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Adaptive Channel Allocation for OFDM-Based Smart Antenna Systems with Limited Transceiver Resources

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Adaptive Channel Allocation for OFDM-based Smart Antenna Systems with Limited Transceiver Resources

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***Abstract*— Smart antennas constitute perhaps the most promising means of increasing capacity in wireless systems by allowing intra-cell channel reuse by several users. The employment of smart antennas at the physical layer raises significant issues in medium access control (MAC) layer. In this paper, we study the impact of smart antennas on MAC layer channel allocation in the presence of limited transceiver resources, where a transceiver is a communication unit that is used to set up a distinct beam. The problem is addressed in the context of Orthogonal Frequency Division Multiplexing (OFDM), which is the predominantly proposed signaling scheme for wireless broadband access. Since a beam can only serve users in different subcarriers, the problems of subcarrier and transceiver assignment are coupled. We propose heuristic algorithms to allocate channels to users, adjust beamforming vectors and assign users and channels in beams, with the objective to increase system throughput and provide QoS to users in the form of minimum rate guarantees. Our criteria for resource assignment and beam formation are based on spatial separability properties of users, beam vector cross-correlations and induced interference to the system. This unified cross-layer approach is shown to yield significant throughput benefits.**

I. INTRODUCTION

The increasing demand for advanced services, such as telecommuting, home networking, video conferencing, fast internet access or multimedia and the need for user mobility and flexibility in personal area, local area or wide area networks have necessitated wireless broadband access. Quality of service (QoS) provisioning to users in order to meet anticipated data rates in the volatile wireless medium by using the finite available resources is a challenging issue. At the physical layer, QoS is translated to an acceptable signal-to-interference and noise ratio (SINR) level or bit error rate (BER) at the receiver of each user, while at the MAC layer, QoS is usually expressed in terms of minimum throughput or maximum delay guarantees. The ability of the network infrastructure to fulfil such QoS requirements depends on procedures that span several communication layers. Thus, at the MAC layer, QoS guarantees can be provided by appropriate scheduling [1] or channel allocation methods [2]. At the physical layer, adaptation of transmission power, modulation level or symbol rate helps in maintaining acceptable link quality [3]. Moreover, the employment of smart antennas at the transmitter constitutes perhaps the most promising means of increasing capacity through Space Division Multiple Access (SDMA) [4].

Orthogonal Frequency Division Multiplexing (OFDM) is the predominantly proposed access and signaling scheme for wireless broadband networks [5]. OFDM is included in the IEEE 802.11a [6] and ETSI HIPERLAN/2 standards for wireless local area networks (LANs), as well as in the digital audio/video broadcasting (DAB/DVB) standards in Europe. It has also been proposed by IEEE 802.15 and 802.16 working groups for wireless personal area networks (WPANs) and fixed broadband wireless access (FBWA). In OFDM, the wide-band spectrum is divided into orthogonal narrow-band subcarriers with overlapping spectra. Each user symbol is split into subsymbols, each subsymbol modulates a different subcarrrier and user subsymbols are transmitted in parallel over these low-rate subcarriers. OFDM transmission results in reduction of effective transmission symbol rate, mitigation of inter-symbol interference (ISI) and significant improvements in data rates. With adaptive modulation techniques, the number of transmitted bits of a user subsymbol in a subcarrier can be adapted according to subcarrier quality, so as to maintain acceptable BER per subcarrier [7]. A large number of bits increases rate, but requires higher SINR to maintain a certain BER.

Space division multiple access (SDMA) with smart antennas is recognized as the prominent means of enhancing capacity of wireless networks. It can be combined with any multiple access technique and enables intra-cell channel reuse by several spatially separable users. The smart wireless LAN (SWL) system [8] is proposed for cooperation with the 802.11 protocol. Several companies, e.g. Iospan-wireless, Navini and Arraycomm aim at SDMA commercial products that support certain multiple access schemes. In SDMA, multiple beams are formed by an adaptive antenna array at the base station. The main lobe of each beam is directed towards one user, while interferers are nulled out, so as to maintain an acceptable receiver SINR. In the uplink, the user separation problem is decomposed into independent optimization problems, one for each user, and user beams (filtering vectors) are easily computed. However, user separation in the downlink is cumbersome, due to the fact that beam adaptation for one user affects interference for all users. Moreover, receivers are distributed and are not expected to have multiple antennas and thus they cannot perform joint signal detection. In [9], the authors propose an iterative algorithm for power control and transmit beamforming for a set of cochannel links. The algorithm converges to a feasible solution, if there exists one and minimizes total transmitted power. A similar algorithm in the context of OFDM is presented in [10]. In [11], the authors study downlink beamforming by decoupling beam orientation and power adaptation.

The employment of smart antennas at the physical layer raises significant issues in higher layers. In [12] and [13], some simple heuristics for time slot assignment to users in an SDMA/TDMA system are proposed, with the objective to increase system capacity. In [14], the authors present a framework for joint time slot allocation and packet scheduling based on packet transmission deadlines for an SDMA system. However, adaptive resource allocation in the context of OFDM or other multiple access schemes has predominantly been studied independently from user spatial separation through SDMA and channel reuse. Intra-cell channel reuse is suboptimal and is usually based on static cell sectorization [15] or beam switching methods [16], which do not capture user mobility, channel dynamicity and traffic load variations. Related research on beamforming has mostly focused on beam adaptation for a single channel, so as to maintain an acceptable SINR at each receiver. Channel assignment to users and link adaptation are performed independently for each user, without any consideration of the impact on other users.

Another issue which has not been addressed in literature is that of limited transceiver resources of the antenna array. The hitherto adopted assumption is that a different beam can be formed for each user in each utilized channel. The problem of channel allocation for an SDMA/OFDM system with unlimited transceiver resources was addressed in [17] and a greedy heuristic procedure was presented. However, high implementation complexity and infrastructure cost, physical space inadequacies, or specifications on maximum induced interference to neighboring base stations and users may impose limitations on the number of beams that can be formed. These situations arise more often in WLANs, WPANs, or other indoor environments. Assuming that each beam is formed by an dedicated transceiver, the number of available transceivers at the base station will be limited. This limitation affect channel allocation, since channel assignment to users and user clustering into a limited number of adaptively formed beams are interrelated. The efficiency of a channel assignment to users depends on channel reuse, which in turn is determined by beam formation and distribution of users and channels to different transceivers. With an appropriate combined assignment strategy at the base station, these issues can be jointly considered. Spatial separation of users can be adjusted by transmit beamforming and selective user assignment in channels, while channels and users are appropriately allocated to transceivers, so that an acceptable SINR is ensured at each receiver.

In this paper, we investigate the impact of smart antennas with limited transceiver resources on MAC layer channel allocation, with the objective to increase system capacity and provide minimum rate guarantees to users. We propose two heuristic algorithms to assign spatially separable users in the same channels and distribute users and channels within available transceivers, while appropriately adjusting beam patterns by transmit beamforming. Spatial characteristics of users, induced interference to the system by beam patterns and beam cross-correlation properties are some of the utilized criteria for channel assignment and beamforming vector computation. The main goal of our study is to evaluate the benefits of this cross-layer approach in terms of achievable system rate, identify the tradeoffs that are associated with resource (channel or transceiver) limitations and motivate further research on integrated layer design issues.

The paper is organized as follows. In section II we provide the transmission and channel models and the assumptions used in our approach. In section III, we present the problem, outline the rationale of our approach and describe the proposed algorithms. In section IV, a special case of the problem is examined and in section V numerical results are provided. Finally section VI concludes our study. A few words about the notation before we proceed. Vectors are denoted with boldface letters. The cardinality of set $\mathcal{X}$ is $|\mathcal{X}|$. Superscripts T and H denote transpose and conjugate transpose of a vector or matrix, $\Re(\cdot)$ is the real part of a complex number, while $\|\mathbf{w}\|$ is the ℓ_2-norm of the complex vector $\mathbf{w} = (w_1, \dots, w_n)^T$, i.e, $\|\mathbf{w}\| = \sqrt{\sum_{i=1}^{n} |w_i|^2}$. The dominant generalized eigenvector of matrices $(\mathbf{A}, \mathbf{B})$ is the normalized eigenvector that corresponds to the largest posi-

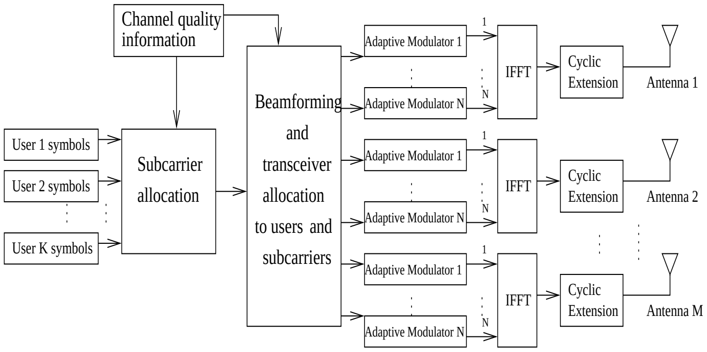

Fig. 1. Block diagram of a SDMA/OFDM transmitter with adaptive subcarrier and transceiver allocation.

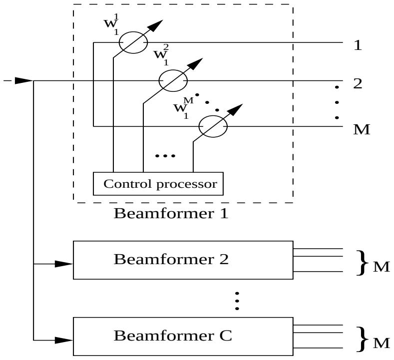

Fig. 2. The structure of the C transceiver modules.

tive eigenvalue of eigenproblem $\mathbf{A}\mathbf{x}{=}\lambda\mathbf{B}\mathbf{x}$. When $\mathbf{A}$, $\mathbf{B}$ are symmetric and positive definite, this is equivalent to eigenproblem $\mathbf{C}\mathbf{y}{=}\lambda\mathbf{y}$, with $\mathbf{C} = \mathbf{L}^{-1}\mathbf{A}(\mathbf{L}^{-1})^H$ and $\mathbf{y} = \mathbf{L}^H\mathbf{x}$, where $\mathbf{L}$ is a non-singular lower triangular matrix that appears in the Cholesky decomposition of $\mathbf{B}$, $\mathbf{B} = \mathbf{L}\mathbf{L}^H$ [18].

II. SYSTEM MODEL

A. Transmitter model

We consider the downlink of a single-cell system, with K users. The receiver of each user has an omni-directional antenna. The base station is equipped with a uniform linear antenna array of M antennas and the distance between two antennas is δ. The base station employs OFDM transmission with N orthogonal subcarriers. An underlying slotted transmission scheme is assumed. Packetized data arrive from higher layer queues for transmission over the radio channel. Each packet occupies a time slot of duration T_s seconds and consists of S symbols. Each OFDM symbol consists of bits that must be assigned to different subcarriers. Let $b_{n,k}$ denote the number of bits of user k in subcarrier n. These bits constitute the n-th subsymbol of k that modulates subcarrier n. User subsymbols can in general consist of different number of bits in different subcarriers, depending on subcarrier quality. This is achieved by using a different modulation level for each subsymbol. Assuming that the channel is invariant for a slot duration, each packet symbol of a user is split into subsymbols over the same set of subcarriers. The transmission rate for k in a slot is then $(S/T_s)\left(\sum_{n=1}^N b_{n,k}\right)$ bits/sec. In this paper, we assume that one modulation level is used, so that b bits are assigned to each used subcarrier. Then, transmission rate for k is $(Sb/T_s)N_k$ bits/sec, where N_k is the number of subcarriers for k. User k must satisfy a minimum rate requirement of r_k bits/sec, which denotes requested QoS of the MAC from the physical layer. Due to single rate transmission, r_k is directly mapped to a minimum number of channels x_k that need to be allocated to k.

When the rate requirements for each user are not given, the objective of the subcarrier and bit allocation algorithm is to maximize total rate for the K-user system with N subcarriers. When the rate requirements are given, the problem is to satisfy users by using the minimum number of subcarriers. In this work, we are concerned with the first problem.

The block diagram of an OFDM/SDMA transmitter is depicted in Fig. 1. User symbols enter the subcarrier allocation module, which determines cochannel sets of users in different subcarriers. Next, beamforming is performed. There exist C transceivers and each of them can form a beam $\mathbf{w}_c = \left(w_c^1, \ldots, w_c^M\right)^T$, for $c = 1, \ldots, C$. Beams are normalized, i.e, $\|\mathbf{w}_c\| = 1$. A set of C transceivers is depicted in Fig. 2. Users and subcarriers are then allocated to transceivers. Computation of beamforming vectors, user assignment to transceivers and subcarrier allocation to users are interdependent and also depend on subcarrier quality of users. Clearly, users that are allocated to the same transceiver (i.e, covered by the same beam in space) must use different subcarriers. Further, if two or more users use the same subcarrier, they must be assigned to different transceivers. The output of this module is forwarded into M parallel modules of N adaptive modulators. Each modulator modulates the corresponding subcarrier with bits of users that are allocated to that subcarrier. Each symbol in the stream is then transformed into N time domain samples by inverse fast Fourier transform (IFFT). A cyclic extension of some time samples is appended to prevent ISI and the signal is transmitted from the M antennas. The baseband transmitted signal for user k before beamforming is,

$$s_k(t) = \sum_{i=-\infty}^{\infty}\sum_{n=1}^{N} d_{n,k}^{(i)}\, g\left(t - i(T+T_g)\right) e^{j\omega_n t}\,, \qquad (1)$$

namely, it is a sum of pulses $g(t)$ of duration $T+T_g$, where T and T_g are the symbol and guard interval durations. Each pulse is multiplied by a complex subcarrier coefficient $d_{n,k}^{(i)}$, which denotes the n-th subsymbol of the i-th symbol of user k that modulates subcarrier n. This subsymbol is shifted in time by i modulation intervals of duration $T + T_g$ and in frequency by $\omega_n = n(\Delta\omega)$, where $\Delta\omega = 2\pi/T$ is the subcarrier bandwidth. Assume that pulse $g(t)$ is normalized to 1. If there is no overlap between pulses, each OFDM symbol can be studied separately. Then, by letting $i = 0$ and $T_g = 0$, the OFDM symbol of user k is $\mathbf{s}_k^0(t) = \sum_{n=1}^{N} d_{n,k} e^{j\omega_n t}$.

B. Channel Model and downlink received signal

The multi-path channel between base station antenna m and user k is,

$$h_k^m(t) = \sum_{\ell=1}^{L} \beta_{k,\ell}\, \delta\left(t - \tau_{k,\ell} + \tau_{k,\ell}^m\right), \tag{2}$$

where L is the number of paths, $\beta_{k,\ell}$ is the complex gain of the ℓ-th path of user k and $\tau_{k,\ell}$ is the delay for that path with respect to a reference antenna element. The gain $\beta_{k,\ell}$ is a complex random variable with variance $A_{k,\ell}$, while $\tau_{k,\ell}$ is uniformly distributed in $[0, T]$. The term $\tau_{k,\ell}^m = (\delta/\tilde{c})(m-1)\cos\theta_{k,\ell}$ captures the delay to the m-th antenna, where $\theta_{k,\ell}$ is the angle of the ℓ-th path of user k and $\tilde{c}$ is the electromagnetic wave propagation speed. The Fourier transform of $h_k^m(t)$ at subcarrier frequency ω_n is $H_{n,k}^m = \sum_{\ell=1}^{L} \beta_{k,\ell} e^{-j\omega_n(\tau_{k,\ell}+\tau_{k,\ell}^m)}$. Let $\xi_{k,\ell}(\omega_n) = \beta_{k,\ell} e^{-j\omega_n \tau_{k,\ell}}$ be the complex gain of the ℓ-th path of user k as a function of the subcarrier frequency ω_n. Define the m-th element of the $M \times 1$ antenna steering vector $\mathbf{v}_n(\theta_{k,\ell})$ at direction $\theta_{k,\ell}$ and frequency ω_n as $v_n^m(\theta_{k,\ell}) = e^{j\omega_n \tau_{k,\ell}^m}$. Then, the vector $\mathbf{a}_{n,k} = \sum_{\ell=1}^{L} \xi_{k,\ell}^*(\omega_n)\mathbf{v}_n(\theta_{k,\ell})$ is called spatial signature of user k at ω_0 and captures spatial and multi-path properties of the user. Then the vector channel of user k at ω_n can be expressed as,

$$\mathbf{a}_{n,k} = \sum_{\ell=1}^{L} \xi_{k,\ell}^*(\omega_n)\mathbf{v}_n(\theta_{k,\ell}), \tag{3}$$

In the sequel, we assume that the major limitation in the system is cochannel interference rather than noise, so that receiver SINR is approximated by SIR. Let user k receive the useful signal from beam c. Then, the received signal for k at subcarrier n is,

$$y_{n,k}^c = \left(\mathbf{a}_{n,k}^H \mathbf{w}_c\right) d_{n,k} + \sum_{b\neq c}\left(\mathbf{a}_{n,k}^H \mathbf{w}_b\right) d_{n,k} + \sum_{b\neq c}\sum_{j\neq k}\left(\mathbf{a}_{n,k}^H \mathbf{w}_b\right) d_{n,j}. \tag{4}$$

The first term denotes the useful power of user k. The second and third terms capture the effect of cochannel interference in subcarrier n, caused by signals intended for user k in other beams, as well as by signals for other users in other beams. The average power received in subcarrier n of user k due to transmission towards user $j \neq$ in beam b is $E\{|\left(\mathbf{a}_{n,k}^H \mathbf{w}_b\right) d_{n,j}|^2\} = \mathbf{w}_b^H \mathcal{H}_{n,k} \mathbf{w}_b$, where

$$\mathcal{H}_{n,k} = \sum_{\ell_1=1}^{L}\sum_{\ell_2=1}^{L} \mathbf{v}_n(\theta_{k,\ell_1})\, \mathbf{v}_n^H(\theta_{k,\ell_2})\, E\left\{\xi_{k,\ell_1}(n)\xi_{k,\ell_2}^*(n)\right\}, \tag{5}$$

where it was assumed that subsymbols are normalized to unit power. We observe that,

$$E\left\{\xi_{k,\ell_1}(n)\xi_{k,\ell_2}^*(n)\right\} = \begin{cases} 0, & \text{if } \ell_1 \neq \ell_2 \\ A_{k,\ell}, & \text{if } \ell_1 = \ell_2 = \ell, \end{cases} \tag{6}$$

assuming that all paths are independent. Then we have,

$$\mathcal{H}_{n,k} = \sum_{\ell=1}^{L} A_{k,\ell}\, \mathbf{v}_n(\theta_{k,\ell})\mathbf{v}_n^H(\theta_{k,\ell}). \tag{7}$$

The matrix $\mathcal{H}_{n,k}$ is called spatial covariance matrix of user k in subcarrier n and in general it has rank$(\mathcal{H}_{n,k}) > 1$. Observe that the received power by user k in subcarrier n due to transmission towards user j in beam b does not depend on the location of user j, but only on matrix $\mathcal{H}_{n,k}$ and beam $\mathbf{w}_b$. The average SIR at the output of the matched filter receiver at subcarrier n of user k is,

$$SIR_{n,k}^c = \frac{\mathbf{w}_c^H \mathcal{H}_{n,k} \mathbf{w}_c}{\sum_{b\neq c} \mathbf{w}_b^H \mathcal{H}_{n,k} \mathbf{w}_b}. \tag{8}$$

Hence, SIR depends only on beams that employ subcarrier n for transmission and not on identities of cochannel users.

By using training symbols, spatial covariance matrices are estimated in the up-link by using N_s samples $\mathbf{x}_{n,k}(q)$, $q = 1, \ldots, N_s$, of the received array signals for each n and k. Then, estimates of $\mathcal{H}_{n,k}$ are obtained by averaging. With time division duplexing (TDD) and reasonable time invariance of the channel, the base station uses these estimates in adapting down-link transmission. Since single rate transmission is assumed, quadrature amplitude modulation (QAM) with one modulation level M_0 is employed. The minimum required SIR to maintain BER $\leq \epsilon$ at the receiver is given by threshold $\gamma = -(\ln(5\epsilon)/1.5)(M_0 - 1)$,[19].

III. CHANNEL ALLOCATION IN SDMA SYSTEM WITH LIMITED TRANSCEIVER RESOURCES

A. Problem Statement

Smart antennas enable intra-cell reuse of a channel by multiple users, by exploiting spatial properties of users, such as angular separation and multi-path conditions. In an

OFDM/SDMA system, channel reuse pertains to simultaneous utilization of a subcarrier by several users.

Two or more users are *spatially separable* in a subcarrier if there exist beamforming vectors, one for each user, such that minimum SIR requirements at corresponding receivers are satisfied. Then, users can simultaneously use the channel and maintain a specified receiver BER. Spatial separability of users in a subcarrier depends on the number and identities of cochannel users through their spatial covariance matrices, which capture angular and multi-path characteristics. Beams using the same subcarrier to transmit to other users also affect SIRs at receivers of cochannel users. Finally, spatial separability depends on subcarriers, in the sense that a set of users that are spatially separable in one subcarrier, may not be separable in a different subcarrier.

Each beamforming vector corresponds to one of the C beams that are formed by C transceivers at the base station. Clearly, users that are illuminated by the same beam must use different subcarriers. In addition, two or more users in different beams may not be allowed to use the same subcarrier, due to high cochannel interference, caused by beam orientations and non-ideal beam patterns. Further, multi-path angles and gains may result in spatial covariance matrices that do not allow reuse in that subcarrier. A simple example is the case of two users with small angular separation and a single line-of-sight path.

Each user in a subcarrier experiences cochannel interference from other beams that use the same subcarrier to transmit to other users. When a larger number of users is assigned to the subcarrier in different beams, the rate for that subcarrier is increased. With higher subcarrier reuse, users require fewer subcarriers to satisfy their rate requirements. Hence, more users can be accommodated in the system and capacity is increased. However, a larger number of cochannel users renders spatial separability difficult, since cochannel interference increases and SIR decreases. Since non-separable users must be assigned to other subcarriers so as to alleviate cochannel interference, capacity is not enhanced from this point of view. This tradeoff between number of assigned users in the same subcarrier and spatial separability affects total achievable rate. In addition, users and subcarriers must be associated with transceivers and beams must be computed for each transceiver, so that users are appropriately clustered in beams, subcarriers are reused in different beams, receiver SIRs for used subcarriers are acceptable and minimum rate requirements of users are satisfied.

The question that arises is whether there exists a way to jointly perform subcarrier and transceiver allocation to users, as well as adaptation of beamforming vectors, so as to maximize the number of assigned users per subcarrier and ultimately increase capacity. For given beamforming vectors, the problem reduces to finding maximum cardinality cochannel user sets for each subcarrier. For each subcarrier, the cochannel set consists of at most one user from each beam. Identifying the maximum cardinality cochannel set is equivalent to finding the maximum clique in an appropriately defined graph, which is an NP-Complete problem [20]. When beamforming vectors are adaptable and the number of transceivers is unlimited, the problem again reduces to finding maximal cochannel sets. When beamforming vectors are adaptable and the number of transceivers is limited, the problem becomes even more challenging.

Consider first the case of unlimited number of available transceivers. First, a large set of spatially separable users must be identified for each subcarrier. Then, beamforming vectors for these users must be computed, so that receiver SIRs are acceptable. The problem is that SIR at a receiver depends on beamforming vectors of other users and therefore it depends on the cochannel set itself. The enumeration of all possible user assignments in a subcarrier is of exponential complexity. In addition, even if the cochannel set of users is fixed, the computation of beamforming vectors that lead to acceptable SIRs is a highly non-linear problem, as will be seen in the sequel. Therefore, some heuristic algorithms should be designed for construction of large cochannel sets of spatially separable users for each subcarrier and computation of corresponding beamforming vectors. When the constraint on the number of available transceivers comes into picture, our goal should be to reduce the number of (at most NK) currently formed beams for each user and subcarrier to C. This can be done by sequentially unifying appropriate sets of two or more beams into single beams, until the desired number of C beams is reached.

In this paper, we address the problem of channel assignment for SDMA systems with limited transceiver resources, with the goal to increase system capacity and provide minimum rate guarantees to users. We propose heuristic algorithms which consist of two stages. In the first stage, spatially separable users are assigned in subcarriers and corresponding beams are computed. In the second stage, sets of formed beams are sequentially replaced with new single beams. The algorithms are executed on a symbol basis and the resulting allocation and beamforming are replicated for all symbols of a user packet. The allocation is updated once every one or more time slots.

B. Proposed Approach

1) The first stage of the algorithm: The basic idea in the first stage is to create large cochannel sets of spatially separable users for each subcarrier. In order to keep complexity to a reasonable level, we consider the class of algorithms, where users are sequentially inserted in the channel and no user reassignments are performed. At each step of the procedure, an appropriate user is assigned to a subcarrier and beamforming vectors of cochannel users are adjusted, so that acceptable SIRs are ensured. In particular, inserted users should cause the least interference to users that are already assigned in the channel and should receive the least interference from them, so that future user assignments in the channel are facilitated.

Fix attention to user assignment in subcarrier n. Let $\mathcal{U}_n$ denote the set of users that are already assigned in n at some point of the algorithm and let k be the user to be inserted next in the channel. For user j in subcarrier n, the beamforming vector is denoted by $\mathbf{w}_{n,j}$. Insertion of a new user creates a new interference scenario for other cochannel users and beamforming vectors of existing users may need to be recomputed so as to maintain acceptable SIRs. For each user $j \in \mathcal{U}_n$, we define the ratio of desired power at its receiver, over undesired power that is caused to other cochannel users, including new user k, by beam $\mathbf{w}_{n,j}$. More precisely, we are interested in the maximum value of this ratio, $Z_{n,j}^{(k)}$, over all directions of beam $\mathbf{w}_{n,j}$:

$$Z_{n,j}^{(k)} = \max_{\mathbf{w}_{n,j}} \frac{\mathbf{w}_{n,j}^H \mathcal{H}_{n,j} \mathbf{w}_{n,j}}{\mathbf{w}_{n,j}^H \Big(\sum_{\substack{i \in \mathcal{U}_n \\ i \neq j}} \mathcal{H}_{n,i} + \mathcal{H}_{n,k} \Big) \mathbf{w}_{n,j}} \,, \tag{9}$$

subject to $\|\mathbf{w}_{n,j}\| = 1$. The vector $\mathbf{w}_{n,j}^*$ that maximizes this ratio is the dominant generalized eigenvector of matrix pair $\left[\mathcal{H}_{n,j}, \left(\mathcal{H}_{n,k} + \sum_{i \in \mathcal{U}_n : i \neq j} \mathcal{H}_{n,i}\right)\right]$ and is found with the method outlined at the end of section I. We also compute the ratio $Z_{n,k}$ for user k that is inserted in n:

$$Z_{n,k} = \max_{\mathbf{w}_{n,k}} \frac{\mathbf{w}_{n,k}^H \mathcal{H}_{n,k} \mathbf{w}_{n,k}}{\mathbf{w}_{n,k}^H (\sum_{j \in \mathcal{U}_n} \mathcal{H}_{n,j}) \mathbf{w}_{n,k}} \,, \tag{10}$$

subject to $\|\mathbf{w}_{n,k}\| = 1$. The denominator of this ratio captures the cochannel interference caused by user k to other cochannel users. With beamforming vectors $\mathbf{w}_{n,k}^*$ and $\mathbf{w}_{n,j}^*$, $j \in \mathcal{U}_n$, we evaluate the SIRs of users as,

$$SIR_{n,j} = \frac{\mathbf{w}_{n,j}^{*H} \mathcal{H}_{n,j} \mathbf{w}_{n,j}^*}{\sum_{\substack{i \in \mathcal{U}_n \\ i \neq j}} \mathbf{w}_{n,i}^{*H} \mathcal{H}_{n,j} \mathbf{w}_{n,i}^* + \mathbf{w}_{n,k}^{*H} \mathcal{H}_{n,j} \mathbf{w}_{n,k}^*} \,, \tag{11}$$

$$SIR_{n,k} = \frac{\mathbf{w}_{n,k}^{*H} \mathcal{H}_{n,k} \mathbf{w}_{n,k}^*}{\sum_{j \in \mathcal{U}_n} \mathbf{w}_{n,j}^{*H} \mathcal{H}_{n,k} \mathbf{w}_{n,j}^*} \tag{12}$$

If all SIRs are above the threshold γ, we proceed by defining a preference factor $F_{n,k}$ for user k. First, the beamforming vector must yield strong desired signal at the receiver. Furthermore, beams and spatial covariance matrices must be such that the interference caused by the beam of user k to other users in the channel, as well as the interference caused to user k by beams of other users is low. These requirements are captured in factor,

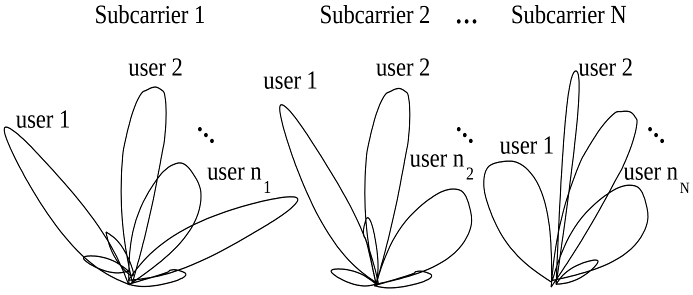

Fig. 3. The beams for each assigned user in each subcarrier after the first stage of the algorithm.

$$F_{n,k} = \frac{\mathbf{w}_{n,k}^{*H} \mathcal{H}_{n,k} \mathbf{w}_{n,k}^*}{I_{n,k}} \tag{13}$$

where,

$$I_{n,k} = \max \left\{ \mathbf{w}_{n,k}^{*H} (\sum_{j \in \mathcal{U}_n} \mathcal{H}_{n,j}) \mathbf{w}_{n,k}^* \,, \sum_{j \in \mathcal{U}_n} \mathbf{w}_{n,j}^{*H} \mathcal{H}_{n,k} \mathbf{w}_{n,j}^* \right\} \tag{14}$$

Clearly, among all user assignments to subcarriers which lead to acceptable SIRs for cochannel users, we prefer the one with the maximum preference factor $F_{n,k}$. User insertion in a subcarrier continues until no further assignment leads to acceptable user SIRs and the procedure is performed for all N subcarriers. At the end of the first stage of the algorithm there will be $\sum_{k=1}^{N} n_k$ beams, where n_k is the number of allocated users that are allocated in subcarrier k. A pictorial view of the situation at this point is shown in Fig. 3.

2) The second stage of the algorithm: In the first stage of the algorithm, the limitation in transceiver resources was not considered. In the second stage, the goal is to reduce the number of beams to C, while maintaining high subcarrier reuse. Each beam will then be formed by a dedicated transceiver. Thus, system rate is high and minimum rate requirements for users are easier to satisfy. The calculated beams are characterized by large subcarrier reuse and low cochannel interference, based on the criteria by which they were constructed. If these beams are appropriately combined, the new beams resulting from the unification are more likely to maintain desirable properties of old beams.

It is clear that only beams belonging to different subcarriers can be combined to one new beam, since the new beam cannot serve two users in the same subcarrier. In addition, although every possible combination of $2, 3, \ldots, N$

beams from different subcarriers is eligible for merging into a single beam, we will consider for unification only pairs of beams, so as to reduce complexity. Thus, at each iteration of the algorithm, the key idea is to select the appropriate pair of beams from different subcarriers and replace it with a single beam that encompasses users in the initial beams. Different criteria for selection of beam pairs and subsequent computation of the new beam can be applied.

Fix attention to beams $(b_k, b_\ell) \equiv (k, \ell)$ that belong to subcarriers n and m respectively and have beamforming vectors $\mathbf{w}_{n,k}$ and $\mathbf{w}_{m,\ell}$. For now, assume that each beam covers one user, as is the case after termination of the first stage of the algorithm. Thus, assume that users k and ℓ are covered by beams b_k and b_ℓ respectively. Note that b_k and b_ℓ may cover the same user in subcarriers n and m. The objective is to replace beams $\mathbf{w}_{n,k}$ and $\mathbf{w}_{m,\ell}$ with a new beam $\mathbf{w}_c$.

The rationale for the selection of a beam pair is to combine beams of different subcarriers with similar orientations. Desirable properties of original beams are thus more likely to be maintained for the new beam as well. Then, SIRs of users in these subcarriers will be high and cochannel interference to users residing in other beams will be low. Furthermore, since the spatial covariance matrix of each user does not change significantly in neighboring subcarriers, it is likely that cochannel user sets in these subcarriers consist of the same users and corresponding beams of users have similar orientations. Our algorithm should then combine beams of one user in different subcarriers, so that users occupy several subcarriers and transmit with few beams, or ideally with one beam (recall that limited number of beams can be formed). The algorithm each time selects the pair of beams (b_k, b_ℓ) with the minimum Euclidean distance $d(k, \ell)$ among all beam pairs, i.e, it selects pair (k^*, ℓ^*) such that,

$$(k^*, \ell^*) = \arg\min_{(k,\ell)} d(k, \ell) = \arg\min_{(k,\ell)} \|\mathbf{w}_{n,k} - \mathbf{w}_{m,\ell}\|^2 . \tag{15}$$

Now, $\|\mathbf{w}_{n,k} - \mathbf{w}_{m,\ell}\|^2 = \|\mathbf{w}_{n,k}\|^2 + \|\mathbf{w}_{m,\ell}\|^2 - 2\Re(\rho_{k\ell})$, where $\rho_{k\ell} = \mathbf{w}_{n,k}^H \mathbf{w}_{m,\ell}$ is the cross-correlation of beams $\mathbf{w}_{n,k}$ and $\mathbf{w}_{m,\ell}$. For normalized beams, (15) reduces to,

$$(k^*, \ell^*) = \arg\max_{(k,\ell)} \Re(\rho_{k\ell}) \tag{16}$$

Next, the beamforming vector of beam $\mathbf{w}_c$ that replaces beams $\mathbf{w}_{n,k}$ and $\mathbf{w}_{m,\ell}$ must be determined. In the sequel, we present two methods to perform this computation.

Approach A: Maximum new/old beam cross-correlation

The new beam $\mathbf{w}_c$ should have the least Euclidean distance from beams $\mathbf{w}_{n,k}$ and $\mathbf{w}_{m,\ell}$. Equivalently it should have high cross-correlation with these beams. Thus, $\mathbf{w}_c$ is the solution of the following optimization problem:

$$\max_{\mathbf{w}_c} \Re\left(\mathbf{w}_c^H \mathbf{w}_{n,k} + \mathbf{w}_c^H \mathbf{w}_{m,\ell}\right), \text{ s.t. } \|\mathbf{w}_c^H\| = 1 . \tag{17}$$

By applying Lagrange multipliers we have,

$$\mathbf{w}_c^* = \frac{\mathbf{w}_{n,k} + \mathbf{w}_{m,\ell}}{\sqrt{2\left(1 + \Re(\rho_{k\ell})\right)}} \tag{18}$$

After computing $\mathbf{w}_c^*$, we (tentatively) replace $\mathbf{w}_{n,k}$ and $\mathbf{w}_{m,\ell}$ with $\mathbf{w}_c^*$ and evaluate the SIRs of users k, ℓ and of users in $\mathcal{U}_n$ and $\mathcal{U}_m$. Note that only users in subcarriers n and m are affected by this replacement. If all SIRs exceed γ, we replace these two beams with the beam $\mathbf{w}_c^*$ and proceed to the selection of the next beam pair. If SIRs of some users are not satisfied, some existing beams (and thus users in these beams) from subcarriers n and m must be eliminated, so that cochannel interference to users in these subcarriers is reduced and SIRs are increased. However, the elimination of a beam (user) from subcarrier n or m results in throughput decrease by Sb bits, since the corresponding user will be removed. Ideally, we would like to keep the number of removed beams as low as possible. Thus, we must select appropriate beams to remove.

Let $V(k, \ell)$ be the set of users in subcarriers n or m (where the initial beams b_k, b_ℓ belonged) whose SIR is violated (i.e, it is less than γ) after computing new beam $\mathbf{w}_c^*$ and replacing beam pair (k, ℓ). Assume now that a user $\kappa \in V(k, \ell)$ is removed (together with its beam). If $\kappa \in \mathcal{U}_n$, then SIRs of users in $j \in \mathcal{U}_n$ will change to new values,

$$SIR_j(\kappa) = \frac{\mathbf{w}_{n,j}^H \mathcal{H}_{n,j} \mathbf{w}_{n,j}}{\sum\limits_{\substack{i \in \mathcal{U}_n \\ i \neq \{j,\kappa,k\}}} \mathbf{w}_{n,i}^H \mathcal{H}_{n,j} \mathbf{w}_{n,i} + \mathbf{w}_c^{*H} \mathcal{H}_{n,j} \mathbf{w}_c^*} . \tag{19}$$

Note that user k is not included in the sum above, since beam $\mathbf{w}_{n,k}$ is removed, but its presence is implied in beam $\mathbf{w}_c$. Similarly, if $\kappa \in \mathcal{U}_m$, SIRs of users $j \in \mathcal{U}_n$ are affected. We choose to remove the beam b_{κ^*} (user κ^*) that leads to maximization of the minimum SIR of remaining users in the two subcarriers. Thus,

$$\kappa^* = \arg\max_{\kappa \in V(k,\ell)} \min_{\substack{j \in (\mathcal{U}_n \cup \mathcal{U}_m) \\ j \neq \kappa}} SIR_j(\kappa), \tag{20}$$

By maximizing the minimum SIR, we intend to balance SIRs and thus increase the number of users with acceptable SIRs. The process of beam elimination according to criterion (20) above continues until all SIRs of users are acceptable. The algorithm then proceeds with the selection of the next pair of beams with criterion (16) and the procedure terminates when the number of beams is reduced to C.

Approach B: Maximum signal strength/minimum induced interference

According to a second criterion, the new beam $\mathbf{w}_c$, which is the outcome of unification of beams $\mathbf{w}_{n,k}$ and $\mathbf{w}_{m,\ell}$, must result in high desired signal strength at receivers of users k and ℓ in the original beams. It should also cause low interference to other users in subcarriers n and m. In fact, we are interested in the beam $\mathbf{w}_c$ that maximizes desired signal power at user receivers and minimizes induced interference to other users. This requirement is captured by ratio,

$$Z(k,\ell)=\max_{\mathbf{w}_c}\frac{\mathbf{w}_c^H\left(\mathcal{H}_{n,k}+\mathcal{H}_{m,\ell}\right)\mathbf{w}_c}{\mathbf{w}_c^H(\sum\limits_{\substack{j\in\mathcal{U}_n\\ j\neq k}}\mathcal{H}_{n,j}+\sum\limits_{\substack{j\in\mathcal{U}_m\\ j\neq\ell}}\mathcal{H}_{m,j})\mathbf{w}_c} \tag{21}$$

After computing $\mathbf{w}_c^*$, SIRs of users are calculated and users are sequentially eliminated according to (20), until acceptable SIRs are ensured. Then, the procedure is similar to the one described above.

The general algorithm can be summarized as follows:

- **STEP 1**: Run the first stage of the algorithm. Find a beam $\mathbf{w}_{n,k}$ for each user k in subcarrier n.
- **STEP 2**: For each pair of beams (k,ℓ) in different subcarriers, compute cross-correlation $\rho_{k,\ell}$. Select pair (k^*,ℓ^*) with maximum ρ_{k^*,ℓ^*}.
- **STEP 3**: Find new beam $\mathbf{w}_c^*$ with approach A or B.
- **STEP 4**: Perform elimination process based on (20), until all user SIRs exceed γ.
- **STEP 5**: If number of beams is C, terminate the algorithm. Else, go to Step 2 and repeat the procedure.

The complexity of finding generalized eigenvectors of a $M\times M$ matrix is $O(M^3)$. The first stage of the algorithm involves generalized eigenvector computation for cochannel users at each user insertion in a subcarrier and in all N subcarriers and thus it has complexity $O(NK^2M^3)$. The second stage involves selection of the pair of beams with maximum cross-correlation ($O(K^2)$), computation of new beam ($O(1)$ for approach A and $O(M^3)$ for approach B), elimination of users ($O(K^2)$) and beam merging ($O(\log K)$). Thus, the second stage has complexity $O\left(K^2\log K\right)$ for approach A and $O\left((K^2+M^3)\log K\right)$ for approach B.

C. Further Considerations and Extensions

1) Unification of beams with more than one users: As the algorithm progresses, one or both of the beams that are selected for unification will not include just one user in one subcarrier, as was the case in the previous subsection. A beam may contain several subcarriers of a user, or users with different subcarriers. These beams are the outcome of the merging process earlier in the algorithm. The algorithm should be slightly modified to deal with these cases as well.

Consider a beam pair (b_k,b_ℓ) with beamforming vectors $\mathbf{w}_k$ and $\mathbf{w}_\ell$. Let beam b_k contain users $k_1,\ldots,k_t$, where user k_i resides in subcarrier n_i, $i=1,\ldots,t$ and let beam b_ℓ contain users $\ell_1,\ldots,\ell_s$, where ℓ_i uses subcarrier m_i, $i=1,\ldots,s$. The problem is again to compute a new beam $\mathbf{w}_c^*$ that will replace beams b_k and b_ℓ.

When approach A is applied, $\mathbf{w}_c^*$ depends only on vectors $\mathbf{w}_k$ and $\mathbf{w}_\ell$ and not on the users that reside in the beams. Thus, $\mathbf{w}_c^*=(\mathbf{w}_k+\mathbf{w}_\ell)/\sqrt{2(1+\Re(\rho_{k,\ell}))}$, similarly to (18). When approach B is considered, some changes in (21) are required. This ratio must now capture the requirement that the new beam $\mathbf{w}_c^*$ should yield high desired signal power at all $t+s$ user receivers within beams b_k and b_ℓ and low interference towards users in subcarriers n_i, $i=1,\ldots,t$ and m_i, $i=1,\ldots,s$. The following changes are needed:

$$\mathcal{H}_{n,k}\text{ and }\mathcal{H}_{m,\ell}\quad\text{become}\quad\sum_{i=1}^{t}\mathcal{H}_{n_i,k_i}\text{ and }\sum_{i=1}^{s}\mathcal{H}_{m_i,\ell_i}$$

$$\sum_{\substack{j\in\mathcal{U}_n\\ j\neq k}}\mathcal{H}_{n,j}\quad\text{becomes}\quad\sum_{i=1}^{t}\sum_{\substack{j\in\mathcal{U}_{n_i}\\ j\neq k_i}}\mathcal{H}_{n_i,j}$$

$$\sum_{\substack{j\in\mathcal{U}_m\\ j\neq \ell}}\mathcal{H}_{m,j}\quad\text{becomes}\quad\sum_{i=1}^{s}\sum_{\substack{j\in\mathcal{U}_{m_i}\\ j\neq \ell_i}}\mathcal{H}_{m_i,j}$$

Next, SIRs for users in beams b_k and b_ℓ are computed. If all SIRs exceed γ, we replace $\mathbf{w}_k$ and $\mathbf{w}_\ell$ with $\mathbf{w}_c^*$ and proceed to the selection of the next beam pair. The SIR of a user that uses a certain subcarrier in a beam does not depend on other subcariers, but only on beamforming vectors of beams that use that subcarrier to serve users. Hence, if some SIRs of users in certain subcarriers are not acceptable, some users that use the same subcarriers need to be eliminated, so that user SIRs are increased. Let $\mathcal{X}$ be the set of users in beams b_k and b_ℓ, i.e, in subcarriers n_i, $i=1,\ldots,t$ and m_i, $i=1,\ldots,s$. Again, let $V(k,\ell)$ denote the set of users with unacceptable SIR. Similarly to (19), let $SIR_j(\kappa)$ be the SIR of user $j\in\mathcal{X}$ if user $\kappa\in V(k,\ell)$ is removed. The criterion for removal of a user is again SIR balancing for remaining users, i.e,

$$\kappa^*=\arg\max_{\kappa\in V(k,\ell)}\min_{\substack{j\in\mathcal{X}\\ j\neq\kappa}}SIR_j(\kappa),\ . \tag{22}$$

Since $\mathcal{X}=\left\{\cup_{i=1}^{t}\mathcal{U}_{n_i}\right\}\cup\left\{\cup_{i=1}^{s}\mathcal{U}_{m_i}\right\}$, (22) can be seen as a generalization of (20). Note that only users and not beams are removed at each step of the procedure. However, if all users that belonged to a beam are gradually eliminated to create acceptable SIRs for utilized subcarriers, that beam is finally removed from the system.

2) Minimum rate requirements for users: Assume that each user k has minimum rate requirements r_k bits/sec. Then, the described methods need to be modified. First, assume that each beam contains one user in one subcarrier and that merging has been performed. If SIRs of some beams (i.e, users) in affected subcarriers are violated, some beams need to be removed, until SIRs are acceptable. During this process, users must continue to satisfy their minimum rate requirements after beam elimination. Thus, if τ_k is the rate associated with user k before a beam elimination, the condition $\tau_\kappa - r_\kappa > Sb$ must be added to criterion (20), so that elimination of beam κ and subsequent rate reduction of user κ by Sb bits do not cause κ to violate x_k. The same condition should be added in (22).

3) Extensions to the Algorithm: In step 2 of the proposed procedure, the pair of beams for merging was selected according to a maximum cross-correlation criterion. Then, the new beam was computed by using approach A or B. We now explain a more efficient but more complex method for beam selection. Assume that a new beam $\mathbf{w}_c^*$ is computed with (21). If SIRs of some users are not acceptable, some users will be removed. After removing a user with criterion (20) or (22), we can compute a new beam $\hat{\mathbf{w}}_c{}^*$ with (21). Clearly, $\hat{\mathbf{w}}_c^*$ is different than $\mathbf{w}_c^*$, since the denominator of (21) now does not include removed users. If SIRs are not satisfied, another user is removed and a new beam is computed. The procedure terminates when acceptable SIRs are ensured for all users. Then, beam pair (b_k, b_ℓ) can be associated with cost $C(k, \ell)$ equal to the number of removed users. The beam pair with the minimum cost should be considered for merging, since it results in minimum throughput decrease. Among beam pairs with the same cost, we can select the one for which the minimum SIR of users is maximum.

IV. OPTIMAL SOLUTION FOR A SPECIAL CASE

We consider the case of $C = 2$ transceivers with fixed beamforming vectors $\mathbf{w}_1$ and $\mathbf{w}_2$. Each user is assigned to a transceiver and uses subcarriers in the corresponding beam. We assume that a set of subcarriers constitutes a subband, within which spatial covariance matrix for each user is fixed and equal to $\mathcal{H}_k$. The minimum number of required subcarriers for user k is $x_k = \lceil r_k T_s / Sb \rceil$. The problem is to satisfy these requirements with utilization of minimum number of subcarriers and with two transceivers.

Let U_i be the set of users in beam $\mathbf{w}_i$, $i = 1, 2$. At most two users assigned in different transceivers can use the same subcarrier simultaneously. Then, we have to find the maximum number of such user pairs, where each pair occupies a subcarrier. The problem is equivalent to maximum matching on a bipartite graph. A bipartite graph $G = (U \cup V, E)$ is constructed as follows. One node for each required subcarrier of a user is added to the graph. Thus, $|U| = \sum_{i \in U_1} x_i$ and $|V| = \sum_{i \in U_2} n_i$. An edge (i, j) is added between nodes $i \in U$ and $j \in V$ (which denote subcarriers of users $\alpha \in U_1$ and $\beta \in U_2$ respectively) if SIRs of these users are acceptable, i.e, if

$$\min\left\{ \frac{\mathbf{w}_1^H \mathcal{H}_\alpha \mathbf{w}_1}{\mathbf{w}_2^H \mathcal{H}_\alpha \mathbf{w}_2}, \frac{\mathbf{w}_2^H \mathcal{H}_\beta \mathbf{w}_2}{\mathbf{w}_1^H \mathcal{H}_\beta \mathbf{w}_1} \right\} \geq \gamma, \tag{23}$$

A matching $\mathcal{M}$ in a graph G is a subset of edges of G, such that no two edges in $\mathcal{M}$ share the same node. Each edge of $\mathcal{M}$ is called a matched edge. A maximum matching $\mathcal{M}^*$ is a matching of maximum cardinality. The assignment that minimizes the number of required subcarriers is as follows. We start by finding $\mathcal{M}^*$. Each edge in $\mathcal{M}^*$ corresponds to a cochannel pair of users. Assign each such pair to a separate subcarrier. Then, for each user corresponding to a node that is not incident to a matched edge, consider a new subcarrier and assign the user to it. The minimum number of subcarriers to satisfy rate requirements of users equals $|\mathcal{M}^*|$, plus the number of nodes that are not incident to a matched edge.

V. SIMULATION RESULTS

A. Simulation setup

We simulate a single-cell system with $K = 15$ users that are uniformly distributed in the cell area. An antenna array with M elements and $\delta = \lambda/2$ is used for transmission. The base station uses OFDM transmission at 5 GHz. For illustrative reasons, we restrict ourselves to a system with $N = 10$ subcarriers. The received power decays with distance d from the base station as d^{-4}. For each such link corresponding to an antenna and a receiver, multi-path fading is simulated with an L-ray model. The angle of each path is uniformly distributed in $[0, \pi]$. The delay between paths is uniformly distributed in $[0, T]$. The complex gain of each path is an independent log-normal random variable with standard deviation $\sigma = 6$dB, which accounts for shadow fading. Results were averaged over 100 random experiments with different channel conditions and user locations.

B. Comparative results

The objective of the simulations is to evaluate and compare the performance of the proposed approaches for our problem. It is also desirable to quantify the impact of different parameters on performance. First, we do not consider minimum rate requirements and focus on system rate. In order to have a fair comparison for the proposed heuristics, the following approaches are simulated:

- Approach A: The first stage of the algorithm is initially executed. At the second stage, the pairs of beams are selected for unification based on criterion (16). The new beam vector is computed with (18). Next, beams are sequentially eliminated, based on (20) or (22), until SIRs of remaining users are acceptable.

- Approach B: The first stage of the algorithm is again executed and the beam pair selection is based on (16). The new beam vector is calculated with (21). After beam elimination with (20) or (22), a new beam is calculated again with a modified ratio in (21). This iterative process of beam elimination and new beam computation terminates when SIRs are acceptable.

The performance metric is average channel throughput, which is defined as the number of users per subcarrier. In Figures 4 and 5, average channel throughput is illustrated as a function of number of available transceivers for the case of $M = 4$ and $M = 8$ antennas, SIR threshold $\gamma = 10\,\mathrm{dB}$ and different multi-path scenarios. For $M = 4$, we observe that for the same multi-path channel conditions (number of paths L), approach B always performs better than approach A. This is explained by the iterative structure of approach B, where beam vectors are iteratively updated, as opposed to approach A, where beam vectors are computed once. In addition, different criteria were utilized for beam computation in the two approaches. For $L = 1$, the difference in the performance of the two approaches is almost constant and independent of the number of transceivers, while for $L = 2$, the difference decreases as the number of transceivers increases. An important observation is that the channel throughput with $L = 2$ is larger than that for $L = 1$ for both approaches A and B, due to the additive effect of traversing the two paths. Similar conclusions can be drawn for the case of $M = 8$ antennas. The results for $M = 8$ antennas and a multi-path channel with $L = 3$ and $L = 5$ paths are shown in Figure 6. The difference in performance of the two approaches is more notable here. Furthermore, the throughput increase with increasing number of paths is also more evident. This increase is explained by the diversity attributes that exist in a channel with rich multi-path. Figures 4 - 6 can also be used to quantify the incurred throughput loss of the beam merging procedure.

When minimum throughput requirements for users come into play, a meaningful performance measure is the residual throughput of users. For user k with minimum number of required channels x_k and attainable number of channels n_k at the end of the algorithm, the residual throughput is $\tilde{r}_k = (x_k - n_k)^+$, where $(x)^+ = x$ when $x \geq 0$ and 0 otherwise. We assume that the minimum number of required channels is uniformly distributed in $\{1, 2, 3, 4, 5\}$. Residual throughput is measured as the average number of additional subcarriers that a user needs so as to satisfy minimum throughput requirements. An efficient algorithm should result in low residual throughput. In Figure 7 the total residual throughput of users is shown as a function of the number of transceivers C, for $M = 4$ antennas, with $\gamma = 10\,\mathrm{dB}$ and $L = 2$. It can be deduced that approach B performs better than approach A, when $C < 15$. For $M = 8$ antennas, B performed better than A when $C < 31$. These observations seem to suggest that the number of transceivers beyond which the two approaches exhibit the same performance is almost directly proportional to the number of antennas.

Finally, we evaluate the performance of the greedy assignment method at the first stage of the algorithm. This constitutes a meaningful methodology for performing subcarrier assignment and beamforming when transceiver resources are unlimited. Since approaches A and B are not used in the first stage, we are interested in the impact of multi-path on performance. In Figure 8, we plot the average throughput as a function of the SIR threshold γ for different multi-path conditions, where a high γ corresponds to a stringent BER requirement. We observe that for $L = 1$, throughput decays almost exponentially with increasing γ, while when $L = 2$ the rate of decay is smaller. This is another evidence for the fact that transmission over a multipath channel can improve performance. For larger number of paths, e.g, $L = 3, 4, 5$ or 6, only minor differences in performance can be seen. However, average throughput for $L > 1$ is superior to that for $L = 1$ when $\gamma > 10\,\mathrm{dB}$.

Although in a realistic system the number of subcarriers N will be larger, subcarrier reuse will depend on spatial properties of users, beamforming and resource (subcarrier and transceiver) allocation policy. Similar tendencies are thus anticipated in a larger system, with larger number of subcarriers and users. Our results manifest the necessity for a sophisticated system design, so as to provide QoS to users and increase system performance. For a given BER requirement (and value of γ) and a number of antennas, there exists a crucial number of transceivers C_0, beyond which no further benefits can be obtained. Viewed differently, the number of transceivers can be made as small as C_0 with no throughput loss.

VI. DISCUSSION

In this paper, we addressed the joint problem of adaptive space division multiplexing and channel allocation in an OFDM-based system, with a view towards increasing system capacity and providing QoS to users in the form of minimum rate guarantees. We considered a realistic scenario with limited transceiver resources and proposed heuristics for subcarrier and transceiver assignment to users and adaptive beamforming to achieve our objectives. Our primary goal is to identify the impact of smart antennas with limited transceiver resources on MAC layer channel allocation.

A transceiver was perceived as a communication unit that is used to set up a distinct beam. Since a beam can serve several users in different subcarriers, the problems of subcarrier and transceiver assignment are coupled. Hence, the determination of the assignment that yields maximum subcarrier reuse with acceptable user SIRs is a hard optimization problem. In section IV the optimal solution was provided for a simple case. In a system with many users and transceivers, the corresponding task would be to identify all possible spatially separable sets of users that can reuse a subcarrier simultaneously in different transceivers. Determination of spatial separability is not straightforward, since SIR at a receiver depends on beams of all cochannel users. Even if a cochannel set of users is given, the computation of beams for acceptable SIRs is a highly non-linear problem. Furthermore, the consideration of limited number of transceivers creates an even more cumbersome situation, since users must be essentially clustered within transceivers.

Therefore, heuristic algorithms need to be adopted, which capture desired properties of a good solution and can provide performance bounds for more general algorithms. Our results indicate that the method which employs iterative beam computation based on maximum signal strength/minimum interference performs remarkably well. Moreover, there exists a crucial number of transceivers, beyond which performance cannot be improved. The degree of channel reuse and the incurred throughput losses at the second stage of the algorithm respectively quantify the impact of smart antennas and limited transceiver resources on the performance of MAC layer resource allocation. The proposed policies can thus serve as benchmarks and the illustrated plots can provide useful design criteria.

There exist several directions for future study. A more general treatment of the topic could include transmission power adaptation. The issue of modulation adaptation on a subcarrier basis is another possible extension. There exists an inherent tradeoff between achievable rate and sustainable amount of interference. A high modulation level for a user increases channel rate, so that the user needs fewer subcarriers to satisfy rate requirements. Hence, more users can be accommodated in the system and capacity is increased. However, a high modulation level renders user spatial separability and resource reuse more difficult, since it requires higher SIR (lower interference) to maintain a given BER.

We conclude the paper by drawing the analogy between the assignment problem that was studied and the corresponding scheduling problem that arises at the packet level. Packets arrive at user queues and need to be transmitted according to a scheduling policy that is applied in every time slot. Due to bit assignment in the frequency domain, it can be shown that the use of k subcarriers by a user in each packet symbol is equivalent to the removal of k fixed length packets from the corresponding user buffer. The C transceivers can be viewed as a set of C servers that serve user packets. In each slot, a server can serve several user queues, under the constraint that corresponding users use different subcarriers. In addition, several fixed length packets from the served user queues can be scheduled at each slot by virtue of OFDM. For a given subcarrier, C user queues can be served by the C servers, with space division multiplexing. However, cochannel interference between beams poses constraints on the eligible user sets for scheduling. A scheduling policy at each slot consists of queue allocation to a server and channel allocation within each slot. The extension of ideas and proposed stabilizing scheduling policies for simple systems to such generalized joint scheduling and channel allocation problems is a topic that warrants further investigation.

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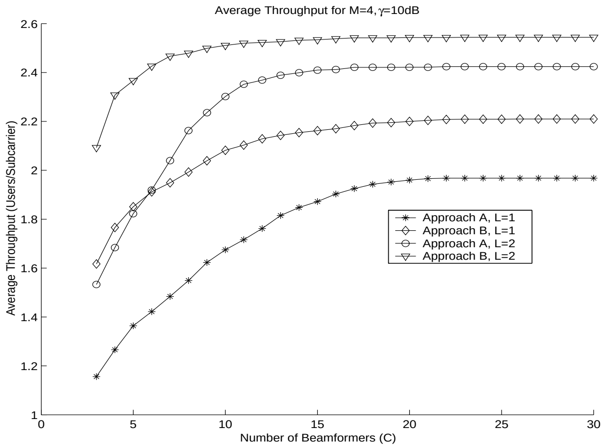

Fig. 4. Average throughput vs. number of transceivers for approaches A and B, for multi-path with $L = 1$ and $L = 2$ paths and $M = 4$ antennas.

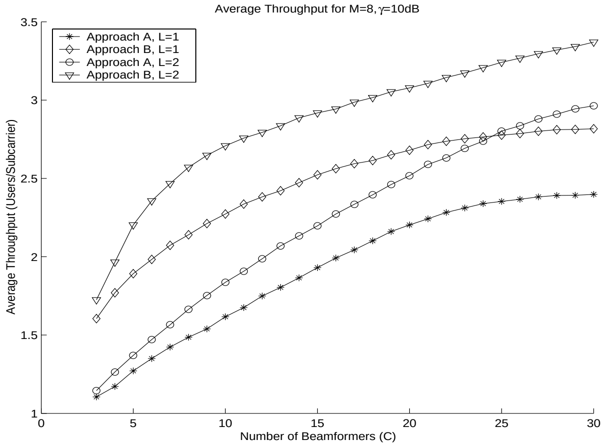

Fig. 5. Average throughput vs. number of transceivers for approaches A and B, for multi-path with $L = 1$ and $L = 2$ paths and $M = 8$ antennas.

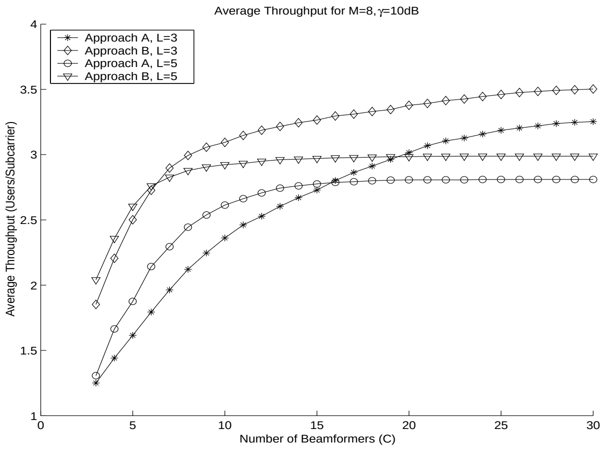

Fig. 6. Average throughput vs. number of transceivers for approaches A and B, for multi-path with $L = 3$ and $L = 5$ paths and $M = 8$ antennas.

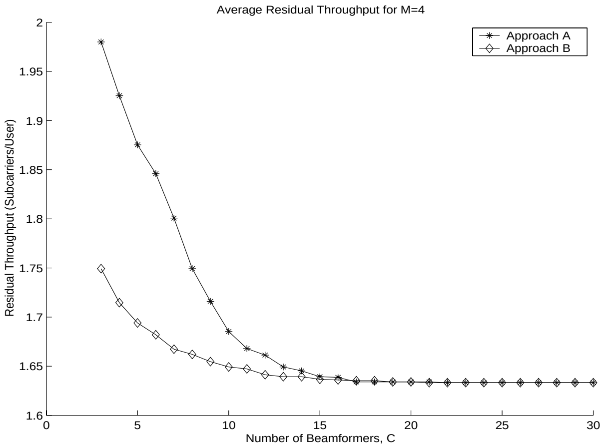

Fig. 7. Residual throughput vs. number of transceivers for approaches A and B, for $M = 4$ antennas.

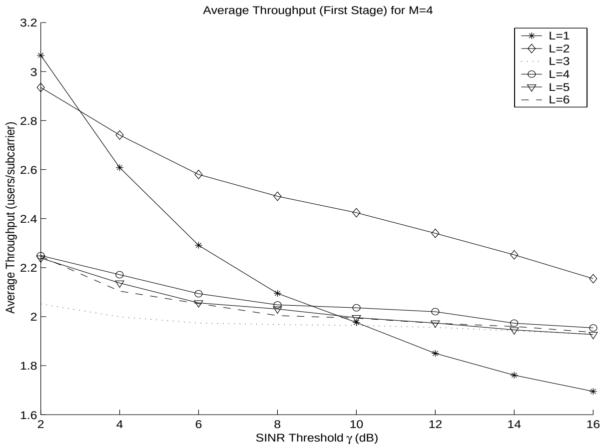

Fig. 8. Average throughput vs. SIR threshold, for unlimited number of transceivers and $M = 4$ antennas.