bank asset purchases at the beginning of the ELB episode. This result is not due to CMP having particularly strong stimulus effects.[84] Rather, it is due to the weak initial stimulus effects of QE

[84] As shown in Figure B.6, interest rate shocks have a modest amount of stimulus effect, in line

in the model.[85] First, QE directly affects only small fraction of households that hold equity. As these equity holders are mostly rich and have below-average marginal propensities to consume, the direct stimulus effects of QE on household consumption is relatively weak compared to those of CMP.[86] Moreover, QE *crowds out* private investment, especially of banks. An increase in equity prices boosts banks' net worth but decreases the expected gross rate of return on equity, i.e., banks' profitability, which discourages banks' investment. In contrast, CMP does not directly increase banks' net worth, but in the short run it increases the profitability of banks by lowering their financing costs. Thus, CMP *crowds in* banks' investment.[87][88]

Overall, CMP has smaller average stimulus effects than QE because of the larger stimulus effects of QE in later periods. On average, the effects of CMP on equity prices and unemployment rate are about 20% and 30% smaller than those of QE.[89] In contrast, the effects of CMP on profits are only about 3% lower than those of QE. This is because CMP strongly positively affects banks' net worth initially, and such effects propagate through the financial accelerator channel embedded in the model. Finally, the average effect on the real wage is greater in the case of CMP

with the findings of the literature.

[85] Note that the central bank's asset purchases in 2009 Q1 are equivalent to about 6.5% of steady state output in the model. Thus, if all of the asset purchases are translated into stimulus on output without any offsetting effects, output should have increased by as much as 6.5%. However, in the model, the initial impact on output is less than 1% of the steady state output, which implies that a substantial proportion of QE's stimulus effects are offset by general equilibrium responses.

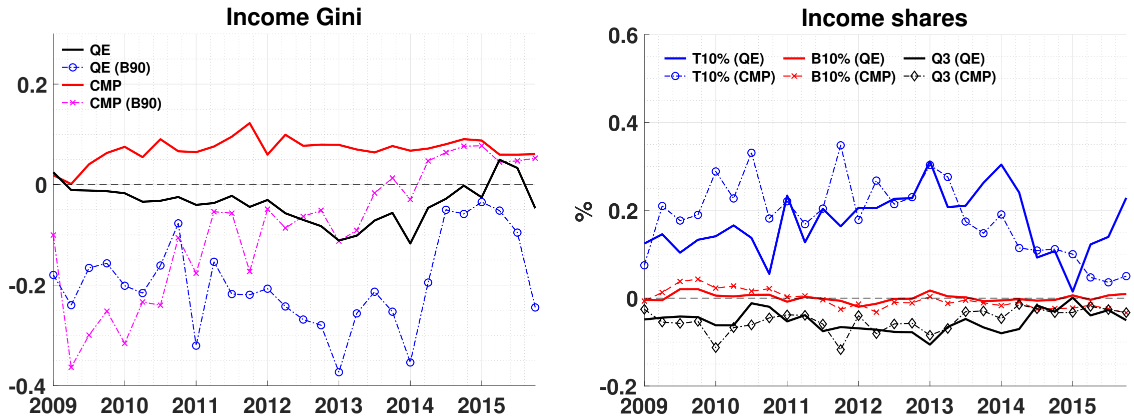
[86] Adjustment frictions also contribute to small direct consumption responses out of QE as they cause only a subset of households to adjust their equity holdings.

[87] The general equilibrium effects of QE and CMP on equity prices and equity premia are similar. However, higher equity prices and lower equity premia are the consequences of banks' expansion in the case of CMP. In contrast, in the case of QE, a large part of higher equity prices and lower equity premia is due to the central bank's action.

[88] Figure B.19 in the appendix shows that CMP has much stronger positive impacts than QE on banks' net worth and investment.

[89] For instance, QE reduces the unemployment rate by 1.4%, while CMP reduces the unemployment rate only by 1% on average during the ELB episode.

Figure 2.11: Distributional effects of QE and CMP: Gini index and income shares



Notes: The left panel shows the differences in income Gini indices between the case of QE or CMP and the counterfactual case of no policies. The black and red solid lines shows income Gini in the case of QE and CMP, respectively. The pink line with crosses and blue line with circles show the Gini indices among the bottom 90% households. The right panel shows the impact of policy on the income share of the top 10%, the bottom 10% and the middle quintile in the case of QE and CMP.

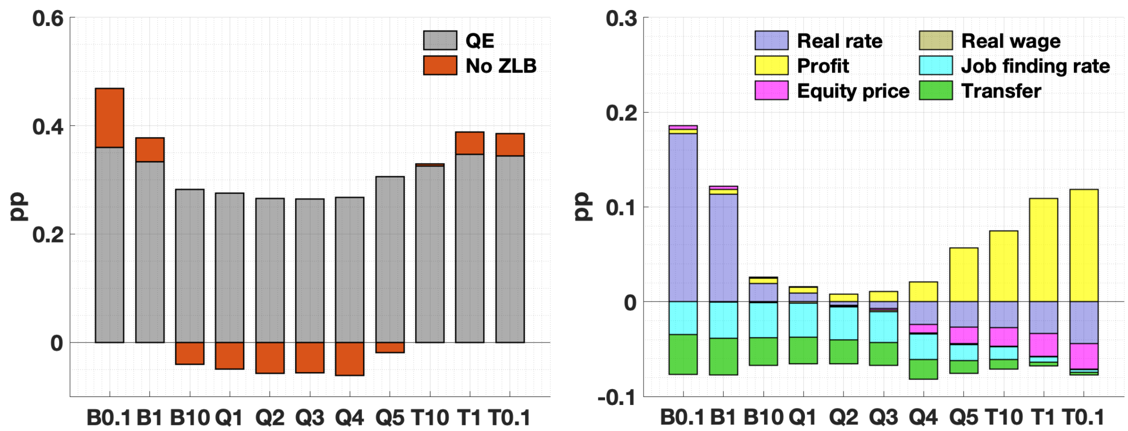
because of the larger initial impact of CMP combined with wage rigidity.[90]

2.5.2 Distributional effects

Figure 2.11 compares the distributional effects of QE and CMP in terms of the income Gini index and the income shares across different wealth groups. Note that, for the first two years of the ELB episode, the unemployment rate is lower, and profits and equity prices are higher for the reasons discussed in the previous section. The initial increase in the real wage is also larger under CMP, though still relatively modest. The associated distributional consequences show more contrast between the top 10% and the bottom 90% under CMP than under QE. The top 10% and the bottom 10% are both higher from 2009 to 2011 in the case of CMP than in the case of QE. Accordingly, the overall income Gini is higher under CMP compared to

[90]Because of high wage rigidity, the increase in the real wage does not quickly dissipate. As the real wage is maintained at a relatively high level, the unemployment rate returns to ints counterfactual level more quickly under CMP than QE.

Figure 2.12: Welfare gain comparison: QE vs CMP



Notes: The figure compares the welfare gains from QE and conventional monetary policy in terms of the consumption equivalent. The gray bars show welfare gains from QE across households. The dark organge bars show additional gains or losses from conventional monetary policy compared to welfare gains from QE. The right panel shows the decomposition of the additional gains or losses from CMP, relative to gains from QE.

either the counterfactual case witn no policy interventions or QE, but the Gini index among the bottom 90% is lower in the case of CMP. That is, the income distribution becomes more skewed in the case of CMP: while the top 10% moves farther away from other households, the distribution among the rest of households becomes more compressed. Overall, I conclude that CMP has more disequalizing effects than QE: CMP increases the income Gini index, whie QE decreases the income Gini index compared to the counterfactual case with no policy iterventions.[91]

In terms of welfare gains, the non-linear effects of CMP are also stronger than those of QE. Since the average stimulus effects of CMP are smaller, most households experience smaller welfare gains under CMP than under QE. Only the top 1% and the bottom 1% of households enjoy higher gains from CMP than QE, as shown in Figure 2.12. The magnitudes of welfare losses from CMP relative to QE are largest

[91]In the case of wealth inequality, both CMP and QE reduces the wealth Gini index. However, the magnitudes of the decrease is smaller under CMP than under QE.

for the middle 60% and smallest for the fifth quintile among five quintiles, which confirms CMP's more adverse effects on inequality than QE.

Decomposing the effects shows that for poor households at the bottom of the wealth distribution, a lower debt burden provides almost all of the additional gains under CMP, which offsets the losses from (relatively) higher unemployment rates and lower transfers.[92] In contrast, for wealthy households, the response of profits provides all of the additional benefits from CMP.[93] And larger effects on profits, relative to those on other variables, is partly due to the benefits of lower real rates for banks in the case of CMP. That is, there is an additional redistribution towards the financial sector in the case of CMP, which cancels out part of the welfare losses from lower rates for wealthy households.

2.6 The effects of forward guidance

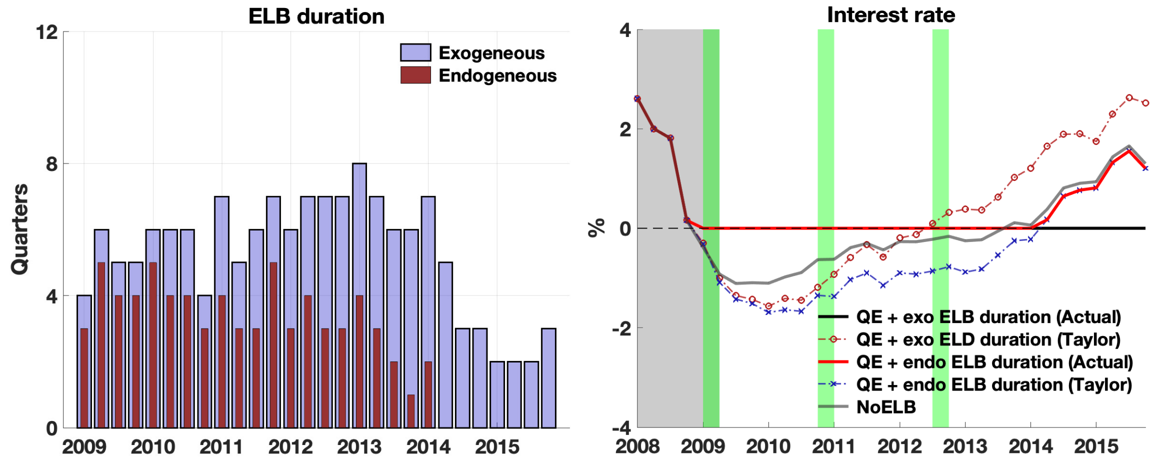
The results presented so far for QE are due not only to the central bank's asset purchases but also to exogenous ELB durations and maintaining the policy rate at zero for longer. In this section, I isolate the effects of asset purchases (pure QE) by simulating the model under the assumption that the expected ELB durations are endogenously determined and that the central bank sets a positive rate as soon as that is prescribed by the Taylor rule.[94]

[92]Since CMP lowers the return on liquid assets and tax revenues are lower in the case of the CMP, lump-sum transfers from the MMMF is smaller.

[93]The average magnitudes of the effects are similar in both cases. However, in the case of the CMP, the initial effects are larger. Since households discount the value of future consumption, larger initial effects on profits lead to welfare gains for wealthy households.

[94]The endogenous ELB durations are computed, using the OccBin method of Guerrieri and Iacoviello (2015).

Figure 2.13: ELB durations and interest rates



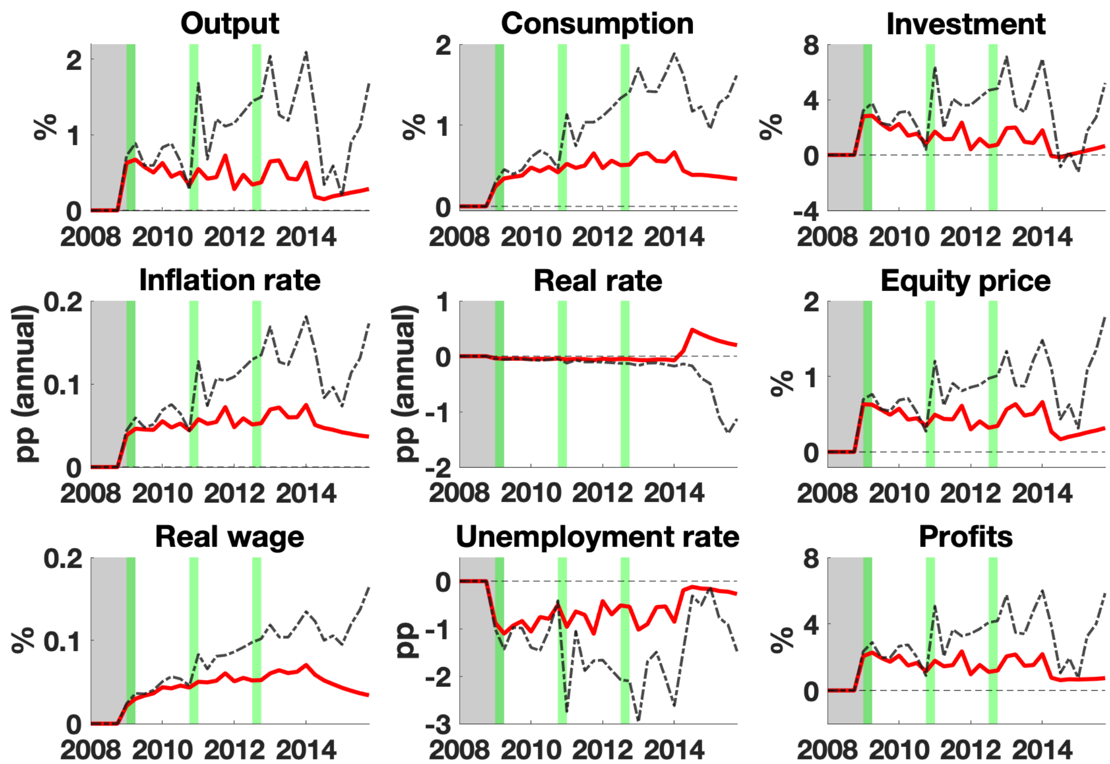
Notes: The left panel shows the expected ELB durations in the baseline case with QE and exogenous ELB durations and in the alternative case with QE and endogenous ELB durations. The dark red bars shows the endogenous ELB durations while the light purple bars show the exogenous ELB durations. The right panel shows the dynamics of the actual policy rate and the policy rate prescribed by Taylor rule in the baseline and the counter-factual case. The thick black line and the red dotted line with circles show the actual policy rate and the Taylor rule prescription in the baseline case. The thick red line and the blue dotted line with crosses show the actual and the Taylor rule prescription in the alternative case.

2.6.1 Aggregate effects

Figure 2.13 shows that exogenous durations are longer than endogenous durations during the entire ELB episode, which implies that there was an additional stimulus from forward guidance. The difference between the endogenous and exogenous durations starts to increase in 2011, which is consistent with the finding of Jones (2017). These longer expected ELB durations work similarly to expected future expansionary monetary policy shocks. Moreover, by maintaining the policy rates at zero, the central bank brings about a further stimulus to the economy around the end of the ELB episode.

When the additional stimulus effects are removed, asset purchases turn out to have much smaller aggregate effects. As discussed in the previous section, this is

Figure 2.14: Aggregate effects of forward guidance



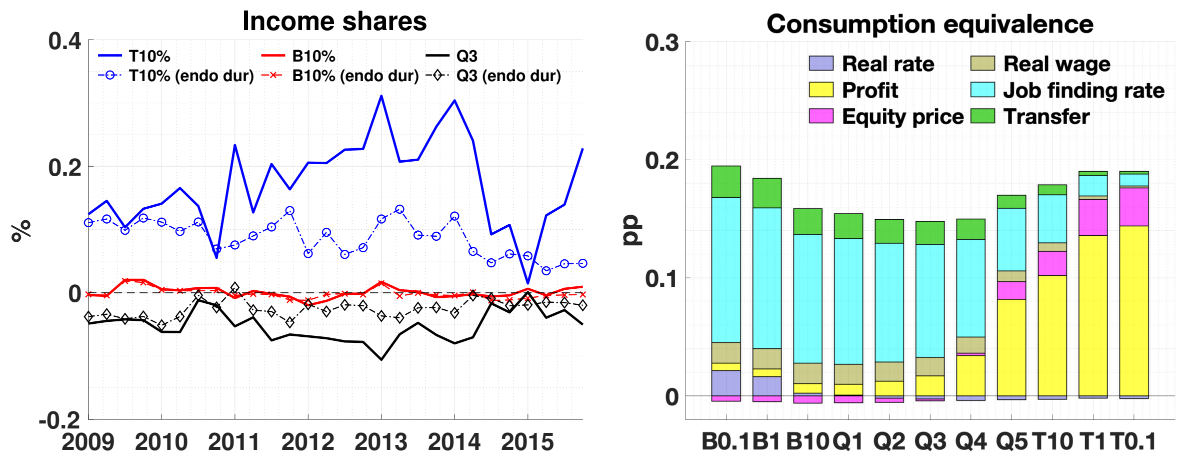
Notes: The solid red and dashed blak lines show the effects of asset purchases with endogenous ELB durations and asset purchases with exogenous durations. Except for inflation, the real interest rate, and the unemployment rate, variables are shown as percentage differences from the corresponding values in the alternative case with no policy interventions. The inflation rate, the real rate, and the unemployment rate are shown as the percentage point differences from their corresponding value in the alternative case.

because asset purchases do not have strong direct stimulus effects on households and crowd out banks' investment.[95] Overall, the central bank's asset purchases account for about 45% of the total aggregate effects of unconventional monetary policies. Unlike the case of CMP, the effects of forward guidance on key aggregate variables are very similar to those of the central bank's asset purchases in terms of the relative magnitudes since, in both cases, the policy rate is fixed at the ELB except for a few quarters at the end of the ELB episode.[96]

[95]Compared to the net effects of QE, conventional monetary policy has stronger overall real effects during the ELB episode.

[96]In the case of CMP, relative magnitudes of the responses of variables are similar to those of

Figure 2.15: Distributional effects of forward guidance: income share and CE



Notes: The left panel shows the income share of different wealth groups as the differences, relative to the corresponding shares in the counterfactual case with no policy interventions. The blue, red, and black straight lines show the shares in the case of QE and exogenous ELB durations. The blue, red, and black dashed lines with marks show the shares in the case of asset purchases and endogenous ELB durations. The right panel shows the additional welfare gains from forward guidance and their decomposition. The units are percentage points.

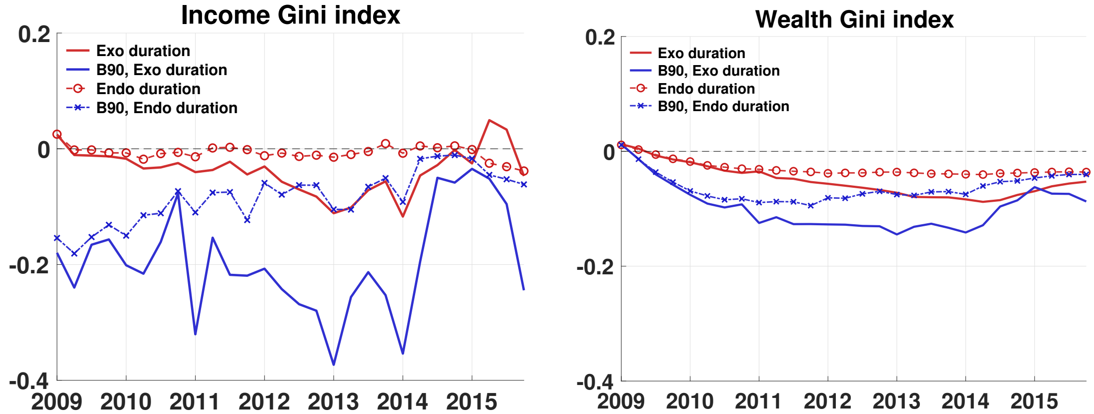
2.6.2 Distributional effects

Next I examine the effects of the additional stimulus from exogenous ELB durations on inequality and the distribution of welfare. The left panel of Figure 2.15 shows the differences in the income shares of different wealth groups between the case of full QE versus pure QE, without the additional stimulus coming from FG and lower rates for longer. Full QE amplified the distributional effects of the central bank's asset purchases both in terms of top income shares and in terms of the Gini index. The top 10% income share increased due to forward guidance, especially in later periods. In contrast, the income share of the middle quintile fell, while the income share of the bottom decile remained largely unchanged.

In the case of the Gini index, Figure 2.16 shows that forward guidance also

QE except for the real interest rate response.

Figure 2.16: Distributional effects of forward guidance: Gini index



Notes: The figure shows the evolution of the income and wealth Gini index under the different assumptions on the expected ELB durations. The blue and red lines show income and wealth Gini index as differences from their respective level in the counterfactual case with no policy interventions. The blue and red dashed lines with circles show income and consumption Gini index when the ELB durations are endogenous.

amplifies the dynamics of the Gini indices induced by asset purchases. As the previous results imply, the Gini index is affected more by the dynamics among households in the bottom 90% of the wealth distribution. Since forward guidance further lowers the unemployment rate, it reduces the wealth and income gap between the bottom 10% and the middle. Accordingly, the degree of inequality, measured by the Gini indices, is lower under full QE, augmented by forward guidance, than under pure QE.

Finally, the welfare effect result is consistent with the result of inequality measures. Additional stimulus from forward guidance benefits both ends of the wealth distribution more than the middle. Specifically, forward guidance preserves the relative ranking of welfare gains across wealth groups and amplifies non-linear welfare effects resulting from asset purchases. I conclude that forward guidance amplifies the distributional effects of asset purchases, operating along the same channels.

2.7 Robustness check

The results presented so far can vary, depending on the parameter values or the specific assumptions on the model structure. In this section, I show that this paper's main results regarding the non-linear distributional effects of QE and sensitivity of the Gini index to dynamics in the lower 90% of the distribution are robust to variations in parametrizations and modeling assumptions.

Table 2.7 summarizes a set of robustness test results, including the effects on key aggregate variables, the Gini index and the top 10% income share, and welfare effects on a set of households wealth groups. The aggregate effects are shown as the ratios to the effects on output. For instance, 0.81 in the third row and the second column (from the left) implies that, in the baseline case, equity prices are, on average, 0.81% higher when output is, on average, 1% higher during the ELB episode compared to when the central bank did not conduct QE. In the case of unemployment rates and real interest rates, the effects are measured as percentage points. The second column reproduces the baseline resultsfrom Section 4.

In the third column, I double the vacancy posting costs to make extensive margin adjustment of labor more costly. In the fourth column, the degree of wage rigidity is set to 0.65, versus the baseline value of 0.8. In the fifth column, I assume that profits from the financial sector are not distributed to equity holders or business owners. Thus, I remove any direct effects of banks' profits on households' inequality

Table 2.7: Robustness check 1/2

	Baseline	(1)	(2)	(3)	(4)	(5)	(6)
Aggregate variables							
Equity prices	0.81	0.80	0.82	0.72	0.71	0.77	0.84
Profits	3.04	2.81	2.77	1.93	1.51	3.01	3.09
Real wage	0.08	0.18	0.20	0.08	0.21	0.08	0.09
Unemployment rates	-1.34	-1.26	-1.27	-1.34	-1.25	-1.34	-1.33
Real rate	-0.23	-0.27	-0.28	-0.23	-0.28	-0.23	-0.25
Gini index							
Wealth	-0.05	-0.07	-0.08	-0.07	-0.10	-0.10	0.03
	(-0.09)	(-0.13)	(-0.14)	(-0.11)	(-0.18)	(-0.19)	(0.09)
Income	**-0.04**	**-0.05**	**-0.06**	**-0.13**	**-0.18**	**-0.25**	**0.15**
	(-0.12)	**(-0.15)**	**(-0.16)**	**(-0.25)**	**(-0.36)**	**(-0.54)**	**(0.26)**
Consumption	0.05	0.04	0.04	0.00	-0.03	-0.08	0.11
	(0.10)	(0.08)	(0.08)	(-0.03)	(-0.07)	(-0.17)	(0.18)
Top 10% share							
Wealth	-0.01	-0.03	-0.03	-0.03	-0.06	-0.05	0.05
	(-0.03)	(-0.06)	(-0.07)	(-0.06)	(-0.11)	(-0.11)	(0.12)
Income	**0.17**	**0.17**	**0.16**	**0.08**	**0.04**	**0.15**	**0.17**
	(0.31)	**(0.32)**	**(0.31)**	**(0.16)**	**(0.12)**	**(0.24)**	**(0.31)**
Consumption	0.06	0.06	0.06	0.02	0.01	0.04	0.07
	(0.10)	(0.10)	(0.09)	(0.05)	(0.03)	(0.07)	(0.10)
CE (Δ C during the ELB episode)							
T10	0.32	0.38	0.37	0.22	0.25	0.39	0.25
	(1.16)	(1.35)	(1.35)	(0.88)	(0.99)	(1.23)	(1.05)
Q3	0.26	0.33	0.34	0.24	0.32	0.41	0.15
	(0.81)	(0.99)	(1.01)	(0.75)	(0.95)	(0.96)	(0.64)
B10	0.28	0.36	0.37	0.27	0.36	0.49	0.15
	(0.84)	(1.03)	(1.05)	(0.79)	(1.00)	(1.05)	(0.65)
Average	0.27	0.34	0.67	0.24	0.31	0.42	0.17
	(0.90)	(1.07)	(1.09)	(0.78)	(0.96)	(1.03)	(0.74)

Notes: (1): High vacancy posting costs ($\iota = 0.063$), (2): Less rigid wage ($\rho_w = 0.65$), (3): Excluding banks' profit from aggregate profit, (4): (3) + Less rigid wage ($\rho_w = 0.65$), (5): Replacement ratio = 10% ($\upsilon = 0.1$), (6) Replacement ratio = 70% ($\upsilon = 0.7$). The table shows the average effects on aggregate variables, the Gini index, and the top 10% shares in each case. Except for the specified parameter value in each case, all other parameter values are set to values at the posterior mode. In all cases, shocks are re-filtered for a given set of parameters and observables. The average effects are shown as ratios to the effects on output. The Gini index, the top 10% shares, and consumption equivalents are shown as percentage points differences. The values in the parenthesis are the maximum or minimum of the corresponding variable during the ELB episode in the case of the Gini index and the top 10% shares. In the case of CEs, the values in the parenthesis show the amount of consumption increase during the ELB episode as percentage differences relative to the level of consumption in the counterfactual case of no unconventional monetary policies.

or welfare. In the sixth column, I assume that the wage is less rigid, i.e., $\rho_w = 0.65$, and exclude banks' profits from the aggregate profits. In the last two columns, the replacement ratio for the unemployment benefit is set to 10 and 70%, respectively, while maintaining other parameter values at the posterior mode.

Table 2.8: Robustness check 2/2

	Baseline	(7)	(8)	(9)	(10)
Aggregate variables					
Equity prices	0.88	0.88	0.88	0.88	0.88
Profits	3.28	3.28	1.65	1.65	3.28
Real wage	0.09	1.09	0.09	1.09	0.09
Unemployment rates	-1.45	-1.45	-1.45	-1.45	-1.45
Real rate	-0.25	-0.25	-0.25	-0.25	-0.25
Gini index					
Wealth	-0.05 (-0.09)	-0.09 (-0.14)	-0.06 (-0.11)	-0.10 (-0.16)	-0.06 (-0.10)
Income	**-0.04 (-0.12)**	**-0.12 (-0.20)**	**-0.16 (-0.34)**	**-0.25 (-0.42)**	**-0.05 (-0.13)**
Consumption	0.05 (+0.10)	-0.06 (-0.12)	-0.02 (-0.05)	-0.12 (-0.16)	0.04 (0.09)
Top 10% share					
Wealth	-0.01 (-0.03)	-0.06 (-0.10)	-0.03 (-0.06)	-0.07 (-0.13)	-0.02 (-0.03)
Income	**0.17 (0.31)**	**0.06 (0.20)**	**0.05 (0.09)**	**-0.06 (-0.13)**	**0.15 (0.26)**
Consumption	0.06 (0.10)	-0.02 (-0.06)	0.01 (0.04)	-0.07 (-0.08)	0.06 (-0.1)
CE (Δ C during the ELB episode)					
T10	0.32 (1.16)	0.81 (1.67)	0.23 (0.87)	0.72 (1.37)	0.32 (1.16)
Q3	0.26 (0.81)	1.30 (1.89)	0.25 (0.78)	1.29 (1.86)	0.27 (0.82)
B10	0.28 (0.84)	1.37 (1.98)	0.28 (0.82)	1.37 (1.95)	0.29 (0.86)
Average	0.27 (0.90)	1.23 (1.84)	0.25 (0.80)	1.20 (1.74)	0.28 (0.90)

Notes: (7): 1 percentage point higher real wage, (8) 50% lower profits, (9) 1 percentage point higher real wage + 50% lower profits, (10) Assuming the steady state distribution in 2009 Q1. The table shows the average effects of QE on the Gini index and the top 10% shares during the ELB episode. All the effects are computed from the micro-level simulation only. That is, without estimating the aggregate effects, assumed effects are applied to households' distribution. The Gini index, the top 10% shares, and consumption equivalents are shown as percentage points differences. The values in the parenthesis are the maximum or minimum of the corresponding variable during the ELB episode in the case of the Gini index and the top 10% shares. In the case of CEs, the values in the parenthesis show the amount of consumption increase during the ELB episode as percentage differences relative to the level of consumption in the counterfactual case of no unconventional monetary policies.

In Table 2.8, I evaluate the distributional consequences of QE by assuming different paths of profits and real wages. In the third column, I assume that the effects of QE on real wages are one percentage point greater than in the baseline case. In the fourth column, I instead assume that the effects of QE on profits are

50% smaller. In this case, profits are only 1.65% higher than the counterfactual case with no unconventional monetary policies. In the fifth column, I adopt both assumptions. Thus, real wages are, on average, 1.09% higher, while profits are only 1.65% higher than in the counterfactual case. In the last column, I use the steady state households' distribution instead of the model's distribution at the beginning of 2009 Q1 to see if the differences in households' distribution have a significant impact on the main results of the paper.

As the tables show, the main result of this paper is maintained in most cases: QE increases the top 10% income and consumption share while also reducing overall income and wealth inequality, as measured by the Gini index. When vacancy posting costs are higher, or the degree of wage rigidity is lower, the relative magnitudes of the increase in real wages are larger than in the baseline case.[97] As a result, income inequality, as measured by the Gini index, declines by a greater amount than in the baseline case. However, even in these cases, the top 10% income and consumption shares increase because of QE relative to a case without unconventional monetary policies. Similarly, when profits from the financial sector are excluded from aggregate profits, relative increases in profits are smaller. Thus, the income Gini index decreases by a larger amount. When the wage is less rigid, the income Gini falls, on average, by 0.18 percentage points, which is almost four times larger magnitude than that in the baseline case. However, the top 10% income and consumption shares still rise even though the magnitudes are smaller than in the baseline case.

[97] When the vacancy posting cost is high, firms utilize capita more, which increases labor demand by the complementarity between inputs. Thus, the real wage rises by more compare to when the vacancy posting cost is smaller.

In terms of welfare gains, U-shaped effects are not preserved when banks' profits are excluded from aggregate profits, and thus, not distributed to business owners or equity holders (case (3) & (4)). In those cases, consumption equivalents are decreasing in wealth, unlike in the baseline case. However, even in these cases, the top 10% households experience a higher consumption increase than the middle quintile during the ELB episode. That is, income and consumption gains are U-shaped during the ELB episode, but the expected contractionary effects of tapering mostly offset the welfare gains of wealthy households. Accordingly, the long-run welfare effects become monotonic in these cases. In cases (7) and (9), welfare effects are also monotonically decreasing in wealth. However, in these cases, the assumed wage increases are implausibly high given the degree of wage rigidity that I find via estimation.

Interestingly, I find that fiscal policy, specifically the extent of the unemployment benefit, matters for the distributional consequences of QE. When the replacement rate is only 10%, QE reduces the income Gini index by a much larger amount compared to other cases. Also, welfare gains from QE for the bottom 10% households are much larger than their welfare gains in the baseline case, even though the magnitudes of the unemployment rate and real wage responses are similar. This is because, when the unemployment benefit is relatively smaller, income gain from being employed is much larger. For the same reasoning, if the unemployment benefit is much larger, then income gain from being employed is substantially smaller, and thus, QE mostly increases income and consumption inequality without providing significant benefits to the bottom 90% households.

To recapitulate, the main results that I find in this paper hold across different parametrizations and modeling assumptions unless the unemployment benefit is improbably high or the relative magnitudes of profits and wage responses implausibly similar. Also, the deviations of the households' distribution from the steady state distribution do not have any significant impact on the main results on the distributional consequences of QE.

2.8 Conclusion

In this paper, I examine the distributional consequences of QE during the ELB episode that followed the Great Recession in the U.S. To this end, I develop a medium-scale HANK model that features portfolio choice, wage rigidity, labor market frictions, banks, and a zero lower bound on the policy rate. I model quantitative easing as central bank private asset purchases, as in Gertler and Karadi (2011), and forward guidance as exogenous expected ELB durations, as in Jones (2017). I parametrize the model to match the micro-data on households' wealth and income composition. Moreover, to discipline the model's parameters associated with the dynamics of key aggregate variables, such as the real wage, the unemployment rate, and profits, I estimate the model with the macro data on the U.S. economy using Bayesian methods.

The estimated model generates empirically plausible dynamics of wages, unemployment, and profits to exogenous shocks. In particular,it generates a procyclical response of profits to an expansionary monetary policy shock, unlike most existing

New Keynesian models. Because of this, the model uncovers wealthy households' substantial benefits from expansionary monetary policy that existing New Keynesian models cannot capture.

A counterfactual analysis reveals that QE reduced the wealth and income Gini indices during the ELB episode, mainly via its positive impacts on employment. However, at the same time, QE widens the income gap between the top 10% and the bottom 90% by substantially increasing profits and equity prices.

The results of this paper suggests that both the criticism regarding the adverse effects of QE on inequality and the counterargument based on QE's positive impacts on labor markets can be justified, depending on the focus. If one focuses on the gap between the top 10% and all other households, QE can be seen as increasing inequality. If one focuses on the improvement of welfare at the bottom, QE can be seen as reducing inequality, as it reduces the gap between the bottom 10% and the middle of the wealth distribution. Importantly, the result also implies that if a model fails to capture wealthy households' benefit from monetary policy, an analysis based on it can lead to a misleading or incomplete conclusion on the effects of monetary policy on inequality.

Chapter 3: Distributional Consequences of a Higher Inflation Target

3.1 Introduction

The low and falling real interest rates observed over the past few decades in the U.S. and other advanced countries have raised concern for the conduct of monetary policy, as for a given inflation target, a lower real interest rate reduces the nominal interest rate and increases the potential incidence of a binding ELB constraint.[1] In response, several economists have suggested an increase in the inflation target.[2] Since a higher inflation target raises the steady-state nominal interest rate, it can reduce the probability of hitting the ELB.

In this chapter, I examine such benefits of a higher inflation target as well as the transition path of the economy towards a higher inflation target. To this end, I use the HANK (Heterogeneous Agent New Keynesian) model developed and estimated in Lee (2021) and raise the inflation target and the steady-state nominal interest rate from 2% to 4% in annualized terms. Then, I examine the transition from the steady-state with the 2% inflation target to the steady-state with the 4% inflation target. Moreover, I investigate fluctuations of the economy around the

[1] For discussion of the secular decline of the real interest rate, see Laubach and Williams (2016) and Holston et al. (2017).

[2] See, among others, Blanchard et al. (2010) and Ball (2014).

two steady-states over an extended period, by taking into account possible ELB episodes.

The analysis in this chapter shows that the economy experiences a significant expansion during the transition when the Phillips curve is forward-looking and the nominal interest rate adjusts gradually. The effects are particularly strong on the unemployment rate, equity price, and profits. In contrast, the real wage barely increases during the transition, and falls in some parametrizations due to the rapid increase in the inflation rate during the transition.

Such aggregate effects of the transition lead to non-linear distributional consequences, similar to those emphasized in Lee (2021); both ends of the wealth distribution enjoy higher income, consumption, and long-term welfare gains than the middle class. Quantitatively, the benefit is the biggest at the bottom of the wealth distribution. Thus, the overall degree of inequality, as measured by the Gini index, falls during the transition despite larger income and consumption gains at the top compared to the middle.

To study the long-run stochastic properties of the economy around different steady-states, the model is simulated for 100,000 quarters with a set of exogenous shocks. The simulation results show that the higher inflation target reduces both the frequency and duration of ELB episodes. At the higher target, the simulated economy stays at the ELB for around 3% of the time, while the fraction of the time spent at the ELB is more than 8 percentage points higher in the baseline case. Moreover, the average ELB duration is about 10.5 quarters in the baseline case, while it is only about 7.5 quarters in the model with the higher inflation target.

By making the ELB episodes less frequent and relatively short-lived, the 4% inflation target boots and stabilizes the aggregate economy. The average levels of output, consumption, and the aggregate capital stock are noticeably higher in the model with the higher inflation rate. In particular, the unemployment rate is lower and less volatile in the model with the higher inflation target than in the lower inflation baseline case.

Lower unemployment risks in the higher inflation target case enable households at the middle and the bottom of the wealth distribution to consume more goods and purchase a larger amount of equity than in the baseline case. This increase in aggregate demand leads to higher profits. However, greater demand for equity increases equity prices and reduces dividend rate on equity holdings.[3] Consequently, wealthy households hold a relatively smaller amount of wealth compared to the baseline case and thus, enjoy relatively smaller levels of income and consumption compared to other households in the economy. In short, the higher inflation target reduces the degree of wealth, income, and consumption inequality in the economy.

The remainder of the chapter is organized as follows. The second section discusses the analysis framework briefly. The third section shows and discusses the results regarding the transition path of the economy. The fourth section presents the results from the simulation and discusses the aggregate and distributional consequences of the higher inflation target. The fifth section concludes.

[3]In the model, the dividend is a fraction of profit per capital. Thus, if the aggregate capital stock is higher, then the dividend rate can be lower despite higher profits.

3.2 The model

I use the model developed and parametrized in Lee (2021) as the baseline model. The only difference is that the steady-state nominal interest rate is set to 2% in annualized terms, while maintaining the same steady-state real variables by modifying the annuity arrangement regarding bond holdings in the model.[4] For more detailed description, discussion of the estimation, and fit of the model, see Lee (2021).

Since there are no inefficiency costs that arise from price or wage rigidity at the steady-state of the model, real variables remain unchanged when the inflation target and the nominal interest rate increase by the same amount. However, if the inflation target and the steady-state nominal interest rate change, the model can experience a transition path as economic agents in the model re-optimize their behavior based on the new inflation target. Among others, the real wage, the inflation rate, and the nominal policy rate should adjust based on the new inflation target and the new

[4] Since a certain fraction of households dies each period, an annuity arrangement should be put in place to process the deceased households' asset holdings. In Lee (2021), I assume that the deceased households' bond holdings are discarded. Instead, in this chapter, I assume that the deceased households' bond holdings are distributed to surviving households in proportion to their bond holdings. Hence, the effective rate of return on bond holdings in the model is the same in Lee (2021) and this chapter.

steady-state nominal interest rate, according to the following equations.

$$\text{Wage formula:}\quad \frac{w_t}{w} = \left(\frac{1}{\epsilon_t^w}\frac{r_t^l}{r^l}\right)^{1-\rho_w}\left\{\frac{w_{t-1}}{w}\times\left(\frac{\pi_{t-1}}{\pi_t}\right)^{\iota_w}\times\left(\frac{\pi}{\pi_t}\right)^{1-\iota_w}\right\}^{\rho_w} \tag{3.1}$$

$$\text{NKPC:}\quad \log\pi_t - \log\pi_{t-1}^{\iota_p}\pi^{1-\iota_p} = \mathbb{E}_t\Big[\Lambda_{t,t+1}\big(\log\pi_{t+1} - \log\pi_t^{\iota_p}\pi^{1-\iota_p}\big)\Big] + \kappa\left(\text{MC}_t - \frac{1}{\Psi_t^p}\right) \tag{3.2}$$

$$\text{Taylor rule:}\quad i_{t+1} = i + \rho_R(i_t - i) + (1-\rho_R)\{\phi_p i\log(\pi_t/\pi) + \phi_u(u_t - \bar{u})\} + \epsilon_{R,t} \tag{3.3}$$

As I will explain in more detail, the transition dynamics crucially depends on the relevant parameter values in the above equations.

3.3 Transition dynamics

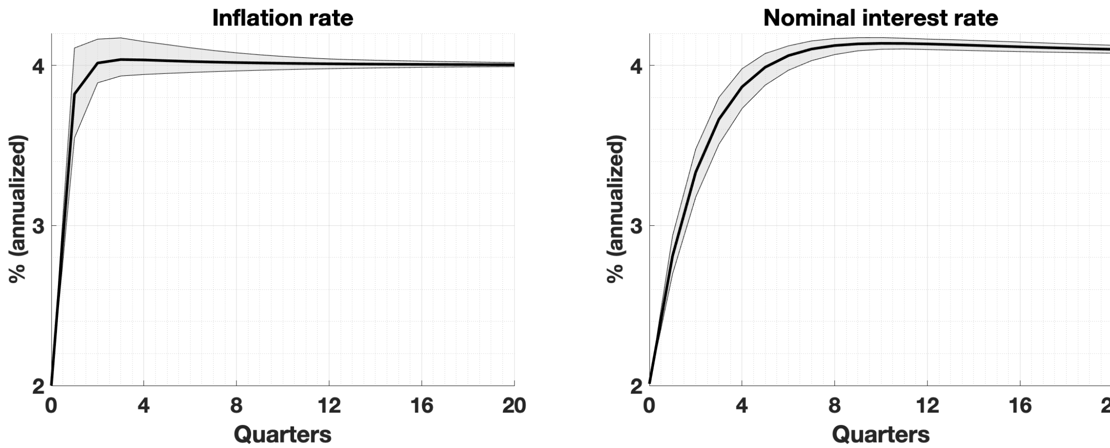
To examine the aggregate and distributional effects of transitioning toward a higher inflation target, I assume that the central bank announces that the new inflation target is higher than the previous level by two percentage points in annualized terms. That is, the central bank raises the inflation target from 2% to 4% in period 1. Also, I assume that economic agents, i.e., households and firms, fully trust the central bank's announcement. Since there is no inefficiency that arises from price or wage rigidity in the model, real variables in the two different steady-states, i.e., one with a lower inflation target and the other with a higher inflation target, are identical.

The dynamics of the economy during the transition path crucially depend on how the nominal interest rate, the inflation rate, and the real wage adjust based on the new target. First, under the new higher inflation target, the steady-state nominal rate is also higher by the same amount, as the real interest rate should be identical across two regimes. Hence, the central bank needs to increase its policy rate during the transition. Depending on the degree of interest rate smoothing and the aggregate status of the economy, the pace of the interest rate adjustment can vary. Also, goods producers increase their prices under the new regime. If the degree of backward-looking behavior in the Phillips curve is strong, i.e., if the fraction of firms that index their prices to the previous inflation rate is large, then the aggregate inflation rate adjusts relatively slowly. In contrast, if the Phillips curve is significantly forward-looking, then the inflation rate adjustment is relatively fast. Finally, the pace of real wage adjustment depends on the degree of the indexation to previous inflation in the wage function. If a higher fraction of the real wage is indexed to the new inflation target, the real wage can increase significantly once the new regime is announced. However, if the real wage is mostly indexed to previous inflation, the real wage adjusts relatively slowly. If the interest rate adjusts smoothly while the inflation response is immediate, the economy is likely to experience an expansion as the real interest rate falls significantly in the first few periods of the transition. If the interest rate adjusts faster than the inflation rate, the economy can experience a contraction during the transition.[5] In the following subsections, I use parameter values that are drawn from the Markov chain computed in Lee (2021).[6]

[5]If a larger fraction of the real wage is indexed to the new inflation target, the economy still can enjoy an expansion because of higher real wages.

[6]The central bank can choose a path of nominal interest rate that it considers desirable to achieve its goal during the transition. I do not assume that the central bank has any intention to affect the economy in a way that it wants. In other words, the conduct of monetary policy is the same as before the transition.

Figure 3.1: Transitional dynamics of the inflation and the nominal rate



Notes: The figure shows how the inflation and the nominal interest rate change during the transition path. Transitional dynamics are evaluated with 5,000 sets of parameters that are randomly drawn from the chain estimated in the chapter 2. The grey areas show the 10 to 90 percentile region of the responses. The black lines are the mean responses.

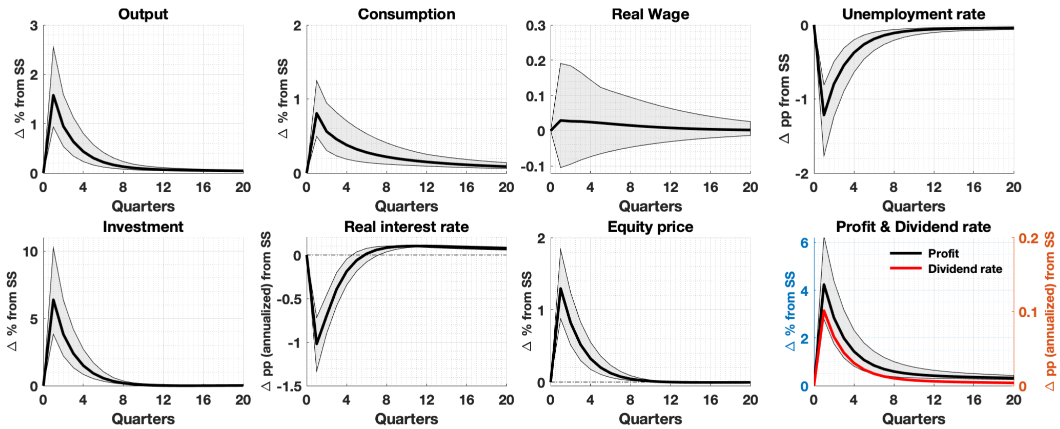
Results based on different assumptions on the inflation rate, the nominal interest rate, and the real wage behavior are presented in the appendix.

3.3.1 Aggregate dynamics

Figure 3.1 shows the dynamics of inflation and the nominal interest rate during the transition towards a higher inflation target. As shown in Table 2.6, the degree of backward-looking pricing behavior is relatively weak, i.e., ι_p is small, while the degree of interest smoothing is quite large. Accordingly, the inflation rate quickly jumps, but the nominal rate rises only gradually. Hence, the real interest rate falls, and the economy experiences a substantial expansion at the beginning of the transition.

As shown in Figure 3.2, the real interest rate falls by about one percentage point in annualized terms in the first period. Since the real interest rate falls signif-

Figure 3.2: Aggregate effects of raising the inflation target

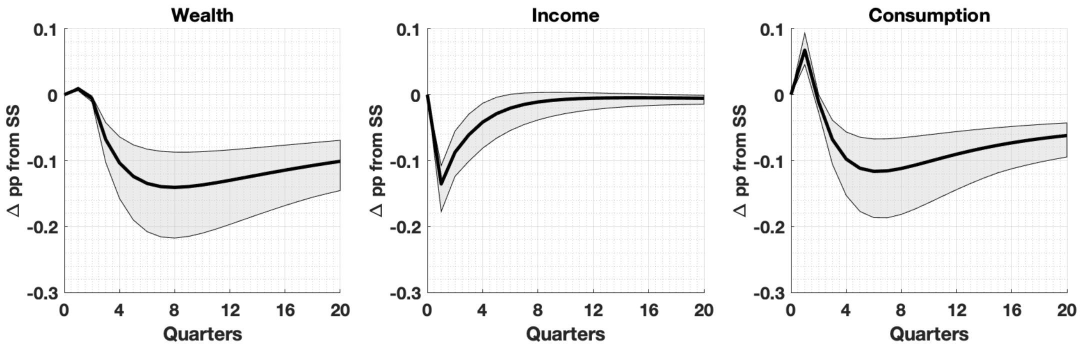


Notes: The figure shows how aggregate variables change during the transition path. Transitional dynamics are evaluated with 5,000 sets of parameters that are randomly drawn from the chain estimated in the chapter 2. The grey areas show the 10 to 90 percentile region of the responses. The black lines are the mean responses.

icantly, consumption increases by inter-temporal substitution. Also, as households adjust their portfolio composition based on assets' relative returns, investment rises. This increase in consumption and investment stimulates production and increases asset prices. As firms expand their production, labor demand increases, which leads to a fall in the unemployment rate. However, due to a rapid rise in the inflation rate and nominal wage rigidity, the real wage increases only slightly. Under some parametrizations, the real wage actually falls during the transition. Thus, the unemployment rate falls substantially during the transition. As firms enjoy higher output demand and relatively flat marginal costs, their profits go up.

The magnitudes of the responses are quite significant. Output, consumption, and investment increase by about 2%, 1%, and 6% on average in the first period, respectively. Equity prices and profits also rise substantially, by about 1% and 4% in the first period. The unemployment rate also falls by more than one percentage point on average in the first period. In contrast, the average response of the real

Figure 3.3: Inequality dynamics during the transition: Gini index



Notes: The figure shows the changes in wealth, income, and consumption Gini index during the transition dynamics. The unit is perecentage point.

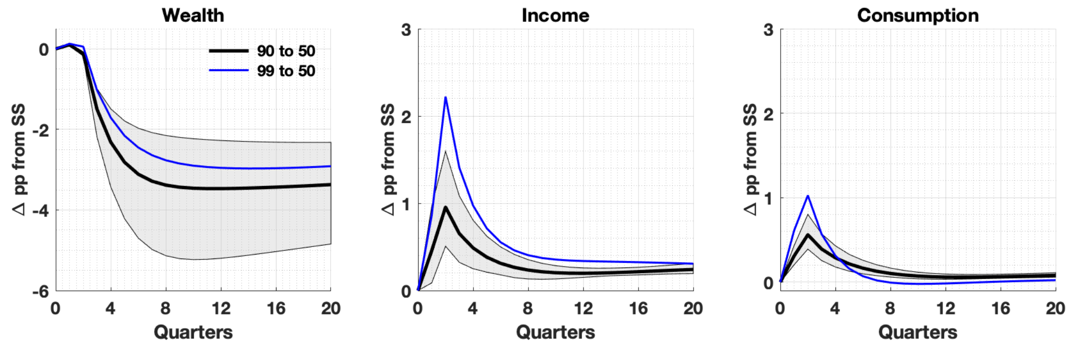
wage is less than 0.1%, and the magnitude of change is sometimes negative, as shown in the right panel of Figure 3.2.

3.3.2 Distributional consequences

Given that households at the bottom of the wealth distribution are mostly in debt and a large fraction of them are unemployed, the lower unemployment rate and real interest rate are likely to benefit them substantially, which reduces inequality. However, higher profits and equity prices during the transition are likely to benefit households at the top of the wealth distribution, which increases inequality. Thus, the net distributional consequences of the transition to a higher inflation target are *ex-ante* unclear. To investigate how inequality evolves during the transition, I compute the changes in two measures of inequality; the Gini index and the 90 to 50 ratio. The first measure examines the overall degree of inequality, while the latter one focuses on the dynamics at the top relative to the middle.

First, Figure 3.3 shows the changes in the wealth, income, and consumption

Figure 3.4: Inequality dynamics during the transition: log 90 to 50 percentile ratio



Notes: The figure shows the changes in the log 90 to 50 ratio of wealth, income, and consumption during the transition dynamics. The unit is perecentage point.

Gini indices during the transition. As shown in the figure, all of Gini indices fall during most of the transition, which implies that the effects of the lower unemployment rate and real rate dominate the countervailing effects of higher profits and equity prices on inequality. The consumption Gini initially increases, because the significant fall in the real interest rate motivates relatively wealthy households to consume more. However, the initial decrease in savings due to lower real interest rates leads to less consumption of wealthy households in the future, which reduces overall consumption inequality. Meanwhile, higher equity prices increase overall wealth inequality slightly at first. However, as newly employed households reduce their debt and/or accumulate assets, wealth inequality also falls as the transition continues. The effects of the transition on wealth and consumption inequality are persistent while the effects on income inequality are transitory.[7]

Though the overall degree of inequality, as measured by Gini index, decreases during the transition, it may still be the case that households at the top of the wealth

[7]Wealth inequality dynamics are much more persistent than income inequality dynamics in HANK models with two assets since, in these models, adjusting one of the assets are costly, and thus, portfolio adjustment occurs very gradually.

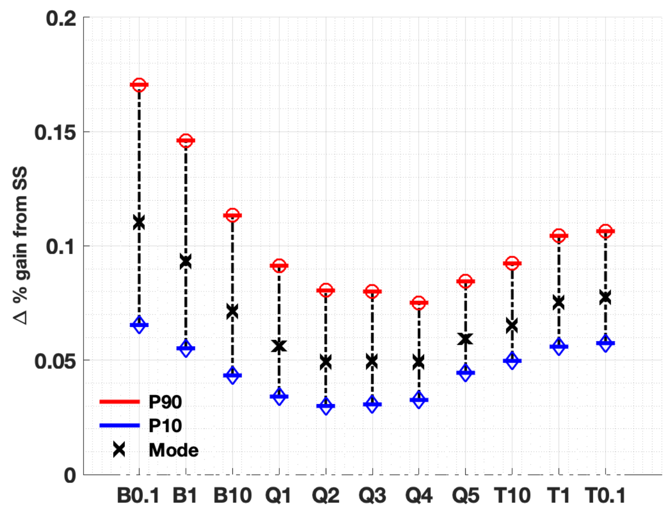
distribution enjoy larger gains than households at the middle. Figure 3.4 confirms that the top 10% wealthiest households enjoy higher income and consumption gains than the median household. According to the figure, households at 90th percentile of initial net worth enjoy 0.5 to 1.5 percentage point larger income growth in the first period compared to the households at the middle. Though the magnitudes are smaller, the top 10% wealthiest households also enjoy about 0.5% larger consumption growth in the first period of the transition.[8] These results show that, as in Lee (2021), there are non-linear distributional consequences during the transition; both ends of the wealth distribution enjoy higher income and consumption gains than the middle.[9]

Figure 3.5 shows the welfare gains from the transition across wealth groups, as measured by consumption equivalents. That is, 0.1 on the y-axis implies that households are indifferent between enjoying 0.1% higher consumption each period at the steady-state and enjoying the expansion during the transition. As shown in the figure, the results are consistent with the results regarding the Gini index and the log 90 to 50 ratios. On average, households at the bottom 10% and below of the wealth distribution enjoy the largest welfare gains compared to other groups, which is consistent with the decrease in the overall degree of inequality during the transition. Also, households at the top 10% of the wealth distribution enjoy higher welfare gains compared to households at the middle. The result is consistent with

[8]The 90 to 50 percentile wealth ratio still falls. This is because wealthy households either sell their equity holdings at a higher price or purhcases a less *amount* of equity holdings due to higher prices.

[9]The 50 to 10 income and consumption ratio (not shown) significantly falls during the transition. That, is the gaps between the bottom and the middle significantly shrinks.

Figure 3.5: Welfare gains across wealth groups during the transition



Notes: The figure shows welfare gains from the transition, measured by consumption equivalents, across households' wealth groups. Welfare gains are computed using 5,000 sets of parmaeter draws from the estimated Markov chain. The blue diamonds represent the 10 percentile of computed welfare gains for each group while the red circles represent the 90 percentile of welfare gains. The black x marks show the mode of welfare gains for each wealth group. B0.1, B1, and B10 represent the bottom 0.1%, 1%, and 10% wealth groups. T10, T1, and T0.1 represent the top 10%, 1%, and 0.1% represent the top 10%, 1%, and 0.1% wealthiest household groups, respectively.

higher log 90 to 10 income and consumption ratios during the transition.

The results in this section show that the transition towards a higher inflation target reduces overall inequality by benefiting households at the bottom of the wealth distribution the most. However, households at the top of the wealth distribution also enjoy above-average welfare gains, because of higher profits and equity prices. Households at the middle of the wealth distribution enjoy the least gains, as higher inflation rates reduce both the real interest rate on bond holdings and the value of their labor income.

3.4 Long-run stochastic behavior

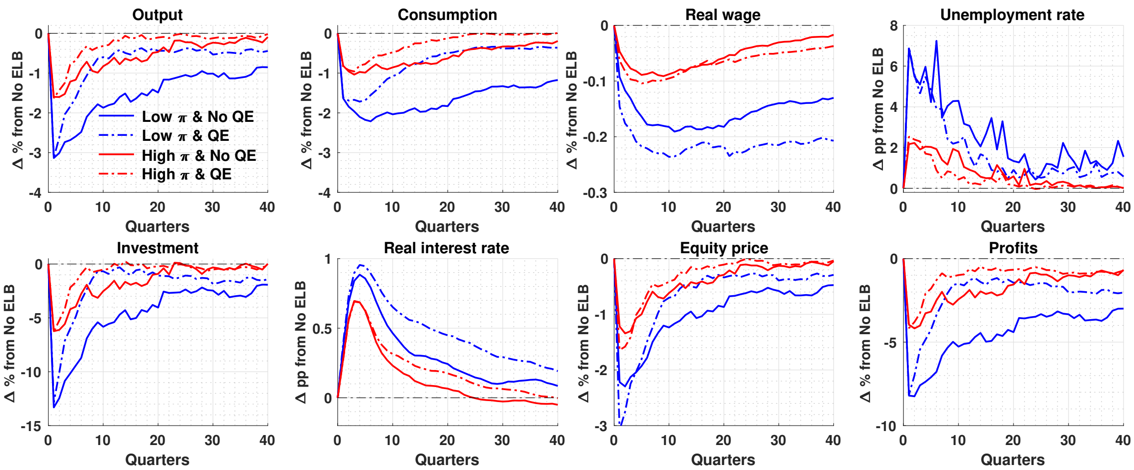
One of the reasons some economists have advocated a higher inflation target is to avoid ELB episodes, which can lead to severe recessions. By having a higher inflation target, accompanied by a higher steady-state nominal interest rate, the central bank can have more room to lower its policy rate to fight a recession. That is, the probability of hitting the ELB is likely to be lower if the inflation target is higher. In this section, I examine this benefit of a higher inflation target by simulating models with different target inflation rates. Specifically, I simulate models with productivity, risk premium, investment technology, price markup, and wage shocks for 100,000 quarters and check whether the ELB binds each period for a given set of exogenous shocks. If the ELB binds, I compute a perfect foresight transition path from the ELB regime using the OccBin methodology. I also measure the effects of QE during the simulations by comparing simulations with and without QE operations. I use the following QE policy rule, which is similar to Cui and Sterk (2018).

$$A_{t+1}^{\mathrm{CB}} = \chi_t^{\mathrm{QE}} A^{\mathrm{CB}} \tag{3.4}$$

$$\chi_t^{\mathrm{QE}} = \rho_{\mathrm{QE}} \chi_{t-1}^{\mathrm{QE}} + (1 - \rho_{\mathrm{QE}}) \big\{ \phi_\pi^{\mathrm{QE}} \log(\pi/\pi_t) + \phi_u^{\mathrm{QE}} (u_t - u) \big\} \tag{3.5}$$

where A^{CB} is the central bank's asset holdings, with the notation without time subscripts representing the corresponding steady-state value. The autocorrelation and the responsiveness to the inflation and unemployment gaps are denoted by ρ_{QE}, ϕ_π^{QE}, and ϕ_u^{QE}, respectively. As in Lee (2021), the central bank finances its asset

Figure 3.6: Consequences of a binding ELB constraint



Notes: The figure shows the average effects of a binding ELB constraint on aggregate variables during the simulation, as percentage or percentage point differences relative to hypothetical cases in which the ELB does not bind. The red and blue solid lines show the cost of a binding ELB constraint in a lower and higher long-run inflation cases, respectively. The dashed lines show the effects of QE operations on aggregate variables.

purchases by issuing bonds. When the model is out of the ELB, the central bank restores its asset holdings to its steady-state level as follows.

$$A_{t+1}^{CB} = \rho_{\text{CB}} A_{t-1}^{\text{CB}} + (1 - \rho_{\text{CB}}) A^{CB} \tag{3.6}$$

For the simulation, I set ρ_{QE}, ρ_{CB}, ϕ_{π}^{QE}, and ϕ_{u}^{QE} to 0.95, 0.95, 50, and 50, respectively.[10]

3.4.1 Aggregate dynamics

When the ELB constraint binds, the central bank cannot lower the policy rate to the level that it needs to cope with exogenous shocks. As a consequence, the real interest rate is higher than the desired level and the economy experiences

[10]Different parameter values for the QE policy rule do not qualitatively change the results presented in this section.

contraction. If the nominal interest rate were higher because of a higher inflation target, then the central bank can lower the real interest rate further. Thus, it is less likely for the economy to be constrained by the ELB, and even if the constraint binds, the difference between the actual real interest rate and its desired level is smaller. Thus, the magnitude of a contraction is, in general, smaller when the long-run inflation rate is higher.

Figure 3.6 shows the average magnitudes of contraction during ELB episodes in different cases. To examine the cost of a binding ELB constraint, I compute the evolution of aggregate variables in a hypothetical economy in which the ELB does not bind. The economy is at the same initial state, but the nominal interest rate can become negative in the hypothetical case. Figure 3.6 shows the percentage or percentage point differences of aggregate variables compared to the level of the same variables in the hypothetical case. The first period in the figure is the beginning of ELB episodes.

As shown in the figure, when the inflation target is lower, the economy experiences more severe recessions when the ELB binds, because the relative level of the real interest rate is higher at the ELB when the inflation target is lower. On average, output, consumption, and investment are lower by 3, 2, and 13% relative to the hypothetical no ELB case, while the magnitudes of contraction are only about 1.5, 1, and 6% when the inflation target is higher. The effects on real wage, unemployment rate, equity price, and profits are also similar. A higher inflation target reduces the effects of a binding ELB constraint on these variables substantially. In short, a higher inflation target reduces the cost of a binding ELB constraint significantly.

Table 3.1: Aggregate effects of the higher inflation target 1

	inc ELB	frac ELB	dur ELB	E(Y)	std(Y)	E(C)	std(C)	E(K)	std(K)
Low π (No QE)	4.24 (-)	11.10 (-)	10.45 (-)	0.59 (-)	3.46 (-)	1.12 (-)	1.67 (-)	-1.66 (-)	0.29 (-)
Low π (QE)	5.02 (0.78)	9.13 (1.97)	7.27 (3.18)	0.81 (0.22)	2.17 (1.29)	1.43 (0.31)	1.22 (0.45)	4.89 (6.55)	0.26 (0.03)
High π (No QE)	1.60 (2.64)	3.03 (8.07)	7.53 (2.92)	1.13 (0.54)	1.82 (1.64)	1.94 (0.50)	1.15 (0.52)	8.95 (10.6)	0.23 (0.06)
High π (QE)	1.74 (2.50)	2.62 (8.48)	6.03 (4.42)	1.18 (0.59)	1.69 (1.77)	2.00 (0.88)	1.11 (0.56)	9.87 (11.5)	0.23 (0.06)

Note: inc ELB, frac ELB, and dur ELB stand for the incidence of ELB episodes per 100 years, the fraction of the quarters during which the model stays at the ELB, and the average duration of ELB episodes during the simulation, respectively. Y, C, and K denote output, consumption, and capital stock, respectively. $E(\cdot)$ and $std(\cdot)$ stand for the mean and standard deviation. No QE and QE refer to cases in which the central bank does and does not conduct QE when it is constrained by the ELB, respectively. Low (high) π represents the model with the low (high) inflation target. The mean values are expressed as % differences relative to its steady-state values. Standard deviations are the standard deviations of the quarterly growth rates of the corresponding variables. Values in the parenthesis are the differences relative to the baseline case with the low inflation target and no QE operation. The red numbers indicate minus values, while blue numbers indicate positive values.

QE operations at the ELB also have similar effects, as shown in the figure.

Tables 3.1 and 3.2 show that raising the inflation target lowers the incidence, frequency, and duration of ELB episodes. In the baseline setup with a 2% inflation target and no QE operations, the economy is at the ELB about 11% of the time, and the average duration of the ELB episodes is about 10.5 quarters. In terms of the incidence, the model economy expirences 4.2 ELB episodes per 100 years. Raising the inflation target reduces incidens, frequency, and duration of ELB episodes substantially. With the 4% inflation target, the economy is at the ELB for about 3% of the time, and the average duration of the episodes is only about five quarters. Conducting QE operations when the economy reaches the ELB also reduces the duration of the ELB episodes, but the magnitudes of the effects are much smaller compared to those of raising the inflation target.[11] In the model with the 2% inflation target, QE operations reduce the frequency and the duration only by 0.5

[11] The incidence of ELB episodes is slightly higher when the cental bank conducts QE operations at the ELB, because the economy escapes the ELB relatively quickly, and thus can be constrained by the ELB more frequently.

Table 3.2: Aggregate effects of the higher inflation target 2

	E(u)	std(u)	E(w)	std(w)	E(Q)	std(Q)	E(Π)	std(Π)	E(r_a)	std(r_a)
Low π (No QE))	5.97 (-)	2.91 (-)	0.69 (-)	0.55 (-)	-0.52 (-)	0.47 (-)	0.30 (-)	9.54 (-)	2.39 (-)	0.22 (-)
Low π (QE)	5.67 (0.30)	1.67 (1.24)	0.61 (0.08)	0.55 (0.00)	-0.20 (0.32)	0.35 (0.12)	2.13 (1.83)	6.96 (2.58)	2.28 (0.11)	0.16 (0.06)
High π (No QE)	5.58 (0.39)	1.30 (1.61)	0.81 (0.12)	0.55 (0.00)	-0.13 (0.39)	0.27 (0.20)	4.20 (3.90)	6.19 (3.35)	2.23 (0.16)	0.15 (0.07)
High π (QE)	5.54 (0.43)	1.16 (1.75)	0.80 (0.11)	0.55 (0.00)	-0.09 (0.43)	0.25 (0.22)	4.46 (4.16)	5.97 (3.57)	2.21 (0.18)	0.14 (0.08)

Note: u, w, Q, Π, and r_a denote the unemployment rate, real wage, equity price, profits, and dividend rate, respectively. $E(\cdot)$ and $std(\cdot)$ stand for the mean and standard deviation. No QE and QE refer to cases in which the central bank does and does not conduct QE when it is constrained by the ELB, respectively. Low (high) π represents the model with the low (high) inflation target. The mean values are expressed as % differences relative to its steady-state values except for the inflation rate, whose mean values are shown as raw values. Standard deviations are the standard deviations of the quarterly growth rates of the corresponding variables. The unit for the standard deviation of the inflation rate is percentage points. Values in the parenthesis are the differences relative to the baseline case with the low inflation target and no QE operation. The red numbers indicate minus values, while blue numbers indicate positive values.

percentage points and one and half quarters, respectively.

By making the ELB episodes less frequent and relatively short-lived, the higher inflation target boosts and stabilizes the aggregate economy. In the baseline case with the 2% inflation target and no QE operations, the average levels of output, consumption, and capital stock are lower than the corresponding non-stochastic steady-state values, since the economy experiences severe recessions when the central bank is constrained by the ELB. With the higher inflation target, the economy suffers fewer severe recessions as a result of facing a binding ELB constraint less frequently, and thus the average levels of aggregate variables are higher. Compared to the baseline case, output, consumption, and the capital stock are higher on average by about 0.54, 0.50, and 10.6 percentage points of the corresponding steady-state values, respectively. Moreover, the higher inflation target makes aggregate variables less volatile. The standard deviations of output and consumption are non-negligibly lower in the case with the higher inflation target.

Regarding the aggregate variables that directly affect households' welfare, the higher inflation target has more subtle effects. The higher inflation target lowers the average unemployment rate and makes it substantially less volatile. On average, the unemployment rate and its standard deviation are lower by about 0.39 and 1.61 percentage points, respectively, compared to the baseline case. The higher inflation target increases and stabilizes the equity price and profits in the economy as well. Compared to the baseline case with the lower inflation target and no QE operations, the equity price is 0.39 percentage points higher on average, while its standard deviation is about 0.2 percentage points lower. The effects on profits are more substantial. Under the higher inflation target, the level of profits is higher by about 3.90 percentage points of its steady-state level, while its standard deviation is less by more than 2 pecentage points. However, the level and the volatility of the real wage are slightly higher in the cases with the higher inflation target due to relatively higher and more volatile inflation rate in those cases.[12] Also, despite the higher level of profits, the level of the dividend rate is lower on average in the cases with the higher inflation target, as the level of capital stock is much higher in those cases.[13]

3.4.2 Distributional dynamics

Tables 3.3 and 3.4 summarize the distributional effects of the higher inflation target during the simulation. First, the higher inflation target lowers wealth, income, and consumption inequality, as measured by the Gini index. Also, the higher

[12]Relative to the level of the inflation target, the volatility of the inflation rate is lower in the higher inflation target cases, but in the absolute levels, the volatility is higher in those cases.

[13]The dividend rate is aggregate profits per unit capital stock.

Table 3.3: Distributional effects of the higher inflation target 1

	E(Gini W)	std(Gini W)	E(Gini I)	std(Gini I)	E(Gini C)	std(Gini C)
Low π (No QE)	0.45 (-)	0.18 (-)	0.61 (-)	0.26 (-)	1.12 (-)	0.29 (-)
Low π (QE)	0.18 (0.27)	0.15 (0.03)	-0.24 (0.85)	0.23 (0.03)	-0.21 (1.33)	0.23 (0.06)
High π (No QE)	0.25 (0.20)	0.12 (0.06)	-0.52 (1.13)	0.23 (0.03)	-0.67 (1.79)	0.21 (0.08)
High π (QE)	0.21 (0.24)	0.12 (0.06)	-0.64 (1.25)	0.23 (0.03)	-0.86 (1.98)	0.17 (0.12)

Note: Gini W, Gini I, and Gini C represent wealth, income, and consumption Gini index, respectively. $E(\cdot)$ and $std(\cdot)$ stand for the mean and standard deviation. No QE and QE refer to cases in which the central bank does and does not conduct QE when it is constrained by the ELB, respectively. Low (high) π represents the model with the low (high) inflation target. The mean values are percentage point differences relative to its steady-state values. The units for the standard deviations are percentage points. Values in the parenthesis are the differences relative to the baseline case with the low inflation target and no QE operation. The red numbers indicate minus values, while blue numbers indicate

Table 3.4: Distributional effects of the higher inflation target 2

	$E(\frac{w_{999}}{w_{50}})$	$E(\frac{w_{99}}{w_{50}})$	$E(\frac{w_{90}}{w_{50}})$	$E(\frac{I_{999}}{I_{50}})$	$E(\frac{I_{99}}{I_{50}})$	$E(\frac{I_{90}}{I_{50}})$	$E(\frac{C_{999}}{C_{50}})$	$E(\frac{C_{99}}{C_{50}})$	$E(\frac{C_{90}}{C_{50}})$
Low pi (No QE)	31.8 (-)	28.9 (-)	30.5 (-)	-0.79 (-)	-0.51 (-)	-2.45 (-)	-2.54 (-)	2.36 (-)	7.43 (-)
Low pi (QE)	4.33 (27.5)	5.72 (23.2)	6.81 (23.7)	-2.99 (2.20)	-2.58 (2.07)	-2.22 (0.23)	-5.59 (3.05)	-0.61 (2.97)	6.43 (1.00)
High pi (No QE)	0.37 (31.4)	0.40 (28.5)	-0.48 (30.1)	-4.05 (3.26)	-3.42 (2.91)	-2.08 (0.37)	-6.97 (4.43)	-1.82 (4.18)	6.08 (1.35)
High pi (QE)	-3.59 (35.5)	-2.90 (31.8)	-3.80 (34.3)	-4.36 (3.57)	-3.71 (3.20)	-2.07 (0.38)	-7.42 (4.88)	-2.26 (4.62)	5.91 (1.52)

Note: The table shows the log 99.9, 90, and 90 to 50 wealth, income, and consumption ratios. W, I, and C stand for wealth, income, and consumption, respectively. $E(\cdot)$ stand for the mean. No QE and QE refer to cases in which the central bank does and does not conduct QE when it is constrained by the ELB, respectively. ELB, respectively. Low (high) π represents the model with the low (high) inflation target. Values in the parenthesis are the differences relative to the baseline case with the low inflation target and no QE operation. The red numbers indicate minus values, while blue numbers indicate

inflation target makes the degree of inequality less volatile. This is because, ELB episodes typically raise inequality substantially by increasing the unemployment rate. By preventing episodes of the higher unemployment rate and inequality, a higher inflation target reduces the overall degree of inequality, as measured by the Gini index.

Table 3.4 compares the level of wealth, income, and consumption of households at the right end of the wealth distribution with the median households. As shown in the previous section, lower inequality does not necessarily imply that gains from a higher inflation target are monotonically decreasing in households' wealth. However, unlike during the short-rutn transition path, the ratio of wealthy households' wealth, income, and consumption relative to the median household is smaller in the cases with a higher inflation target than in the baseline case. The second, third, and fourth columns show that, by making severe recessions less frequent and short-lived, the higher inflation target facilitates the median household's asset accumulation. The average level of the median household's wealth is much higher in relative terms when the inflation target is higher. In contrast, the wealthy household's average wealth is relatively lower when the inflation target is higher. This is because other households' asset accumulation increases equity prices and reduces the dividend rate in the higher inflation rate cases. Since the wealthiest households hold less wealth, their income and consumption gains from transitioning from the lower inflation target and no QE case to the higher inflation target and no QE case are smaller than the median household's income and consumption gains, as shown as lower log ratios.[14]

In short, the higher inflation target enables households at the middle and the bottom of the wealth distribution to consume more goods and accumulate more wealth by making ELB episodes less severe and frequent, which makes the unemployment rate lower and more stable compared to the baseline case. As households face lower unemployment risks, they shift their savings to equity and increase the

[14]Business owners enjoy higher income gains than other households because of much higher profits. Thus, the log 90 to 50 income ratios are slightly higher in the cases with a higher inflation target. However, since the fraction of business owners is small, the effect of lower dividend rates dominates at the very top of the wealth distribution, and thus the log 99.9 and 90 to 50 income ratios are smaller in the cases with a higher inflation target.

capital stock.[15] The increase in equity demand boosts equity prices but lowers the gross rate of return on equity even though it increases overall profits in the economy. Consequently, wealthy households hold relatively less wealth compared to the baseline case, which leads to relatively less income and consumption for the wealthy relative to the average household in the economy in the higher inflation target cases. That is, the higher inflation target reduces wealth, income, and consumption gaps compared to the lower inflation target cases.

3.5 Conclusion

In this chapter, I evaluate the aggregate and distributional consequences of raising the inflation target by examining the transition path towards a higher inflation target and the long-run stochastic behavior of the model with different inflation targets. First, I find that the economy experiences a substantial expansion during the transition when the Phillips curve is forward-looking and the Taylor rule features a significant degree of interest rate smoothing.

The transition also has non-linear distributional effects. Both ends of the wealth distribution enjoy a higher income, consumption, and long-term welfare gains than households at the middle of the wealth distribution. The reason is that the unemployment rate, equity prices, and profits respond more strongly than the real wage during the transition. Quantitatively, the welfare gain is the biggest at the bottom of the wealth distribution, and the transition reduces overall wealth, income,

[15] In the baseline case with a lower inflation target and no QE operations, the liquid asset share in the household's portfolio is about 15%. In contrast, in the case with a higher inflation target, the liquid asset share is only about 9%.

and consumption inequality despite large income and consumption gains at the top compared to the gains of the middle.

Over the business cycle, the higher inflation target boosts and stabilizes the aggregate economy by making ELB episodes less frequent and less severe. Moreover, the higher inflation target lowers the level of inequality and makes it more stable. The higher inflation target lowers the average unemployment rate over the business cycle and makes it less volatile. Thus, households at the middle and the bottom of the wealth distribution consume more and purchase more equity. The overall increase in aggregate demand increases profits in the economy, but it increases the equity price and reduces the gross rate of return on equity holdings. Consequently, wealthy households hold less wealth, which leads to lower degrees of wealth, income, and consumption shares of the wealthy in the economy.

The analysis in this chapter emphasizes the benefits of having a higher inflation target for the aggregate economy as well as inequality. However, the framework used in this chapter does not feature possible permanent costs of the higher steady-state inflation rate, such as production inefficiencies that arise from increased price dispersion at the steady-state. I leave a more balanced comparison of the costs and benefits of a higher inflation target for future research.

Appendix A: Appendix for Chapter 1

A.1 Market clearing conditions

In the model, there are five markets. The market clearing condition for the final good market is given as follows.[1]

$$S_t = \int \left[c(a_t, b_t) + \underline{\mathrm{R}} \times \mathbb{1}_{\{b_t<0\}} b_t\right] d\mu_t + \int_0^1 \left[\left\{\frac{\eta}{2\kappa}\left(\log\frac{\pi_{j,t}}{\bar{\pi}}\right)^2 + \frac{\phi_x}{2}\left(\log\frac{x_{j,t}}{\bar{x}}\right)^2\right\} S_t \right.$$
$$\left. + (R_{t+1}-1)(r_t^l L_{j,t} + r_t^k K_{j,t})\right] dj + \Xi + I_t + \Phi(K_{t+1}, K_t) + G_t \tag{A.1}$$

where $I_t = K_{t+1} - \{1-\delta(v_t)\}K_t$ is aggregate investment. Equation (A.1) reflects the assumption that the household borrowing premium and the interest on working capital are not a part of the government revenue but represent a waste of resources. As shown in the above, the final good is used for consumption, investment, government purchases and other miscellaneous costs in the model.

Market clearing for capital rental services, the capital stock utilized in the current period must equal the capital services demanded by the intermediate good

[1] For the ease of notation, I drop idiosyncratic states in household's optimal policies.

firms.

$$v_t K_t = \int_0^1 K_{j,t} dj \tag{A.2}$$

Likewise, the labor supplied by households (via labor agencies) must equal the labor services demanded by the intermediate good firms.

$$\int \mathbb{1}_{\{e_t=1\}} s_t n_t d\mu_t = \int_0^1 L_{j,t} dj \tag{A.3}$$

The market clearing condition for the equity market is given as follows.[2]

$$K_{t+1} = (1-\zeta) \int a_{t+1}(a_t, b_t) d\mu_t \tag{A.4}$$

That is, the sum of the surviving households' equity holding should be equal to the next period aggregate capital stock.

Finally, the market clearing condition for the bond market is given as follows.

$$B_{t+1} = \bar{B} \left(\frac{R_t/\pi_t \times B_t}{\bar{R}/\bar{\pi} \times \bar{B}} \right)^{\rho_B} = \int b_{t+1}(a_t, b_t) d\mu_t \tag{A.5}$$

where the government's bond issuance rule is taken into account.

[2] To introduce a stochastic death and a hypothetical annuity arrangement into the model, I assume that the capital service provider accumulates the capital stock that is equal to the equity holding of surviving households. Similarly, the mutual fund only receive the payment from households that will survive next period. The annuity arrangement keeps the remaining payments and use them for the annuity payment next period. Since there are infinitely many households and the law of large numbers holds, the amounts of these payments are known without uncertainty.

A.2 A recursive equilibrium

Economic agents in the model, such as households and firms, need to forecast future variables that are relevant for their optimal decision. Especially, households and firms need to know the evolution of the households' distribution across their idiosyncratic states, since it determines the aggregate capital stock, bond holding, and the size of labor force. Let $\mu_t(\tilde{a}, \tilde{b}, \tilde{s}, \tilde{e}, \tilde{\chi})$ denote the mass of households with $(\tilde{a}, \tilde{b}, \tilde{s}, \tilde{e}, \tilde{\chi})$ in period t, where $\tilde{a}$ is the equity holding, $\tilde{b}$ is the bond holding, $\tilde{s}$ is idiosyncratic productivity, $\tilde{e}$ is the working status in period t, and $\tilde{\chi}$ is the equity adjustment cost respectively. Then, the law of motion for the distribution is described as follows.[3]

$$\mu_{t+1}(\tilde{a}, \tilde{b}, \tilde{s}, \tilde{e}, \tilde{\chi}) = \sum_{s \in \mathbb{S}} P_s(s_{t+1} = \bar{s} | s_t = s) \times \sum_{e \in \{1,2,3\}} P_e(e_{t+1} = \bar{e} | e_t = e) \times$$
$$P_\chi(\chi_{t+1} = \tilde{\chi}) \times \int \mathbb{1}_{\{a_{t+1}(a_t, b_t | s_t, e_t) = \bar{a}\}} \times \int \mathbb{1}_{\{b_{t+1}(a_t, b_t | s_t, e_t) = \bar{b}\}} d\mu_t(a_t, b_t, s_t, e_t) \quad \text{(A.6)}$$

where P_e is the probability of working status transition, which is determined by the stochastic process for s_t, the exogenous transition probability in and out of the entrepreneurial state, the exogenous separation rate, and the endogenous job-finding rate.

With the market clearing conditions and the law of motion for the households' distribution, a recursive equilibrium in the model is defined as follows.

[3] I describe the law of motion as if the adjustment costs are drawn from a discretized process though they are assumed to follow a continuous distribution.

Definition (Recursive equilibrium): A recursive equilibrium is a set of variables and functions $\{a_{t+1}$, b_{t+1}, c_t, n_t, q_t, w_t, r_t^a, r_t^l, r_t^k, v_t, u_t, V_t, M_t, $V(a_t, b_t|s_t, e_t, \chi_t, X_t)$, $J_L(s_t|\tilde{X}_t)$, $J_K(K_t|X_t)$, K_{t+1}, Π_t, Ψ_t, π_t, $\{J_I(i_{j,t-1}, P_{j,t-1}|X_t)$, $Y_{j,t}$, $L_{j,t}$, $K_{j,t}$, $\pi_{j,t}$, $S_{j,t}$, $a_{j,t}$, $i_{j,t}\}$, MC_t, $f(\tilde{X}_t)$, S_t, G_t, L_t, D_t, T_t, R_{t+1}, $\Lambda_{t,t+1}\}$ and laws of motion such that:

1. Given the laws of motion, w_t, q_t, r_t^a, R_t and π_t, the value function $V(a_t, b_t)$ is a solution to the household's problem. $a_{t+1}(a_t, b_t)$, $b_{t+1}(a_t, b_t)$, $c(a_t, b_t)$, $n(a_t, b_t)$ are the associated optimal decision rules.

2. Given r_t^k, r_t^l, R_{t+1}, S_t, P_t, MC_t, and $\Lambda_{t,t+1}$, value function $J_I(i_{j,t-1}, P_{j,t-1}|X_t)$ solves the problem of an intermediate good producer. $K_{j,t} = K(i_{j,t-1}, P_{j,t-1}|X_t)$, $L_{j,t} = L(i_{j,t-1}, P_{j,t-1}|X_t)$, and $P_{j,t} = P(i_{j,t-1}, P_{j,t-1}|X_t)$ are the associated optimal decision rules. Sales $S_{j,t}$ and new production $Y_{j,t}$ are determined by (1.25) and (1.27) respectively. The profit of the intermediate good firm is given by $S_{j,t} - \{\frac{\eta}{2\kappa}\Big(\log\frac{\pi_{j,t}}{\bar{\pi}}\Big)^2 + \frac{\phi_x}{2}\Big(\log\frac{x_{j,t}}{\bar{x}_j}\Big)^2\}S_t - MC_tY_{j,t} - \Xi$.

3. The final good sale S_t and the price index P_t are given by (1.20) and (1.23).

4. The marginal cost of production MC_t and the gross inflation rate π_t are given by (1.31) and (1.32).

5. Given r_t^l, w_t and $\Lambda_{t,t+1}$, the value of labor agency $J_L(s_t|\tilde{X}_t)$, the number of matching M_t, vacancies V_t, the job-finding rate $f(\tilde{X}_t)$, and the unemployment rate u_t are determined by (33), (35), and (36).

6. For given r_t^k, the capital service firm decides v_t following (41).

7. The equity price q_t and return r_t^a are given by equation (1.45) and (1.46).

8. The fiscal authority decides the bond supply according to equation (1.48).
9. The monetary authority decides the nominal return on bond R_{t+1} according equation (1.51).
10. G_t (or T_t, τ_a, τ_y) adjusts to balance the government budget constraint.
11. The law of motion for μ_t is consistent with the relevant optimal decision rules.
12. All firms make the same choices (a symmetric equilibrium).
13. All markets clear.

A.3 Numerical solution

The model is solved using a perturbation method developed by Reiter (2009). The method has been further improved by Winberry (2018), Bayer and Luetticke (2020), and others. The method relies on a linearized system of the model around its steady state. The method adopted in this paper is based on the method of Bayer and Luetticke (2020).

The steady state of the model is solved using the endogenous grid method of Carroll (2006). Because of stochastic equity adjustment costs, the household value functions are globally concave, and the endogenous grid method can be used to find a solution to the household's problem. Once households' decision rules over the state space are determined, I compute a stationary distribution, following a histogram approach developed by Young (2010). That is, the distribution is expressed as a histogram and, if a mass falls between the grid points, it is allocated to the nearest grid points based on the distance to such points.

After solving for the steady state, I linearize the model around it using Bellman equations, laws of motions for the distribution and exogenous processes, and other equilibrium conditions. The most critical problem associated with applying a perturbation method to a HANK model is the size of the state space. Since the model has many idiosyncratic states, including holdings of each asset, idiosyncratic productivity, and working status, it is almost infeasible to compute the Jacobian of the linearized system without a state-space reduction even though a relatively small number of grid points are used to approximate each dimension of the state space. Thus, for a state space reduction, I adopt a methodology used by Bayer and Luetticke (2020). Specifically, I use Chebyshev polynomials to approximate the value functions and a fixed copula to reduce the dimension of the distribution. Once I reduce the dimension of the linearized system, I solve the model by using the perturbation method developed by Schmitt-Grohé and Uribe (2004). Under the parameterizations explored in this paper, the model has a unique perturbed solution around the steady state.

Appendix B: Appendix for Chapter 2 and 3

B.1 Further details on the model description

B.1.1 Households

Let V_a and V_b denote the partial derivative of the value function with respect to illiquid and liquid asset holding, respectively. Similarly, u_c denotes the partial derivative of the utility function with respect to consumption. By Envelope Theorem, I have the following expressions for the partial derivatives of the value function.

$$V_a(a_t, b_t) = \begin{cases} (q_t + r_t^a)u_c(c_t^A, n_t) & \text{if adjust} \\ r_t^a u_c(c_t^N, n_t) + \beta(1-\zeta)\mathbb{E}\Big[V_a(a_t, b_{t+1})\Big] & \text{if not adjust} \end{cases} \tag{B.1}$$

$$V_b(a_t, b_t) = \begin{cases} \left(\frac{1+\tilde{i}_t}{\pi_t}\right)u_c(c_t^A, n_t) & \text{if adjust} \\ \left(\frac{1+\tilde{i}_t}{\pi_t}\right)u'(c_t^N, n_t) & \text{if not adjust} \end{cases} \tag{B.2}$$

where c_t^A and c_t^N are the optimal consumption when the household chooses to adjust its illiquid asset holding or not, respectively.[1] Households choose to adjust their

[1]Households' optimal hours worked is not affected by the household's portfolio choice.

equity holdings if the following conditions are satisfied.

$$V^A(a_t, b_t) - \chi_t \geq V^N(a_t, b_t) \tag{B.3}$$

where V^A and V^N denote the value of households when they adjust and do not adjust their illiquid asset holding respectively. Then, the probability of adjustment $P^*(a_t, b_t)$ can be computed as follows.

$$\begin{aligned} P^*(a_t, b_t) &= P\Big[\chi_t \leq V^A(a_t, b_t) - V^N(a_t, b_t)\Big] \\ &= F\Big[V^A(a_t, b_t) - V^N(a_t, b_t)\Big] \end{aligned} \tag{B.4}$$

Given the probability of adjustment, the household's Euler equation with respect to each asset holding can be described as follows.

$$\begin{aligned} q_t u_c(c_t, n_t) \geq \beta \mathbb{E}\Big[&P^*(a_{t+1}, b_{t+1})\big\{q_{t+1} + r^a_{t+1}\big\} u_c(c^A_{t+1}, n_{t+1}) \\ &+ \big\{1 - P^*(a_{t+1}, b_{t+1})\big\} r^a_{t+1} u_c(c^N_{t+1}, n_{t+1}) + \big\{1 - P^*(a_{t+1}, b_{t+1})\big\}\mathbb{E}\big[V_a(a_{t+1}, b_{t+2})\big]\Big] \\ &\text{with equality if } a_{t+1} > 0 \quad \text{and} \quad a_{t+1} \neq a_t \end{aligned} \tag{B.5}$$

$$\begin{aligned} u_c(c_t, n_t) \geq \beta \mathbb{E}\Big[&P^*(a_{t+1}, b_{t+1})\Psi^l_t \left(\frac{1 + \tilde{i}_{t+1}}{\pi_{t+1}}\right) u_c(c^A_{t+1}, n_{t+1}) + \{1 - P^*(a_{t+1}, b_{t+1})\}\Psi^l_t \\ &\times \left(\frac{1 + \tilde{i}_{t+1}}{\pi_{t+1}}\right) u_c(c^N_{t+1}, n_{t+1})\Big] \quad \text{with equality if } b_{t+1} > 0 \end{aligned} \tag{B.6}$$

Note that, as explained in the main text, households' optimality condition regarding liquid assets is perturbed by liquidity preference shocks.

B.1.2 Banks

As long as the expected equity premium $R^a_{t+i} - R_{t+i}$ is positive, a bank's optimal choice is to purchase assets to the extent possible. If there is no limit in taking deposits, either a bank expands its assets indefinitely, or the premium becomes zero. To limit the bank's ability to borrow, I assume a moral hazard/costly enforcement problem, as in Gertler and Karadi (2011). Specifically, at the beginning of the period, a bank can divert the fraction Δ of the bank's asset and transfer it to business owners. Once the bank diverts the funds, the depositors force the bank into bankruptcy but can recover only the remaining $1-\Delta$ fraction of assets. It is too costly for the depositors to recover all the funds that the banker diverted. Taking into account this incentive problem, investors will make deposits only to the point the following constraint holds.

$$J^b(N_{jt}) \geq \Delta q_t A^b_{jt+1} \tag{B.7}$$

where the left-hand side is the cost for the bank when it diverts a fraction of assets, i.e., the franchise value of the bank. The right-hand side is the value of diverting. To further specify the above condition, one needs to compute the value of the bank. Using the guess and verify approach, one can show that the bank j's value $J^b(N_{jt})$ is linear in its assets and net-worth.

$$J^b(N_{jt}) = \vartheta^a_t q_t A^b_{jt+1} + \vartheta^n_t N_{jt} \tag{B.8}$$

with

$$\vartheta_t^a = \mathbb{E}_t\Big[(1-\theta_b)\Psi_t^b\Lambda_{t,t+1}(R_{t+1}^a - R_{t+1}) + \theta_b\Psi_t^b\Lambda_{t,t+1}x_{t,t+1}\nu_{t+1}\Big] \tag{B.9}$$

$$\vartheta_t^n = \mathbb{E}_t\Big[(1-\theta_b)\Psi_t^b\Lambda_{t,t+1}R_{t+1} + \theta_b\Psi_t^b\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1}\Big] = (1-\theta_b) + \mathbb{E}_t\Big[\theta_b\Psi_t^b\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1}\Big] \tag{B.10}$$

where $x_t = q_{t+1}A_{jt+2}^b/q_tA_{jt+1}^b$ is the gross growth rate in assets between t and $t+1$ and $z_t = N_{jt+1}/N_{jt}$ is the gross growth rate of net worth. Ψ_t^b is the aggregate risk premium shock, which follows an AR(1) process as below.

$$\log\Psi_t^b = \rho_b\log\Psi_{t-1}^b + \epsilon_{b,t} \quad , \ \epsilon_{b,t} \ \sim \ N(0,\sigma_b^2) \tag{B.11}$$

where $\epsilon_{b,t}$ is a normally distributed shock, and σ_b is its standard deviation. An increase in Ψ_t^b leads to an increase in the value of banks' assets and net-worth by making banks value future more. Thus, a positive shock to Ψ_t^b leads to an expansion of banks' balance sheet.

With the value function derived above, I can re-write the incentive constraint as follows.

$$\vartheta_t^a q_t A_{jt+1}^b + \vartheta_t^n N_{jt} \geq \Delta q_t A_{jt+1}^b \tag{B.12}$$

If the constraint binds, the value of assets that the banker can purchase will be determined by the level of his or her net worth. By re-arranging the above equation, we have

$$q_t A^b_{jt+1} = \frac{\vartheta^n_t}{\Delta - \vartheta^a_t} N_{jt} = \Theta_t N_{jt} \tag{B.13}$$

where Θ_t is the bank's leverage ratio, i.e., the ratio of assets to its net worth.[2] When the constraint binds, I can express the law of motion for net worth as follows.

$$N_{jt+1} = \big\{(R^a_{t+1} - R_{t+1})\Theta_t + R_{t+1}\big\} N_{jt} \tag{B.14}$$

In addition, it follows that

$$z_{t,t+1} = N_{jt+1}/N_{jt} = \big\{(R^a_{t+1} - R_{t+1})\Theta_t + R_{t+1}\big\} \tag{B.15}$$

$$x_{t,t+1} = q_{t+1} A^b_{jt+2}/q_t A^b_{jt+1} = \Theta_{t+1} N_{jt+1}/\Theta_t N_{jt} = (\Theta_{t+1}/\Theta_t) z_{t,t+1} \tag{B.16}$$

Note that all components of Θ_t do not depend on bank-specific variables. Thus, I can sum across banks to obtain

$$q_t A^b_{t+1} = \Theta_t N_t \tag{B.17}$$

where A^b_{t+1} is the aggregate quantity of the equity held by banks and N_t denote the aggregate bank net worth.

Finally, I describe a law of motion for N_t. First, note that N_t is the sum of the net worth of surviving banks, N_{ot} (old), and the net worth of entrants, N_{nt} (new).

[2] Note that, given $N_{jt} > 0$, the constraint binds only if $0 < \vartheta^a_t < \Delta$. Under the parametrizations used in this paper, the constraint always binds.

Regarding the latter, I assume that the value of start-up funds for new bank is equal to the value of assets that exiting banks had intermediated in the previous period, which equals $(1-\theta_b)q_{t-1}A_t^b$. Specifically, for each new bank, the equity mutual fund gives $\omega/(1-\theta_b)$ fraction of this value. Then, I have

$$N_t = N_{ot} + N_{et} = \theta_b\{(R_t^a - R_t)\Theta_{t-1} + R_t\}N_{t-1} + \omega q_{t-1}A_t^b \tag{B.18}$$

Finally, profits from the financial sector are the sum of net-worth of existing banks, net of start-up funds for new banks.

$$\Pi_t^b = (1-\theta_b)\{(R_t^a - R_t)\Theta_{t-1} + R_t\}N_{t-1} - \omega q_{t-1}A_t^b \tag{B.19}$$

B.2 Numerical method

B.2.1 Solution method

For the calibration, I solve for the steady state of the model globally. Specifically, I use value function iteration combined with the endogenous grid method of Carroll (2006) to compute households' policy functions. Then, I find the invariant distribution using the non-stochastic simulation method of Young (2010) with the representation of the idiosyncratic distribution as histograms. The solution method captures the precautionary motive associated with idiosyncratic shocks as they are still present even though the model is at the steady state, and there are no aggregate shocks.

Once the steady state is found, I solve for the dynamics of the model using a perturbation method developed by Reiter (2009) with a state-space reduction technique proposed by Bayer and Luetticke (2020).[3] The methodology enables a fast solution that is necessary for Bayesian estimation. However, since the state-space is much larger compared to a representative model even after the reduction, estimating the model by solving the dynamics in full each time during the process is still not feasible.[4] Thus, one needs a way to accelerate the solution process.

On this regard, I follow Bayer et al. (2020) and update only a subset of the Jacobian during the estimation process. The system of equations that characterize an equilibrium can be expressed as follows.

$$\mathbb{E}_t\Big[\mathcal{F}(X_{t+1}, Y_{t+1}, X_t, Y_t)\Big] = 0 \tag{B.20}$$

where $\mathcal{F}$ is a non-linear function that consists of equilibrium conditions and laws of motion for relevant objects including the idiosyncratic distribution. $\mathbb{E}_t$ is the expectation operator conditional on the information available at period t. $X_{t+1} = (X_{1t+1}, X_{2t+1}, X_{3t+1}, \epsilon_{t+1})'$ is the vector of pre-determined or state variables. Specifically, X_{1t+1} is the vector of "idiosyncratic" state variables. In my model, X_{1t+1} consists of households' idiosyncratic state distribution at the end of period t.[5] X_{2t+1} is the vector of "summary" variables, which includes aggregate bond and equity hold-

[3] Bayer and Luetticke (2020) approximate the deviation of value functions from their steady state values using Chebyshev polynomials, and use a fixed copula for the approximation of changes in the idiosyncratic distributions.

[4] On a workstation computer with 10 cores (20 threads), it takes about 40 seconds to solve the dynamics model when 17,600 ($40 \times 40 \times 11$) points were used to represent the idiosyncratic state space.

[5] Note that the endogenous state variables for period $t+1$ are determined in period t.

ing of households. Variables X_{2t+1} summarize the idiosyncratic decision of households into one scalar variable. Importantly, the relationship between idiosyncratic state and variables in X_{2t+1} is not affected by parameter values. X_{3t+1} is the vector of purely "aggregate" variables in the sense that idiosyncratic variables do not appear in the equations that define these variables. ϵ_{t+1} is the vector of all exogenous shocks. Y_t is the vector of endogenous control variables and further decomposed into Y_{1t+1}, Y_{2t+1}, and Y_{3t+1}. Y_{1t+1} is the vector of "idiosyncratic" control variables, which include the value functions and their derivatives. Y_{2t+1} is the vector of "summary" variables. Finally, Y_{3t+1} is the vector of "aggregate" variables.

The key idea of Bayer et al. (2020) is that one does not need to update the Jacobian with respect to "idiosyncratic" variables during the estimation if the estimated parameters are only relevant for the dynamics and do not affect households' problem. To this point more clearly, I write down the system of equations (B.20) as follows.

$$\mathbb{E}_t\Big[\mathcal{F}(X_{t+1}, Y_{t+1}, X_t, Y_t)\Big] = \big[\mathcal{F}_{1,t}, \mathcal{F}_{2,t}, \mathcal{F}_{3,t}, \mathcal{F}_{4,t}, \mathcal{F}_{5,t}, \mathcal{F}_{6,t}, \mathcal{F}_{7,t}\big]' \tag{B.21}$$

where $\mathcal{F}_{1,t}$ is the set of equations that describe relations among idiosyncratic state variables, i.e., between X_{1t} and X_{1t+1}. $\mathcal{F}_{2,t}$ is summary equations that aggregate individual variables into aggregate state variables. Note that $\mathcal{F}_{1,t}$ is affected only by parameters that alter households' optimal behaviors. Likewise, $\mathcal{F}_{2,t}$ is not affected by parameter choice as they are aggregation of individual variables over idiosyncratic state space. $\mathcal{F}_{3,t}$ is the set of equations for aggregate variables. Importantly, idiosyncratic state variables, i.e., ones in X_{1t}, do not appear in $\mathcal{F}_{3,t}$. Instead, variables in $X_{2,t}$ may appear in $\mathcal{F}_{3,t}$. $\mathcal{F}_{4,t}$ is the exogenous stochastic processes.

The remaining three sets of equations describe relations regarding control variables. $\mathcal{F}_{5,t}$ is the set of equations on idiosyncratic control variables. In the model, such variables include value functions and their derivatives. Again, parameters that are not relevant for households' problem do not affect these equations. $\mathcal{F}_{6,t}$ is summary equations regarding control variables.[6] Again, changes in parameters that are not relevant for households' problem do not affect these two sets of equations. Finally, $\mathcal{F}_{7,t}$ is the set of equations on aggregate variables. Note that idiosyncratic state and control variables appear in $\mathcal{F}_{7,t}$ only through summary variables.

From equation (B.21), we know that the Jacobian has the following form.

$$\mathcal{J}_t = \begin{bmatrix} \frac{\partial \mathcal{F}_{1,t}}{\partial X_{t+1}} & \frac{\partial \mathcal{F}_{1,t}}{\partial Y_{t+1}} & \frac{\partial \mathcal{F}_{1,t}}{\partial X_t} & \frac{\partial \mathcal{F}_{1,t}}{\partial Y_t} \\ \frac{\partial \mathcal{F}_{2,t}}{\partial X_{t+1}} & \frac{\partial \mathcal{F}_{2,t}}{\partial Y_{t+1}} & \frac{\partial \mathcal{F}_{2,t}}{\partial X_t} & \frac{\partial \mathcal{F}_{2,t}}{\partial Y_t} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathcal{F}_{7,t}}{\partial X_{t+1}} & \frac{\partial \mathcal{F}_{7,t}}{\partial Y_{t+1}} & \frac{\partial \mathcal{F}_{7,t}}{\partial X_t} & \frac{\partial \mathcal{F}_{7,t}}{\partial Y_t} \end{bmatrix} \tag{B.22}$$

where $\frac{\partial \mathcal{F}_{j,t}}{\partial X_l} = \left[\frac{\partial \mathcal{F}_{j,t}}{\partial X_{1l}}, \frac{\partial \mathcal{F}_{j,t}}{\partial X_{2l}}, \frac{\partial \mathcal{F}_{j,t}}{\partial X_{3l}}, \frac{\partial \mathcal{F}_{j,t}}{\partial \epsilon_l} \right]$, and $\frac{\partial \mathcal{F}_{j,t}}{\partial Y_l} = \left[\frac{\partial \mathcal{F}_{j,t}}{\partial Y_{1l}}, \frac{\partial \mathcal{F}_{j,t}}{\partial Y_{2l}}, \frac{\partial \mathcal{F}_{j,t}}{\partial Y_{3l}} \right]$ for $l = t$ and $t+1$. During Bayesian estimation, we need to update the Jacobian to compute a likelihood of the model for given data and for a given set of parameters. Since the dimension of the Jacobian is very large, updating the Jacobian is time-consuming.

[6] For instance, the aggregate consumption and saving are the sum of individual consumption and saving.

However, we do not need to update all the blocks in the Jacobian every time if we estimate parameters and shock processes that are only relevant for the dynamics of the model and do not directly affect households' optimal behaviors. Specifically, we only need to update the following derivatives: $\frac{\partial \mathcal{F}_{3,t}}{\partial X_{2t+1}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial X_{3t+1}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial \epsilon_{t+1}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial Y_{2t+1}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial Y_{3t+1}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial X_{2t}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial X_{3t}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial \epsilon_{t}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial Y_{2t}}$, $\frac{\partial \mathcal{F}_{3,t}}{\partial Y_{3t}}$, $\frac{\partial \mathcal{F}_{4,t}}{\partial \epsilon_{t}}$, $\frac{\partial \mathcal{F}_{7,t}}{\partial X_{2t+1}}$, $\frac{\partial \mathcal{F}_{7,t}}{\partial X_{3t+1}}$, $\frac{\partial \mathcal{F}_{7,t}}{\partial \epsilon_{t+1}}$, $\frac{\partial \mathcal{F}_{7,t}}{\partial Y_{2t+1}}$, $\frac{\partial \mathcal{F}_{7,t}}{\partial Y_{3t+1}}$. Then, the number of equations that we need to evaluate is close to the number of equations in a representative model with the same features. Thus, estimating the model using Bayesian method is possible.

B.2.2 Inversion filter

In this paper, I use an inversion filter to back out the structural shocks, following Guerrieri and Iacoviello (2017) and Cuba-Borda et al. (2019). Let $\mathbb{Y}_{\{1:\mathrm{T}\}} = \{Y_1, Y_2, ..., Y_T\}$ denote the set of observables, where Y_j is the $n_y \times 1$ vector that contains the data on n_y observables in period j for j = 1, ... T. Also, denote the set of all the endogenous variables of the model in period t with the $n_x \times 1$ vector X_t. Similarly, ϵ_t is the $n_\epsilon \times 1$ vector of structural shocks in period t. With these notations, one can describe a general form of the solution of the model in period t as follows.

$$X_t = P_t X_{t-1} + D_t + Q_t \epsilon_t \tag{B.23}$$

where P_t, D_t, and Q_t are the matrices of coefficients in the solution. As time subscripts imply, the coefficients in the solution can be time-varying. However, when the model is at the reference regime, i.e., when the ZLB is not binding in the data, these coefficients are not time-varying and one can compute them by applying a standard perturbation method. Specifically, we have

$$X_t = PX_{t-1} + Q\epsilon_t \tag{B.24}$$

when the ZLB is not binding. Let H_t be a $n_y \times n_x$ vector that selects the variables in the model that correspond to the observables.[7] Then,

$$Y_t = H_t X_t = H_t P X_{t-1} + H_t Q \epsilon_t \tag{B.25}$$

From the above equation, one can easily compute the set of structural shocks ϵ_t as follows given that the matrix $H_t Q$ is invertible.

$$\epsilon_t = (H_t Q)^{-1}(Y_t - H_t P X_{t-1}) \tag{B.26}$$

During the ELB periods, finding ϵ_t can be more demanding task since the matrices P_t, D_t, and Q_t depend not only on the state and structural shocks but also on the expectation on the duration of the ZLB episodes. However, if one assumes an exogenous duration of the ZLB, one can easily compute ϵ_t as follows.

$$\epsilon_t(\tilde{T}_t) = \{H_t Q(\tilde{T}_t)\}^{-1}\{Y_t - H_t P(\tilde{T}_t) X_{t-1} - H_t D(\tilde{T}_t)\Big\} \tag{B.27}$$

[7] As the data on the central bank's asset is only available since 2003, I include the variable as an observable only during those periods. Accordingly, I only introduce QE shocks during the same periods as well.

where $\tilde{T}_t$ is the expected ZLB durations in period t. Note that the solution and the corresponding structural shocks are conditional on the duration T of the ZLB episodes. Once I find the series of shocks using the filter, I compute the likelihood of the model given the data as follows.

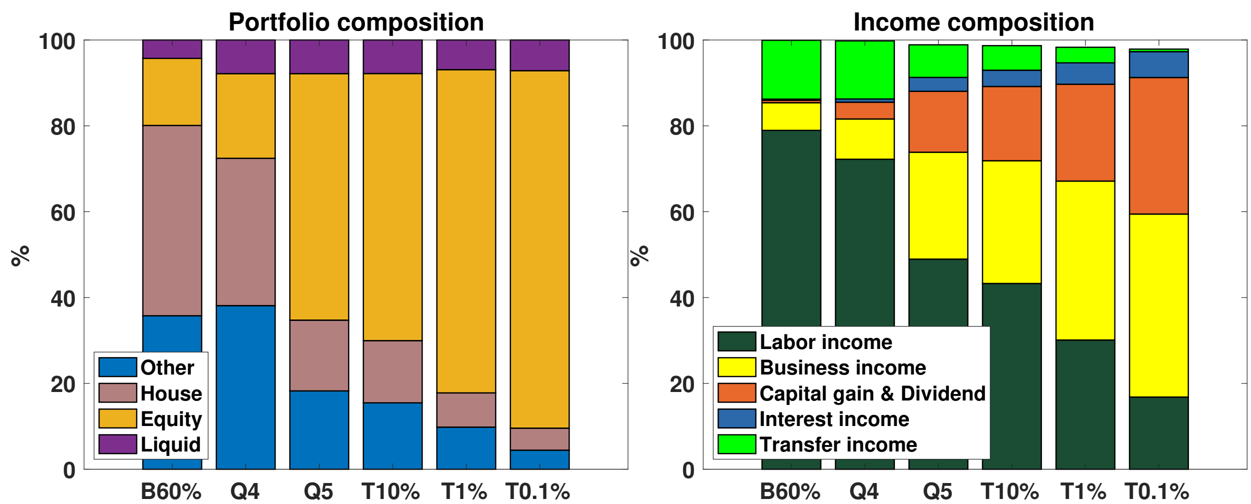
$$\log p\big(\mathbb{Y}_{\{1:T\}}\big) = -\frac{Tn_y}{2}\log(2\pi) - \frac{T}{2}\log(\det(\Sigma)) - \frac{1}{2}\sum_{t=1}^{T}\epsilon_t'\Sigma^{-1}\epsilon_t + \sum_{t=1}^{T}\log\left(|\det\frac{\partial\epsilon_t}{\partial Y_t}|\right) \tag{B.28}$$

where $\frac{\partial\epsilon_t}{\partial Y_t} = \big\{H_t Q_t\big\}^{-1}$.[8]

[8]The result is based on the local linearity of the solution. For more details, see Guerrieri and Iacoviello (2015).

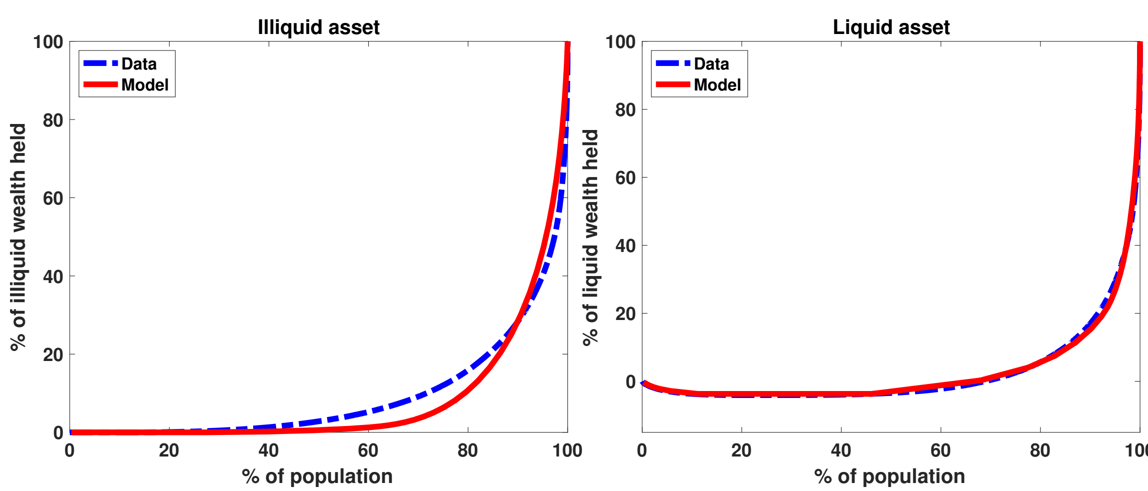
B.3 The fit of the model

Figure B.1: Portfolio and income composition in the data



Notes: The figure shows more detailed decomposition of households portfolio and income composition in the data. For the description of each item, see the main text.

Figure B.2: Lorenz curves in the data and the model



Notes: The figure shows asset holding inequality in the data and in the model using Lorenz curves. For the definition of liquid and illiquid asset in the data, see the main text.

B.4 Further details on the estimation

B.4.1 Observables and a mapping between the data and the model

For the estimation, I use the following data. The most of the data were collected from FRED or BEA. The data period is from 1992 Q1 to 2018 Q4, except for the central bank's assets, of which data is only available since 2003.

1. Output
   - Model : $\tilde{Y}_t^{\text{obs}} = \log\left(\frac{Y_t}{Y_{t-1}}\right)$
   - Data : Nominal GDP (FRED, GDP), divided by GDP deflator (FRED, GDPDEF) and civilian non-institutionalized population (FRED, CNP16OV), log-transformed, first-differenced and de-meaned.
2. Consumption
   - Model : $\tilde{C}_t^{\text{obs}} = \log\left(\frac{C_t}{C_{t-1}}\right)$
   - Data : The sum of PCE on non-durable goods and services (BEA NIPA Table 2.3.5, item 8 & 13), divided by GDP deflator (FRED, GDPDEF) and civilian non-institutionalized population (FRED, CNP16OV), log-transformed, first-differenced and de-meaned.
3. Investment
   - Model : $\tilde{I}_t^{\text{obs}} = \log\left(\frac{I_t}{I_{t-1}}\right)$

- Data : The sum of private fixed investment (BEA NIPA Table 5.3.5, all types) and PCE on durable goods (BEA NIPA Table 2.3.5, item 3), divided by GDP deflator (FRED, GDPDEF) and civilian non-institutionalized population (FRED, CNP16OV), log-transformed, first-differenced and de-meaned.

4. Inflation rate

   - Model : $\tilde{\pi}_t^{\text{obs}} = \log\left(\frac{\pi_t}{\pi}\right)$
   - Data : Log difference of GDP Implicit Price Deflator (FRED, GDPDEF) minus 0.5 percentage point.

5. Interest rate

   - Model : $\tilde{i}_t^{\text{obs}} = \log\left(\frac{R_t}{R}\right)$
   - Data : Effective Federal Funds Rate, divided by 400 to express in quarterly units minus logarithm of the model's steady state nominal rate.

6. Real wage

   - Model : $\tilde{w}_t^{\text{obs}} = \log\left(\frac{w_t}{w_{t-1}}\right)$
   - Data : Average hourly earnings of production and non-supervisory employees in total private sector (FRED, AHETPI), divided by GDP deflator (FRED, GDPDEF), log-transformed, first-differenced and de-meaned.

7. Unemployment rate

   - Model : $\tilde{u}_t^{\text{obs}} = \log\left(\frac{u_t}{u}\right)$

- Data : Unemployment as the number of unemployed as a percentage of the labor force (FRED, UNRATE) minus minus 5 percent divided by 100.

8. Lump-sum transfer

   - Model : $\tilde{T}_t^{\text{obs}} = \log\left(\frac{T_t^g}{T_{t-1}^g}\right)$
   - Data : The sum of government's current transfer payment (BEA NIPA table 3.2, item 26), capital transfer payments (item 22), net of current transfer receipts (item 19), capital transfer receipts (item 42), and unemployment benefit (NIPA underlying table 3.12U, item 7), divided by GDP deflator (FRED, GDPDEF) and civilian non-institutionalized population (FRED, CNP16OV), log-transformed, first-differenced and de-meaned.

9. Profits

   - Model : $\tilde{\Pi}_t^{\text{obs}} = \log\left(\frac{\Pi_t}{\Pi_{t-1}}\right)$
   - Data : Corporate profits after tax with inventory valuation adjustment and capital consumption adjustment (BEA account code: A551RC), divided by GDP deflator (FRED, GDPDEF), and civilian non-institutionalized population (FRED, CNP16OV), log-transformed, first-differenced and de-meaned.

10. Central bank's assets

   - Model : $\tilde{A}_{t+1}^{\text{CB,obs}} = \log\left(\frac{A_{t+1}^{\text{CB}}}{A_{2007}^{\text{CB}}}\right)$

- Data : All Federal Bank's assets (FRED, WALCL), divided by GDP deflator (GDP deflator), civilian non-institutionalized population (CNP16OV), and its end of 2007 level. Log-transformed

B.4.2 Structural shocks

1. Total factor productivity shock

$$\log Z_t = \rho_z \log Z_{t-1} + \epsilon_{Z,t} \ , \epsilon_{Z,t} \ \sim \ N(0, \sigma^2_{\epsilon_Z,t}) \tag{B.29}$$

2. Risk premium shock (a shock to banks' discount factor)

$$\Lambda^b_{t,t+1} = \Psi^b_t \Lambda_{t,t+1} \tag{B.30}$$

$$\log\left(\frac{\Psi^b_t}{\Psi^b}\right) = \rho_b \log\left(\frac{\Psi^b_{t-1}}{\Psi^b}\right) + \epsilon_{b,t} \ , \epsilon_{b,t} \ \sim \ N(0, \sigma^2_{b,t}) \tag{B.31}$$

3. Price mark-up shock

$$\Psi^p_t = \frac{\eta_t}{\eta_t - 1} \tag{B.32}$$

$$\log(\Psi^p_t) = \rho_p \log(\Psi^p_{t-1}) + \epsilon_{p,t} \ , \ \epsilon_{p,t} \ \sim \ N(0, \sigma^2_p) \tag{B.33}$$

4. Investment technology shock

$$\log(\Psi^k_t) = \rho_k \log(\Psi^k_{t-1}) + \epsilon_{k,t} \ \ , \sim \ N(0, \sigma^2_k) \tag{B.34}$$

5. Liquidity preference shock

$$\log(\Psi_t^l) = \rho_l \log(\Psi_{t-1}^l) + \epsilon_{l,t} \ , \ \epsilon_{l,t} \ \sim \ N(0, \sigma_l^2) \tag{B.35}$$

6. Wage shock

$$\frac{w_t}{w} = \left(\epsilon_{w,t}\frac{r_t^l}{r^l}\right)^{\vartheta_w(1-\rho_w)} \left\{\frac{w_{t-1}}{w} \times \left(\frac{\pi}{\pi_t}\right)\right\}^{\rho_w} \quad , \quad 0 < \rho_w < 1 \ , \ \vartheta_w > 0 \tag{B.36}$$

7. Lump-sum transfer shock

$$T_t^g = \left(1 - \frac{1}{\Psi_t^g}\right)Y \tag{B.37}$$

$$\log\left(\frac{\Psi_t^g}{\Psi^g}\right) = \rho_g \log\left(\frac{\Psi_{t-1}^g}{\Psi^g}\right) + \epsilon_{g,t} \ , \epsilon_{g,t} \ \sim \ N(0, \sigma_g^2) \tag{B.38}$$

8. Monetary policy shock

$$1 + \hat{i}_{t+1} = (1+\hat{i})\left(\frac{1+\hat{i}_t}{1+\hat{i}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left\{\exp(u_t - u)\right\}^{\phi_u}\right]^{1-\rho_R} \exp(\epsilon_{R,t}) \quad , \ \epsilon_{R,t} \ \sim \ N(0, \sigma_R^2) \tag{B.39}$$

$$i_{t+1} = \min\{0, \hat{i}_{t+1}\} \tag{B.40}$$

9. Fixed cost shock

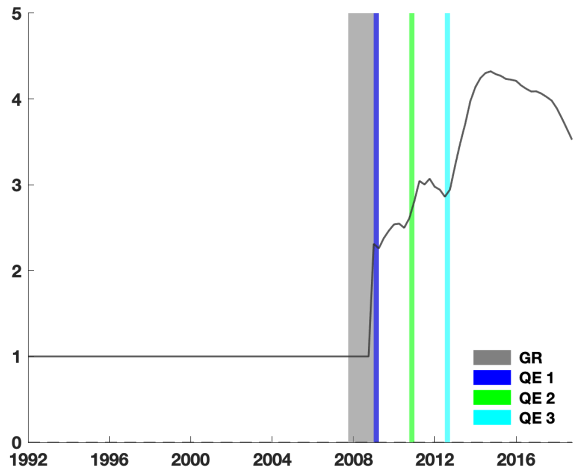
$$\Psi_t^F = \rho_F \Psi_{t-1}^F + (1-\rho_F)\Psi^F + \epsilon_{F,t} \quad , \ \epsilon_{F,t} \ \sim \ N(0, \sigma_F^2) \tag{B.41}$$

10. QE shock

$$A^{\text{CB}}_{t+1} = \Psi_{\text{QE},t} Y \ , \quad \log(\Psi^{\text{QE}}_{t}) = \rho_{\text{QE}} \log(\Psi^{\text{QE}}_{t-1}) + \epsilon_{\text{QE},t} \quad \epsilon_{\text{QE},t} \ \sim \ N(0, \sigma^2_{\text{QE}}) \tag{B.42}$$

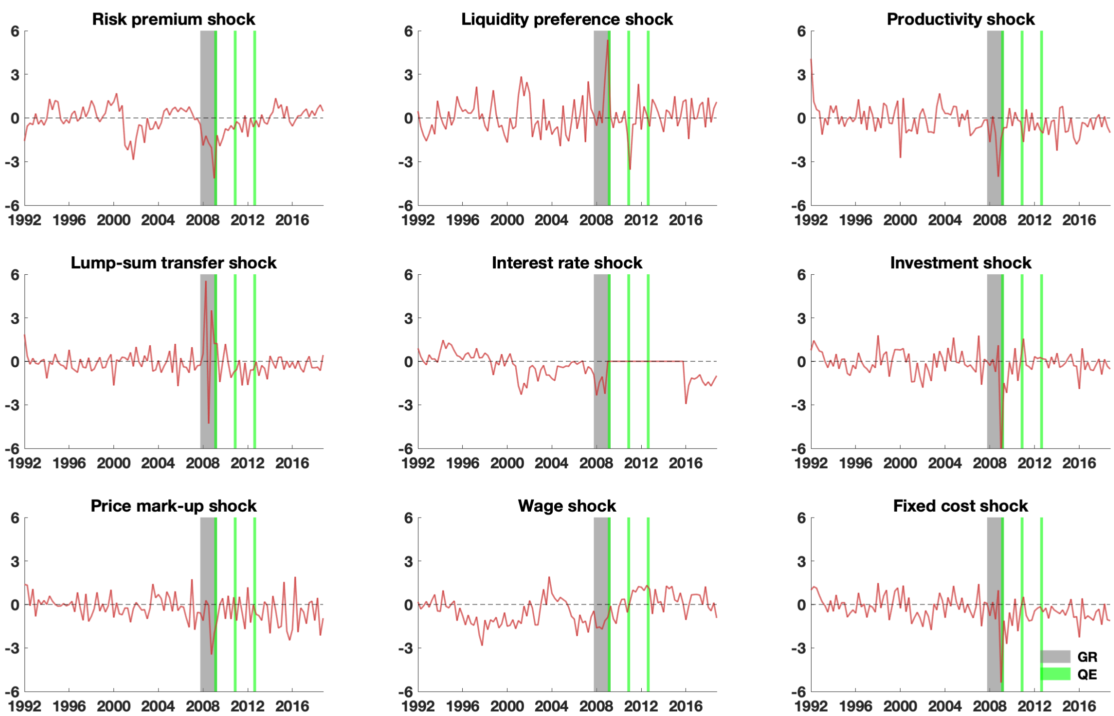
B.4.3 Additional figures and tables

Figure B.3: The central bank's assets



Notes: The figure shows the central bank's asset as the ratio to its end of 2007 level. Green, blue, green, and sky blue area depict the Great Recession periods, the period in which QE 1, 2, and 3 are announced.

Figure B.4: Filtered shock series



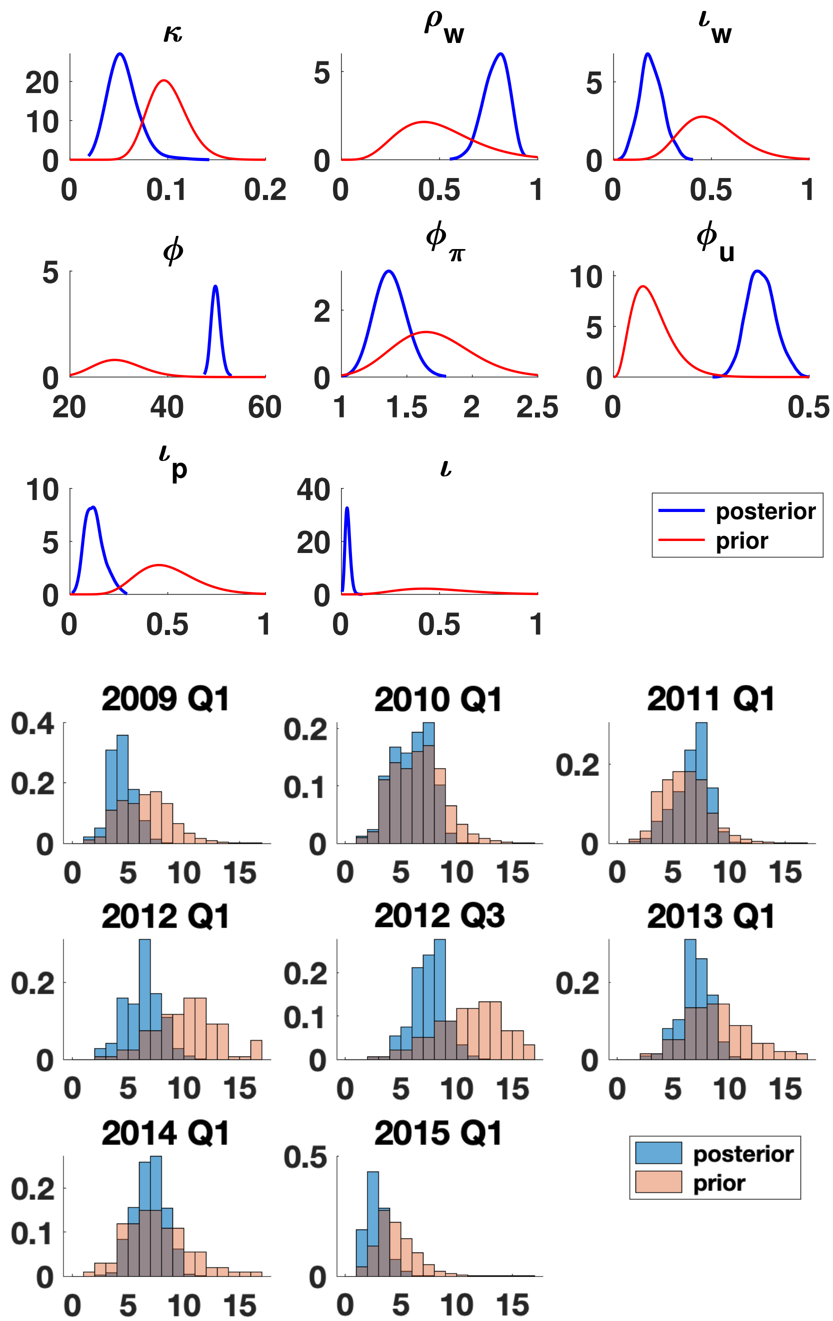
Notes: The figure shows the time series of the filtered shocks during the sample periods as a ratio to its standard deviation. The shaded gray area represents the periods of the Great Recession. The transparent green bars represent the quarters in which QE 1, 2, and 3 are announced or implemented.

Table B.1: Prior and posterior distributions of expected ELB durations

	Prior			Posterior		
	Mode	10%	90%	Mode	10%	90%
2009 Q1	5	2	8	4	3	5
2009 Q2	5	2	8	6	3	8
2009 Q3	5	2	8	5	3	7
2009 Q4	5	2	8	5	3	7
2010 Q1	5	2	8	6	3	8
2010 Q2	5	2	8	6	3	8
2010 Q3	5	2	8	6	3	8
2010 Q4	5	2	8	4	3	6
2011 Q1	4	2	7	7	4	8
2011 Q2	4	2	6	5	3	7
2011 Q3	8	5	11	6	4	8
2011 Q4	8	5	11	7	6	9
2012 Q1	9	5	12	6	4	8
2012 Q2	10	5	14	7	6	9
2012 Q3	10	5	13	7	5	9
2012 Q4	11	7	14	7	6	9
2013 Q1	9	5	13	8	5	9
2013 Q2	7	3	12	7	5	8
2013 Q3	7	4	12	6	5	8
2013 Q4	8	4	11	6	4	8
2014 Q1	6	3	10	7	5	8
2014 Q2	6	3	9	5	4	6
2014 Q3	3	1	5	3	2	5
2014 Q4	2	1	4	3	2	4
2015 Q1	1	1	3	2	1	4
2015 Q2	1	1	3	2	2	3
2015 Q3	1	1	2	2	2	3
2015 Q4	1	1	1	3	1	3

Notes: The unit is one quarter.

Figure B.5: Posterior distributions of estimated parameters and expected ELB durations



B.5 Model Dynamics

A countercyclical response of profits to demand shocks is a common feature of New Keynesian models. Since the factor prices are relatively flexible while the price is assumed to be rigid, a markup of the price over marginal cost is countercyclical in New Keynesian models when demand shocks, such as monetary policy and government spending shocks, occur. Consequently, profits fall after an increase in aggregate demand.[9] Though this feature is not consistent with the existing empirical evidence, the literature has not paid much attention since, in representative agent New Keynesian models, the response of profits did not seem to matter for the model's implications on the aggregate dynamics of the economy.

However, recently, the literature started to challenge this feature of New Keynesian models. Broer et al. (2019) pointed out that a fall in profits is a key amplification channel through which an expansionary monetary policy shock leads to a strong output response. Specifically, a fall in profits induces households to increase their labor supply by generating a negative wealth effect. Alves et al. (2019) also demonstrate that the way profits are distributed affects the aggregate consequences of monetary policy shocks. In particular, when a larger share of profits is allocated to liquid assets, monetary policy shocks have greater amplification in their model. These recent findings in the literature show the importance of profit responses in determining the aggregate dynamics of New Keynesian models.

[9] A lower markup does not necessarily imply lower profits since, in principle, the response of the quantity sold can be large enough to offset the negative effect of markups on profits. However, in standard New Keynesian models, the effect of markup dominates as the quantity response is relatively moderate. As a result, profits decrease despite an increase in demand.

In this paper, I emphasize the importance of profit dynamics for the distributional consequences of monetary policy. Since profits constitute a substantial portion of wealthy households' income, the way that profits respond to monetary policy determines their welfare gains/losses from the policy. In short, when profits respond strongly procyclically to monetary policy as in the data, wealthy households can enjoy a considerable amount of welfare gains from an expansionary monetary policy shock.

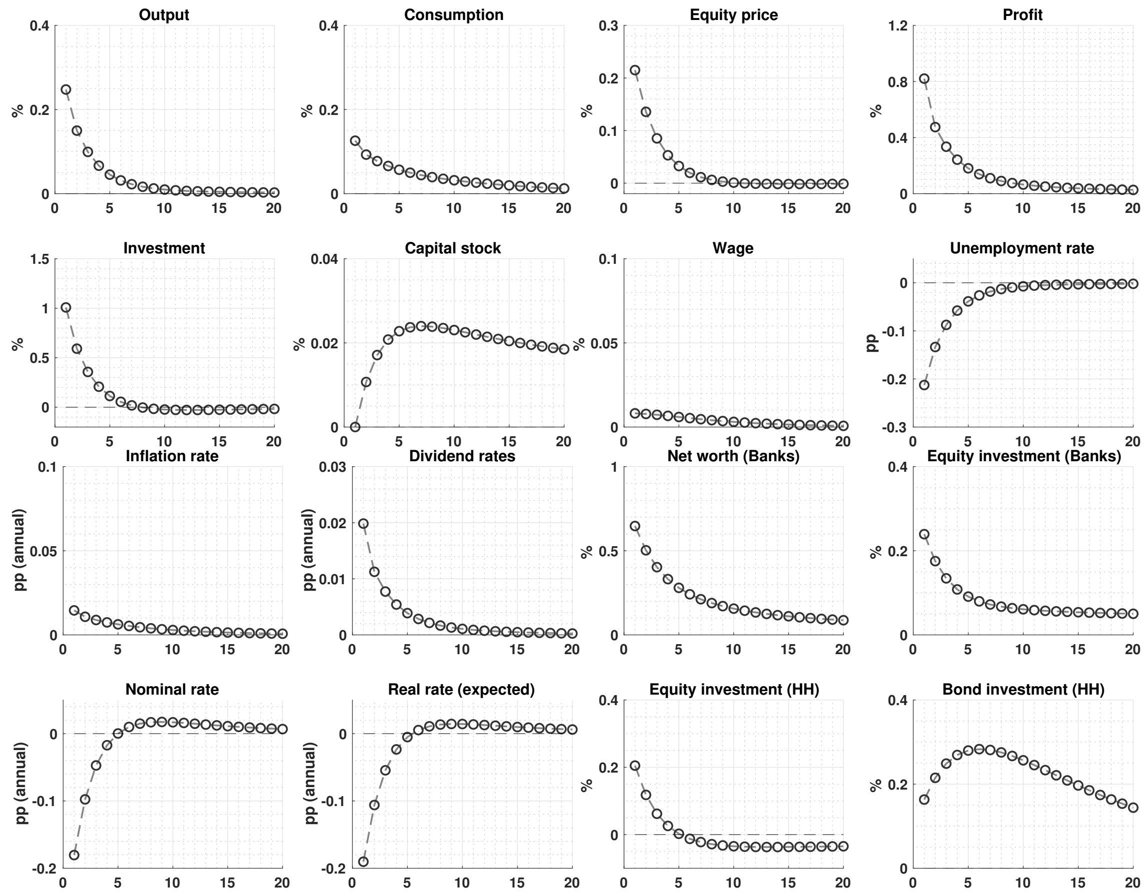
In the following subsections, I show the model's impulse responses, including a procyclical response of profits, to an expansionary monetary policy shock, and discuss how the model generates such a response.

B.5.1 Procyclical profits

Figure B.6 shows the responses of the model's aggregate variables to an expansionary monetary policy shock at the posterior mode of parameter values. The figure shows that, when a negative interest rate shock occurs, profits substantially increase in the model. This feature of the model contrasts starkly with existing New Keynesian models in which profits exhibit strong countercyclicality in response to monetary policy shocks. More importantly, such responses are consistent with empirical evidence; a monetary SVAR model presented in the appendix generates similar profits, wage, and unemployment rate responses in terms of the direction and the relative magnitudes.[10]

[10] A noticeable feature of the model, relative to the SVAR model, is the lack of the hump-shaped responses, which is a common feature of most of the existing HANK models. Since models do not feature internal delaying mechanisms, such as habits, the responses are immediate when there is an exogenous shock. Recently, Auclert et al. (2020b) develop a HANK model that incorporates

Figure B.6: Impulse responses to an expansionary monetary policy shock

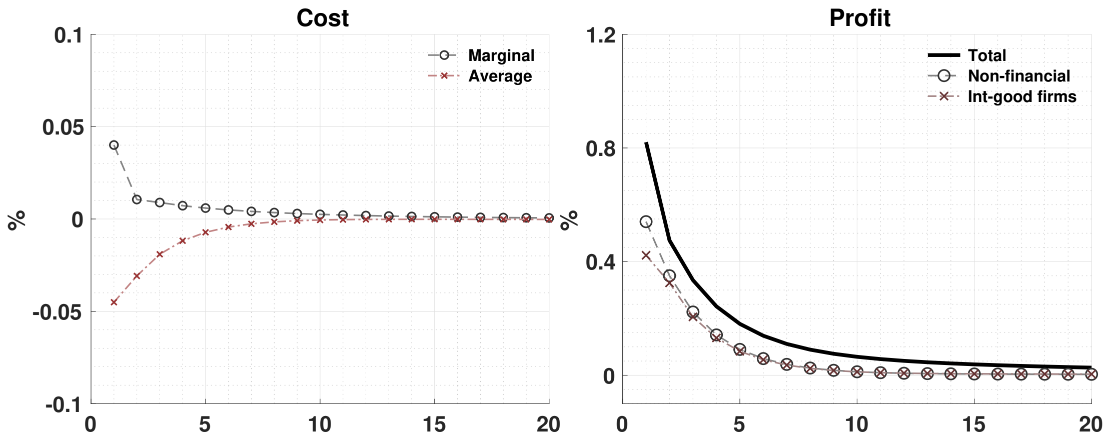


Notes: The figure shows the model's impulse responses to a negative 25 basis points (annualized) interest rate shock. All variables are shown as the percentage deviations from their respective steady state values except for the nominal rate, the inflation rate, the dividend rates and the unemployment rate. The nominal rate, the inflation rate, and the dividend rates are expressed in terms of the annualized percentage point difference from the steady state values. The unemployment rate is shown as the percentage point difference from the steady state unemployment rate.

How does the model generate a procyclical profit response to changes in demand while existing models could not? First, wage rigidity and labor market frictions dampen the response of the real marginal cost. When the aggregate demand increases, firms expand their production by hiring more labor and capital services. In a standard New Keynesian model, such an increase in factor demand leads to an

sticky expectations and generate delayed responses of the aggregate variables to exogenous shocks in their model.

Figure B.7: Responses of different types of costs and profits



Notes: The figure shows the response of different kinds of costs and profits to an expansionary monetary policy shock. The left panel shows the response of the intermediate good firms' marginal cost, which is shown with the gray dotted line with circles, and the average cost of the non-financial sector, which is shown with the red dotted line with crosses. The right panel shows the response of the total profits, total non-financial sector's profits, and the intermediate good firms' profits. The black solid line shows the response of the aggregate profits while the gray dotted line with circles and the red dotted line with crosses show the non-financial sector profits and the intermediate good firms' profits, respectively.

increase in the real marginal cost, or equivalently, a fall in markups. Thus, profits fall.[11] However, in the model, the real wage does not respond much because of wage rigidity. If labor supply adjusts only through intensive margin, little changes in the real wage imply little changes in the labor supply. Then, to increase output, firms need to utilize the capital more intensively, which results in a substantial increase in the capital rental rate or the variable depreciation. An extensive margin adjustment of labor supply via frictional labor markets allows firms to increase labor inputs without increasing the real wage and the capital rental rate much. Consequently, the real marginal cost does not respond strongly to an increase in demands in the model. [12]

[11] In a standard New Keynesian model, the degree of price rigidity should be high for a monetary policy shock to have real effects. A high degree of price rigidity implies, in the absence of the factor price rigidity, a strong countercylicality of profits or markups, the latter of which has been often challenged in the literature.

[12] Note that the marginal cost for intermediate good firms is determined by the capital and labor rental rate, and I do not impose any rigidity on the labor rental rate. However, wage rigidity and

Besides, based on a recent finding of Anderson et al. (2018), I assume that the fixed cost accounts for a significant proportion of the total production cost.[13] The presence of the fixed cost helps the model generate a procyclical profit response as well. What matters for firms' profit is not the marginal cost per se but the average production cost. When the fixed cost accounts for a substantial proportion of the total cost, the average cost can fall even though the marginal cost increases. Moreover, as the production sector is decentralized in the model, the sector-wide cost is lower than the cost of intermediate good firms.[14] Thus, as Figure B.7 shows, while the marginal cost of intermediate good firms mildly increases, the average cost of the entire non-financial sector decreases, which results in a substantial increase in non-financial firms' profits.

Finally, the presence of banks also helps the model generate a substantial increase in profits. First, an increase in banks' net-worth contributes to higher profits.[15] When the interest rate falls and investment increases, the equity price

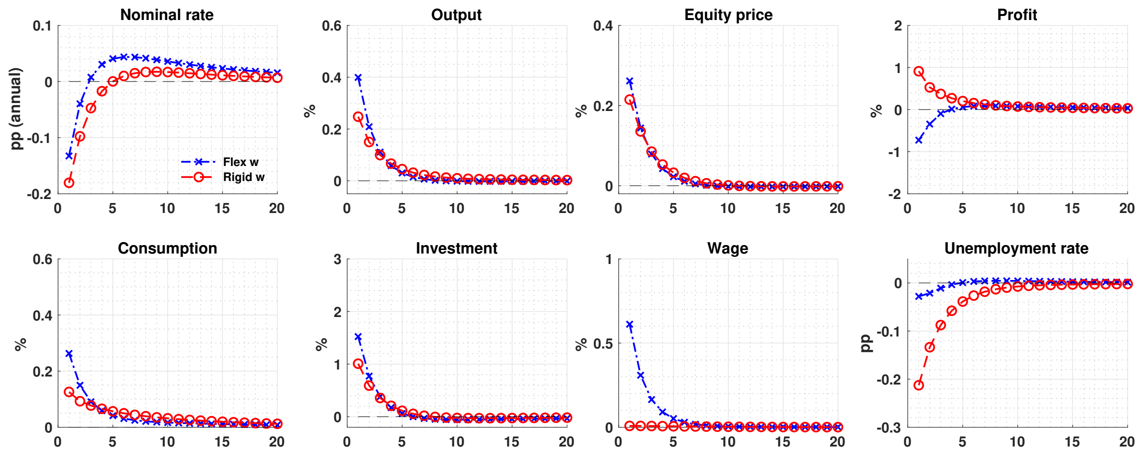
labor supply via labor agencies effectively increase the elasticity of labor supply with respect to changes in the labor rental rate. Thus, to achieve the same amount of an increase in labor input, a smaller magnitude of the rental rate increase is required.

[13]Anderson et al. (2018) show that, using confidential retail sector transactions data, gross margin, which can be interpreted as markups in the model, is acyclical or mildly procyclical while net operating profits are highly procyclical. They interpret the latter result as suggesting the presence of fixed costs.

[14]Ignoring miscellaneous adjustment costs, the intermediate good firms' total cost can be expressed by $\Gamma_t Y_t + \Xi$, where Γ_t is the real marginal cost and Ξ is the fixed cost. In contrast, the total cost of the non-financial sector as a whole is $\delta(v_t)K_t + w_t L_t + \iota V_t + \Xi$. Because of accelerated depreciation and the wage rigidity, the latter is smaller than the former during an expansion unless ι is too high.

[15]The empirical evidence on the effects of monetary policy on banks' profitability is mixed and not conclusive. Borio et al. (2017) concluded that low interest rates and flat term structure erodes banks' profitability mainly through their negative impacts on banks' net interest income. However, they solely focused on the trend changes in the interest rate structure and, importantly, did not take into account any effects of monetary policy on the aggregate economy in their analysis. A more recent work by Altavilla et al. (2018) showed that an expansionary monetary policy shock does not reduce banks' profitability once they control for the endogeneity of the policy measures. Finally, Zimmerman (2019) showed, using the panel data of more than 100 countries for more

Figure B.8: Wage rigidity and the IRFs to an expansionary monetary policy shock



Notes: The figure shows the impulse responses of variables in models with different assumptions on the wage rigidity. The blue dotted lines with crosses show IRFs from the model with flexible wage ($\rho_w = 0$), and the red dotted lines with circles show IRFs from the baseline model with wage rigidity. All parameters take on values at their respective posterior mode in each model. The unit for the nominal interest rate and the unemployment rate is percentage point. The unit for all other variables is the percentage deviation from the corresponding steady state value.

increases, and thus the gross return on banks' net-worth substantially increases on impact. The effects of an increased net worth propagate through a financial accelerator channel and persist for a long time, leading to higher aggregate profits.[16] In the process, banks also lead to strong investment responses. Thus, even though consumption response is relatively small due to a weak redistribution and the wage rigidity, the overall demand of goods can increase significantly because of banks' investment demand.

Table B.2: Posterior mode under the rigid and flexible wage assumption

	κ	ι_p	ρ_w	ι_w	ι	ρ_R	ϕ_π	ϕ_u
Rigid wage	0.0525	0.1219	0.7982	0.1835	0.0317	0.7927	1.3101	0.3748
Flexible wage	0.1114	0.0564	0	0	0.0929	0.8405	2.5354	0.1590

B.5.2 Comparison with a model with the flexible wage

Figure B.8 shows the impulse responses of variables in the baseline model and the model with the flexible wage. For a fair comparison, I re-estimate the model by assuming that the wage is flexible, i.e., $\rho_w = 0$. Table B.2 shows the values of key parameters at the posterior mode.

Two things are noticeable in the figure. First, depending on the assumption of wage rigidity, the response of profits is entirely different. When the wage is assumed to be flexible, profits exhibit strong countercyclicality in response to monetary policy shocks. While profits fall substantially, the real wage soars after an increase in the aggregate demand. Due to a strong real wage response, the unemployment rate changes little in the model with the flexible wage. However, as I show in the appendix, these responses are not consistent with the empirical evidence.

The other result that is noticeable in the comparison is that, when the real wage is flexible, an expansionary monetary policy shock has stronger initial stimulus effects compared to a model with wage rigidity. For instance, an annualized 25 bp falls in the policy rate leads to 0.4% increase in output on impact when the

than 100 years, the importance of loan losses and credit growth for bank profits and shows that a monetary policy tightening leads to a fall in banks' profits in contrast with the previous findings.

[16]Due to the incentive problem characterized by Gertler and Karadi (2011), the total amount of deposits that a bank can take is limited to a certain fraction of the bank's net worth. Thus, an increase in the bank's net worth allows the bank to purchase more assets by taking more deposits, which leads to a further increase in its net worth.

wage is flexible. In contrast, the corresponding magnitude of the impact is only 0.25% in the baseline model. Given that the parameter values at the mode imply much smaller real effects of monetary policy shocks, i.e., a steeper Philips curve and stronger responsiveness of the policy rate to the inflation gap, the magnitude of the initial response under the flexible wage is substantial. Two channels are working behind this result. The first one is redistribution. When profits are strongly countercyclical, an expansionary monetary policy shock leads to a stronger redistribution from wealthy to working-class households. Since the latter has a higher marginal propensity to consume than the former, the aggregate consumption response from the monetary policy shock is larger when the wage is flexible. The other one is an amplification that arises from the complementarity between consumption and labor in GHH preference. When the real wage goes up, households supply more labor under the GHH preference. Then, they also demand more consumption since consumption and labor are complementary. Such an increase in demand for goods further stimulates the production and increases the real wage, creating a substantial amount of amplification. Auclert et al. (2020a) argue that, based on earlier findings of Monacelli and Perotti (2008) and Bilbiie (2009), such an amplification due to the complementary between consumption and labor results in unrealistically high fiscal multipliers in New Keynesian models with the flexible wage.

To recapitulate, the model with the flexible wage generates impulse responses of key aggregate variables that are not consistent with the data in terms of both direction and magnitude. Such results support the modeling approach adopted in this paper, which emphasize the role of wage rigidity and frictional labor markets.[17]

B.6 Structural VAR analysis

In this section, I provide an empirical evidence on the effects of monetary policy on real wage, unemployment rates, and profits, which motivated a new HANK model that I develop in this paper. Specifically, I conduct a structural vector autoregression (VAR) analysis. The specification of the SVAR model is based on a standard monetary VAR model that apperar in Christiano et al. (1999) and Christiano et al. (2005). Specifically, I augment a 7 variable VAR model in Christiano et al. (1999) with the variables of interest in this paper, i.e., real wage, unemployment rates, and profits. In addition, to have a better understanding of the fiscal responses, I include the lump-sum transfer variable in the VAR model as well.

As is standard, it is assumed that the policy instrument, i.e., the Fed Funds rate, denoted by FF_t, is determined as follows.

$$\mathrm{FF}_t = f(\Omega_t) + \epsilon_{r,t} \tag{B.43}$$

where f is the feedback rule, Ω_t is the information set available to the central bank in period t, and $\epsilon_{r,t}$ is an exogenous shock to the policy decision. Let $\mathcal{Y}_t$ denote the vector of the variables included in the VAR model.

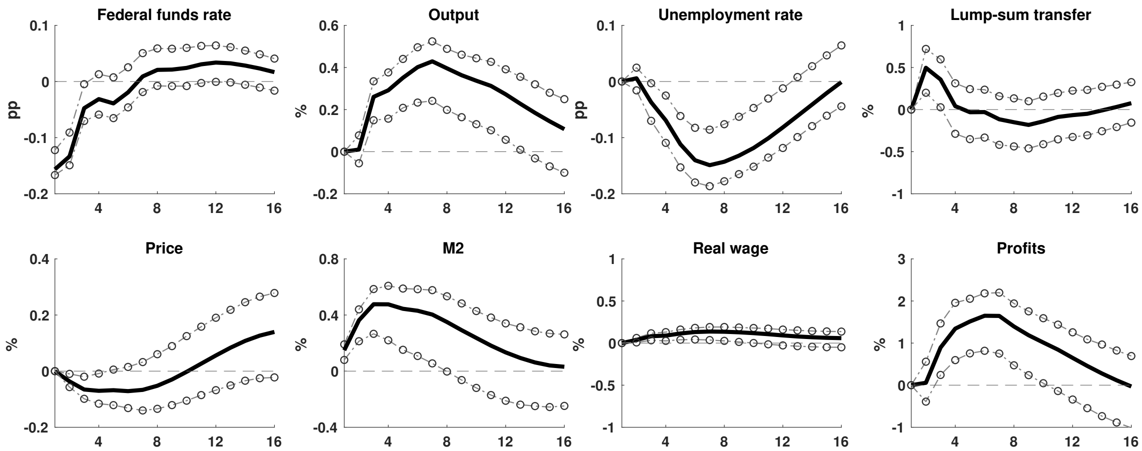
[17] The role of the wage rigidity recently regained attention in the literature. Broer et al. (2019) advocate focusing on the wage stickiness rather than the price stickiness because of its implications on the redistribution and the amplification in New Keynesian models. Nekarda and Ramey (2020) also do so based on their findings on the cyclicality of markups.

$$\mathcal{Y}_t = \begin{bmatrix} \log(\text{Output}_t) \\ \log(\text{Price index}_t) \\ \log(\text{Commodity price index}_t) \\ \log(\text{Real wage}_t) \\ \text{Unemployment rate}_t \\ \log(\text{Profits}_t) \\ \log(\text{Lump-sum transfer}_t) \\ \text{FF}_t \\ \log(\text{Total reserves}_t) \\ \log(\text{Non-borrowed reserves}_t) \\ \log(\text{M2}_t) \end{bmatrix} \tag{B.44}$$

The information set available to the monetary authority includes the data on output, price index, commodity price, index, real wage, unemployment rate, profits, and lump-sum transfer. As in Christiano et al. (1999), I assume that the innovation $\epsilon_{r,t}$ is orthogonal to all variables in the central bank's information sect. Thus, the monetary policy shock is identified using a standard recursive identification strategy.

For the data, I use the same data that I used for the estimation of my model. The exceptions are commodity price index, total reserve, non-borrowed reserve, and M2, which are not included in the set of observaables for the estimation. For the commodity price index, I use the World Bank non-energy commodity price

Figure B.9: Impulse responses to a shock to FFR: 1960 Q1 to 2007 Q4



Notes: The figure shows the impulse responses of variables to a negative one standard deviation fall in the Federal Funds rate in a SVAR model. The Federal funds rate and the unemployment rate are shown as the percentage point difference from the pre-shock levels. All other variables are shown as the percentage deviation from the pre-shock levels. The dotted lines with circles show 90% boot-strapped confidence intervals with 5,000 runs for each impulse response.

index, smoothing the quarterly change by taking a three quarter average.[18] For the number of lags, I use 4 lags, and the data period is from 1960 to 2007. For the robustness check, I also used 1) average hourly earnings of production and non-supervisory workers, and 2) profits before tax without investment valuation and capital consumption adjustment. Also, I compute impulse responses, using a short sample periods, i.e., from 1979 Q1 to 2007 Q4. Across different specifications, data, and sample periods, the results are similar.

Figure B.9 shows the impulse responses of variables to a 11 basis point expansionary monetary policy shock. As shown in the figure, in response to an expansionary monetary policy shock, the unemployment rate decreases substantially while the real wage responds little. The real wage responses are barely statistically significant. In contrast, profits rises significantly. The lump-sum transfer responds

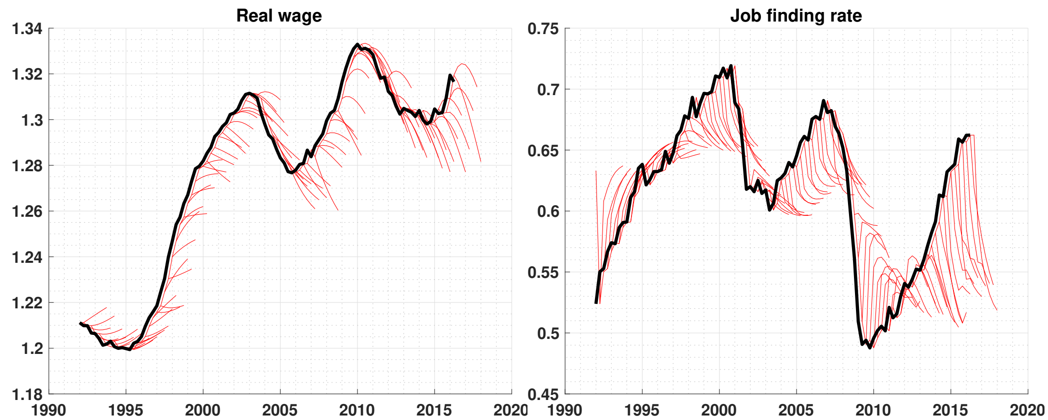
[18]The commodity price index is included to alleviate the 'price puzzle' phenomenon.

Figure B.10: Impulse responses to a shock to FFR: 1979 Q1 to 2007 Q4

Notes: The figure shows the impulse responses of variables to a negative one standard deviation fall in the Federal Funds rate in a SVAR model. The Federal funds rate and the unemployment rate are shown as the percentage point difference from the pre-shock levels. All other variables are shown as the percentage deviation from the pre-shock levels. The dotted lines with circles show 90% boot-strapped confidence intervals with 5,000 runs for each impulse response.

procyclically for the first few periods after the shock, but the responses are mostly statistically insignificant. The corresponding variables in the model exhibit similar dynamics except for the lack of hump-shaped responses, which is a common limitation of the most of existing HANK models in the literature.[19] Most of variables in the SVAR model peaks between 4th and 8th quarters after the shock. In contrast, in the model, responses are immediate.

Figure B.11: Realized and expected paths of the real wage and the job-finding rate



Notes: The figure shows the realized values of the equity price in the sample along with its expected path in each period. The thick black line shows the realized path and the red 'hairs' are the expectations.

B.7 Further details on the results

B.7.1 The decomposition method

To evaluate the relative contribution of various channels to the evolution of inequality and heterogeneous welfare effects, I compute foresight paths of the following variables each period in the sample .

$$\{\ w_{t,t+j}\ ,\ i_{t,t+j}\ ,\pi_{t,t+j}\ ,\ q_{t,t+j}\ ,\ r^a_{t,t+j},\ \Pi_{t,t+j}\ ,T_{t,t+j}\ ,f_{t,t+j}\}_{j=1}^{N} \tag{B.45}$$

where $x_{t,t+j}$ is the expected value of x in period $t+j$ given the information in period t. N is a very large number that ensures that $x_{t,t+N}$ converges to its steady state value in N periods. The above eight variables, i.e., real wage, nominal rate, inflation rate, equity price, dividend rates, total lump-sum transfer, and the job-finding rate,

[19]The only exception in the current literature is the model of Auclert et al. (2020b). They develops a HANK model with sticky expectations and generates hump-shaped responses of aggregate variables in a full-fledged HANK model.

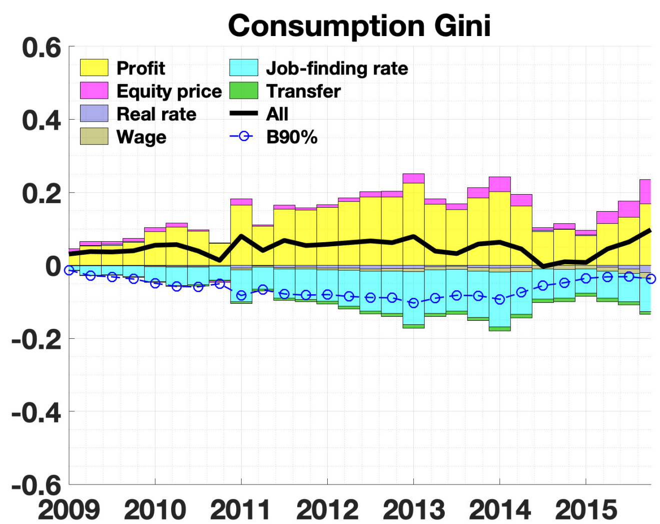
are what determine the household's optimal decisions and welfare together with the expected future value (utility) of households' choices. Exploiting the fact that the expected future shocks are zero each period in the model, I compute the expected paths of the above variables both in the baseline and the alternative cases.[20] Using different combinations of these paths, I solve the household's problem from $t + N$ periods backwardly and compute households' optimal decisions and values (utility). For instance, in one path, I assume that only the job-finding rate follows the path in the baseline case, and all other variables follow the path in the alternative case. By computing households' optimal decisions and the associated utility in this path and comparing them with optimal decisions and values in the alternative case, I can compute the contribution of the job-finding rate on the behavior and expected welfare of households in a given period in the baseline case.[21] Figure B.11 shows the realized path of the real wage and the job-finding rate along with each period's household expectations on it.

[20]The number of the expected paths is equal to the number of periods in the sample multiplied by the number of variables. The starting value of each path, i.e., $x_{t,t}$, coincides with the realized value as it is observed, but all the future expectations are not necessarily correct because of unexpected shocks in the future. That is, $x_{t,t+1}$ is, in general, different from $x_{t+1,t+1}$.

[21]For the complete decomposition of the households' behavior and the associated welfare, I examine the following eight combinations. In the first combination, all variables follow the path in the counter-factual case. In the second combination, all variables follow the paths in the baseline case. In the third case, only the profit and dividend rates follow the paths in the baseline case, while all others follow paths in the counter-factual case. In the fourth combination, only the nominal rate and the inflation rate follow the baseline paths. In the fifth combination, only the real wage follows the baseline path. In the sixth combination, only the job-finding rate follows the baseline path. In the seventh combination, only the equity price follows the baseline path. Finally, in the eighth combination, only the total transfer follows the baseline path, and all others follow the paths in the counter-factual case.

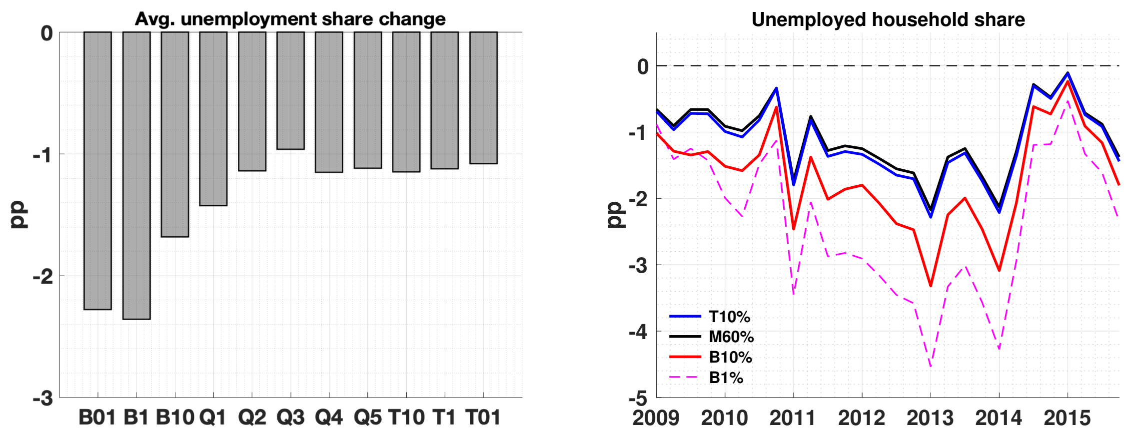
B.7.2 Additional figures

Figure B.12: Distributional effects of QE: Gini index



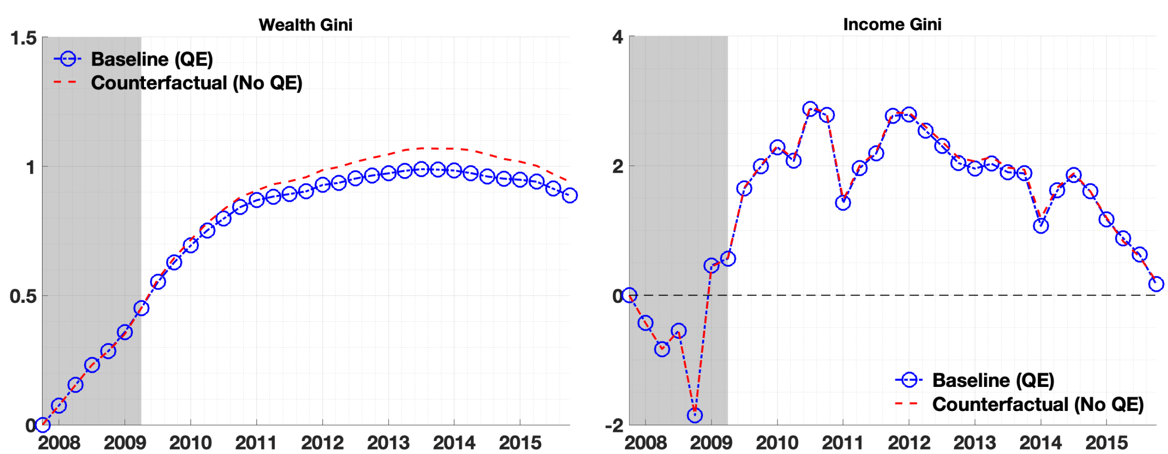
Notes: The figure shows relative degrees of inequality in the model during the ELB episode as differences in the Gini index between the baseline and the counterfactual case. The thick black line shows the overall effects of QE, while each bar shows the contribution of each variable to the overall effects. The blue dotted line with circles shows the Gini index computed from households at the bottom 90% of the wealth distribution. The Y-axis unit is the difference in the Gini index, which is on a zero to 100 scale.

Figure B.13: Unemployed household shares across wealth groups



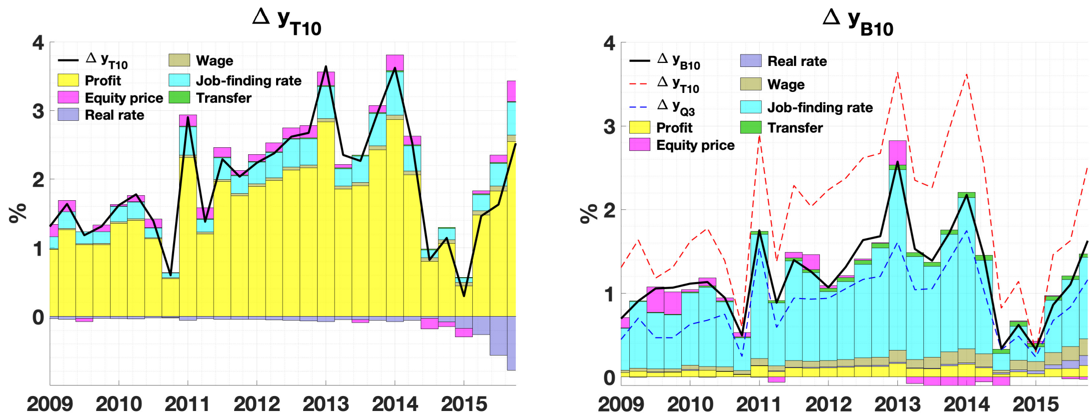
Notes: The left panel shows average changes in the share of unemployed households induced by QE across wealth groups. The right panel shows the evolution of unemployed household shares during the ELB episode, as percentage point difference from the corresponding values in the counterfactual case with no unconventional policy interventions. The blue, black, red, and dashed pink lines show the share of unemployed households in the top 10%, the middle 60%, the bottom 10%, and the bottom 1% of the wealth distribution, respectively.

Figure B.14: Wealth and income inequality during the ELB period: Gini index



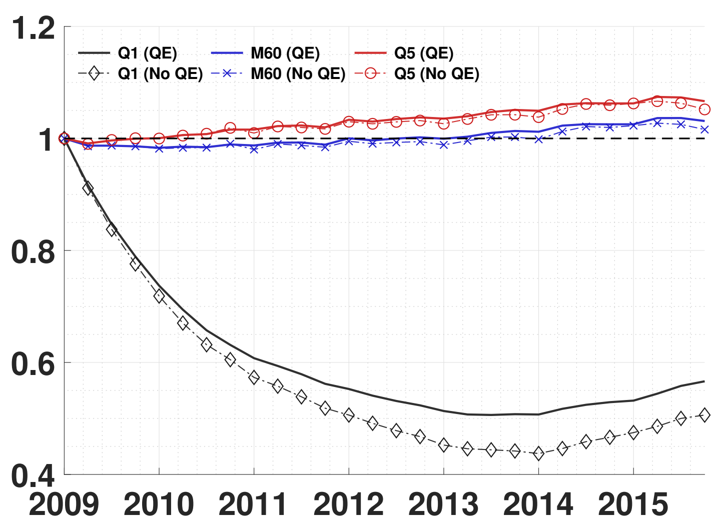
Notes: The figure shows the evolution of the wealth and income Gini indices during the ELB episode, as differnces of the index relative to its 2007 Q4 level. The blue lines with circles show the Gini indices in the baseline case. The dashed red lines show the Gini indices in the counterfactual case.

Figure B.15: Top 10% vs Bottom 10%: Income effects decomposition



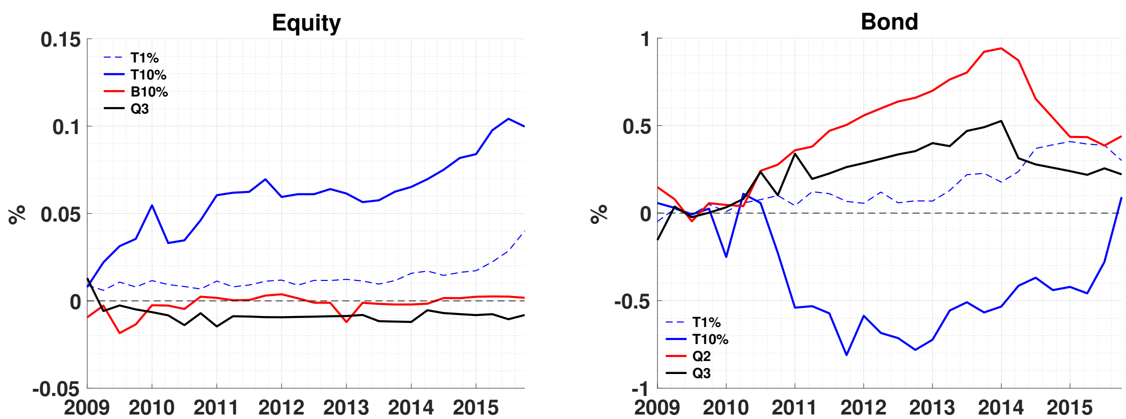
Notes: The figure shows the decomposition of income growth due to QE, for the top 10% and the bottom 10% of the wealth distribution. The black line in each panel shows the growth rate of total income. Each bar shows the contribution of each income component to the total income growth attributable to QE during the ELB episode. In the right panel, the red and the blue dotted line show the income growth rates of the top 10% and the middle quintile, respectively.

Figure B.16: Effects of QE on households' wealth



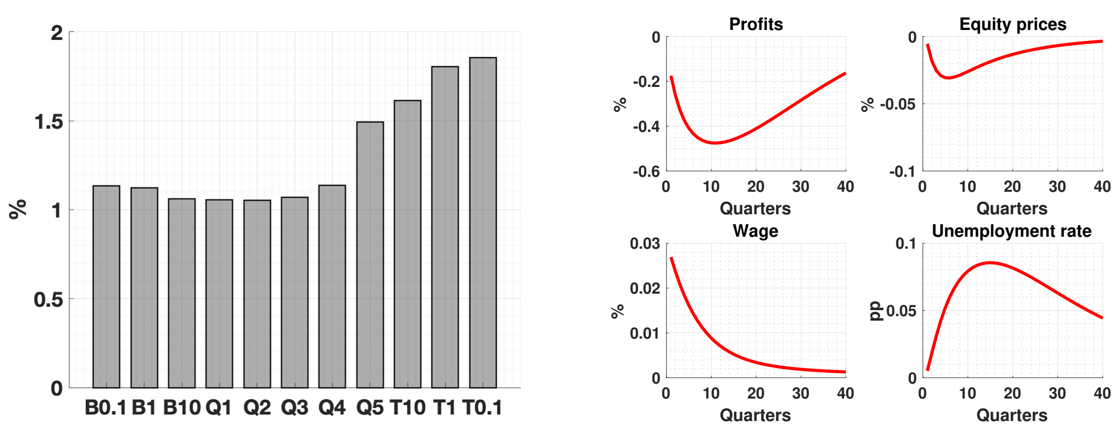
Notes: The black, blue, and red straight lines show the ratio of households' wealth during the ELB episode, relative to their 2009 Q1 level, in the baseline case with QE operation. The black, blue, and red dashed lines with diamond, crosses, and circles show the ratio in the counterfactual case with no QE.

Figure B.17: Effects of QE on equity and bond shares



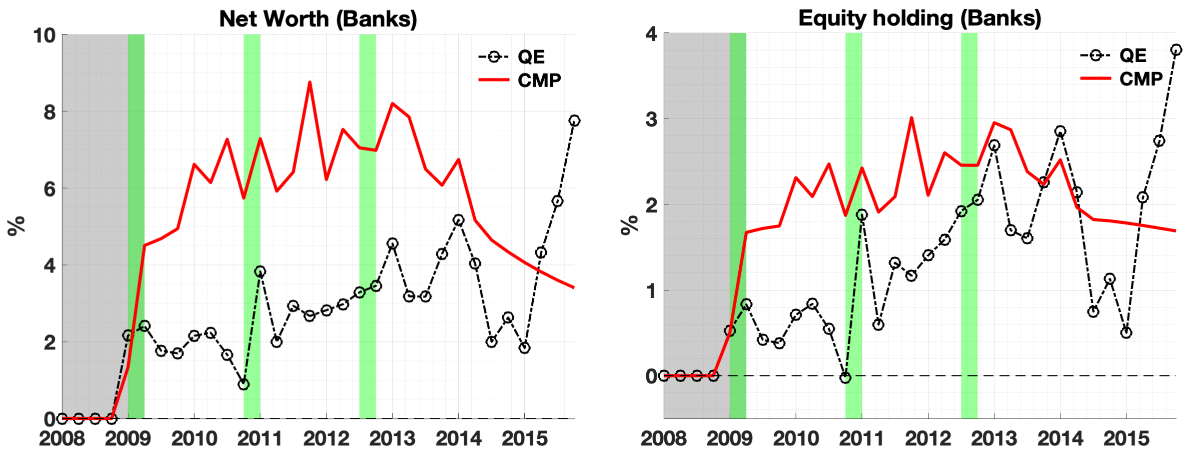
Notes: The figure shows different wealth groups' equity and bond shares during the ELB episode. T1%, T10%, B10%, Q2 and Q3 refers to the top 1% and 10%, the bottom 10%, the second and the middle quintile, respectively. The unit is the difference in the share of equity and bond between the baseline and the counterfactual case.

Figure B.18: Average consumption gain and households' expectations beyond the sample



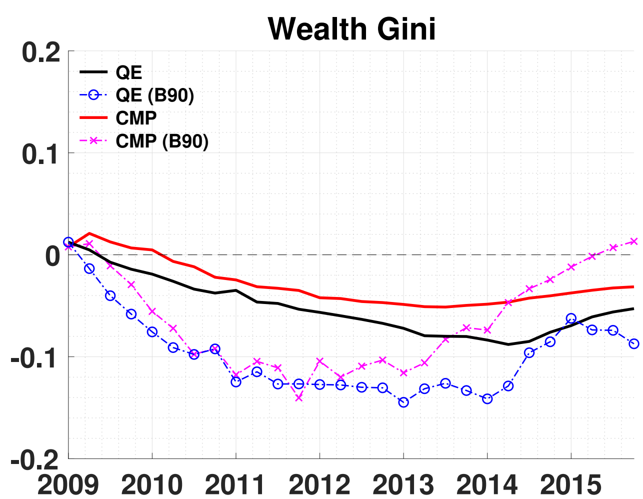
Notes: The left panel shows relative levels of consumption in the baseline case of QE during the ELB episode, relative to the corresponding consumption levels in the counterfactual case of no QE across households' wealth groups. The right panel shows households' expectations on profits, equity prices, wages, and unemployment rates from 2019 Q1 onwards.

Figure B.19: Effects of monetary policy on banks: QE vs CMP



Notes: The red lines show banks' net worth and equity holdings in the case of CMP, relative to their respective level in the counterfactual case with no policy interventions. The black dotted lines with circles show the corresponding values in the case of QE. QE refers to quantitative easing. CMP refers to conventional monetary policy.

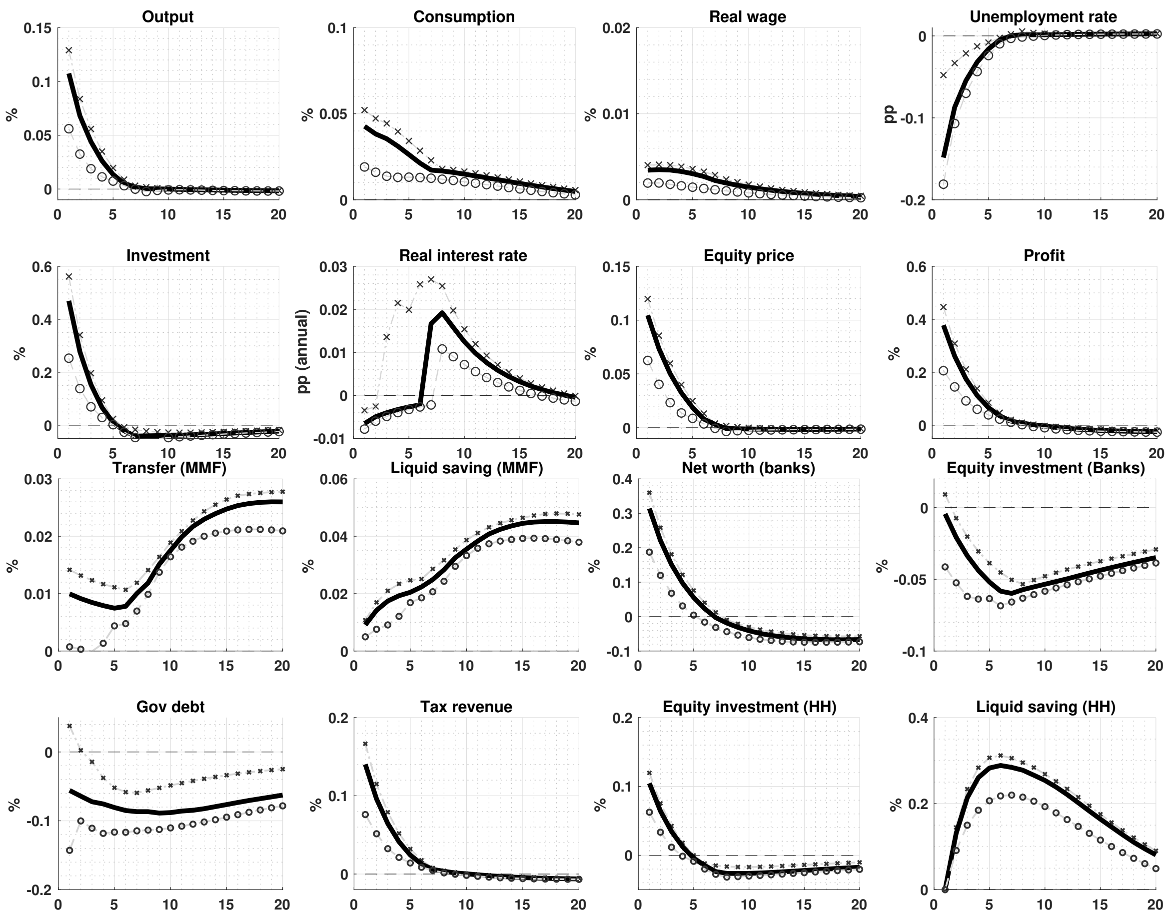
Figure B.20: Distributional effects of QE and CMP: the wealth Gini index



Notes: The figure shows the differences in wealth Gini indices between the case of QE or CMP and the counterfactual case of no policies. The black and red solid lines shows the wealth Gini index in the case of QE and CMP, respectively. The pink line with crosses and blue line with circles show the Gini indices among the bottom 90% households.

B.7.3 Effects of QE shocks: aggregate effects

Figure B.21: Aggregate effects of QE shocks



Notes: The figure shows the effects of an unexpected central bank's asset purchases on key aggregate variables during the ELB episodes. The thick black lines show the mean effects on each aggregate variable while the dashed lines with cross and circles show the 90 percentile and 10 percentile effects. The effects are calculated using the expected path of the economy each period.

The results presented in the main text are compounded by different factors, including QE, exogenous ELB durations, shocks, and different states of the economy. In this section, I evaluate the effects of QE by fixing the latter three factors. That is, I assume the same expected ELB durations and the same state of the economy, and vary only the amount of the central bank's asset purchases in a given period.

Hence, the sole difference between the two cases that I will examine is the QE shock.

Figure B.21 shows the average effects of unexpected central bank's asset purchases that are equivalent to 1.1% of the steady state output on aggregate variables.[22] Overall, the effects are very similar to the effects of an expansionary monetary policy shock in terms of the sign and relative magnitudes, except for the dynamics of the real interest rate.[23] When the central bank increases its asset holdings by about 20% of its steady state asset holding, output and investment increase by, on average, 0.1 and 0.5% on impact, respectively.[24] The effect on consumption is relatively small, with an average increase of about 0.05% on impact.

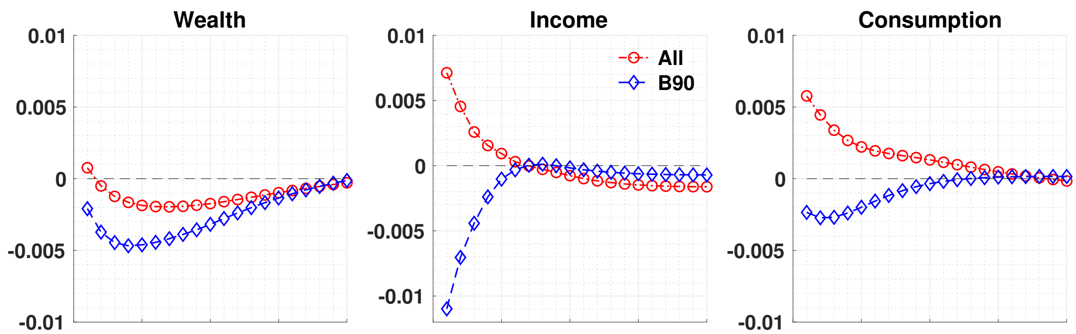
Relative magnitudes of the real wage, unemployment rate, equity price, and profit responses are also similar to those of an expansionary interest rate shock. On impact, profits and the equity price rise up to 0.4% and 0.1% on average. The effects on unemployment rates are also significant, with the average effect of 0.15% fall on impact. In contrast, the effects on the real wage are almost negligible. In short, QE has aggregate effects that are very similar to those of the conventional monetary policy, except for the real interest rate, which is slightly lower initially but expected

[22]The effects are computed along the expected path of the economy each period. That is, the figure shows what would have happened in the economy if there were no other shocks except for the first period. That is, the first period's effects are realized effects while the subsequent periods' effects are expected ones. As the economy experiences different sets of exogenous shocks and the expected ELB durations are different each period, the effects of QE shocks are also different each period. The average amount of unexpected increase in the central bank's asset is about 20% of the steady state central bank asset during the ELB episodes.

[23]In the case of the real interest rate, a QE shock lowers the real rate while the economy stays at the ELB by inducing higher inflation rates. However, when the exogenous ELB durations end, the real interest rate jumps to a higher level than when there was no QE as the economic conditions are better under QE. This is because, without QE, the policy rate prescribed by Taylor rule is lower.

[24]Relative to the size of unexpected asset purchases, the aggregate effects are quite small. There are two reasons for this result. First, QE crowds out private investment by lowering equity premium. Second, an expected increase in the future real rate partially offsets initial stimulus effects.

Figure B.22: Distributional effects of QE shocks: Gini index



Notes: The figure shows the average effects of QE shocks on wealth, income, and consumption Gini index during the ELB episodes. The red dotted lines with circles show Gini indices computed from the whole distribution. The blue dotted lines with circles show Gini indices computed from the bottom 90% household distribution.

to be higher in the later periods when the ELB duration ends.

B.7.4 Effects of QE shocks: distributional effects

Figure B.22 shows the effects of QE shocks on wealth, income, and consumption Gini indices. In contrast to the previous result, an expansionary QE shock increases income inequality in the short-run, i.e., up to 6 quarters after the shock. This result implies that a lower income inequality under QE with endogenous ELB durations results from different economic states, rather than the asset purchase itself. Even though QE shocks tend to increase income inequality, a measured degree of inequality is lower because of the persistent effects of lower unemployment rates on income distribution. In short, the cumulative effects of a series of QE shocks can lower income inequality even though a QE shock per se tends to increase it.

More importantly, when the degree of inequality is measured only among the bottom 90% households, the effects of QE shocks on inequality are qualitatively different. As shown in the figure, QE shocks lower income and consumption inequality among the households that belong to the bottom 90% wealth group. This result implies that, if a data or a model fails to capture the dynamics of wealthy households' income and consumption, an analysis based on it can lead to an incomplete and misleading conclusion on the effects of QE on the degree of inequality.

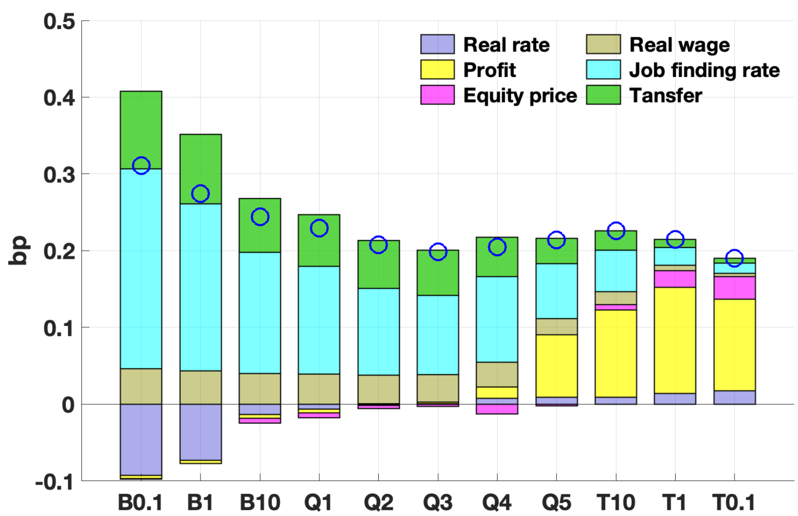
Finally, I examine the welfare gains from an expansionary QE shock across different household groups. Figure B.23 shows how much of their life-time consumption under no QE shock would households give up to benefit from an expansionary QE shock.[25] Overall, an expansionary QE shock has non-linear welfare effects, benefiting both ends of the wealth distribution more than it benefits the middle class. The figure shows a clear u-shape pattern across five quintiles of wealth groups. I call this result 'hollowing out' of the distribution.

Regarding the decomposition of the welfare effects, the results are consistent with the previous results, except for the additional effects form the transfers.[26] QE benefits the majority of households by improving labor market conditions, but at the same time, benefits the wealthy substantially with higher profits. In the model, the top 1% and 0.1% household groups benefit the most from an expansionary monetary policy shock.

[25]Note that, in this section, CE is computed over two different life-time paths of (expected) consumption, one under an expansionary QE shock and the other under no QE shock, instead of using realized paths of consumption during the ELB episodes. Also, I compute CE each period with newly-defined wealth groups, using the period's wealth distribution. The figure shows the average CE for each group during the ELB episodes.

[26]As shown in Figure B.21, MMMF increases its transfers six quarters after the shock. This is because the real rate increases substantially as the exogenous ELB durations end around that time. A higher real rate has both income and substitution effects. In the model, as MMMF holds a large amount of deposits, income effects are dominant. An increase in the transfers disproportionately benefits relatively poor households, strengthening the u-shape pattern shown in the figure.

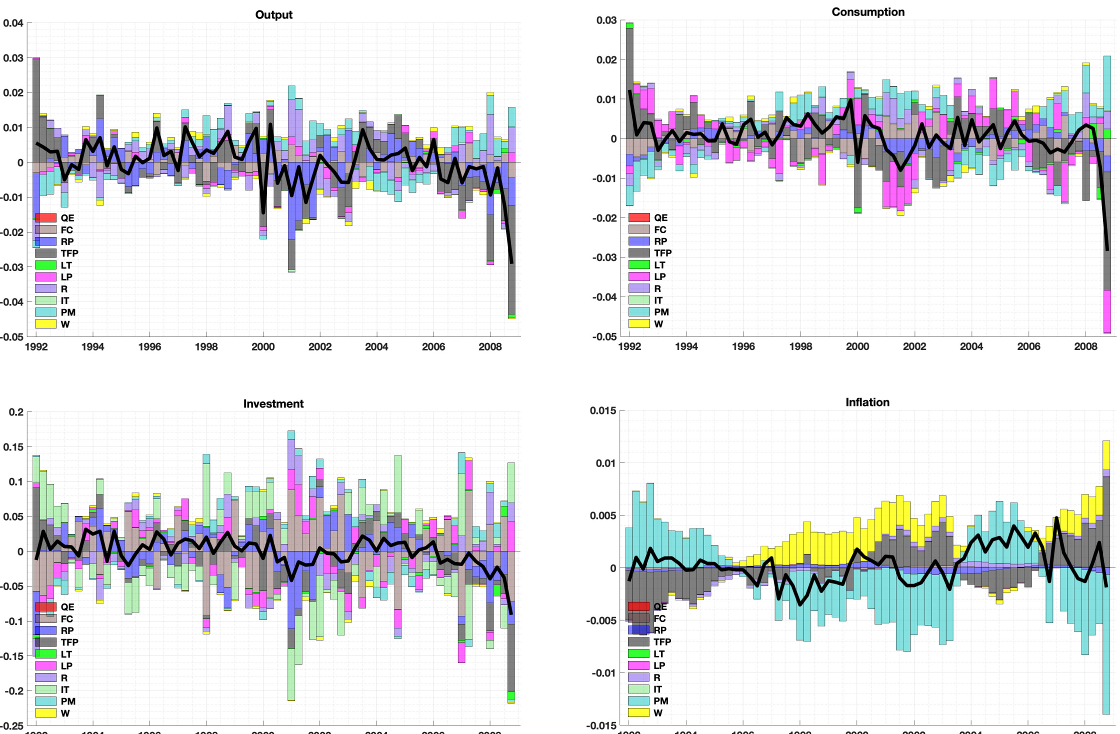
Figure B.23: Distributional effects of QE shocks: CE



Notes: The figure shows the welfare effects of QE shocks in terms of CE. Each bar represents the contribution of each variable to the total welfare gain. Bars in the negative region represents welfare losses. The unit is basis points.

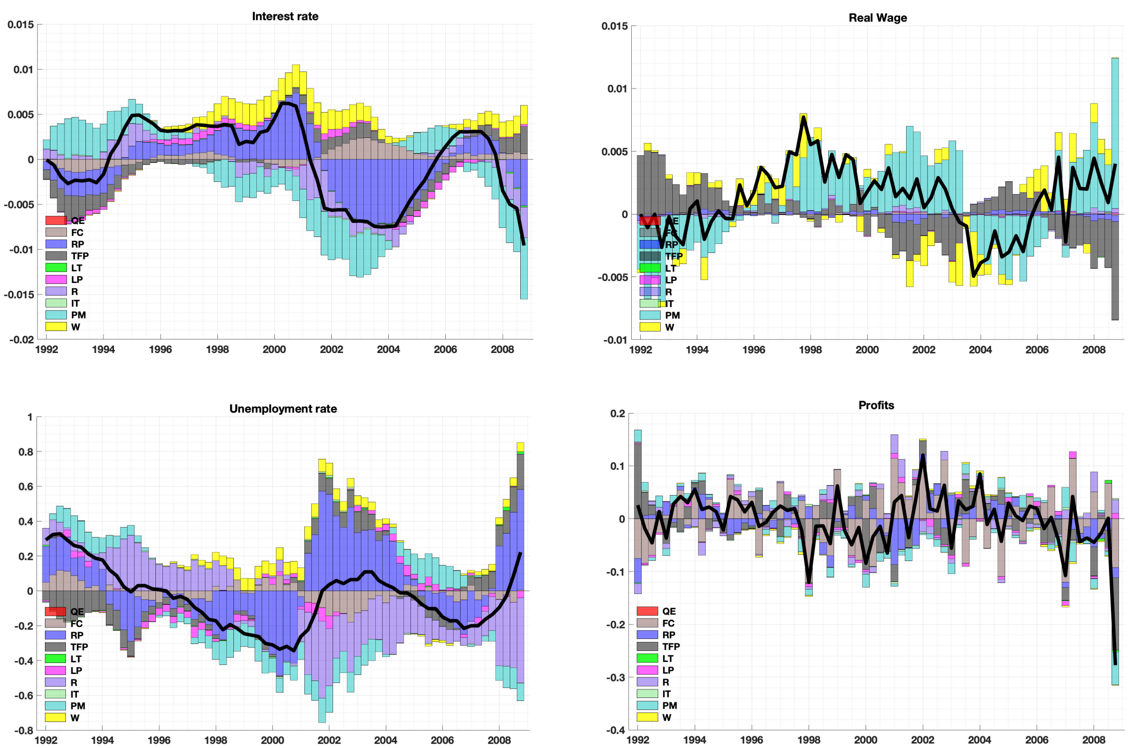
B.7.5 Historical decomposition

Figure B.24: Historical decomposition of the data before the Great Recession 1/2



Notes: FC, RP, TFP, LT, LP, R, IT, PM, and W stand for QE, risk premium, productivity, lump-sum transfer, liquidity preference, interest rate, investment technology, price mark-up, and wage shock, respectively.

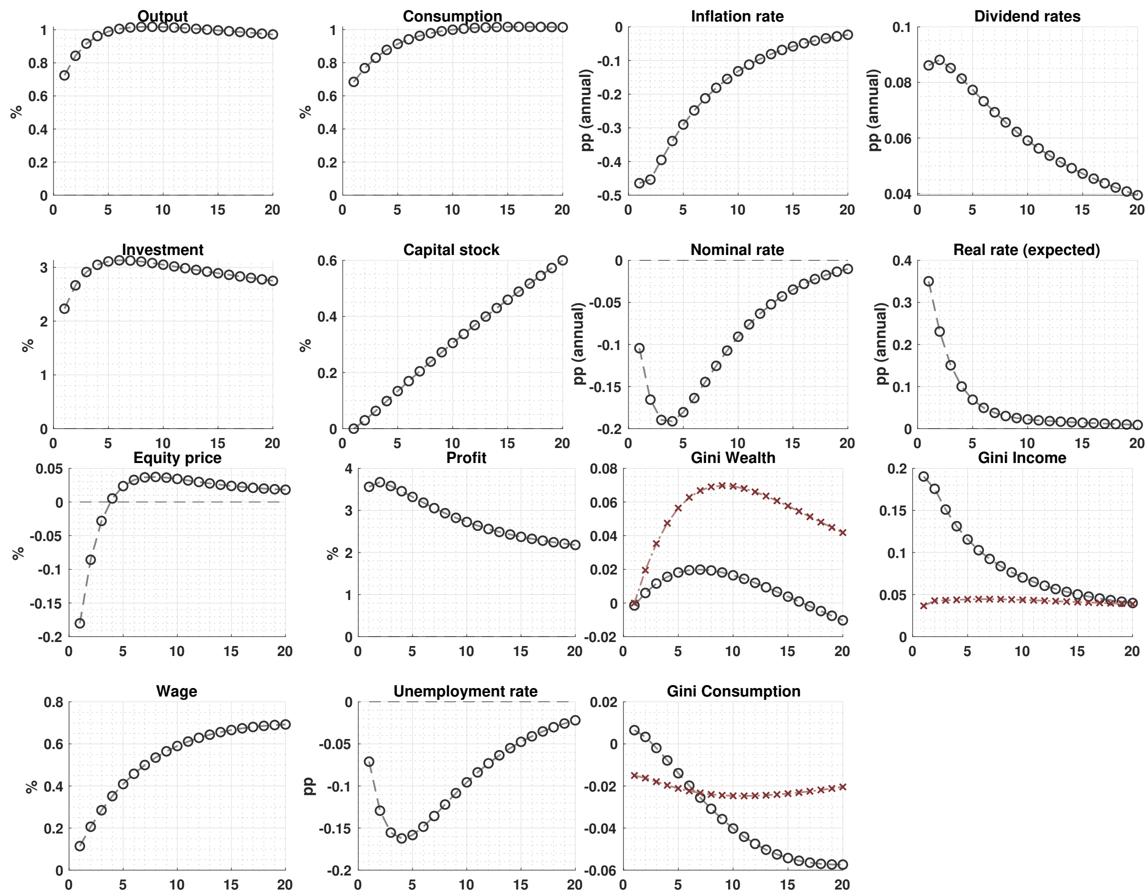
Figure B.25: Historical decomposition of the data before the Great Recession 2/2



Notes: FC, RP, TFP, LT, LP, R, IT, PM, and W stand for QE, risk premium, productivity, lump-sum transfer, liquidity preference, interest rate, investment technology, price mark-up, and wage shock, respectively.

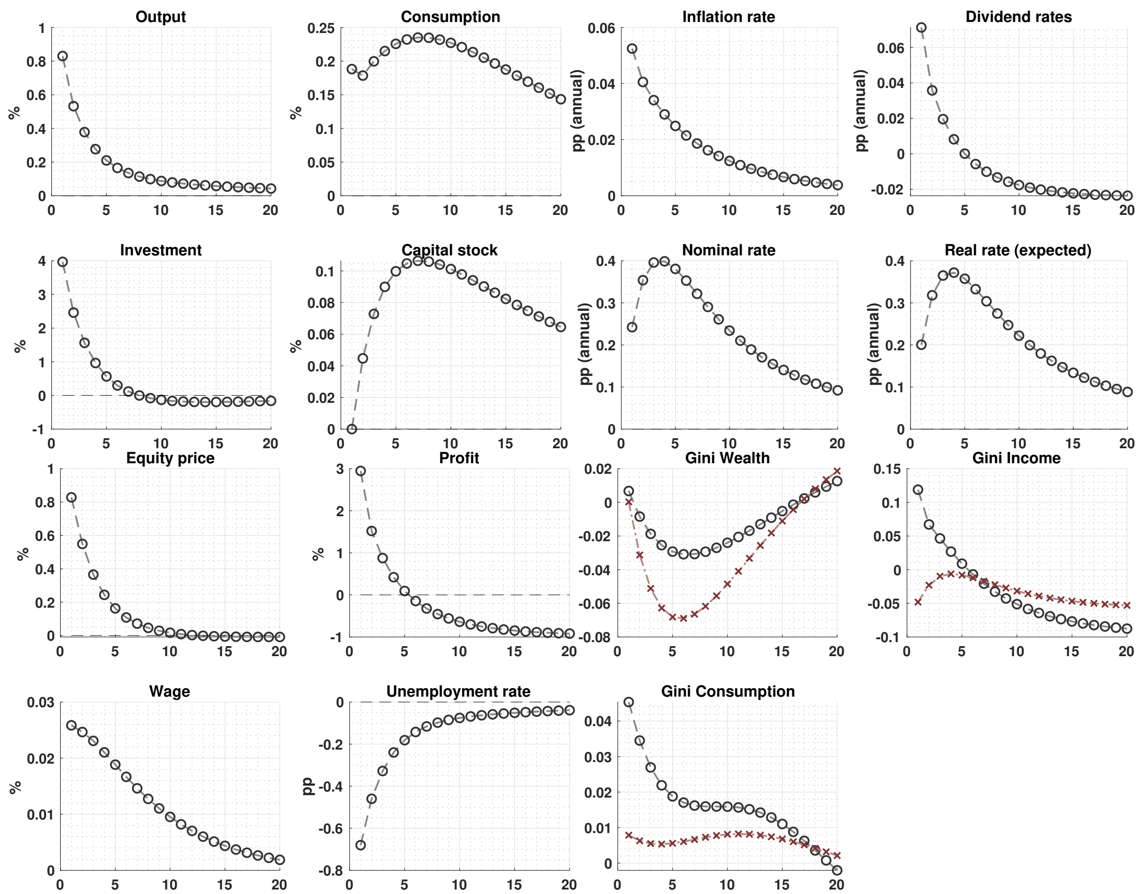
B.7.6 Impulse responses to exogenous shocks

Figure B.26: IRFs to a TFP shock



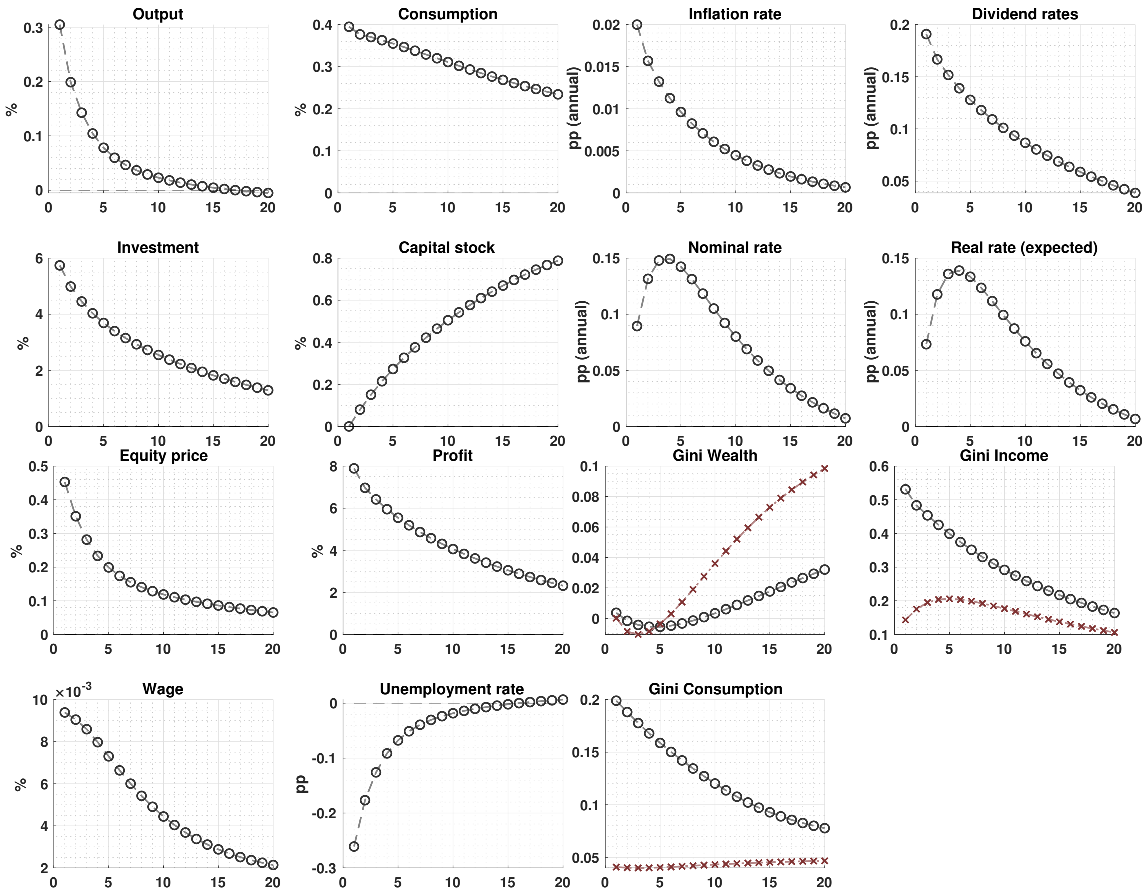
Notes: The figure shows impulse responses of variables to one standard deviation expansionary TFP shock. The red dashed lines with crosses show Gini index among the bottom 90% households.

Figure B.27: IRFs to a risk premium shock



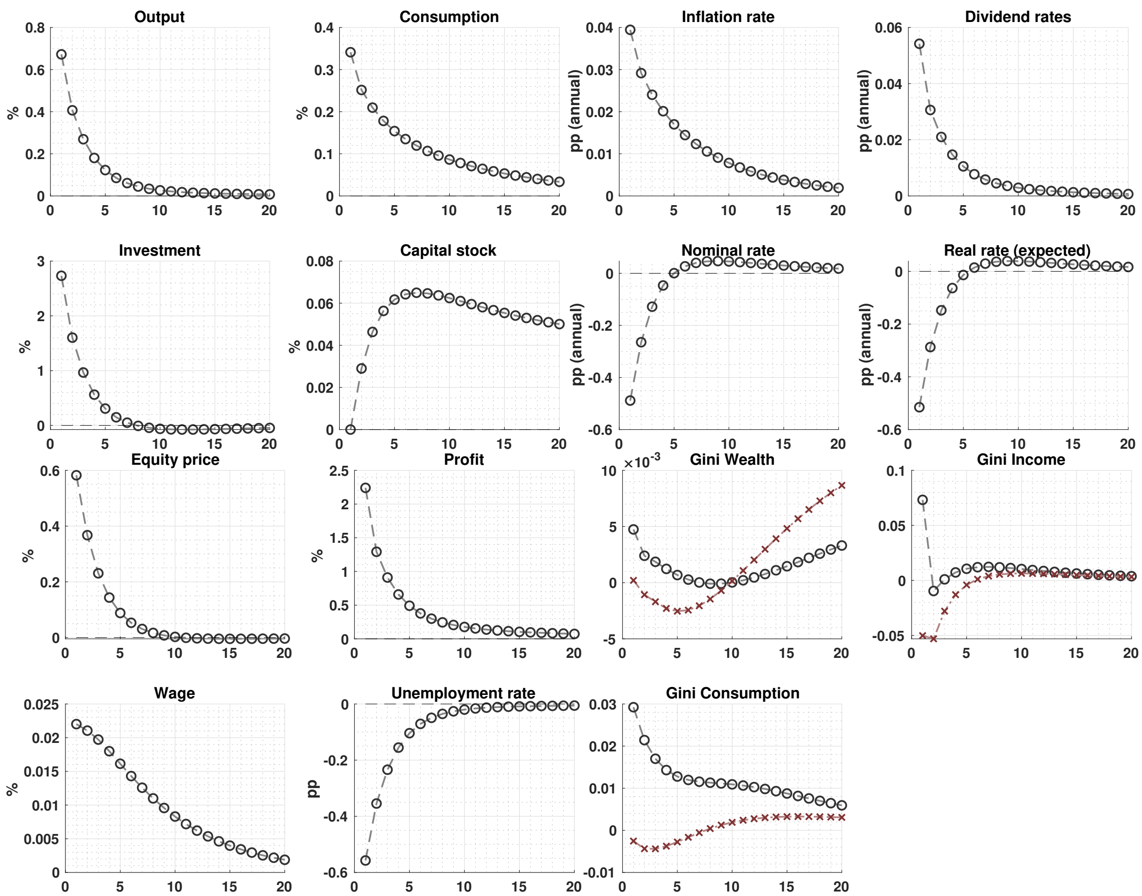
Notes: The figure shows impulse responses of variables to one standard deviation expansionary risk premium shock. The red dashed lines with crosses show Gini index among the bottom 90% households.

Figure B.28: IRFs to a fixed cost shock



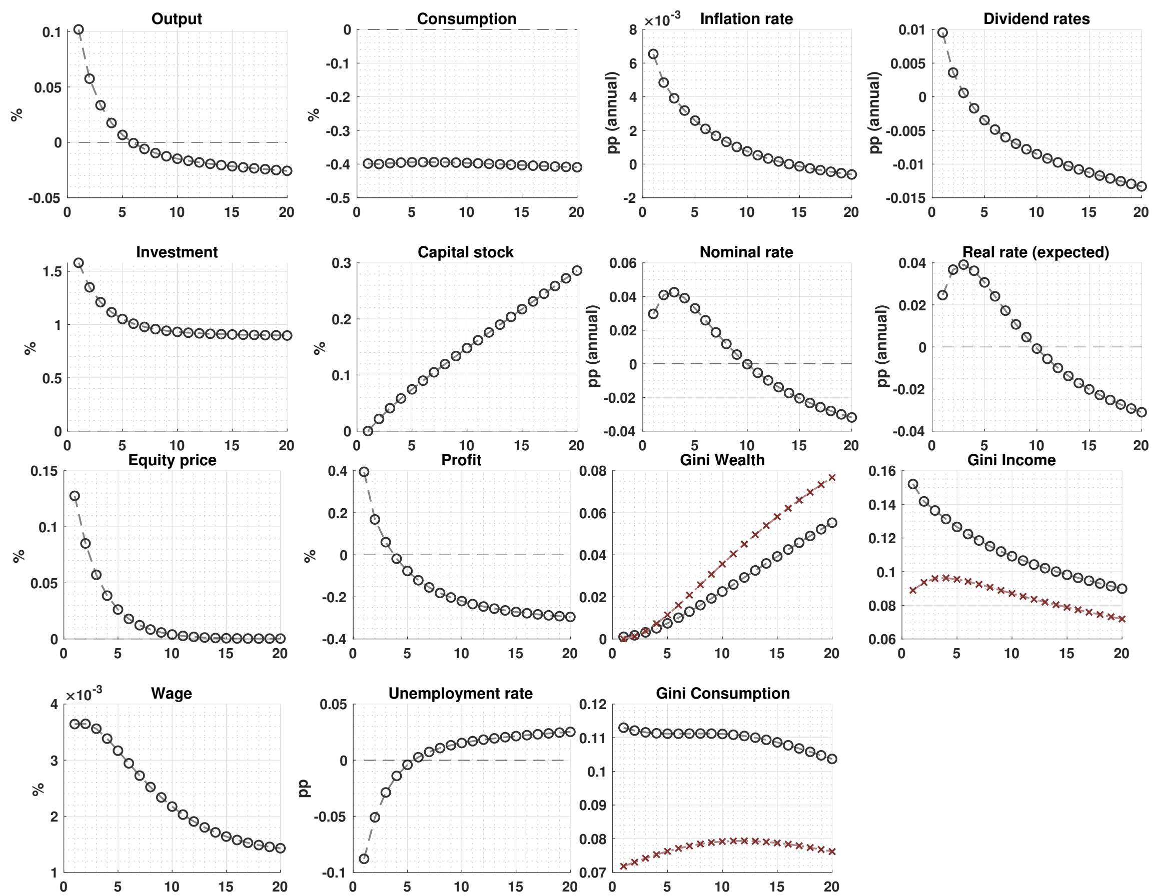
Notes: The figure shows impulse responses of variables to one standard deviation expansionary fixed cost shock. The red dashed lines with crosses show Gini index among the bottom 90% households.

Figure B.29: IRFs to an interest rate shock



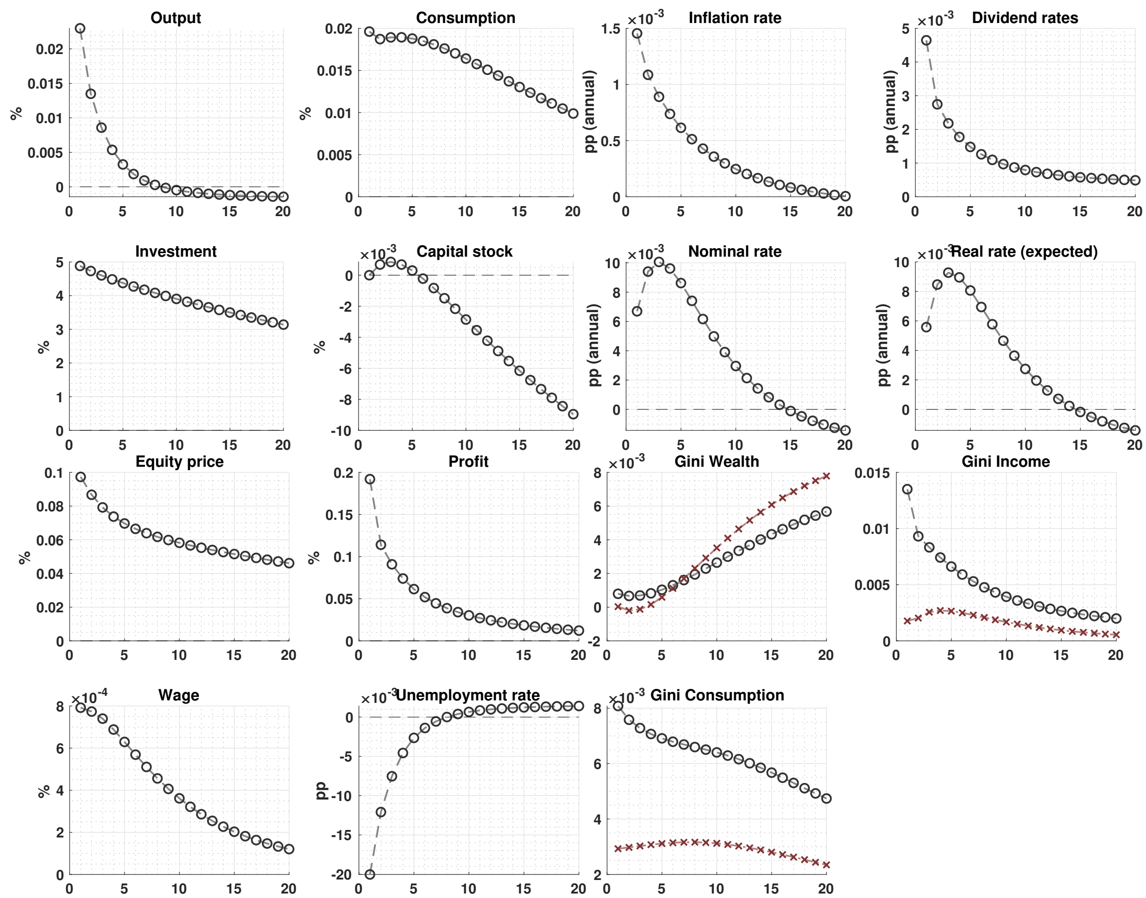
Notes: The figure shows impulse responses of variables to one standard deviation expansionary interest rate shock. The red dashed lines with crosses show Gini index among the bottom 90% households.

Figure B.30: IRFs to a liquidity preference shock



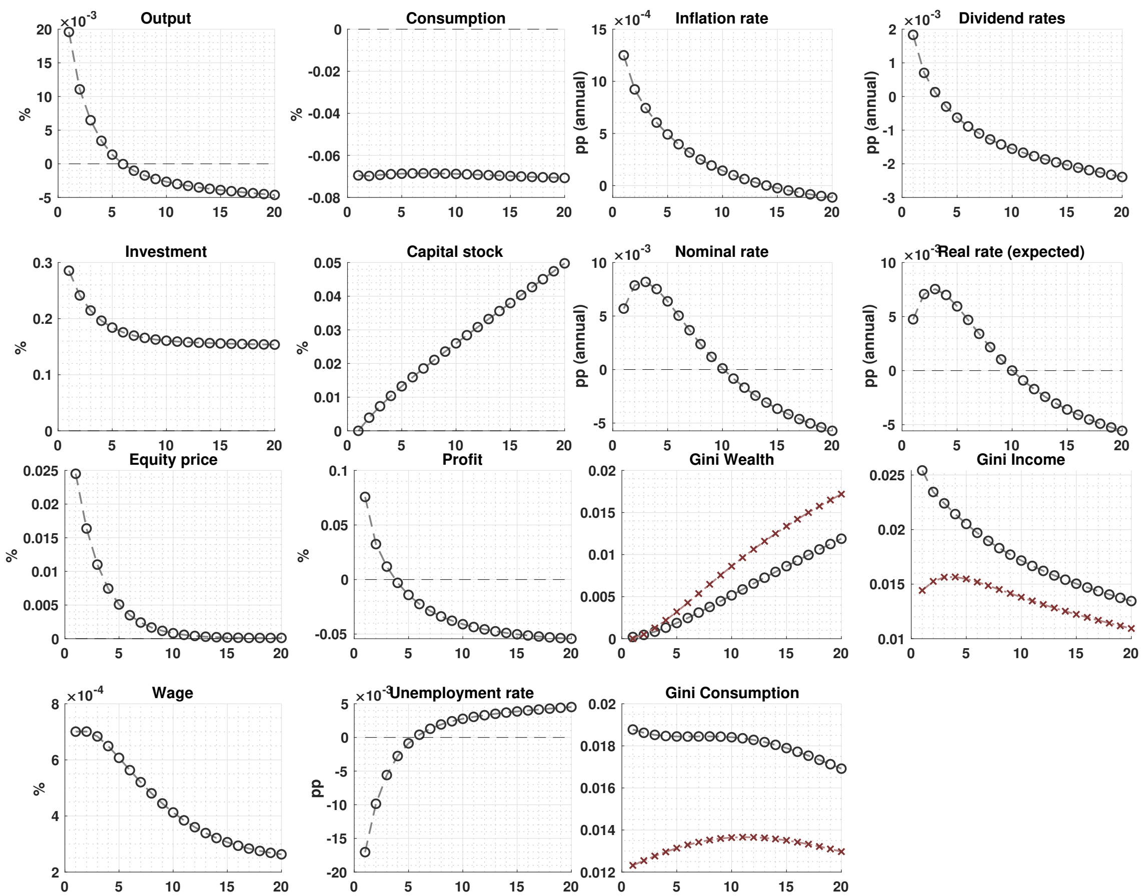
Notes: The figure shows impulse responses of variables to one standard deviation expansionary liquidity preference shock. The red dashed lines with crosses show Gini index among the bottom 90% households.

Figure B.31: IRFs to an investment technology shock



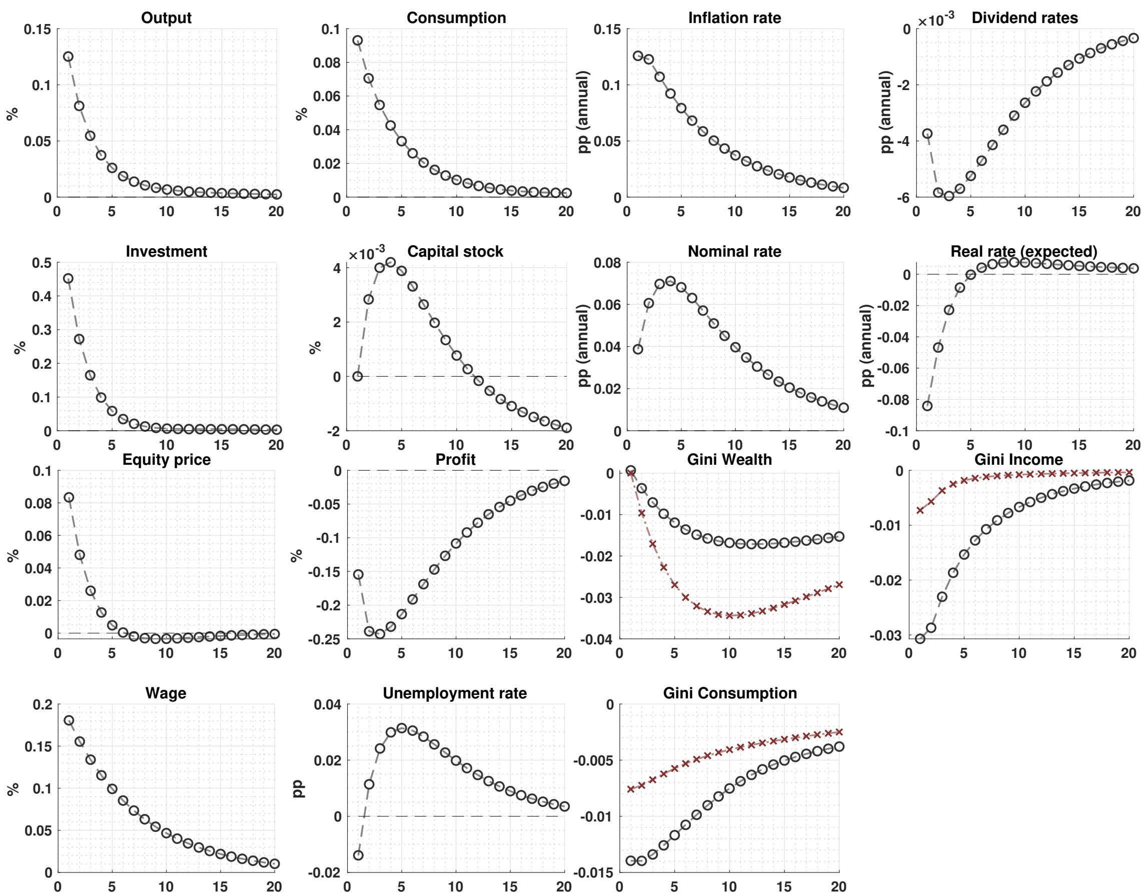
Notes: The figure shows impulse responses of variables to one standard deviation expansionary investment technology shock. The red dashed lines with crosses show Gini index among the bottom 90% households.

Figure B.32: IRFs to a lump-sum transfer shock



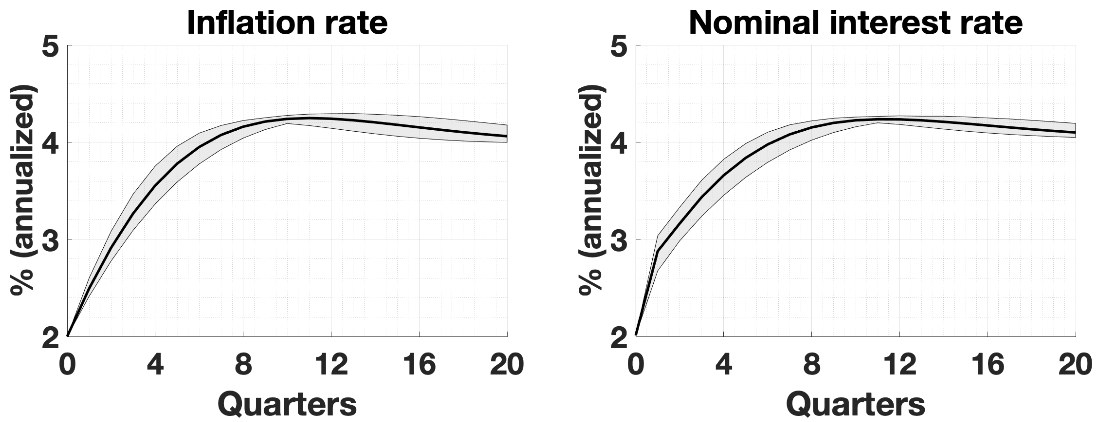
Notes: The figure shows impulse responses of variables to one standard deviation expansionary lump-sum transfer shock. The red dashed lines with crosses show Gini index among the bottom 90% households.

Figure B.33: IRFs to a real wage shock



Notes: The figure shows impulse responses of variables to one standard deviation expansionary wage shock. The red dashed lines with crosses show Gini index among the bottom 90% households.

Figure B.34: Transitional dynamics of the inflation and the nominal rate



Notes: The figure shows how the inflation and the nominal interest rate change during the transition path. Transitional dynamics are evaluated with 5,000 sets of parameters that are randomly drawn from the chain estimated in the chapter 2. The grey areas show the 10 to 90 percentile region of the responses. The black lines are the mean responses.

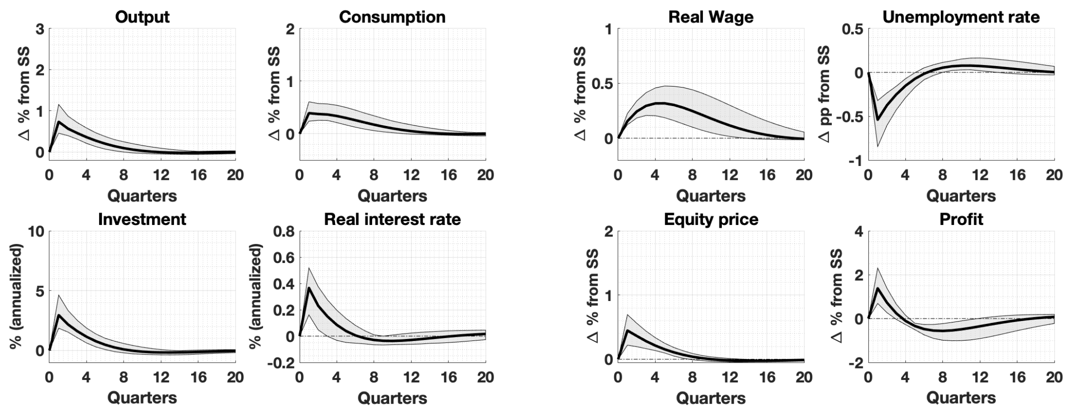
B.8 Transition path with alternative assumptions

B.8.1 Aggregate dynamics

In this subsection, I show the transition dynamics of the model economy with the backward-looking price-setting behavior and no interest rate smoothing. Figure B.34 shows that, with a backward-looking price setting, the inflation rate increases gradually to the new inflation target over the two years. Also, as the central bank does not smooth the interest rate path, the nominal interest rate increases relatively quickly.[27] Consequently, the real interest rate rises in contrast to the baseline case in the main document. The economy still experiences expansion because the real wage increases because of the indexation to the new target, but the magnitudes are much

[27]Though there is no smoothing, the interest rate increases still gradually since the inflation rate remains lower compared to the new target for substantial amount of time and the central bank responds to relatively lower inflation rates with lower policy rates.

Figure B.35: Aggregate effects of raising the inflation target



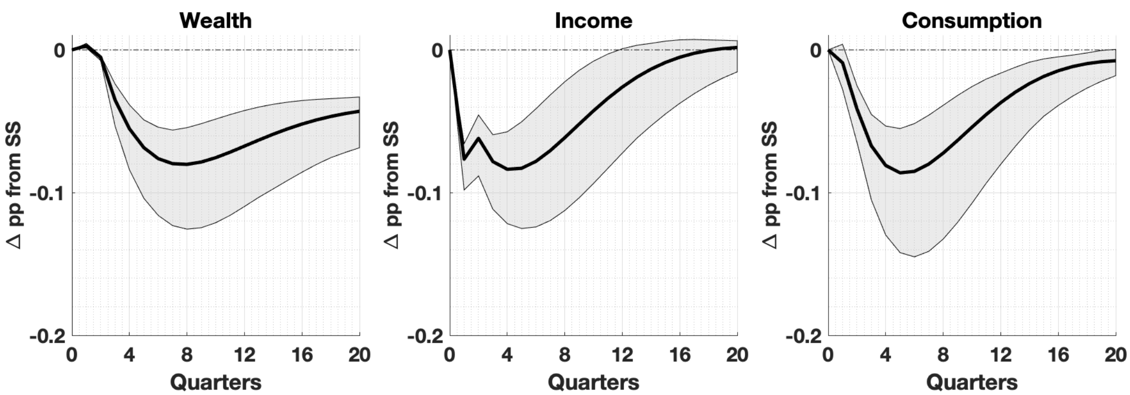
Notes: The figure shows how aggregate variables change during the transition path. Transitional dynamics are evaluated with 5,000 sets of parameters that are randomly drawn from the chain estimated in the chapter 2. The grey areas show the 10 to 90 percentile region of the responses. The black lines are the mean responses.

smaller compared to the baseline case in which the real interest rate falls during the transition. Besides, profits fall after a year, as shown in Figure B.35, because of higher real wages during the transition.

B.8.2 Distributional consequences

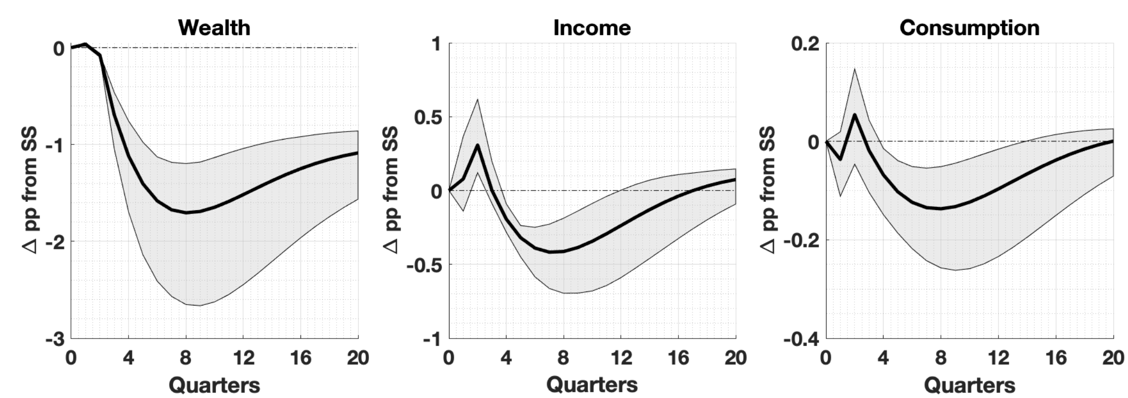
Figure B.36 and B.37 show that, regardless of the measures, inequality falls during the transition. This is because real wages are substantially higher during the transition, and profit falls below its steady-state level. In terms of the log 90 to 50 ratios, inequality initially rises while profits are higher due to expansion. But, once profits fall, then the degree of inequality is lower even when it is measured by the log 90 to 50 ratios.

Figure B.36: Inequality dynamics during the transition: Gini index



Notes: The figure shows the changes in wealth, income, and consumption Gini index during the transition dynamics. The unit is perecentage point.

Figure B.37: Inequality dynamics during the transition: log 90 to 50 percentile ratio



Notes: The figure shows the changes in the log 90 to 50 ratio of wealth, income, and consumption during the transition dynamics. The unit is perecentage point.

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