

ABSTRACT

Title of dissertation: **ESSAYS ON SOVEREIGN DEBT STRUCTURE,
DEFAULT AND RENEGOTIATION**

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Emerging market economies have witnessed recurrent large-scale sovereign debt crises. Many of these crises shared two features: a significant shift towards short-term debt before a crisis, and a prolonged debt renegotiation afterwards. This dissertation studies each phenomenon in a chapter. The first chapter constructs a dynamic model of sovereign debt default and renegotiation in which the shift towards short-term debt before a default results from two inefficiencies in the sovereign debt market: no legal enforcement of debt repayment and no explicit seniority structure. These inefficiencies give rise to “default risk” and “debt dilution risk”. The mechanism by which default risk favors short-term debt is well understood: long-term bonds incorporate additional default risk, and hence bear higher risk premia than short-term bonds, giving the government the incentive to rely more on short-term debt before a crisis. “Debt dilution” occurs because the absence of an explicit seniority structure implies that new short-term debt issuances can reduce the amount recoverable by existing long-term debt-holders, and thus “dilute” existing long-term

debt. This effect is more severe as default and debt restructuring become more likely, resulting in higher risk premia on long-term bonds as a crisis approaches. Quantitative analysis shows that the model generates a large shift to short-term debt before a default. The fraction of this shift due to debt dilution is significantly larger than the fraction due to default risk. In the long run, debt dilution tends to increase default frequency, decrease debt-to-GDP ratio and make capital inflows less procyclical.

The second chapter provides an explanation to the observed length of delays in debt restructuring negotiations. Contrary to the common wisdom that delays are costly and inefficient, this chapter argues that delays can be beneficial in that they allow the economy to recover from a crisis and make more resources available to settle the defaulted debt. As a result, the negotiating parties can be better off by waiting and then dividing a larger “cake.” By introducing a stochastic bargaining game, based on Merlo and Wilson’s (1995) framework, into a sovereign default model, this chapter shows quantitatively that our argument can generate an average delay length comparable to that experienced by Argentina in its most recent debt restructuring. In addition, the model successfully accounts for the observed volatility of bond spreads.

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DEDICATION

To my parents and my fiance.

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Chapter 1

Debt Dilution and the Maturity Structure of Sovereign Bonds

1.1 Introduction

The past few decades have witnessed recurrent large-scale sovereign debt crises in emerging market economies. Many of these crises were preceded by a significant change in the affected countries' foreign debt maturity structures: an increasing reliance on short-term debt as a crisis drew near¹. Table 1.1 reports the weighted average maturities for some pre-crisis episodes²: Argentina from 2000Q2 to 2001Q2, Brazil from 1997Q3 to 1998Q3, Mexico from 1993Q2 to 1994Q2, Russia from 1997Q1 to 1998Q1 and Turkey from 2000Q2 to 2001Q2. For comparison, we also report these countries' weighted average maturities from the mid 1990s to around 2002³, excluding the pre-crisis periods. In all the cases, average maturities shortened more than one third in the pre-crisis episodes.

Excessive reliance on short-term debt is widely regarded as a key factor behind emerging markets crises: it not only increases a country's vulnerability to financial crisis, but also affects the depth of a crisis when it occurs⁴. Why, then, do emerging

¹This stylized fact is well documented in the literature. In particular, Broner, Lorenzoni and Schmukler (2005) report such a shift toward shorter maturities in emerging markets debt issuances before crises, using a newly-constructed database of sovereign bond prices and issuance.

²In this paper, a crisis refers to an episode with high default risk, but not necessarily an explicit default.

³The time periods are: Argentina from 1993Q2 to 2001Q2, Brazil from 1994Q2 to 2002Q2, Mexico from 1991Q1 to 2002Q2, Russia from 1993Q2 to 2000Q3, and Turkey from 1992Q1 to 2003 Q1. Data Source: Broner, Lorenzoni and Schmukler (2005).

⁴See, for example, Calvo and Mendoza (1996), and Cole and Kehoe (1996).

Table 1.1: Weighted Average Maturities in Pre-Crisis Episodes

Country	Pre-Crisis Episode	Pre-Crisis Average Maturity (Years)	Average Maturity Excluding Pre-Crisis Periods (Years)
Argentina	2000Q2 - 2001Q2	6.46	9.77
Brazil	1997Q3 - 1998Q3	9.23	17.87
Mexico	1993Q2 - 1994Q2	4.87	11.59
Russia	1997Q1 - 1998Q1	8.79	14.67
Turkey	2000Q2 - 2001Q2	4.90	7.91

market economies near crisis rely more on short-term debt even when they end up paying a high price for doing so? An obvious answer to this question is “default risk”: since there is no legal enforcement of sovereign debt repayment, the borrowing country has an option to default when its financial situation worsens. Since long-term bonds are less likely to mature before default than short-term bonds, long-term bonds have higher default risk and bear higher risk premia. As a consequence, long-term bonds may become “too expensive” as a country approaches financial distress, giving the government the incentive to rely more on short-term debt. Arellano and Ramanarayanan (2006) employ this argument to show that debt issuances near crisis are mainly in short-term maturities.

Default risk, however, is only part of the story. This paper emphasizes another inefficiency in sovereign debt markets that also plays an important role in driving the shift to short-term debt before crises. This inefficiency is the lack of an explicit seniority structure among different sovereign debt claims. Modern sovereign bond contracts usually contain a *pari passu* clause, defining the priorities of debt obligations. It typically takes the following formulation:

The Notes rank, and will rank, pari passu in right of payment with all other present and future unsecured and unsubordinated External Indebtedness of the Issuer. (Buchheit and Pam (2003))

The Latin phrase “*pari passu*” means “equally”⁵, implying that this clause precludes preferential treatment of debt obligations in a debt restructuring. While a few exceptions, to be discussed shortly, do exist in practice, this paper focuses on the “*pari passu*” case by assuming that all the debt holders share the debt recovery value *pro rata* in a post-default debt restructuring. In these circumstances, “debt dilution risk” arises. This source of inefficiency in sovereign debt contracts is less well-known than the default risk problem, and is therefore the main focus of this paper. Debt dilution occurs when a sovereign issues new debt that does not improve the country’s repayment ability proportionally. As a result, these new debts not only increase default probability, but also reduce the amount that can be recovered by existing debt-holders in a post-default debt restructuring, and thus “dilute” existing debts⁶. This dilution effect is in play as long as default risk exists. It does not depend on whether the sovereign ends up defaulting. The dilution effect is most severe when default and debt restructuring are near and the sovereign issues new debts primarily to meet its growing financial needs. Consequently, long-term

⁵See Buchheit and Pam (2003), Bratton (2004) and Financial Markets Law Committee (2005) for detailed discussions on the interpretations and applications of this clause.

⁶A simple numerical example can illustrate this idea: Suppose a sovereign has 100 dollars of existing long-term debt (held by investor A) at the point of default, and the sovereign is able to repay 80 dollars. Then the debt recovery rate is 80%. However, if the sovereign borrows another 100 dollars from investor B on the eve of crisis and after default, it is able to repay 100 dollars (i.e., the new borrowing does not increase the repayment ability proportionally), then the debt recovery rate for both debt-holders is only 50%. Clearly, the new debt dilutes the existing long-term debt, and investor A suffers a capital loss.

lenders charge additional risk premia to cover the “dilution risk”, which leads to a shift towards the relatively “cheap” short-term bonds when the government is near financial distress.

The main purpose of this paper is to show, quantitatively, that debt dilution plays an important role in driving the shift towards short-term debt near crisis, and that debt dilution also has long-run implications for debt levels, default frequency and cyclical dynamics. To this end, the paper constructs a dynamic model of default and renegotiation of sovereign debt of short and long maturities and shows that default risk and the debt dilution effect, together, bias the maturity structure toward short-term bonds before crisis episodes. We then disentangle the effect of debt dilution by comparing setups with different degrees of the dilution effect. Such a comparison shows that the fraction of the shift in maturity structure due to the debt dilution effect is significantly larger than the fraction due to default risk. This result has an important policy implication: if emerging market economies want to improve their maturity structures by issuing bonds of longer term, then the inefficiency of no explicit seniority structure should be addressed first. Furthermore, the comparison of setups with different degrees of dilution highlights the long-run effects of the dilution problem: debt dilution increases long-run default frequency, decreases the debt-to-GDP ratio and makes capital inflows less procyclical.

The analysis begins with a benchmark model, in which a risk-averse sovereign borrows from a group of risk-neutral, competitive international investors using non-contingent, one-period (short-term) and two-period (long-term) bonds. Facing a stochastic output stream, the sovereign has the option to default, and default results

in exclusion from international capital markets and a proportional output loss. The country can re-enter the credit market through renegotiation with the creditors over debt reduction and repayment. In this post-default debt renegotiation, all debt holders are treated equally and hence have the same debt recovery rate, which is endogenously determined in a Nash bargaining game. In equilibrium, sovereign bonds are priced to compensate creditors for both default risk and the debt dilution effect, with the latter reflected in the endogenous debt recovery rate.

We characterize the equilibrium debt recovery rate analytically. The debt recovery rate is decreasing in the amount of defaulted debt, indicating that the debt dilution effect is more severe the more the country issues new debt on the eve of a crisis. Furthermore, we demonstrate that for a given debt stock, the maturity structure plays an important role in determining a country's default probability: a larger proportion of maturing debt leads to a higher default probability in the near future.

The quantitative analysis shows that the model successfully accounts for some of the key features of Argentina's business cycle and debt experience, including the volatility of consumption, the countercyclical current account, and countercyclical bond spreads. We find that, by introducing the debt dilution effect, the model can account for the default frequency and debt-to-GDP ratio observed in Argentina without employing unusually high rates of time preference, as has been done in previous quantitative studies of sovereign debt. The model generates a large shift to short-term debt before crises. Moreover, by comparing the benchmark case of "*pari passu*" (i.e., a moderate degree of dilution) with two alternative setups reflecting no

dilution and a maximum degree of dilution, we show that the dilution effect plays a more important role than default risk in driving the shift in maturity structure. This comparison also evaluates the long-run impacts of the dilution problem, as mentioned above.

Although “*pari passu*” is the typical legal requirement, in practice there are a few cases in which creditors are treated differently in terms of payment. International financial institutions, such as the IMF, enjoy *de facto* seniority. They are fully repaid even if the borrowing country is in default with its private lenders. This “*de facto* seniority” issue has been recognized in the literature. For example, Saravia (2003) studies the conflict between the official and private lenders in the competition for repayments, given the *de facto* seniority of the former. This paper focuses on privately-held foreign bonds issued by emerging market governments, and thus abstracts from the “*de facto*” seniority problem⁷. Another counter example to equal treatment is the “holdout” problem⁸. By refusing to participate in debt restructuring and filing lawsuits against the debtor country, holdout creditors hope to be repaid as specified in the original bond contracts. The holdout problem has been widely regarded as disruptive to the normal debt restructuring process. Consequently, different solutions have been proposed and adopted in recent years to address this problem⁹. This paper abstracts from the “holdout” problem and main-

⁷Even if a *de facto* official senior lender is added to this model, the key result remains the same: the existence of a *de facto* senior lender changes the interest rates charged for both short and long-term bonds, but the *shift* towards short-term among the privately-held sovereign bonds is not affected.

⁸The holdout problem is not a violation of the *pari passu* clause and can actually be an opportunistic use of that clause. See Buchheit and Pam (2003) and Bratton (2004) for further discussions.

⁹For example, the Sovereign Debt Restructuring Mechanism (SDRM) was proposed by the IMF

tains a key assumption in our benchmark model that all debt holders are legally equal and share debt haircuts *pro rata* in the debt restructuring process.

Debt dilution is a familiar theme in the corporate finance literature¹⁰. These studies generate considerable insights, which unfortunately may not hold in the sovereign debt context because the debt dilution problem in corporate debt does not result from the absence of an explicit seniority structure. Seniority of creditors does exist at the corporate level and is established by contracts and statutes.

The importance of an explicit seniority structure in sovereign debt has become the focus of recent studies. Roubini and Setser (2004, Chapter 7) provide a thorough discussion of how sovereign debt restructurings could be made more orderly if a precise system of priority rules were in place. Sachs and Cohen (1982) argue that without bond seniority provisions, a sovereign tends to overborrow, and the new borrowing increases the default risk born by the long-term debt in effect. Thus, short-term debt is more favorable under default risk. Bolton and Jeanne (2005) use a 3-period model to show how creditors can achieve implicit seniority by lending in debt instruments that are hard to restructure, resulting in a shift from easy-to-restructure instruments (bank loans) to hard-to-restructure ones (bonds) in equilibrium. The above papers are based on a static borrowing framework, in which

in 2002, providing a statutory approach to solve the holdout problem. Collective Action Clauses (CACs), aiming to align bondholders' incentives by specifying a majority rule, help to eliminate the hold out problem by contractual means. These CACs have been recently adopted in many emerging market bond contracts after Mexico's pioneering inclusion of such clauses in February 2003. Even without the SDRM and without CACs in a large amount of defaulted debt issued under New York Law, Argentina successfully avoided disruptions caused by holdout creditors by taking legal action, which gained support from the United States government. See Bratton (2004) and Gelpern (2005) for more details on Argentina's experiences in dealing with holdout problems.

¹⁰See Fama and Miller (1972), White (1980), Schwartz (1989), Bizer and DeMarzo (1992), Hart and Moore (1995), and Bisin and Rampini (2004).

the defaulting country's desire for future access to capital markets doesn't affect the outcome of renegotiation. In contrast, the model studied in this paper incorporates endogenous default and renegotiation into an infinite-horizon dynamic model, in which the debt dilution effect is introduced via the issuance of debt instruments of multiple maturities. This setup allows us to study quantitatively the effects of debt dilution before crisis and in the long run.

The present paper builds on recent quantitative models of sovereign debt default and renegotiation by Aguiar and Gopinath (2006), Arellano (2007) and Yue (2006). These papers make significant additions to the pioneering theoretical framework of Eaton and Gersovitz (1981) to explain empirical regularities of sovereign debt and business cycles in emerging economies. To date, however, this literature has not explored the quantitative implications of debt dilution. We show that the debt dilution effect can be very useful in filling important gaps that remain. In particular, debt dilution plays a more important role than default risk in driving the shift toward short-term debt before a crisis, and also the dilution effect helps substantially in accounting for observed default frequencies and debt-to-GDP ratios.

This paper is also related to the literature on the optimal sovereign debt maturity structure. Broner, Lorenzoni and Schumukler (2005) study this problem by emphasizing changes in the lenders' risk aversion and their liquidity needs. Arellano and Ramanarayanan (2006) focus on the impact of default risk on bond term structures. Our paper differs from the latter paper in that we take into consideration the debt dilution effect, an inefficiency associated with debt restructuring. We find that dilution risk can explain a much larger fraction of the shift in maturity structures

before crises than default risk.

The remainder of this paper is organized as follows. We describe the model in section 2. Section 3 defines the model’s equilibrium and characterizes some of its properties. Model calibration and simulation results are provided in section 4. In section 5, we compare the benchmark model with two alternative setups to explore the significance of debt dilution. This section also presents the results of the sensitivity analysis. Finally we offer concluding remarks in section 6. The proofs and the computation algorithm are in Appendix A.

1.2 The Model

This model is based on the canonical framework of Eaton and Gersovitz (1981), and follows Yue (2006) in modeling the debt renegotiation process. The model differs from Yue (2006) in two main aspects: introducing multiple bond maturities¹¹ and assuming a “*pari passu*” treatment of different debt holders in post-default renegotiations. These modifications are critical in studying the debt dilution effect. Dilution of the earlier long-term debt by the later debt does not arise if only one-period bonds are traded and only one investor participates in debt renegotiations, as in Yue (2006). In this section, we briefly describe the model environment and then present the sovereign government’s problem, the renegotiation problem and the international investors’ problem in detail.

The model studies a small open economy with a stochastic stream of a non-

¹¹Arellano and Ramanarayanan (2006) also introduce multiple maturities to study the term structure of sovereign bonds. But they do not model the renegotiation problem after default, and hence the debt dilution effect is absent from their model.

storable consumption good y_t , drawn from a compact set Y . $\mu(y_t|y_{t-1})$ is the probability distribution function of y_t conditional on the previous realization y_{t-1} . In this economy, a risk-averse benevolent government maximizes the expected lifetime utility of a representative domestic resident. The preferences of the sovereign government are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1.1}$$

where $0 < \beta < 1$ is the discount factor, and c_t denotes consumption in period t . The period utility function $u(c_t)$ is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions.

In each period, the sovereign solves different problems depending on its credit standing s at the beginning of that period. A good credit standing $s = 0$ indicates that the country is current on its debt service and the government can then choose whether to repay the maturing debt or to default. If the former is chosen, the government maintains a good credit standing and maintains access to credit markets. On the other hand, if the government decides to default, its credit standing deteriorates to $s = 1$ in the next period, indicating that the country is in default. Default carries two penalties that are common in the sovereign debt literature, a proportional output loss¹² and exclusion from international capital markets. During this exclusion period, the borrower initiates a renegotiation on debt reduction with its debt holders. Once an agreement has been reached and the renegotiated amount of debt has been repaid in full, the sovereign's reputation is restored and it is no

¹²This assumption is empirically relevant. Sturzenegger (2002), for example, estimates the post-default output loss using a panel of 100 countries, and finds that the percentage of output contraction is equal to 2%.

longer excluded from international capital markets.

International investors are passive in this model. They are risk neutral and behave competitively in international capital markets. They can borrow as needed in capital markets at the world risk-free interest rate r , which we assume to be constant and unaffected by the size of investors' borrowing and lending. Investors have perfect information on the country's asset holdings and current endowment realizations. They always commit to repay their debts.

Capital markets are incomplete in that only non-contingent zero-coupon bonds with short (one-period) and long (two-period) maturities are traded. The face value of a discount bond issued in period t and maturing in period τ is denoted as b_τ^t , specifying the amount to be repaid in period τ . If b_τ^t is positive, this reflects government's saving; otherwise, it reflects the government's debt. We denote the price of a bond with face value b_τ^t as q_τ^t . This price is determined endogenously in equilibrium.

1.2.1 Sovereign Government's Problem

At the beginning of period t , an endowment shock y_t is realized, and the country inherits a credit standing s_t and a vector of existing assets B_t from the last period. B_t consists of 3 assets: the long-term bond issued 2 periods ago, b_t^{t-2} , the short-term bond issued in the previous period, b_t^{t-1} , and the long-term bond issued in the previous period, b_{t+1}^{t-1} . Let $V(y_t, s_t, B_t)$ be the country's lifetime value function from period t on when the current endowment is y_t , the credit standing is s_t and the set of existing assets is B_t . The government makes decisions depending on the current state.

If the sovereign enters the current period with good credit standing ($s = 0$), then the country may choose whether to repay the maturing debt or to default on it. Its value function is given by

$$V(y_t, 0, B_t) = \max\{V^R, V^D\} \quad (1.2)$$

where V^R is the value function for repaying debt, and V^D is the value function for default. If the former option is chosen, the sovereign fully repays the maturing debt and then solves a standard consumption-saving problem by trading in short-term and long-term bonds. Moreover, the government enters the next period with good credit standing: $s_{t+1} = 0$. Thus, the value of repaying debt is given by

$$V^R(y_t, 0, B_t) = \max_{b_{t+1}^t, b_{t+2}^t} u(c_t) + \beta EV(y_{t+1}, 0, B_{t+1}) \quad (1.3)$$

$$s.t. \quad c_t = y_t - q_{t+1}^t(y_t, B_{t+1})b_{t+1}^t - q_{t+2}^t(y_t, B_{t+1})b_{t+2}^t + b_t^{t-1} + b_t^{t-2}$$

$$B_{t+1} = \{b_{t+1}^t, b_{t+1}^{t-1}, b_{t+2}^t\}$$

where b_{t+1}^t and b_{t+2}^t are the short-term and the long-term bonds traded in this period. $q_{t+1}^t(y_t, B_{t+1})$ and $q_{t+2}^t(y_t, B_{t+1})$ are prices corresponding to the short-term and the long-term bonds. B_{t+1} is the set of existing assets at the beginning of period $t + 1$.

If the government defaults in this period, we assume that it defaults on all existing debts. Following default, the economy suffers a proportional output loss γy_t and it is unable to borrow or save during default. The defaulting government then enters the subsequent period with bad credit standing and a quantity of defaulted debt, B_{t+1} . The value function for default, V^D , is thus given by

$$V^D(y_t, 0, B_t) = u(c_t) + \beta EV(y_{t+1}, 1, B_{t+1}) \quad (1.4)$$

$$s.t. \quad c_t = (1 - \gamma)y_t$$

$$B_{t+1} = (b_t^{t-1} + b_t^{t-2})(1 + r) + b_{t+1}^{t-1}$$

To keep the model tractable, we use the risk free interest rate r , instead of a bond-specific interest rates, to compute the present value of the defaulted debt, B_{t+1} . The expression $(b_t^{t-1} + b_t^{t-2})(1 + r)$ captures the opportunity cost to the creditors of not getting repaid on time.

When the sovereign starts a period with bad credit standing $s = 1$, it has to settle its defaulted debt through renegotiation with the creditors on debt reduction. Debt renegotiation determines an endogenous debt recovery rate $\alpha(y, B) \in [0, 1]$. If agreement is reached in the current period, we assume that the country immediately repays its reduced debt arrears. It is then able to enter the next period with an upgraded credit standing of $s_{t+1} = 0$ and no outstanding debt:

$$V(y_t, 1, B_t) = u((1 - \gamma)y_t + \alpha_t(y_t, B_t)B_t) + \beta EV(y_{t+1}, 0, 0) \quad (1.5)$$

If no agreement is reached in this period, the defaulted debts remain unsettled and interest payments accrue. Both parties continue debt renegotiation in the following period. In this case, the value function of the government is

$$V(y_t, 1, B_t) = u((1 - \gamma)y_t) + \beta EV(y_{t+1}, 1, (1 + r)B_t) \quad (1.6)$$

Default is preferable when $V^D \geq V^R$. Thus, we can characterize the default policy of a sovereign government in good standing by a default set $y^D(B) \subseteq Y$, defined as follows

$$y^D(B) = \{y \in Y : V^D(y, 0, B) \geq V^R(y, 0, B)\}$$

This default set specifies a set of endowments, for which default is optimal given the debt position B . Consequently, we can compute the sovereign's default probability in period $t+1$, θ_{t+1} , as the probability that tomorrow's endowment shock falls in the default set, given the government's debt position in period $t+1$, B_{t+1} and today's endowment realization, y_t .

$$\theta_{t+1}(y_t, B_{t+1}) = \int_{y^D(B_{t+1})} d\mu(y_{t+1}|y_t) \quad (1.7)$$

1.2.2 Debt Renegotiation Problem

The debt renegotiation problem is modeled as a Nash Bargaining game. Due to the static nature of Nash Bargaining, agreement is always reached in one period in equilibrium. Therefore this model does not generate delays in the renegotiation process.

We choose the threat point to be eternal autarky with a proportional output loss of γy each period for the economy, and no repayment for the debt-holders. We choose this threat point instead of one that allows future renegotiations because it gives the sovereign the lowest possible reservation value, providing an incentive for the government to agree to the highest possible debt recovery rate α . Note that a larger debt recovery rate implies a smaller capital loss suffered by the existing long-term debt holders, and thus a less severe debt dilution effect. Therefore, this choice of threat point stacks the model against the dilution effect.

The sovereign's expected value of staying in autarky from this period on is

given by

$$V^A(y_t) = E \sum_{i=t}^{\infty} \beta^{i-t} u((1-\gamma)y_i)$$

The country's surplus in the Nash bargaining game, Δ_B , is the difference between the expected value of accepting a debt recovery rate, α_t , and the expected value of staying in autarky forever¹³:

$$\Delta_B(y_t, B_t, \alpha_t) = u((1-\gamma)y_t + \alpha_t B_t) + \beta EV(y_{t+1}, 0, 0) - E \sum_{i=t}^{\infty} \beta^{i-t} u((1-\gamma)y_i) \quad (1.8)$$

The sovereign can gain from settling the defaulted debt in two ways: avoiding the proportional output loss once the defaulted debt has been settled, and regaining access to capital markets after debt restructuring, which provides the opportunity to smooth consumption in the future.

Regarding bond holders, we assume that they share the value of debt recovery pro rata. Hence all enjoy the same debt recovery rate in the debt restructuring process. This is consistent with our main assumption that all the debt holders rank equally. Consequently, bond holders' interests are aligned and we assume that they act as a representative bond holder in the renegotiation process. We define Δ_L to be the representative bond holder's surplus, which is the amount of recovered debt:

$$\Delta_L(y_t, B_t, \alpha_t) = -\alpha_t B_t \quad (1.9)$$

The amount of surplus each party extracts from debt renegotiation depends on its bargaining power. A bargaining power parameter ϕ captures the institutional arrangements governing the bargaining process in reduced form. The higher

¹³Alternatively, the surplus can be defined in terms of consumption compensation.

bargaining power one party has, the more surplus it can extract from this game. We denote the country's bargaining power as $\phi \in [0, 1]$, so that the representative bond holder's bargaining power is $(1 - \phi)$. The equilibrium debt recovery rate α^* is determined by the following Nash Bargaining problem

$$\alpha^*(y_t, B_t) = \operatorname{argmax} \Delta_B(y_t, B_t, \alpha_t)^\phi \Delta_L(y_t, B_t, \alpha_t)^{1-\phi} \quad (1.10)$$

$$s.t. \quad \Delta_B(y_t, B_t, \alpha_t) \geq 0$$

$$\Delta_L(y_t, B_t, \alpha_t) \geq 0$$

Note that the borrower's surplus is expressed using cardinal utility functions, so a change in units may lead to a change in the value of the equilibrium debt recovery rate. However, the equilibrium *properties* of the debt recovery rate are not affected by the units used, ensuring that the characteristics of dilution and its impact on maturity structure are independent of the particular units used in the utility functions.

1.2.3 International Investors' Problem

The last part of the model specifies the international investors' problem. An infinite number of risk neutral and competitive international investors are randomly chosen to trade with the sovereign government via one-period and two-period bonds¹⁴. The trading investor chooses the face values of bonds to maximize its expected profits from both short-term and long-term bonds, taking bond prices as

¹⁴It is important to assume that the investors are randomly chosen. In these circumstances, the sovereign trades with *different* investors in each period and hence the new investors do not internalize the dilution effect on the existing long-term bonds.

given

$$\max_{b_{t+1}^t, b_{t+2}^t} E(\pi_t^s + \pi_t^l) \quad (1.11)$$

where π_t^s and π_t^l are profits from bonds with short and long maturities, respectively.

More explicitly, the expected profit from the short-term bond $E(\pi_s)$ can be expressed

as

$$E(\pi_t^s) = \begin{cases} q_{t+1}^t b_{t+1}^t - \frac{b_{t+1}^t}{1+r} & \text{if } b_{t+1}^t \geq 0 \\ q_{t+1}^t b_{t+1}^t - \left[\frac{1-\theta_{t+1}}{1+r} + \frac{\theta_{t+1}E(\alpha_{t+2})}{1+r} \right] b_{t+1}^t & \text{if } b_{t+1}^t < 0 \end{cases} \quad (1.12)$$

When $b_{t+1}^t \geq 0$, the government saves by purchasing bonds. Since international investors commit to repaying debts, there is no default risk on investors' borrowing.

Therefore, the bond price is risk free. When $b_{t+1}^t < 0$, the government borrows by

issuing new bonds, and default and debt restructuring are possible future events. In

this case, the expected present value of repayment consists of two terms. The first coincides with repayment in the following period, while the second reflects default.

As noted in the government's problem, the probabilities of these events are $(1 - \theta_{t+1})$

and θ_{t+1} , respectively. In the case of default, debt renegotiations ensue in period

$t + 2$. α_{t+2} is then the debt recovery rate agreed upon in the same period, due to

the static Nash bargaining assumption.

Similarly, expected profits from the long-term bond $E(\pi_l)$ can be expressed

as:

$$E(\pi_t^l) = \begin{cases} q_{t+2}^t b_{t+2}^t - \frac{b_{t+2}^t}{(1+r)^2} & \text{if } b_{t+2}^t \geq 0 \\ q_{t+2}^t b_{t+2}^t - \frac{E(H)}{(1+r)^2} b_{t+2}^t & \text{if } b_{t+2}^t < 0 \end{cases} \quad (1.13)$$

where $E(H)$ is the expected proportion of repayment

$$H = \prod_{i=1}^2 (1 - \theta_{t+i}) + \theta_{t+1} \alpha_{t+2} + (1 - \theta_{t+1}) \theta_{t+2} \alpha_{t+3}$$

From the first order conditions of the above maximization problem, we obtain the price functions of short-term bonds as follows:

$$q_{t+1}^t = \begin{cases} \frac{1}{1+r} & \text{if } b_{t+1}^t \geq 0 \\ \frac{1-\theta_{t+1}}{1+r} + \frac{\theta_{t+1}E(\alpha_{t+2})}{1+r} & \text{if } b_{t+1}^t < 0 \end{cases} \quad (1.14)$$

By the same token, the price function of long-term bonds is given by:

$$q_{t+2}^t = \begin{cases} \frac{1}{(1+r)^2} & \text{if } b_{t+2}^t \geq 0 \\ \frac{\prod_{i=1}^2 (1-\theta_{t+i})}{(1+r)^2} + \frac{\theta_{t+1}E(\alpha_{t+2})}{(1+r)^2} + \frac{(1-\theta_{t+1})\theta_{t+2}E(\alpha_{t+3})}{(1+r)^2} & \text{if } b_{t+2}^t < 0 \end{cases} \quad (1.15)$$

Finally, note that default probabilities and debt recovery rates both lie in the interval $[0, 1]$, and hence the prices of both bonds range from 0 to the price of a risk free bond. Consequently, the interest rates of sovereign bonds are always higher than or equal to the risk free rate. We define the spreads of both bonds as the differences between the country interest rates and the risk free interest rate.

1.3 Equilibrium

The recursive equilibrium of the model is defined as follows:

Definition 1 A *recursive equilibrium* is defined by (i) a sequence of allocations of consumption $c(y, s, B)$, short-term bond holdings $b^1(y, B)$, long-term bond holdings $b^2(y, B)$, and a default set $y^D(B)$; (ii) a pricing function for short-term bonds $q^1(y, B')$, in which B' is the set of bonds existing in the subsequent period, and a pricing function for long-term bonds $q^2(y, B')$; (iii) a debt recovery rate $\alpha(y, B)$ such that:

1. Given the short-term and long-term bonds' pricing functions $q^1(y, B')$ and

$q^2(y, B')$ and the debt renegotiation outcome $\alpha(y, B)$, the country's asset holdings $b^1(y, B)$, $b^2(y, B)$, consumption $c(y, s, B)$, and default set $y^D(B)$ satisfy the government's optimization problem.

2. Given the bond pricing functions $q^1(y, B')$ and $q^2(y, B')$, the debt recovery rate $\alpha(y, B)$ solves the post-default debt renegotiation problem.
3. Given the renegotiation outcome $\alpha(y, B)$ and the country's optimal policy, the bond pricing functions $q^1(y, B')$ and $q^2(y, B')$ satisfy the investors' maximization problem.

1.3.1 Properties of the Recursive Equilibrium

We first characterize the equilibrium properties of the debt recovery value and debt recovery rate conditional on default.

Proposition 1 *For an endowment shock y and a defaulted debt stock $B < 0$, the equilibrium debt recovery value $\alpha^*(y, B)B$ satisfies*

$$\alpha^*(y, B)B = \begin{cases} \bar{B}(y) & \text{if } B \leq \bar{B}(y) \\ B & \text{if } B > \bar{B}(y) \end{cases}$$

where $\bar{B}(y)$ is a threshold debt recovery value as a function of the endowment y .

Proof. See Appendix A. ■

This proposition establishes the relation between the value of recovered debt and the level of defaulted debt. If the country defaults at a relatively low debt level ($B > \bar{B}(y)$)¹⁵, no debt is forgiven. If the country defaults at a sufficiently high

¹⁵Since the defaulted debt B has a negative sign by model construction, $B > \bar{B}(y)$ is equivalent to $|B| < |\bar{B}(y)|$.

debt level, it repays the threshold debt recovery value regardless of the amount of defaulted debt. Intuitively, such a threshold debt recovery value optimally splits the total surplus between the sovereign and the investors in the bargaining game. Thus, the government will always be willing to repay this debt recovery value as long as it owes more than that amount ($B \leq \bar{B}(y)$). If the country owes less than that amount, it fully repays its defaulted debt B . As for bond holders, they are no doubt satisfied with full repayment when the level of defaulted debt is relatively low. In the case of a high level of defaulted debt, the bond holders have to accept the threshold debt recovery value, since it is the largest repayment that the government would accept.

Proposition 1 also highlights the country's *incentive* to borrow more before crisis, if it already has a high debt level and default seems to be unavoidable. In these circumstances, more borrowing doesn't increase the amount of repayment in the post-default debt restructuring, but may serve to postpone the crisis, which can be beneficial to the country. A higher level of borrowing, however, incurs a larger capital loss to the earlier existing long-term lenders through the dilution effect.

The next corollary establishes the existence of the debt dilution effect.

Corollary 1 *For an endowment shock y and **defaulted** debt levels $B^1 \leq B^2 < 0$, the equilibrium debt recovery rate satisfies $\alpha^*(y, B^1) \leq \alpha^*(y, B^2)$.*

Proof. See Appendix A. ■

Corollary 1 follows immediately from Proposition 1. It demonstrates that the debt recovery rate is (weakly) decreasing in the amount of defaulted debt. So the

larger is the quantity of defaulted debt, the (weakly) smaller is the proportion the sovereign needs to repay in a debt restructuring process, leading to a more severe dilution of its earlier existing long-term debt. This result is obtained due to the *pari passu* ranking of the debt holders. Given the equilibrium debt recovery value characterized in Proposition 1, the larger is the number of investors that share this recovered debt repayment pro rata, the smaller is the proportion each investor finally gets.

As a crisis approaches, new investors perceive that they can effectively obtain a share of recovered debt that would otherwise go to the earlier existing investors in the debt restructuring. Therefore, the new lenders will not charge an interest rate as prohibitive as would be the case if the new debt were legally subordinated. This relatively low interest rate on new lending enables the sovereign to borrow more. Equipped with both the incentive and the ability to borrow more, the government may do so at the expense of the existing long-term creditors.

Even in tranquil times, when crisis is not on the horizon, the potential risk of debt dilution impacts long-term borrowing. Long-term debt may be diluted before maturing if bad shocks occur and the country issues new debt to postpone a crisis. Thus, the possibility of future debt dilution discourages long-term borrowing even in good times.

Although the risk of debt dilution exists in both booms and recessions, it is more severe when a crisis is unfolding. Since this risk impacts sovereign borrowing through the expected outcome of debt renegotiations, it has larger effects when default and debt restructuring become more likely.

Lastly, we present the relationship between the amount of maturing debt (given the total debt stock) and the probability of default in Proposition 2.

Proposition 2 *Given the equilibrium debt recovery rate $\alpha^*(y, B)$ and a debt stock with the face value of B^T , for **maturing** debt $B^{1M} \leq B^{2M} < 0$, if default is optimal for B^{2M} then default is also optimal for B^{1M} . Thus, given B^T , the default set for B^{1M} , $y^D(B^1)$, is (weakly) larger than the default set for B^{2M} , $y^D(B^2)$: $y^D(B^2) \subseteq y^D(B^1)$.*

Proof. See Appendix A. ■

For a given debt stock, our model predicts that the government is more likely to default as the quantity of maturing debt is higher. The intuition is that a larger amount of maturing debt forces the government to seek higher levels of new financing in the near term, at the cost of higher interest rates. When interest rates are sufficiently high, the country's total debt stock increases in the subsequent period, leading to a higher default probability.

In our model, the ratio of maturing debt to the total debt stock, $\frac{B^M}{B^T}$, is fully captured by the ratio of short-term to long-term debt. The higher is the ratio of short-term to long-term debt issuances, the larger is the maturing debt for a given debt stock in the subsequent period, and thus the larger is the probability of default. A higher default probability, in turn, affects borrowing costs and hence the choices of debt maturities by the government.

1.4 Quantitative Analysis

We solve the model numerically to evaluate the quantitative significance of the debt dilution effect.

1.4.1 Calibration

This model is calibrated to match various features of the Argentine economy. We define one period as a quarter.

We use a constant relative risk aversion (CRRA) utility function:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \quad (1.16)$$

where σ is the risk aversion coefficient. This parameter is set to 2, a commonly used value in real business cycle studies. We set the parameter of proportional output loss γ to be 2%, based on Sturzenegger's (2002) estimate, and the quarterly risk-free interest rate r is set to 1%, following Yue (2006).

We assume that the exogenous endowment stream follows a log-normal AR(1) process:

$$\log(y_t) = \log(\bar{y}) + \rho(\log(y_{t-1}) - \log(\bar{y})) + \varepsilon_t, \quad \varepsilon \sim N(0, \sigma_\varepsilon^2) \quad (1.17)$$

where \bar{y} is mean output and is normalized to 1. The correlation coefficient ρ and standard deviation σ_ε are computed from the quarterly real GDP data of Argentina from 1993 Q1 to 2005 Q4. GDP data are seasonally adjusted and are taken from the Ministry of Economy and Production (MECON). The data are detrended using a Hodrick-Prescott filter with a smoothing parameter of 1600. We then construct

a discrete Markov representation of the endowment process as a 21 state Markov chain, employing the quadrature procedure of Hussey and Tauchen (1991).

In the last part of the calibration, we set the time discount factor β and the sovereign government's bargaining power θ simultaneously to match the frequency of default and the average debt recovery rate of Argentina using the simulated method of moments (SMM). A higher discount factor decreases the frequency of default and increases the average debt recovery rate, since a more patient sovereign government cares more about its future consumption smoothing, and thus defaults less frequently to avoid default penalties, including temporary exclusion from the international capital markets and the proportional output loss following default. Conditional on default, a more patient government is willing to accept a higher debt recovery rate since regaining access to capital markets is greatly valued. A higher borrower's bargaining power raises the default frequency and lowers the average debt recovery rate. With a stronger position in debt renegotiation, the government is able to acquire more of the surplus and hence settles its defaulted debt with a lower debt recovery rate. Consequently, defaults are less costly to the economy, giving the government an incentive to default more frequently.

Argentina is a so-called "serial defaulter", defaulting 4 times from 1824 to 1999 as documented in Reinhart, Rogoff and Savastano (2003), and then defaulting a fifth time in late 2001, resulting in an average quarterly default frequency of 0.70%. The average debt recovery rate in the most recent Argentine debt restructuring was approximately 35%¹⁶. As shown in Table 1.3, our calibrated model generates a

¹⁶In September 2003, the Kirchner government made an offer to bondholders involving a write

default frequency of 0.72% and a debt recovery rate of 0.38%, which closely match the target data moments.

The time discount factor is found to be 0.99%, which is in the range of standard values used for this parameter in quantitative RBC models and *significantly higher* than those found in recent quantitative studies on sovereign default¹⁷. By incorporating the debt dilution effect, our model is able to match the observed default frequency without resorting to unusually high rates of time preference. Debt dilution amplifies default risk through its impact on the maturity structure: with a moderate default probability, the potential risk of dilution aggravates the shift towards short-term bonds, resulting in a larger ratio of maturing debt to the total debt stock in the subsequent period, which leads to a higher default probability. A higher default risk, in turn, makes the dilution problem a bigger concern. As the equilibrium default probability is higher than in the case without dilution, the default frequency is higher in the long run. This intuition is also confirmed by a comparison of the benchmark model with an alternative setup with default risk only, as presented in Section 5.

The bargaining power parameter is found to be 0.666, implying a strong position of Argentina in its recent post-default debt renegotiations, which is largely consistent with our observations. The calibration results are summarized in Table

down of 75% of the defaulted debt's value, non-payment of the past due interest, and the issuance of new low-interest bonds, equivalent to a 90% hair-cut rate in the net present value of the bonds. The Argentine Bondholders Committee made a counter-offer, accepting a write-off of 65% in the nominal value of the defaulted debt and requiring payment of past due interest. The final offer in 2005 was roughly identical to this counter-offer.

¹⁷For example, the time discount factor is found to be 0.74 in Yue (2006) and it is set to 0.8 in Aguiar and Gopinath (2006).

1.2. Table 1.3 compares the target data statistics and the model-generated statistics.

Table 1.2: Model Parameter Values

Parameter	Value	Sources
Coefficient of Risk Aversion	$\sigma = 2$	RBC literature
Output Loss in Default	$\gamma = 2\%$	Sturzenegger (2002)
Risk Free Interest Rate	$r = 1\%$	Yue (2006)
Std. Dev. of Endowment shock	$\sigma_\varepsilon = 0.047$	MECON
Autocorr. Coef. of Endowment	$\rho = 0.83$	MECON
Discount Factor	$\beta = 0.99$	Calibrated
Borrower's Bargaining Power	$\phi = 0.666$	Calibrated

Table 1.3: Target Statistics and Model Statistics

Target Statistics	Data	Model
Default Frequency	0.70%	0.72%
Debt Recovery Rate	35%	38%

1.4.2 Numerical Results on Equilibrium Properties

Figure 1.1 plots the equilibrium debt recovery rate as a function of the endowment state and the ratio of defaulted debt to mean output.

This figure confirms the results of Proposition 1 and Corollary 1. A positive endowment shock causes the government to repay more to settle its defaulted debt. Moreover, for a given endowment realization, the debt recovery rate is negatively related to the defaulted debt ratio. No debt is forgiven at sufficiently low levels of defaulted debt, and debt forgiveness increases as the level of defaulted debt increases.

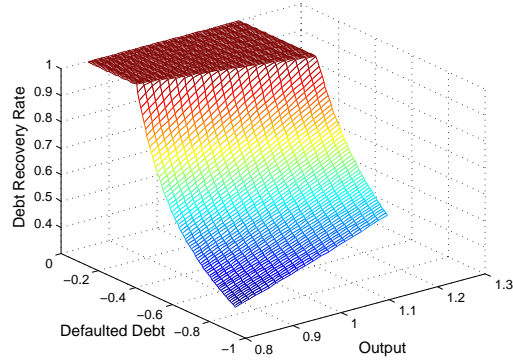


Figure 1.1: Equilibrium Debt Recovery Rate

Figure 1.2 presents default probabilities of the subsequent period given that today's output realization is in the best and the worst endowment states. Once today's endowment is given, the default probability in the next period depends on the debt position, including the total debt stock and the maturity structure. Figure 1.2 shows that the default probability increases as the levels of both maturing and outstanding debt become higher. In addition, default probabilities are (weakly) decreasing in output, implying that defaults are more likely to occur in bad states. This is because in a bad endowment state, it is more costly for a risk averse borrower to repay non-contingent debt, resulting in a higher incentive to default. This result is consistent with observations in the data¹⁸.

Table 1.4 illustrates the relationship between next period's default probability and the debt maturity structure by a numerical example. In this example, the endowment state is comparable to Argentina's GDP in 2001 Q3, when Argentina was near default, and the ratio of the total debt stock to mean output is fixed at 64%.

¹⁸See Arellano (2007) for a detailed discussion on countercyclical default risk.

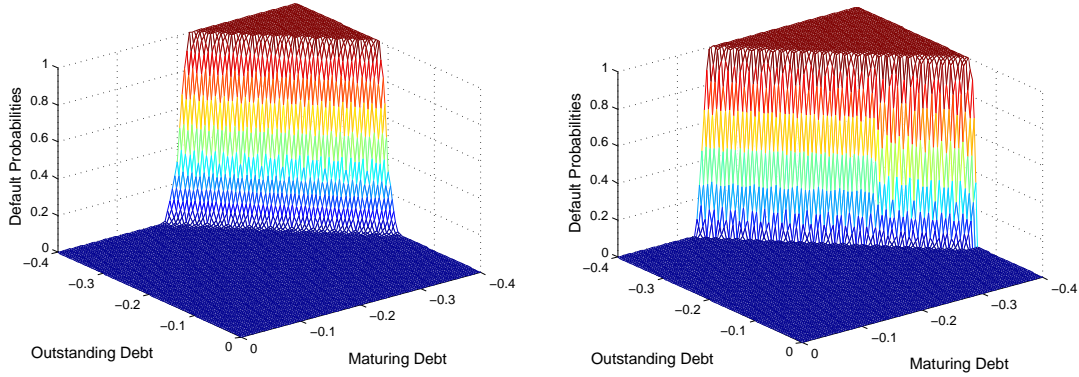


Figure 1.2: Equilibrium Default Probabilities in the Best (Left) and the Worst (Right) Endowment States

We report default probabilities with different ratios of maturing debt to total debt stock. The default probability is as low as 11.35% when the fraction of maturing debt is 35%, but jumps to 48.13% as the fraction of maturing debt reaches 55%, and finally increases to 55.61% when the fraction of maturing debt is 75%. This example conveys a clear message: maturity structure matters for the “*debt tolerance*” of the economy. As documented in Reinhart, Rogoff and Savastano (2003), many emerging market economies experience “extreme duress at overall debt levels that would seem quite manageable by the standards of the advanced industrial economies”. As an example, Argentina defaulted in late 2001 with an external debt-to-GDP ratio of 53.3%. By comparison, Table 1.4 shows that an external debt-to-GDP ratio of 64% may be sustainable (meaning that the default probability is low), as long as the fraction of maturing debt is sufficiently small. This result confirms the common wisdom that excessive reliance on short-term debt is a key factor behind emerging markets crises.

Table 1.4: Default Probabilities and Maturity Structures

Ratio of Maturing Debt to Total Debt	Default Probability
35%	11.35%
55%	48.13%
75%	55.61%

1.4.3 Simulation Results

We conduct 500 rounds of simulations, with 1000 periods per round, and then extract the last 50 periods in each round to study the limiting distribution of the model economy. Table 1.5 reports the business cycle statistics generated by the model and found in the data.

The external debt-to-GDP ratio is computed by dividing Argentina’s external debt by its GDP from 1993 to 2004 using the debt data from the World Bank. Seasonally adjusted consumption and current account data from 1993Q1 to 2005Q4 are taken from MECON. Bond spreads are annualized quarterly spreads on 3-year (short-term) and 9-year (long-term) foreign currency denominated bonds issued by the Argentine government, taken from Broner, Lorenzoni and Schmukler (2005).

Overall, the model accounts for key features of the Argentine economy, including volatile consumption, countercyclical current account and bond spreads, and the relationship between the average short-term and long-term spreads and their relative volatilities.

The model generates an average debt-to-GDP ratio of 21.16%, which is significantly higher than in previous quantitative models of sovereign debt¹⁹. This

¹⁹For example, Yue (2006) has an average debt service-to-GDP ratio of 9.69%. Arellano and

Table 1.5: Business Cycle Statistics from the Model and the Data

Statistics	Data	Model
Average External Debt-to-GDP	64%	21.16%
Consumption Std. Dev./Output Std. Dev.	1.03	1.08
Correlation between CA/Output and Output	-0.91	-0.33
Std. Dev. of CA/Output	5.40%	4.21%
Average Long-term Spreads	5.92%	5.3%
Average Short-term Spreads	5.16%	3.77%
Std. Dev. of S-T spreads /Std. Dev. of L-T spreads	1.45	2.02
Correlation between S-T Spreads and Output	-0.54	-0.14
Correlation between L-T Spreads and Output	-0.66	-0.21

improvement stems from the debt dilution effect in our model. The debt dilution problem affects the average level of debt through two channels. First, it amplifies default risk, resulting in a higher frequency of default, as argued in Section 4.1. With more frequent defaults, the sovereign can sustain a lower level of debt. However, given that both previous models and this model are calibrated to match the default frequency in the data, dilution's impact through the first channel does not make a difference in the debt-to-GDP ratio of our model relative to previous studies. Therefore, dilution affects the average debt-to-GDP ratio in this model mainly through a second channel: it makes capital inflows less procyclical by lowering borrowing costs in recessions and raising borrowing costs in booms, as compared to the case without dilution. In bad times, new creditors know that they can acquire a share of the debt recovery value at the expense of existing lenders in a post-default debt restructuring, and hence new lenders do not charge prohibitively high interest

Ramanarayanan (2006) get an average debt-to-GDP ratio of 7.3%. Aguiar and Gopinath (2006) obtain an average debt-to-GDP ratio of 20% by assuming a trend shock and a discount factor of 0.8.

rates, enabling a country near crisis to borrow relatively more than it would be able to without the dilution effect. In good times, however, dilution risk works in the opposite direction: creditors lending in long-term instruments may suffer dilution before the debt matures, so they require higher interest rates to cover the potential dilution risk, resulting in lower capital inflows to the country. Overall, since dilution risk is more serious during recessions, its effect of raising debt levels before crisis outweighs that of lowering borrowing in tranquil times. As a consequence, the second channel enables the model to support a higher average debt-to-GDP ratio than those found in previous studies. In section 5, when we compare setups with different degrees of dilution using the same set of parameter values, dilution works through both channels and we find that, all else equal, a higher degree of dilution leads to a lower debt level.

The simulation also matches the consumption volatility observed in the data at a business cycle frequency. It is a common feature of emerging economies that consumption is more volatile than output (Neumeyer and Perri (2004)). Aguiar and Gopinath (2006) and Yue (2006) obtain this result by assuming a stochastic trend for the output process, implying that output shocks are permanent. In contrast, in this model, output shocks are transitory and the volatile consumption stream results from both default risk and dilution risk. The possibility of default tightens the implicit borrowing constraint in recessions and hence limits the government's ability to smooth consumption in bad states. Thus, default risk contributes to the volatility of consumption. Debt dilution risk, as we have argued above, works through two channels: increasing default frequency and making capital inflows less procyclical.

The first effect amplifies default risk, contributing to consumption volatility, while the second effect helps to smooth consumption. Overall, forces contributing to consumption volatility dominate quantitatively, leading to more volatile consumption than output.

This model matches the observed countercyclical behavior of the current account, although the model correlation is less negative than in the data. The argument is very similar to that for consumption volatility. Default risk makes the current account more countercyclical by tightening the implicit borrowing constraint in bad states. The debt dilution effect, on the one hand, makes capital inflows less procyclical and hence the current account less countercyclical; on the other hand, it increases the frequency of default, causing more frequent current account reversals following defaults. Again, the effects contributing to current account countercyclicity dominate, so that we get a countercyclical current account in the long run.

The model produces higher average long-term spreads than short-term spreads, which is in line with data. Long-term spreads are generally higher since they incorporate default risks both in the near term and in the future, as well as the potential dilution risk, reflected in the expected debt recovery rate conditional on default. Short-term spreads, in contrast, reflect only default risk and the expected debt recovery rate in the near term. As a result, this model does a better job in matching average long-term spreads than it does in matching short-term spreads. But it should be taken into consideration that we assume risk neutral investors, which lowers average bond spreads for both short and long maturities.

The model captures the relative volatility of short-term and long-term spreads

observed in the data. Short-term spreads are more volatile than long-term spreads, because short-term spreads are generally lower than long-term spreads, but near crises they jump more than long-term spreads do, mainly due to a shift toward short-term bonds in the maturity structure. Such a shift exacerbates the increase in short-term spreads, while dampening the increase in long-term spreads.

Finally, the model generates countercyclical bond spreads. Both short-term and long-term spreads are higher in pre-crisis episodes due to the higher default risk and debt dilution risk. However, this model misses the magnitudes of both correlations.

We then proceed to study the bond maturity structures in booms and recessions. Figure 1.3 displays the time series dynamics of both short-term and long-term bond issuances prior to a default (default occurs in period 21). In plotting this figure, we take the *average* over all default episodes in the simulated data of the bond issuances from 20 to 1 period before default. Therefore, this figure reflects the *general* behaviors of bond issuances before default, not just a special case.

It is clear from Figure 1.3 that the government mainly borrows long-term (i.e., long-term assets are negative) and occasionally saves short-term (i.e., short-term assets are positive) during normal times (from period 1 to 17). In the pre-crisis episode (period 18 to 20), both short-term and long-term borrowing increase dramatically, and the short-term debt rises more sharply than long-term debt, indicating a shift toward short-term bonds before default.

To see this shift toward short-term bonds more clearly, we classify all non-default simulation periods into two groups: tranquil periods and turbulent periods.

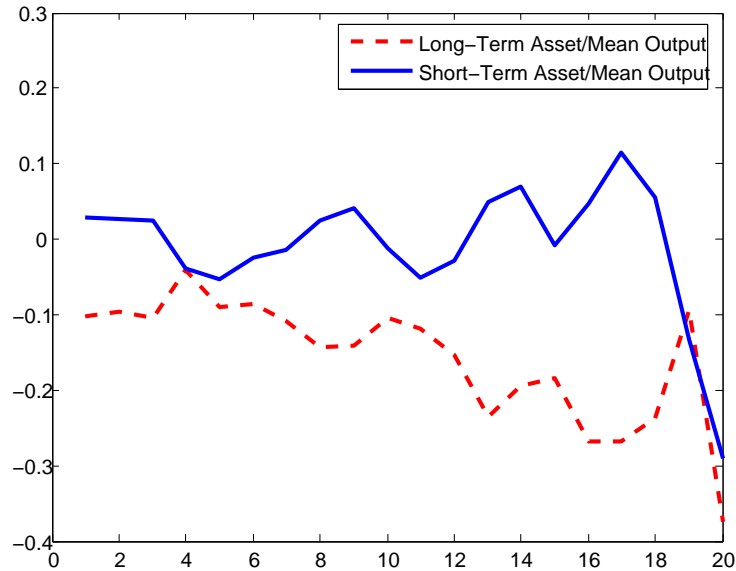


Figure 1.3: Time Series of Bond Issuances Prior to Default

We then compare the maturity structures in each group. Turbulent periods are defined as the four periods before each default event, and all the remaining non-default periods are defined as tranquil²⁰. Table 1.6 presents the average levels of short-term and long-term debt relative to mean output, as well as the ratios of short-term to long-term debt in tranquil and turbulent periods.

In tranquil times, the government generally has savings in short-term, which can be interpreted as accumulation of foreign reserves, and mainly borrows long-term. In turbulent times, the ratios of short-term and long-term debt to mean output increase to 6.3% and 24.6%, respectively. Such an increase in the debt-to-GDP ratio in the pre-crisis episode is consistent with recent experience in Argentina:

²⁰We have done a robustness check by redefining turbulent periods as two periods or six periods before each default event, and the increases in the ratios of average short-term debt to average long-term debt from tranquil to turbulent periods are significant in both cases. We also tried to define turbulent periods as those in which bond spreads are high, and we got similar results.

Table 1.6: Short-Term and Long-Term Debt in Tranquil and Turbulent Times

	Tranquil Periods	Turbulent Periods
(Ave. S-T Debt(-) or Saving(+))/Mean Output	0.8%	-6.3%
(Ave. L-T Debt(-) or Saving(+))/Mean Output	-10.6%	-24.6%
Average S-T Debt/Average L-T Debt	-7.5%	25.6%

its external debt-to-GDP ratio was 53.3% at the time of default in late 2001, which is well above the average external debt-to-GDP ratio of 38.54% from 1993 to 2001. In addition, the average ratio of short-term to long-term debt in the simulated data jumps from -7.5% in tranquil times to 25.6% in turbulent times, indicating an increase of 33.1 percentage points in the ratio of short-term to long-term debt. Thus, this model generates a striking shift in maturity structures toward short-term debt before crisis episodes, as is observed in the data for Argentina and other countries. Such a shift is an outcome of both default risk and the debt dilution effect. What fraction of this shift is due to default risk and how much is due to debt dilution? In the next section, we explore the role of the debt dilution effect in driving the shift in maturity structure, as well as its impacts on other business cycle variables.

1.5 Debt Dilution, Maturity Structure and Long-Run Effects

In this section, we first study the effects of debt dilution on the maturity structure by comparing the benchmark model with two alternative setups with different degrees of dilution. In one setup, we completely eliminate the debt dilution effect,

leaving only default risk, by assuming a “first-in-time” seniority structure²¹. That is, earlier lenders are senior and are repaid first in a post-default debt restructuring. Later lenders, being junior, only receive repayments after the senior lenders have been fully repaid. By assuming such a “first-in-time” seniority structure, existing long-term debt will not be diluted by new debt issuances, since new creditors are unable to obtain earlier lenders’ debt recovery value. In the other setup, we assume the exactly opposite seniority structure: “last-in-time”. That is, later lenders are senior and are repaid before the earlier ones. In this case, we have the maximum degree of debt dilution, since the new creditors get the maximum amount of the earlier lenders’ debt recovery value. Thus, in terms of the degree of debt dilution, our benchmark model is an intermediate case, with the “first-in-time” and “last-in-time” setups being two extreme cases.

To solve the alternative model setups, we change the key assumption in the benchmark renegotiation problem: bondholders no longer get the same debt recovery rate. Instead, the debt recovery rate for each bondholder depends on her seniority. Bond price functions change accordingly to accommodate the relative seniorities of bond issuances. We solve the alternative setups using the same parameter values as in the benchmark model, and conduct the same simulation exercises as in the benchmark case. Again, we separate all simulation periods into tranquil periods and turbulent ones by defining turbulent periods as the four periods before each default event and tranquil periods as all remaining non-default periods. Table 1.7

²¹Bolton and Skeel (2004) propose this seniority structure in order to make debt restructuring more orderly. We follow them in using the term “first-in-time”.

presents the ratio of short-term and long-term debt to mean output, as well as the ratio of short-term to long-term debt in both tranquil and turbulent times in all three setups.

Table 1.7: Comparison of Setups with Different Degree of Debt Dilution

	First-in-time (No dilution)	Benchmark (Moderate dilution)	Last-in-time (Max. dilution)
Ave. S-T Debt in Tranquil Periods	-0.18%	0.80%	0.20%
Ave. S-T Debt in Turbulent Periods	-0.46%	-6.3%	-8.44%
Ave. L-T Debt in Tranquil Periods	-12.24%	-10.6%	-5.07%
Ave. L-T Debt in Turbulent Periods	-26.45%	-24.6%	-18.22%
Ave. S-T/Ave. L-T Debt in Tranquil Periods	1.46%	-7.5%	-3.92%
Ave. S-T/Ave. L-T Debt in Turbulent Periods	1.74%	25.6%	46.33%
Changes in Ave. S-T/Ave. L-T Debt	0.28%	33.1%	50.35%

The upper panel of Table 1.7 shows the average ratios of short-term and long-term debt to mean output in booms and recessions. In turbulent times, the average ratio of short-term debt to mean output increases from 0.46% in the “first-in-time” (no dilution) case to 6.3% in the benchmark case (moderate dilution), and increases even further to 8.44% in the “last-in-time” (maximum dilution) case. Correspondingly, long-term borrowing during turbulent periods decreases as the degree of debt dilution increases, from 26.45% in the “first-in-time” case to 24.6% in the benchmark model and finally to 18.22% in the “last-in-time” setup.

To show the shift in maturity structures more clearly, we present the average ratio of short-term to long-term debt in both tranquil and turbulent times in the lower panel of Table 1.7. In the “first-in-time” case, the shift toward short-term debt in turbulent times is entirely driven by default risk and the increase is only

0.28 percentage points. In the benchmark case, the shift in maturity structure is driven by both default risk and a moderate degree of dilution, and the ratio of short-term to long-term debt rises by 33.1 percentage points in turbulent periods. This comparison shows that the debt dilution effect plays a significantly more important role than default risk in driving the maturity structure toward short-term debt before crisis. In the “last-in-time” case, the increase in the short-term/long-term debt ratio is even larger than in the benchmark case, again confirming our argument that debt dilution can significantly shift the maturity structure towards short-term debt before crises.

The table also shows that the possibility of debt dilution affects the government’s access to long-term debt even in tranquil times. Without debt dilution, the government can sustain a ratio of long-term debt to mean output of 12.24% in booms, while this ratio decreases to 10.6% with a moderate degree of dilution, and drops to only 5.07% with the maximum dilution. As argued earlier, the possibility of future dilution drives up long-term spreads and hence discourages long-term borrowing even in tranquil times.

Table 1.8: Long-run Behaviors of All Setups

	First-in-time	Benchmark	Last-in-time
Default Frequency	0.18%	0.72%	2.92%
Average Debt/Output Ratio	24.93%	21.16%	12.53%
Std. Dev. of Consumption/Std. Dev. of Output	0.87	1.03	1.49
Corr. between CA and Output	-0.51	-0.33	-0.15

Finally, in Table 1.8 we compare the long-run probability of default, the av-

verage debt-to-output ratio and the correlation between the current account and output in all three setups. In the long run, a larger degree of debt dilution leads to an increase in default frequency, from 0.18% in the “first-in-time” case to 2.92% in the “last-in-time” setup. Moreover, the “first-in-time” setup supports the highest average debt-to-output ratio of 24.93%²², while the “last-in-time” setup can only sustain an average debt-to-output ratio of 12.53%. Here debt dilution affects the equilibrium debt-to-output ratio through two channels: by increasing default frequency and by making capital inflows less procyclical, with the first effect decreasing debt levels and the second increasing them, as shown in Section 4.3. The overall effect is a lower average debt-to-output ratio with larger degree of dilution. By the same token, the government is able to achieve smoother consumption streams in setups with a smaller dilution effect.

Lastly, the current account is always countercyclical, even with the maximum degree of dilution, confirming our argument in Section 4.3 in explaining the countercyclicality of the current account. However, we find that a higher degree of debt dilution effect makes the current account less countercyclical, or the capital inflows less procyclical. This result suggests that debt dilution can be desirable to some degree, especially from the perspective of a distressed economy. Debt dilution al-

²²Recall that in the “first-in-time” case, the dilution effect is absent and only default risk is in play. In this sense, the “first-in-time” set-up is similar to standard sovereign default models, such as Arellano (2007) and Yue (2006). However, the debt-to-GDP ratio reported here is not directly comparable to those in the standard models because the high debt ratio here is mainly driven by the low default frequency (0.18% quarterly), much lower than those in the standard models (e.g., 0.7% quarterly in Yue (2006) and 3% annually in Arellano (2007)). Note that in this paper, we calibrate the *benchmark* model, not the “first-in-time” set-up, to match the quarterly default frequency of 0.7%. If we were to re-calibrate the “first-in-time” case to match the quarterly default frequency, we should obtain a much lower debt-to-GDP ratio similar to the standard results.

lows a country near crisis to have more access to new financing, although mainly in short-term form. The country chooses optimally whether to borrow more in short-term or to borrow less in long-term. By choosing the former, the government may be able to postpone the crisis for some time and then default on a larger amount of debt; by choosing the latter, it might need to default immediately but with a lower level of defaulted debt. If borrowing more in short-term is the optimal choice, the sovereign government is willing to pay the price associated with dilution, namely a higher default probability and larger amount of debt arrears conditional on default. From this point of view, debt dilution is very similar to default in that both can be beneficial for a country in distress, even though *ex ante* they may seem inefficient and costly.

1.5.1 Sensitivity Analysis

Table 1.9 presents a sensitivity analysis of the benchmark model, considering different values for the discount factor β and the sovereign's bargaining power ϕ .

The upper panel of Table 1.9 reports the sensitivities of default frequency, the debt recovery rate and the mean debt-to-GDP ratio to different values of the time discount factor, holding all other parameters fixed at their benchmark values. When the country is more patient, it defaults less frequently since it cares more about intertemporal consumption smoothing. Due to a reduced default frequency, a more patient country can sustain a higher mean debt-to-GDP ratio. Also, with a higher discount factor, the equilibrium debt recovery rate is higher, because regaining access to international capital markets is valued more by the government.

Table 1.9: Sensitivity Analysis for the Benchmark Model

	Default Frequency	Debt Recovery Rate	Mean Debt-to-GDP
Time Discount Factor			
$\beta = 0.99$	0.72%	38.54%	21.16%
$\beta = 0.98$	8.12%	28.77%	17.16%
$\beta = 0.95$	17.17%	16.15%	15.44%
$\beta = 0.90$	20.91%	9.78%	14.25%
$\beta = 0.85$	24.13%	7.17 %	13.74%
Borrower's Bargaining Power			
$\phi = 0.46$	0%	N.A.(There is no default)	44.57%
$\phi = 0.666$	0.72%	38.54%	21.16%
$\phi = 0.86$	3.07%	21.95%	7.5%

The lower panel shows the effect of changing the borrower's bargaining power. Stronger bargaining power of the government results in a lower debt recovery rate. Consequently, defaults are not as costly and the government tends to default more frequently. Both a higher default frequency and a lower debt recovery rate shift the bond price functions downward, making government borrowing more costly. As a result, the mean debt-to-GDP ratio decreases as the borrower's bargaining power increases.

1.6 Conclusion

The absence of an explicit seniority structure in sovereign debt has been widely recognized as an important inefficiency in post-default debt restructurings in recent years. One consequence of this inefficiency is the emergence of the debt dilution problem. Debt dilution occurs when a country issues new debt that does not improve the country's repayment ability proportionally. This newly-issued debt may reduce

the amount that can be recovered by the existing long-term debt-holders in a post-default debt restructuring, and thus “dilutes” existing debt. This paper incorporates the debt dilution problem into a dynamic model of sovereign default and renegotiation to explain a well-documented phenomenon: emerging market economies rely more on short-term debt as they are approaching crisis. We find that debt dilution is possible in both booms and recessions, but is more severe as a country’s financial situation worsens and a potential crisis unfolds. The model also predicts that the maturity structure plays an important role in determining default probability. For a given debt stock, the country is more likely to default as the fraction of maturing debt increases.

We explore the quantitative predictions of the model and find that by introducing the dilution effect, our model is able to generate a default frequency and an average debt-to-GDP ratio comparable to those observed in the data, without assuming unusually high rates of time preference for the sovereign borrower. In contrast, previous studies of sovereign debt rely on unusually low discount factors to generate reasonable default frequencies and to support high debt ratios. Also, the dilution effect helps the model generate consumption volatility and countercyclical current account without trend shocks. In addition, the model generates a large shift to short-term debt before crises; the fraction of this shift due to the “debt dilution” effect is significantly larger than the fraction due to default risk.

By comparing setups with different degrees of dilution, we find that the dilution effect increases the equilibrium default frequency and decreases the average debt-to-GDP ratio, suggesting that debt dilution can be welfare-reducing in the long

run. However, this comparison also predicts that the debt dilution effect makes the current account less countercyclical, or capital inflows less procyclical, which is beneficial to the borrowing country. These results shed light on a potential time-inconsistency problem in designing the optimal seniority structure. Ex ante, we may have incentive to eliminate debt dilution in order to lower the frequency of default and increase the sustainable debt level. But ex post, when a country's financial situation worsens and it is in urgent need of new financing, debt dilution can be preferable, since it lowers borrowing costs, enabling a country near default to borrow more and hence postpone crisis. Thus, an interesting direction for future research is to investigate how to overcome such a time-inconsistency problem by designing the debt seniority structure properly.

Chapter 2

“Beneficial” Delays in Debt Restructuring Negotiations

2.1 Introduction

Many sovereign debt crises were followed by prolonged debt renegotiation processes. Table 2.1 presents several episodes of sovereign debt restructurings (including those under the Brady Plan and some recent cases), and reports the length of delay experienced in each case as well as the haircut rate agreed upon in the final outcome.

Table 2.1: Cases of Sovereign Debt Restructurings

Countries	Episode of Restructuring Negotiation	Delay Length (Months)	Hair Cut (Per cent)
18 countries ¹	08/1982 - 05/1994	141	30 - 35
Argentina	12/2001 - 04/2005	40	65
Ecuador	08/1999 - 08/2000	12	6.5
Pakistan	02/1999 - 12/1999	10	30
Russia	11/1998 - 07/2000	20	69

Sources: Cline (1995), Roubini and Setser (2004) and Sturzenegger and Zettelmeyer (2005).

1. Mexico, Argentina, Brazil, Bulgaria, Costa Rica, the Dominican Republic, Ecuador, Ivory Coast, Jordan, Nigeria, Panama, Peru, the Philippines, Poland, Russia, Uruguay, Venezuela and Vietnam.

Such lengthy debt renegotiations are widely regarded as evidences of inefficiencies¹. This paper argues, however, that delays in a debt renegotiation process may be mutually beneficial for the defaulting country and the debt holders. As

¹See Roubini and Setser (2004) for a thorough discussion on the inefficiencies in the existing debt restructuring procedures and the proposed institutional reforms.

documented by Sturzenegger (2002), among others, defaults are usually associated with large output collapses, and few resources are available for repayment following a default. If a debt settlement occurs immediately after the default, then the “cake” to be allocated is quite small. However, if the negotiating parties wait until the economy recovers somewhat to settle the defaulted debt, they may receive higher payoffs by dividing a larger “cake”. Delays that arise due to this consideration can therefore be beneficial.

Our argument is based on two important features of the real world: the output stream is stochastic and no state-contingent debt repayment schedules are available. In the case of a deterministic output process, the defaulting country and the debt holders know exactly what the future “cakes” are, and hence they can reach an agreement right after the default. Similarly, when state-contingent repayment schedules are available, delays are also unnecessary. In the real world, however, the output process after a default is clearly uncertain: although people may expect the economy to recover from the crisis, they do not know exactly what the future output stream will be. Without resort to state-contingent repayment schedules, the negotiating parties may prefer to wait for more information on the economic recovery to be revealed, and then decide if the current state is good enough to settle the defaulted debt.

The main purpose of this paper is to show, quantitatively, that the “waiting-for-a-larger-cake” consideration can explain the delays experienced by a defaulting country during the renegotiation process, and that these delays can help explain the observed volatility of sovereign bond spreads. To this end, the paper constructs

a dynamic model of sovereign default in which renegotiation is modeled using a stochastic bargaining game based on Merlo and Wilson’s (1995) framework. The Merlo-Wilson model extends the Rubinstein (1982) alternating offer game by allowing for a stochastic environment and an option to “pass” by the proposer. These two extensions lie at the heart of our argument and make the Merlo-Wilson framework a good one to study the effects of “beneficial” delays. In this paper, the renegotiation process is modeled as a two-player stochastic bargaining game, the simplest case in the Merlo-Wilson framework. The game is played as follows: in each period, a proposer is randomly selected with a constant probability. Based on the current output state and the defaulted debt level, the proposer can either propose or pass. If she chooses to propose, then the other player decides whether to accept or to reject. If the proposal is accepted, then the government repays immediately according to the agreement; otherwise, both players enter the next period and the game repeats. If the proposer chooses to “pass”, then again the players enter the next period and the game goes on. This is a game with *complete* information, but delays may occur in equilibrium whenever the proposer perceives that a better agreement can be achieved by waiting. Intuitively, given the level of the defaulted debt, if the current output state is good enough, an agreement can be reached; otherwise, the proposer chooses to “pass” and thus delays the renegotiation process.

We embed the above debt renegotiation process in a dynamic sovereign debt model featuring endogenous defaults. This part of the model builds on the recent quantitative analysis of sovereign default, such as Aguiar and Gopinath (2006), Arelano (2007) and Yue (2006). In our model, a risk-averse sovereign borrows from a

group of risk-neutral, competitive international investors using non-contingent, one-period bonds. Facing a stochastic output stream, the sovereign has the option to default, which results in exclusion from international capital markets and a proportional output loss. The country can re-enter the credit market through renegotiating with the creditors over debt reduction and repayment. The debt recovery rate is endogenously determined in the stochastic bargaining game described above. In equilibrium, sovereign bonds are priced to compensate creditors for the default risk, the expected haircut rate and the cost of delays in the renegotiation.

We first characterize the equilibrium “propose/pass” choices made by each player for all states of output and defaulted debt levels. We find that the identity of the proposer is irrelevant to the timing of an agreement decision; instead, it only affects the allocation of the “cake.” For a given defaulted debt level, there is a threshold output level, under which “pass” is always chosen, and above which an optimal debt recovery rate is proposed and accepted. Consequently, given the defaulted debt level, the expected delay length is essentially determined by the output process.

To investigate the quantitative implications of this model, we calibrate it to the Argentine economy. Numerical analysis shows that the model can generate an average delay length comparable to that experienced by Argentina in its most recent debt restructuring. Furthermore, the model successfully accounts for the observed volatility of bond spreads without assuming risk-averse creditors. In our model, the bond spreads capture not only the default probability and the expected hair cut rate, but also the cost of waiting for an *uncertain* period of time. The existence of

delays adds more uncertainty to the renegotiation outcome, and hence contributes to a larger volatility of bond spreads. In addition, the model simulation accounts for some key features of Argentina's business cycles, such as the countercyclical bond spreads.

Finally, we examine how equilibrium delay length is affected by the properties of the output profile. We find that when the output shocks are sufficiently persistent (i.e., the autocorrelation coefficient is larger than 0.3), the average delay length increases in the persistence of shocks. More persistent shocks lead to a slower economic recovery following a collapse, resulting in a longer equilibrium delay length. When the output shocks are negatively correlated (with a large absolute value of the autocorrelation coefficient), renegotiation agreements are never achieved after defaults. Although it is least costly for the sovereign to settle the defaulted debt in a good output state, doing so does not bring much benefit in terms of consumption smoothing. With negatively correlated output realizations, a good output state today implies a bad one tomorrow. Even if the sovereign settles the debt today and regains access to the capital markets, it cannot borrow much to smooth consumption tomorrow due to the high default risk associated with a bad output state. Consequently, the sovereign will find it optimal to stay in autarky forever after default. We then explore the effects of output volatility: as volatility increases, the equilibrium delay length tends to be longer, because a more volatile output stream implies a smaller probability that a good enough state will be realized. As a result, the players have to wait longer for a sufficiently large "cake".

Despite widespread concerns about the prolonged and costly sovereign debt

restructurings, the existing literature contains few models that can quantitatively explain the observed renegotiation delays. Yue (2006), for example, models the debt renegotiation process using a Nash bargaining game, which excludes delays in equilibrium due to the game's static nature. Arraiz (2006) combines an empirically estimated function of debt settlement probability with a sovereign default model and produces long delays in equilibrium. But those delays mainly result from the empirical estimation of the settlement probabilities. Our paper, on the contrary, obtains sufficiently long delays from an explicit model of the renegotiation process.

In a recent paper, Pitchford and Wright (2007) studies renegotiation delays arising from two *inefficiencies*: creditor holdout and free-riding on negotiation effort. They model the multi-creditor renegotiation process as a series of bilateral bargaining games. The resulting average delay length is 7 years for a set of countries, close to the average delay length of 5 years generated by the present paper for Argentina. Although Pitchford and Wright (2007) models renegotiations explicitly, they do not embed the full-fledged game into the sovereign borrowing framework. Instead, the renegotiation outcomes are taken as parameters when the sovereign makes default decisions. As a result, their model is unable to capture the interactions between defaults and debt renegotiations. Our model improves in this aspect by introducing a dynamic bargaining game into a sovereign default model. Furthermore, delays in Pitchford and Wright (2007) stem from coordination failures, and hence their results may depend on the number of creditors used in the quantitative analysis. Delays in this paper mainly depend on the output profile, so the number of creditors is irrelevant.

All the above papers assume explicitly or implicitly that after default, the country has no access to outside financing and the length of exclusion depends on the renegotiation process. Kovrijnykh and Szentes (2007), however, assumes that even if the country is excluded from competitive capital markets, the incumbent creditor, as a monopolist, may continue to invest (though at an inefficiently low level) and the length of exclusion is controlled by the lender. Kovrijnykh and Szentes (2007) shares the same result as this paper: the defaulting country regains access to the competitive capital markets only after good shocks. But their paper is based on a different argument: good productivity shocks make the defaulting country's project profitable *conditional* on efficient investments. The monopolistic lender can then extract more surplus by allowing the defaulting countries to reaccess competitive markets (i.e., reaccess efficient investments) than by retaining the country under her monopolistic power (i.e., with underinvestments). They show that this argument can explain the exclusion from and the returning to competitive capital markets, but the length of exclusion is not quantitatively examined. Our paper, as explained earlier, is based on the “waiting-for-a-larger-cake” argument and we show quantitatively that this argument can generate sufficiently long delays.

The remainder of this paper is organized as follows. We describe the model in Section 2. Section 3 defines the model's equilibrium and characterizes its properties. Model calibration and simulation results are provided in Section 4. In Section 5, we explore how the output process influences the length of renegotiation delays. Effects of other parameters are also reported in this section. Finally we offer concluding remarks in Section 6. The proposition proof and the computation algorithm are in

Appendix B.

2.2 The Model

This model expands Eaton and Gersovitz's (1981) framework on sovereign default by adding a post-default renegotiation process, which is modeled as a stochastic bargaining game. The game structure is built on Merlo and Wilson (1995).

The model features two types of agents: a small open economy and an infinite number of international investors. The economy faces a stochastic stream of a non-storable consumption good y_t , drawn from a compact set Y . We assume that the exogenous endowment stream follows a log-normal AR(1) process:

$$\log(y_t) = \log(\bar{y}) + \rho(\log(y_{t-1}) - \log(\bar{y})) + \varepsilon_t, \quad \varepsilon \sim N(0, \sigma_\varepsilon^2) \quad (2.1)$$

where \bar{y} is the mean output. In this economy, a risk-averse government maximizes the expected lifetime utility of a representative domestic resident. The preferences of the sovereign government are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (2.2)$$

where $0 < \beta < 1$ is the discount factor, and c_t denotes consumption in period t . The period utility function $u(c_t)$ is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions.

The government can trade with the international investors via non-contingent one-period zero-coupon bonds. The face value of a discount bond issued in period t is denoted as b_{t+1} , specifying the amount to be repaid in period $t + 1$. If b_{t+1} is positive, it reflects the government's saving; otherwise, it is a debt. We denote the

price of a bond issued in period t as q_{t+1} . This price is determined endogenously in equilibrium.

When debt matures, the sovereign can either repay the maturing debt or default. The former option enables the government to trade in the current period and hence to smooth consumption, while the latter results in two penalties: an exogenous proportional output loss² and exclusion from international capital markets during default. Following default, the borrower initiates a renegotiation on debt reduction with its debt holders. This post-default renegotiation is a dynamic bargaining game that determines when an agreement can be reached and how much the sovereign needs to repay in order to reaccess the capital markets. Once an agreement has been reached and the reduced amount of debt has been repaid in full, the sovereign's reputation is restored and it returns to the international capital markets.

International investors, on the contrary, always commit to repaying their debts. They are risk neutral and behave competitively in international capital markets. They can borrow as needed in capital markets at the world risk-free interest rate r , which we assume to be constant and unaffected by the size of investors' borrowing and lending. Investors have perfect information on the country's asset holdings and current endowment realizations.

²This assumption is empirically relevant. Sturzenegger (2002), for example, estimates the post-default output loss using a panel of 100 countries from 1974 to 1999, and finds that "the experience of the 80s tends to suggest an accumulated 4% drop in output over the immediately following four years." To keep the model tractable, we assume that this output loss is exogenous.

2.2.1 Sovereign Government's Problem

At the beginning of period t , an endowment shock y_t is realized, and the country inherits an asset position b_t and a credit standing s_t from the last period. The credit standing can be either good ($s_t = 0$), indicating that the sovereign is current on its debt service, or bad ($s_t = 1$), meaning that the economy is in default.

If the current period begins with a good credit standing ($s_t = 0$), then the country either repays the maturing debt or defaults on it. Let $V(y_t, s_t, b_t)$ be the country's lifetime value function from period t on with the current endowment y_t , credit standing s_t and existing asset position b_t . The value function $V(y_t, 0, b_t)$ is then given by

$$V(y_t, 0, b_t) = \max\{V^R, V^D\} \quad (2.3)$$

where V^R is the value function for repaying debt, and V^D the one for default. If the sovereign fully repays the maturing debt, then it can borrow or save by trading in one-period bonds. Moreover, the government enters the next period with a good credit standing: $s_{t+1} = 0$. Thus, the value of repaying debt is given by

$$V^R(y_t, 0, b_t) = \max_{b_{t+1}} u(c_t) + \beta EV(y_{t+1}, 0, b_{t+1}) \quad (2.4)$$

$$s.t. \quad c_t = y_t - q_{t+1}(y_t, b_{t+1})b_{t+1} + b_t$$

where b_{t+1} is the one-period bond traded in this period, and $q_{t+1}(y_t, b_{t+1})$ is the corresponding price.

If default is chosen, a proportional output loss γy_t and exclusion from the capital markets ensue. The defaulting government then enters the subsequent period

with a bad credit standing and a defaulted debt level, $b_{t+1} = (1 + r)b_t$. To keep the model tractable, we use the risk free interest rate r , instead of a bond-specific interest rate, to compute the present value of the defaulted debt. The term $(1 + r)b_t$ captures the opportunity cost to the creditor of not getting repaid on time. The value function for default, V^D , is given by

$$V^D(y_t, 0, b_t) = u(c_t) + \beta EV(y_{t+1}, 1, b_{t+1}) \quad (2.5)$$

$$s.t. \quad c_t = (1 - \gamma)y_t$$

$$b_{t+1} = b_t(1 + r)$$

If the sovereign starts a period with a bad credit standing $s_t = 1$, it stays in autarky, suffers a proportional output loss and may settle the defaulted debt through renegotiating with the debt holder on debt reduction. The renegotiation process is a dynamic game that can last more than one period. In equilibrium, this bargaining game determines an endogenous debt recovery rate $\alpha(y, b) \in [0, 1]$ and *when* the agreement on $\alpha(y, b)$ can be reached. Thus, the value of staying in default and continuing renegotiation from period t on, $V(y_t, 1, b_t)$, is equal to the expected payoff that the borrower can get from the bargaining game from period t on. We denote this payoff as $\Delta_B(y_t, b_t)$:

$$V(y_t, 1, b_t) = \Delta_B(y_t, b_t) \quad (2.6)$$

The determination of this payoff will be discussed in detail in the debt renegotiation problem.

Default is preferable when $V^D \geq V^R$. Thus, we can characterize the default policy of a sovereign government in good standing by a default set $y^D(b) \subseteq Y$, following Arellano (2007). The default set $y^D(b)$ is defined as follows

$$y^D(b) = \{y \in Y : V^D(y, 0, b) \geq V^R(y, 0, b)\}$$

This default set specifies a set of endowments, for which default is optimal given the debt position b . Consequently, we can compute the sovereign's default probability in period $t+1$, θ_{t+1} , as the probability that tomorrow's endowment shock falls in the default set, given the government's debt position in period $t+1$, b_{t+1} and today's endowment realization, y_t .

$$\theta_{t+1}(y_t, b_{t+1}) = \int_{y^D(b_{t+1})} d\mu(y_{t+1}|y_t) \quad (2.7)$$

2.2.2 Debt Renegotiation Problem

Based on Merlo and Wilson (1995), we model the debt renegotiation problem as a two-player stochastic bargaining game with complete information. This is a stochastic bargaining game in that both the endowment process and the identity of the proposer are stochastic. In each period, a state is realized, which determines the cake (i.e., the set of possible utility vectors to be agreed upon in that period), and the proposer is randomly selected. For simplicity, we assume that each player has a constant probability of being selected as proposer in each period. That is, the identity of the proposer is independent of the cake size. Let ϕ denote the probability that the borrower, B , can propose in a period, and $(1 - \phi)$ is the probability that the lender, L , proposes in each period. The frequency with which a player is selected as

proposer is a parsimonious way to capture the bargaining power acquired through one's ability to enjoy the first-mover advantage. The proposer may either propose a debt recovery rate or pass. If she proposes, then the remaining party chooses to accept or to reject the proposal. If the proposal is accepted, then the defaulting country immediately repays its reduced debt arrears, and then enters the next period with an upgraded credit standing of $s_{t+1} = 0$ without any outstanding debt. If the proposal is rejected, a new output state is realized and the game repeats until an agreement is reached. In the case that the proposer chooses to pass, both players enter the next period and continue the game.

We now define some basic concepts of the game. A *stochastic bargaining game* may be indexed by $(C, \beta, \frac{1}{1+r})$, where for each endowment state $y \in Y$, $C(y) \subset R^2$ is a *cake* representing the set of feasible utility vectors that may be agreed upon in that state. β and $\frac{1}{1+r}$ are the discount factors for B and L , respectively³. A *payoff function* is an element $\Delta(y) \in C(y)$, where $\Delta_i(y)$ is the utility to player i , $i = B, L$.

As in Merlo and Wilson (1995), we focus on a game with *stationary strategies*, that is, the players' actions depend only on the current state and the current offer. In equilibrium, the proposer's strategy is to propose when the proposal would be accepted for sure and to pass otherwise. The other player acts passively: she accepts when a proposal is made and waits if the proposer passes. Therefore, we can denote the proposer i 's equilibrium strategy as a simple stopping function τ_i , with $\tau_i =$

³Merlo and Wilson (1995) assume that the players have the same discount factor. But they also mention that "there is no real restriction implied by the assumption that players discount utility at a common constant rate. So long as the discounted size of the cake converges uniformly to 0. . . player dependent discount factors can always be represented by a different cake process with a common fixed discount factor". So in our model we allow the borrower and the lender to have different discount factors.

0 when i proposes and $\tau_i = 1$ when i passes; the other player accepts or waits accordingly. A *stationary subgame perfect* (SSP) equilibrium is then defined as the players' equilibrium stationary strategies τ_B and τ_L , and the payoff functions, Δ_B and Δ_L , associated with these strategies for player B and L .

We then proceed to characterize the SSP strategies and payoffs. The expected payoff for the borrower B in period t , $E\Delta_B(y_t, b_t)$, is given by:

$$E\Delta_B(y_t, b_t) = \phi\Delta_B^B(y_t, b_t) + (1 - \phi)\Delta_B^L(y_t, b_t) \quad (2.8)$$

Here the superscript denotes the identity of the proposer. So Δ_B^B represents the borrower's payoff when the *borrower* himself is the proposer, and Δ_B^L refers to the borrower's payoff when the *lender* is the proposer.

Correspondingly, the expected payoff for the lender L in period t , $E\Delta_L(y_t, b_t)$, is given by:

$$E\Delta_L(y_t, b_t) = \phi\Delta_L^B(y_t, b_t) + (1 - \phi)\Delta_L^L(y_t, b_t) \quad (2.9)$$

where Δ_L^B denotes the lender's payoff when the *borrower* is the proposer, and Δ_L^L refers to the lender's payoff when himself is the proposer.

First consider the case when the borrower B is the proposer. We refer the proposed debt recovery rate as α^B and the value of proposing as V_B^{prop} . When B proposes and the proposal is accepted, the sovereign repays the agreed amount of debt, $\alpha^B b$, immediately and enters the next period with a good credit standing and no outstanding debt. Thus, V_B^{prop} is given by

$$V_B^{prop}(y_t, b_t) = u[(1 - \gamma)y_t + \alpha_t^B b_t] + \beta EV(y_{t+1}, 0, 0) \quad (2.10)$$

And the lender's payoff from acceptance, V_L^{acpt} , is

$$V_L^{acpt}(y_t, b_t) = -\alpha_t^B b_t \quad (2.11)$$

When B chooses to pass, the sovereign stays in autarky, suffers the proportional output loss in this period and then enters the next period continuing the bargaining game. We denote the value of passing as V_B^{pass} :

$$V_B^{pass}(y_t, b_t) = u[(1 - \gamma)y_t] + \beta E\Delta_B(y_{t+1}, (1 + r)b_t) \quad (2.12)$$

The lender's value of rejecting an offer (or waiting) is denoted as V_L^{rejt} , which is given by

$$V_L^{rejt}(y_t, b_t) = \frac{1}{1 + r} E\Delta_L(y_{t+1}, (1 + r)b_t) \quad (2.13)$$

In equilibrium, the proposer B 's strategy is to propose an α^{B*} that solves the following problem

$$\alpha^{B*}(y_t, b_t) = \operatorname{argmax} V_B^{prop}(y_t, b_t) \quad (2.14)$$

$$s.t. \quad V_B^{prop} \geq V_B^{pass}$$

$$V_L^{acpt} \geq V_L^{rejt}$$

The two constraints specify the set of debt recovery rates acceptable for both players to settle the debt in the current period. If this set is nonempty, then an agreement can be reached in the current period. The stopping function $\tau_B(y_t, b_t) = 0$, meaning that the borrower proposes in this period, and the equilibrium debt recovery rate α^{B*} is the smallest element in this set. In other words, the proposer B can extract extra surplus to the extent that the lender is indifferent between accepting and

rejecting. However, if this set is empty, then the borrower would choose to pass ($\tau_B(y_t, b_t) = 1$), and hence to delay the renegotiation by one period. This is often the case when the current endowment is low and the future ones are expected to be higher. In these circumstances, the borrower is only willing to repay a small proportion of the debt arrears in the current period, smaller than what the lender expects to get in the future with a better endowment realization (and when the lender himself may get to be the proposer). Therefore, there is no debt recovery rate that the borrower is willing to offer and the lender is willing to accept. In the end, the borrower just passes and one more period of delay arises.

We now characterize the players' payoff functions. If α^{B^*} exists, then the borrower's payoff, Δ_B^B , is the value of proposing,

$$\Delta_B^B(y_t, b_t) = V_B^{prop}(y_t, b_t) \quad (2.15)$$

And the lender's payoff, Δ_L^B , is the value of accepting,

$$\Delta_L^B(y_t, b_t) = V_L^{acpt}(y_t, b_t) \quad (2.16)$$

Otherwise, the borrower's payoff is the value of passing,

$$\Delta_B^B(y_t, b_t) = V_B^{pass}(y_t, b_t) \quad (2.17)$$

And the lender's payoff is the value of waiting (equivalent to the value of rejecting):

$$\Delta_L^B(y_t, b_t) = V_L^{rejt}(y_t, b_t) \quad (2.18)$$

Similarly, when the lender is the proposer, we denote the proposed debt recovery rate as α_t^L . The value for the lender to propose an acceptable debt recovery

rate is V_L^{prop} , given by

$$V_L^{prop}(y_t, b_t) = -\alpha_t^L b_t \quad (2.19)$$

And the value for the borrower to accept this proposal is V_B^{acpt} :

$$V_B^{acpt}(y_t, b_t) = u[(1 - \gamma)y_t + \alpha_t^L b_t] + \beta EV(y_{t+1}, 0, 0) \quad (2.20)$$

When L chooses to pass or her proposal is rejected, the value of choosing pass is denoted as V_L^{pass} ,

$$V_L^{pass}(y_t, b_t) = \frac{1}{1+r} E\Delta_L(y_{t+1}, (1+r)b_t) \quad (2.21)$$

And the borrower's value in this case, V_B^{rejt} , is given by

$$V_B^{rejt}(y_t, b_t) = u[(1 - \gamma)y_t] + \beta E\Delta_B(y_{t+1}, (1+r)b_t) \quad (2.22)$$

Again, in equilibrium, the proposer L solves the following problem

$$\alpha^{L*}(y_t, b_t) = \operatorname{argmax} V_L^{prop}(y_t, b_t) \quad (2.23)$$

$$s.t. \quad V_L^{prop} \geq V_L^{pass}$$

$$V_B^{acpt} \geq V_B^{rejt}$$

Similar to our argument above, if the minimum debt recovery rate acceptable for L is too large for B , then no α^{L*} exists, and L chooses to pass ($\tau_L(y_t, b_t) = 1$); otherwise, L proposes and $\tau_L(y_t, b_t) = 0$.

When L proposes, the payoffs for both players, Δ_B^L and Δ_L^L , are given by

$$\Delta_L^L(y_t, b_t) = V_L^{prop}(y_t, b_t) \quad (2.24)$$

$$\Delta_B^L(y_t, b_t) = V_B^{acpt}(y_t, b_t) \quad (2.25)$$

When L passes, the payoffs are:

$$\Delta_L^L(y_t, b_t) = V_L^{pass}(y_t, b_t) \quad (2.26)$$

$$\Delta_B^L(y_t, b_t) = V_B^{rejt}(y_t, b_t) \quad (2.27)$$

An SSP equilibrium is a fixed point of the above recursive system (Equations (7) to (26))⁴.

2.2.3 International Investors' Problem

The last part of the model specifies the international investors' problem. There is an infinite number of risk neutral and competitive international investors, one of whom is randomly chosen to trade with the sovereign government in each period. Investors choose the face value of the bond to maximize expected profits, taking the bond prices as given

$$\max_{b_{t+1}} E(\pi_t) \quad (2.28)$$

where π_t is the profit from trading one-period bonds. More explicitly, the expected profit $E(\pi_t)$ can be expressed as

$$E(\pi_t) = \begin{cases} q_{t+1}b_{t+1} - \frac{b_{t+1}}{1+r} & \text{if } b_{t+1} \geq 0 \\ q_{t+1}b_{t+1} - \frac{1-\theta_{t+1}}{1+r}b_{t+1} + \frac{\theta_{t+1}E(\Delta_L(y_{t+2}, b_{t+2}))}{(1+r)^2} & \text{if } b_{t+1} < 0 \end{cases} \quad (2.29)$$

When $b_{t+1} \geq 0$, the government saves by purchasing bonds. Since international investors commit to repaying debts, the government's saving is risk free. When $b_{t+1} < 0$, the government borrows by issuing new bonds, and default and debt restructuring are possible future events. In this case, the expected present value of

⁴Merlo and Wilson (1995) provides a general proof for this characterization of SSP equilibrium in Theorem 1.

repayment consists of two terms. The first represents the repayment in the following period, while the second reflects the expected debt recovery in the case of default. As noted in the government's problem, the probability of repayment is $(1 - \theta_{t+1})$, and that of default is θ_{t+1} . In the case of default, debt renegotiation begins in period $t + 2$. $E(\Delta_L(y_{t+2}, b_{t+2}))$ is the expected present value of the payoff that the debt holder can get from the renegotiation game.

From the zero-profit condition of the investors, we obtain the price function as follows:

$$q_{t+1} = \begin{cases} \frac{1}{1+r} & \text{if } b_{t+1} \geq 0 \\ \frac{1-\theta_{t+1}}{1+r} - \frac{\theta_{t+1}E(\Delta_L(y_{t+2}, b_{t+2}))}{(1+r)^2 b_{t+1}} & \text{if } b_{t+1} < 0 \end{cases} \quad (2.30)$$

Finally, note that the term $-\frac{E(\Delta_L)}{(1+r)b_{t+1}}$ is just the expected present value of the debt recovery rate, which lies in the interval $[0, 1]$, and hence the bond price ranges from 0 to the price of a risk free bond. Consequently, the interest rates of sovereign bonds are always higher than or equal to the risk free rate. We define the bond spread as the difference between the country interest rate and the risk free interest rate.

2.3 Equilibrium

The recursive equilibrium of the model is defined as follows:

Definition 2 *A recursive equilibrium is defined by (i) a sequence of allocations of consumption $c(y, s, b)$, bond holding $b'(y, b)$ (b' refers to the bond issued in the current period and maturing in the next period), and a default set $y^D(b)$; (ii) a pricing function $q(y, b')$; (iii) the borrower's and the lender's stopping functions $\tau_B(y, b)$*

and $\tau_L(y, b)$, debt recovery rates $\alpha^B(y, b)$ and $\alpha^L(y, b)$, and the payoffs $\Delta_B(y, b)$ and $\Delta_L(y, b)$ such that:

1. Given the pricing function $q(y, b')$ and the debt renegotiation outcome, the country's asset holdings $b'(y, b)$, consumption $c(y, s, b)$, and default set $y^D(b)$ satisfy the government's optimization problem.
2. Given the bond pricing function $q(y, b')$, the debt recovery rate $\alpha^B(y, b)$, $\alpha^L(y, b)$ and the strategies of both players solve the post-default debt renegotiation problem.
3. Given the renegotiation outcome and the country's optimal policy, the bond pricing function $q(y, b')$ satisfies the investors' maximization problem.

2.3.1 Property of the Recursive Equilibrium

We can easily establish the relationship between the identity of the proposer and the timing of reaching an agreement.

Proposition 3 *In a given period with endowment y and defaulted debt b , whether an agreement can be reached in this period does not depend on the identity of the proposer.*

Proof. See Appendix B. ■

This proposition says that in equilibrium, given the endowment realization and the defaulted debt level, no matter who is the proposer, the proposing/pass choice would be the same. The intuition is as follows: if the selected proposer,

say the borrower, can find an allocation that makes both parties at least weakly better off by agreeing on the proposal than waiting for one more period, then this allocation is also a feasible choice (but may not be the best one) for the lender to achieve an agreement as the proposer. Therefore, the identity of the proposer does not determine when an agreement is achieved; it only affects the allocation of the “cake” to each player. It is not surprising that being the proposer in an agreement-reaching period is advantageous: the proposer can extract extra surplus to the extent that the other player is indifferent between accepting and rejecting the proposal. Since the identity of the proposer is independent of the state realizations, the player with a higher probability of being proposer in each period is more likely to enjoy the first-mover advantage. Thus, the frequency of being selected as proposer reflects a player’s bargaining power.

2.4 Quantitative Analysis

We solve the model numerically to evaluate the delay lengths that can be generated by this model.

2.4.1 Calibration

The model is calibrated to match features of the Argentine economy. We define one period as a quarter.

We use a constant relative risk aversion (CRRA) utility function:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \tag{2.31}$$

where σ is the risk aversion coefficient. This parameter is set to 2, a standard value in real business cycle studies. We set the parameter of proportional output loss γ to be 2%, based on Sturzenegger's (2002) estimate. The quarterly risk-free interest rate r is set to 1%, following Yue (2006).

As mentioned in the model set up, we assume that the exogenous endowment stream follows a log-normal AR(1) process:

$$\log(y_t) = \log(\bar{y}) + \rho(\log(y_{t-1}) - \log(\bar{y})) + \varepsilon_t, \quad \varepsilon \sim N(0, \sigma_\varepsilon^2) \quad (2.32)$$

where \bar{y} is the mean output and is normalized to 1. The autocorrelation coefficient ρ and the standard deviation of endowment innovations, σ_ε , are computed from the quarterly real GDP data of Argentina from 1980 Q1 to 2005 Q4. The GDP data are seasonally adjusted and are taken from the Ministry of Economy and Production (MECON). The data are detrended using a Hodrick-Prescott filter with a smoothing parameter of 1600. We then construct a discrete Markov representation of the endowment process as a 51 state Markov chain employing the quadrature procedure by Hussey and Tauchen (1991).

In the last part of the calibration, we set the sovereign's time discount factor β and the sovereign's probability of being selected as proposer in each period, ϕ , simultaneously to match the frequency of default and the average debt recovery rate using the simulated method of moments (SMM).

Argentina defaulted 4 times from 1824 to 1999 as documented in Reinhart, Rogoff and Savastano (2003), and then defaulted a fifth time in late 2001. As a consequence, its long-term quarterly default frequency is about 0.70%. In the last

25 years (for which the quarterly GDP data are available), however, Argentina has defaulted twice, once in 1982 and once in late 2001, resulting in an average quarterly default frequency of 1.92%, much higher than the long-term default frequency. In this calibration, we set the parameter values to match the short-term default frequency. The reason is that in our model, the default decisions and the equilibrium delay lengths are interdependent: states in which defaults are chosen largely determine the renegotiation delay lengths, and the expected delay lengths, in turn, affect the default decisions ex ante. Moreover, both default decisions and equilibrium delay lengths are essentially determined by the output process. Given that the available short series of quarterly GDP data are consistent with a quarterly default frequency of 1.92%, if we artificially set parameter values to match the long-term default frequency (i.e., 0.7%), then we may get a biased result for the average delay length in the quantitative analysis. Therefore, we aim to match the short-term default frequency of 1.92%. The average debt recovery rate in the most recent Argentine debt restructuring was approximately 35% (Sturzenegger and Zettelmeyer (2005)).

Table 2.2: Model Parameter Values

Parameter	Value	Sources
Coefficient of Risk Aversion	$\sigma = 2$	RBC literature
Output Loss in Default	$\gamma = 2\%$	Sturzenegger (2002)
Risk Free Interest Rate	$r = 1\%$	Yue (2006)
Std. Dev. of Endowment Innovations	$\sigma_\varepsilon = 0.047$	MECON
Autocorr. Coef. of Endowment	$\rho = 0.83$	MECON
Discount Factor	$\beta = 0.96$	Matched by Calibration
Borrower's Probability of Being Proposer	$\phi = 0.60$	Matched by Calibration

Table 2.3: Target Statistics and Model Statistics

Target Statistics	Data	Model
Default Frequency	1.92%	2.19%
Debt Recovery Rate	35%	38.47%

The time discount factor is found to be 0.96, and the sovereign’s probability of being proposer, ϕ , is found to be 0.60. Our calibrated model generates a default frequency of 2.19% and an average debt recovery rate of 38.47%, which closely match the corresponding data moments. The calibration results are summarized in Table 2.2. Table 2.3 compares the targeted data statistics and the model-generated statistics.

2.4.2 Numerical Results on Equilibrium Properties

In this part, we present some of the equilibrium properties of the model. Figure 2.1 plots the equilibrium default probabilities in the next period for each state of output realization today and debt-to-mean output ratio tomorrow.

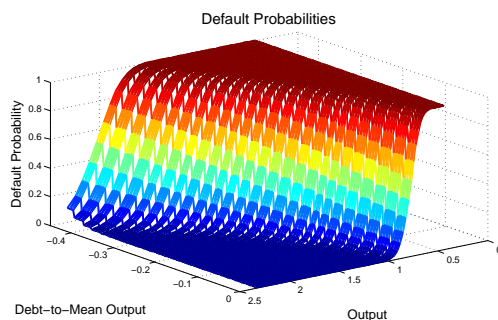


Figure 2.1: Equilibrium Default Probabilities

It is clear that default probabilities are higher with lower endowment realizations today and higher levels of debt tomorrow. Note that a bad endowment shock today implies a large probability of having a low output level tomorrow, given the positively autocorrelated output process. So this result supports our argument that defaults are usually associated with bad output realizations, providing incentives to wait for a larger “cake” in the post-default debt renegotiations.

We then present in Figure 2.2 the equilibrium propose/pass choices made by the borrower and the lender for each endowment state and defaulted debt-to-mean output ratio⁵. On the vertical axes, “Propose” represents the case that a proposal is made and accepted, i.e., an agreement is reached. “Pass”, on the contrary, implies waiting and hence one period of delay.

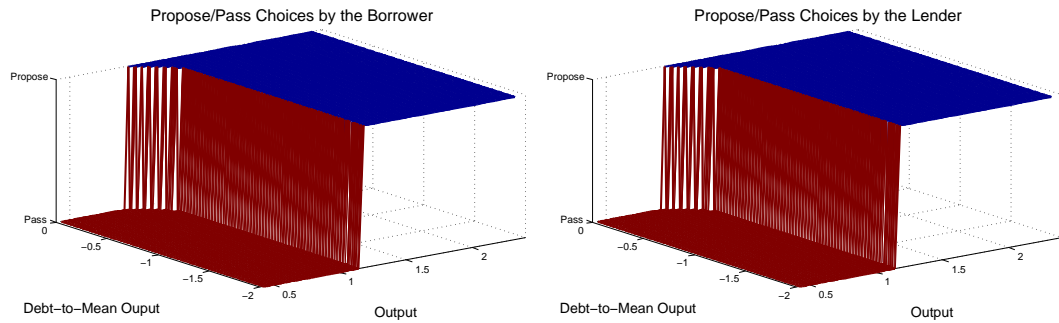


Figure 2.2: Equilibrium Propose/Pass Choices of the Borrower and the Lender

As a confirmation of Proposition 1, the two graphs in Figure 2.2 are identical, meaning that for a given endowment state and a defaulted debt-to-mean output

⁵In the following figures, “debt” refers to the present value of the defaulted debt. Since it may take many periods to reach a renegotiation agreement, the present value of the defaulted debt can be much larger than the original level at the time of default. Therefore, we allow the largest possible defaulted debt-to-mean output ratio to be 200%, which is large enough (i.e., it is never binding) for our benchmark simulations. Since we use negative numbers to denote debt, the lower bound of the defaulted debt-to-mean output ratio is -2.0.

ratio, the borrower and the lender would make the same propose/pass choice no matter who is selected as proposer.

Furthermore, Figure 2.2 shows the states in which an agreement can be attained. We observe three interesting results from this figure: first, for any given debt-to-mean output ratio, there is a threshold output level, under which “pass” is chosen; second, for low levels of defaulted debt (e.g., debt-to-mean output ratios smaller than 0.3), the threshold output state increases in debt; third, when the defaulted debt levels are large enough, the output threshold does not change with the debt level. The first two results are very intuitive: debt settlements only occur in sufficiently good output states, and with a higher defaulted debt level, the sovereign needs a better endowment realization (and hence more available resources) to settle the debt. We will discuss the third result later, after we present the renegotiated debt repayment schedules.

We now plot the equilibrium debt recovery rates proposed by each player at each state (Figure 2.3). In this figure, a zero debt recovery rate implies that the proposer chooses to “pass” in the particular state.

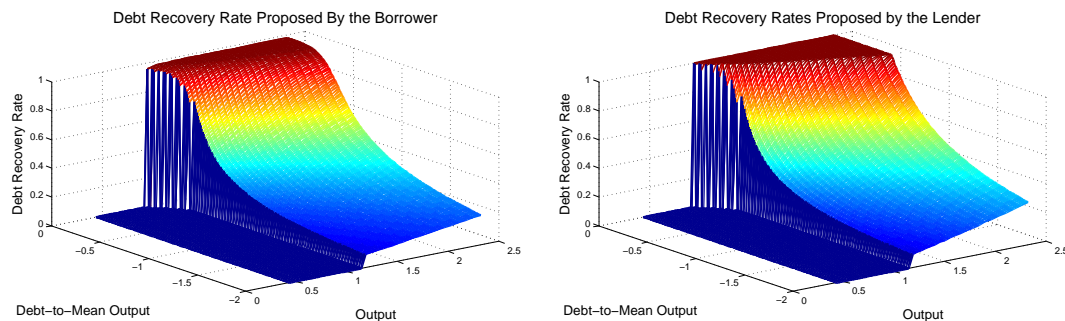


Figure 2.3: Equilibrium Debt Recovery Rates

In general, the debt recovery rates proposed by the borrower are smaller than those proposed by the lender. This is because the proposer enjoys the “first-mover advantage”, which results in lower equilibrium debt recovery rates when the borrower is the proposer and higher ones when the lender is the proposer. In addition, the shapes of the two debt recovery schedules are similar: both of them are convex and decreasing in the defaulted debt level, meaning that a lower level of defaulted debt results in a much larger debt recovery rate. These debt recovery schedules are also similar to that obtained from a Nash Bargaining game, except that in the latter case there is no delay in equilibrium.

Furthermore, we can take a look at the total debt repayments proposed by each player, as shown in Figure 2.4.

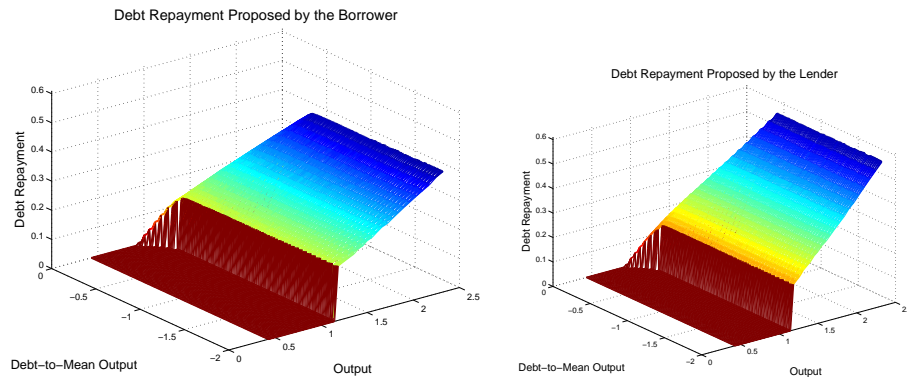


Figure 2.4: Proposed Debt Repayments

When the proposed debt repayment is zero, it means the proposer passes. Again, the debt repayments proposed by the lender are generally larger than those proposed by the borrower. More interestingly, for a given output state, except for the low defaulted debt region (e.g., the debt-to-mean output ratios smaller than 0.3),

the proposed repayment does not change with the debt level. In other words, for any given endowment realization, there is a maximum amount that the sovereign is willing to repay, regardless of the defaulted debt level. This explains why the threshold output state is invariant of the defaulted debt level in the large debt region. And again, this result coincides with that from Nash bargaining, which is not surprising given that the bargaining structure in this paper is a variant of the Rubinstein game. Finally, this maximum amount of repayment increases in the output realization.

2.4.3 Simulation Results

We conduct 500 rounds of simulations, with 2000 periods per round, and then extract the last 200 periods in each round to study the limiting distribution of the model economy.

First, we show the ergodic distribution of the equilibrium delay length in Figure 2.5.

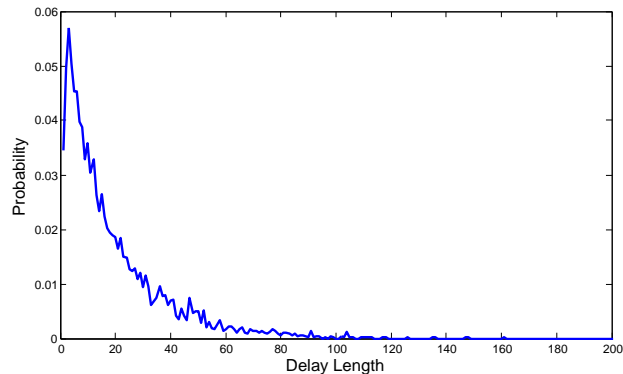


Figure 2.5: Ergodic Distribution of the Equilibrium Delay Length

Equilibrium delay length ranges from zero to about 160 periods (a period is a quarter), while falling between 0 and 20 quarters most frequently. The average delay length, together with other business cycle statistics generated by the model are reported in Table 2.4. For comparison, we also report the corresponding data statistics in that table.

The debt service-to-GDP ratio is computed by dividing Argentina’s short-term debt and debt service for long-term debt by its GDP from 1980 to 2005. The debt data are taken from the Global Development Finance database of the World Bank. Seasonally adjusted consumption and current account data from 1980Q1 to 2005Q4 are taken from MECON. Bond spreads are annualized quarterly spreads on 3-year foreign currency denominated bonds issued by the Argentine government, taken from Broner, Lorenzoni and Schukler (2005).

Table 2.4: Business Cycle Statistics from the Model and the Data

Statistics	Data	Model
Average Renegotiation Delay Length (Quarters)	13.3	19.91
Std. Dev. of Bond Spreads	1.72%	1.37%
Average Bond Spreads	5.16%	3.46%
Correlation between Spreads and Output	-0.54	-0.51
Debt Service-to-GDP	14.88%	9.28%
Std. Dev. of CA/Output	5.40%	4.00%
Correlation between CA/Output and Output	-0.91	0.12
Consumption Std. Dev./Output Std. Dev.	1.03	0.99

The model generates an average delay length of about 20 quarters, slightly longer than the actual delay length experienced by the Argentine government. This result suggests that the incentive to wait for a larger “cake” can indeed result in

long “beneficial” delays in debt renegotiation processes. The simulated average delay length is a bit too long because of our assumption that the sovereign fully repays the reduced debt arrears right after the agreement. In the real world, the debt arrears are not fully repaid in one step; instead, they are restructured or swapped to new ones, and will be repaid in the future. These debt swap and restructuring provide some state contingency to the debt repayment schedules, which can shorten the renegotiation process. In the extreme case with complete market and state-contingent debt repayment schedules, there would be no delays at all. Therefore, our result can be considered as the “upper bound” of the beneficial delay length.

The model simulation closely matches the volatility of bond spreads, which has been hard to explain in the literature. The existing quantitative models on sovereign default either assume exogenous return to the capital markets without repaying anything after default (for example, Model I in Aguiar and Gopinath (2006) and Arellano (2007)), or use Nash Bargaining to model the debt renegotiation process (for example, Yue (2006)). In these two cases, the expected length of exclusion from the capital markets is more or less fixed. In the present model, however, the expected length of exclusion is uncertain and depends on the defaulted debt level and the level of output at the time of default. Therefore, the existence of equilibrium delays adds more uncertainty to the debt settlement outcome and contributes to the larger volatility of bond spreads. The model-simulated average bond spread falls below the data statistic, but it is closer to reality than those obtained by other quantitative analysis of sovereign debt. This improvement stems from both the higher default frequency that we target and the additional “waiting cost” incurred

during delays, which reflects the forgone earnings from re-investing the defaulted debt.

Furthermore, the model captures the countercyclical behavior of the bond spreads. Bond spreads are higher in bad endowment states due to the higher default risk and the larger potential “waiting cost” following default. The latter arises because the players have to wait longer for the economy to recover from a deeper recession before they can settle the defaulted debt.

The model generates an average debt service-to-GDP ratio of 9.28%, lower than the data statistic 14.88%. Note that we include both short-term debt and debt service of long-term debt in calculating the data statistic, while in the model we only have quarterly (short-term) debt. Therefore, it is not surprising that the debt-to-GDP ratio generated by the model is a little lower than that observed in data.

The model simulation matches the current account volatility, but it fails to capture the countercyclical behavior of the current account. In this model, the sovereign faces tighter borrowing constraints in recessions than in booms, and therefore has to rely more on its own savings, rather than outside financing, to smooth consumption in bad times. Consequently, the sovereign has to save more in booms, resulting in a procyclical current account.

Finally, the simulation generates a consumption stream that is a little less volatile than output. It is typical of emerging economies that consumption is more volatile than output (Neumeyer and Perri (2004)). Aguiar and Gopinath (2006) and Yue (2006) match this fact by assuming a stochastic trend for the output process,

implying that output shocks are permanent. The present model has only transitory output shocks and the sovereign aims to smooth consumption over time. Its ability to smooth consumption is quite limited due to the potential default risk, so that the volatility of consumption generated by the simulation is just a little less than that of output.

2.5 Determinants of Renegotiation Delay Length

In this section, we explore the determinants of equilibrium delay length. As aforementioned, the most important parameters are those governing the output process. Hence, we first report the effects of output process on the delay length. We then examine the influence of other parameters, including the discount factor, the probability that the borrower is the proposer, the risk-free interest rate and the proportional output loss following default.

2.5.1 Output Process and Equilibrium Delay Length

In this part, we investigate the effects of output properties on the equilibrium delay length. Table 2.5 reports the average delay lengths under different values of the autocorrelation coefficient ρ and the standard deviation of endowment innovations, σ_ε , holding other parameters at their benchmark values.

The upper panel shows how the delay length varies with the endowment autocorrelation coefficient. A higher autocorrelation implies more persistent endowment shocks, and hence slower recovery after default. Therefore, the players have to wait

longer for a sufficiently large “cake” to materialize. Consistent with this intuition, our results show that as the autocorrelation increases from 0.35 to 0.99, the renegotiation process lengthens from 6.12 to 44.01 quarters on average.

Table 2.5: Delay Lengths under Different Output Processes

Endowment Autocorr. Coef.	Delay Length (Quarters)
$\rho = 0.99$	44.01
$\rho = \mathbf{0.83}$	19.91
$\rho = 0.50$	10.39
$\rho = 0.35$	6.12
$\rho \in [-0.3, 0.3]$	∞
$\rho < -0.3$	∞
Std. Dev. of Endowment Innovations	
$\sigma_\varepsilon = 0.01$	12.32
$\sigma_\varepsilon = \mathbf{0.047}$	19.91
$\sigma_\varepsilon = 0.2$	35.72
$\sigma_\varepsilon = 0.25$	59.44

When the autocorrelation coefficient is too low in absolute value (i.e., when $\rho \in [-0.3, 0.3]$), the output process is close to *i.i.d.* with very small volatility⁶. In this case, it is always optimal for the sovereign to stay in autarky forever after default. Intuitively, when the output stream is not volatile, the benefits of smoothing consumption can be smaller than the cost of repaying the reduced debt arrears. As a result, the renegotiation delay length is infinity. When the output realizations are negatively correlated with large absolute values of ρ (i.e., $\rho < -0.3$), the output process is again volatile, but the sovereign still chooses to stay in autarky forever once

⁶In this exercise, we lower the autocorrelation coefficient ρ , while keeping the standard deviation of endowment innovations, σ_ε , constant. Since $\log(y)$ follows an AR(1) process, its standard deviation $\sigma_y = \sigma_\varepsilon / \sqrt{1 - \rho^2}$. As ρ decreases, σ_y decreases as well.

it defaults. The reason is that on the one hand, it is least costly to settle the defaulted debt in good output states, suggesting that the players should reach an agreement in booms; on the other hand, given the negatively autocorrelated output stream, a good state today implies a bad one tomorrow. A bad output state tomorrow tightens the borrowing constraint and limits the sovereign's ability to smooth consumption. Therefore, a costly debt repayment does not provide much benefit for the sovereign in terms of consumption smoothing. In the end, the economy chooses not to settle the debt even in good states.

The lower panel delivers the effects of endowment volatility on delay length. The equilibrium delay length increases as the volatility increases. This result is intuitive: to settle the defaulted debt, we need large positive shocks to materialize. With a more volatile output stream, it is less likely that a good enough shock will occur. Consequently, the players have to wait longer after default for a sufficiently good realization to settle the defaulted debt.

2.5.2 Other Parameters and Renegotiation Delay Length

Table 2.6 shows the equilibrium delay length under different values for the time discount factor β , the sovereign's probability of being selected as proposer, ϕ , the proportional output loss parameter γ and the risk-free interest rate r .

The upper left panel of Table 2.6 reports the effects of the time discount factor, holding all other parameters fixed at their benchmark values. Interestingly, the average delay length does not move monotonically as the time discount factor changes. The time discount factor affects the equilibrium delay length through two

Table 2.6: Delay Lengths under Different Parameter Values

Discount Factor	Delay Length (Quarters)	Interest Rate	Delay Length (Quarters)
$\beta = 0.99$	16.20	$r=0.01$	19.91
$\beta=0.96$	19.91	$r = 0.05$	8.59
$\beta = 0.85$	13.77	$r = 0.10$	6.51
B 's Prob. of Being Proposer		Output Loss	
$\phi = 0.90$	10.12	$\gamma = 0.005$	23.96
$\phi=0.60$	19.91	$\gamma=0.02$	19.91
$\phi = 0.30$	24.29	$\gamma = 0.10$	8.68

channels: from the perspective of payoff maximization in the bargaining, a more patient sovereign is willing to wait longer, since its patience enables it to be at a stronger position and thus to enjoy a higher payoff; from the perspective of consumption smoothing, however, a more patient government would like to settle the debt sooner so that it can reaccess the capital markets and smooth consumption in the future. With a large discount factor ($\beta = 0.99$), the second effect dominates the first one, so the equilibrium delay length is shorter than the benchmark case ($\beta = 0.96$). When the discount factor is small (e.g., $\beta = 0.85$), on the contrary, the first effect dominates, that is, the sovereign cares more about its bargaining payoff, again resulting in a shorter-than-benchmark delay length. In sum, the equilibrium delay length is short when the sovereign is very patient, then it is longer as the sovereign becomes less patient, and finally it is short again when the sovereign is very impatient.

The lower left panel presents the effect of changing the borrower's probability of being selected as proposer. As argued before, this probability captures the bor-

rower's bargaining power. Stronger bargaining power of the government results in a lower debt recovery rate, making it possible for the sovereign to repay the debt arrears even in a not-so-good state after a shorter period of waiting. Also, settling the debt sooner brings two benefits: regaining access to the capital markets sooner and suffering from the output loss for a shorter period of time. Consequently, as the sovereign's probability of being proposer becomes larger, the average delay length is shorter.

The upper right panel delivers the effects of the world interest rate. As the risk-free interest rate increases, the debt-holder in the bargaining becomes more impatient (her discount factor is $\frac{1}{1+r}$). Therefore, she is eager to reach an agreement and get repaid sooner at the cost of a smaller debt repayment. For the sovereign, repaying less and settling the debt sooner are also beneficial. Therefore, the equilibrium delay length is shorter with a higher risk-free interest rate.

Lastly, the lower right panel reports the delay lengths under different values of post-default proportional output loss. Intuitively, when the loss is larger, staying in default is more costly, forcing the sovereign to settle its debt sooner. As a result, the average delay length decreases in the proportional output loss.

2.6 Conclusion

Delays in post-default renegotiations are common in sovereign debt restructurings. Contrary to the common wisdom that delays are costly and inefficient, this paper argues that delays in reaching a debt restructuring agreement can be

an efficient equilibrium outcome. This is because delays allow the economy to recover from a crisis, enabling the negotiating parties to enjoy a larger “cake”. To investigate whether this argument can explain the actual delay length experienced by emerging market economies, this paper introduces a stochastic bargaining game with complete information, based on Merlo and Wilson (1995), into an otherwise standard dynamic sovereign default model. We find that for a given defaulted debt-to-output ratio, there exists a threshold output level above which an agreement is reached immediately; otherwise, “pass” is chosen by the proposer and one period of delay occurs. Therefore, in equilibrium, the players tend to wait when the output states are bad, and settle the defaulted debt when the economy recovers and better output states are realized.

Quantitative analysis shows that this model is able to generate a delay length close to that experienced by Argentina in its recent debt restructuring event. This result suggests that some delays in the debt restructuring negotiations are beneficial, and we may not need policy reforms to eliminate them. In addition, this model successfully accounts for the volatility of bond spreads observed in the data without assuming risk-averse creditors. The larger volatility of bond spreads can be attributed to the existence of equilibrium delays, which adds additional uncertainty to the renegotiation outcomes.

Furthermore, we examine how the properties of the output process affect the length of delays. When the volatility of output innovations is held fixed, the equilibrium delay length decreases as the output shocks become less persistent. However, when the persistence is too low (or when the output states are negatively autocor-

related), the sovereign prefers to stay in autarky forever after default, resulting in an infinite delay length. As the volatility of output realizations increases, the equilibrium delay length increases as well, because a more volatile output profile makes it less likely that a sufficiently good output will materialize.

This paper is the first in the sovereign debt literature that can quantitatively generate long “beneficial” delays in debt renegotiation processes. It can be made more interesting by assuming a production economy instead of an endowment one, as we do in this paper. In an economy with investment and production, the recovery path after default becomes endogenous: if the sovereign decides to invest more into production (and hence sacrifice more of today’s consumption), then the economy recovers faster; otherwise, the economy may be trapped in recessions for much longer until some exogenous good shock is realized. In the former case, delays are efficient in that the economy is relieved from the debt burden at least temporarily and the government has more resources for investment, which accelerates the economy’s recovery and provides a much larger “cake” for the players to enjoy. In the latter case, however, “waiting” may not be worthwhile because it is almost impossible to predict when a large enough “cake” may materialize and it is quite possible that it will never appear. In such a model set up, multiple equilibria may exist and policies that can induce the government in default onto the “good” track may be welfare-improving.

Appendix A

Appendix to Chapter 1

Proof of Proposition 1. The debt renegotiation problem is

$$\begin{aligned}
 \alpha^*(y_t, B_t) &= \operatorname{argmax} \Delta_B(y_t, B_t, \alpha_t)^\phi \Delta_L(y_t, B_t, \alpha_t)^{1-\phi} \\
 \text{s.t. } \Delta_B(y_t, B_t, \alpha_t) &= u((1-\gamma)y_t + \alpha_t B_t) + \beta \\
 &\quad EV(y_{t+1}, 0, 0) - E \sum_{i=t}^{\infty} \beta^{i-t} u((1-\gamma)y_i) \\
 \Delta_L(y_t, B_t, \alpha_t) &= -\alpha_t B_t \\
 \Delta_B(y_t, B_t, \alpha_t) &\geq 0 \\
 \Delta_L(y_t, B_t, \alpha_t) &\geq 0
 \end{aligned}$$

Taking the first order conditions of the maximization problem, we have

$$\phi \Delta_B^{\phi-1} \Delta_L^{1-\phi} u'((1-\gamma)y_t + \alpha_t^* B_t) B_t = (1-\phi) \Delta_B^\phi \Delta_L^{-\phi} B_t \quad (\text{A.1})$$

Let $B^R = -\alpha_t B_t$ denote the recovered debt repayment. Substituting for Δ_B and Δ_L , we can rewrite equation (A.1) as follows:

$$\phi B^R u'((1-\gamma)y_t - B^R) = (1-\phi) u((1-\gamma)y_t - B^R) + \beta EV(y_{t+1}, 0, 0) - E \sum_{i=t}^{\infty} \beta^{i-t} u((1-\gamma)y_i) \quad (\text{A.2})$$

Taking derivative of both sides with respect to B^R :

$$\frac{\partial LHS}{\partial B^R} = \phi u' + \phi B^R u''(-1) > 0$$

$$\frac{\partial RHS}{\partial B^R} = (1-\phi) u'(-1) < 0$$

Thus, the left-hand-side of equation (A.2) is an increasing function of B^R and the right-hand-side is a decreasing function of B^R . There must exist a unique B^{R*} such that equation (A.2) holds with equality. This B^{R*} is the threshold value \bar{B} . When $B_t \leq \bar{B}$, equilibrium recovered debt repayment is \bar{B} . When $B_t > \bar{B}$, there is no debt forgiveness, since α cannot be larger than 1. This case corresponds with $\Delta_B = 0$ and $\Delta_L > 0$. ■

Proof of Corollary 1. We can rewrite Proposition 1 as:

$$\alpha^*(y, B) = \begin{cases} \frac{\bar{B}(y)}{B} & \text{if } B \leq \bar{B}(y) \\ 1 & \text{if } B > \bar{B}(y) \end{cases}$$

There are 3 possible cases.

Case 1: $B^1 \leq B^2 \leq \bar{B}(y) < 0$, that is both $|B^1|$ and $|B^2|$ are larger than the threshold debt recovery value. In this case, we have

$$\alpha^*(y, B^1) = \frac{\bar{B}(y)}{B^1} \leq \frac{B^2}{B^1} = \alpha^*(y, B^2)$$

Corollary holds for Case 1.

Case 2: $B^1 \leq \bar{B}(y) \leq B^2 < 0$. In this case, we have

$$\alpha^*(y, B^1) = \frac{\bar{B}(y)}{B^1} \leq 1 = \alpha^*(y, B^2)$$

Corollary holds for Case 2.

Case 3: $\bar{B}(y) \leq B^1 \leq B^2 < 0$. In this case, we have

$$\alpha^*(y, B^1) = \alpha^*(y, B^2) = 1$$

Corollary holds for Case 3. ■

Proof of Proposition 2. We first show that the value function $V^R(y, 0, B)$ is an increasing function of maturing debt B^M .

$$\frac{\partial V^R(y, 0, B)}{\partial B^M} = u'(c^*) > 0$$

Therefore, $V^R(y, 0, B^1) \leq V^R(y, 0, B^2)$, given that $B^{1M} \leq B^{2M}$ and $B^{1T} = B^{2T}$.

In addition, given $B^{1T} = B^{2T}$, we have $\alpha^*(y, B^1)B^1 = \alpha^*(y, B^2)B^2$ and $V^D(y, 0, B^1) = V^D(y, 0, B^2)$.

If default is optimal for B^{2M} , then $V^D(y, 0, B^2) \geq V^R(y, 0, B^2)$. Combining all the above results, we have

$$V^D(y, 0, B^1) = V^D(y, 0, B^2) \geq V^R(y, 0, B^2) \geq V^R(y, 0, B^1)$$

So default is also optimal for B^{1M} . Proposition 2 is thus proved. ■

Computation Algorithm

We discretize the spaces of asset holdings and endowment. The limits of the asset space and endowment space are set to ensure that they do not bind in equilibrium and large deviations of shocks are possible. We approximate the distribution of endowment shocks with a discrete Markov transition matrix using a quadrature based procedure (Hussey and Tauchen (1991)). Then we employ the following algorithm:

- I. Guess an initial debt recovery rate. Our guess is $\alpha_0 = 1$.
- II. Guess an initial price for short-term and long-term bonds. Our guesses are the risk-free prices: $q_0^s = \frac{1}{1+r}$ and $q_0^l = \frac{1}{(1+r)^2}$.
- III. Guess an initial value for the value function. Our guess is 0.

IV. Given the initial guesses of α_0 , value function, q_0^s and q_0^l , we solve the country's optimization problem when it repays maturing debt. We use Bellman equation iteration to find the value function V_0^R and the optimal choices for assets. To allow the asset decision rule to be chosen off grid nodes, we use bilinear interpolation to fill in the off-grid values of the value function. We then compute the value function for default, V_0^D . By comparing V_0^R and V_0^D , we are able to obtain the government's default decisions for all y and B . We also obtain the default set by finding the cut-off endowment shock at which $V_0^R = V_0^D$.

V. If the newly-obtained value function is close enough to the old one (we use a numerical tolerance of 10^{-4}), then proceed to the next step; otherwise, return to step IV using the new value function. Iterate the value function until convergence is obtained and then proceed to step VI.

VI. Given the converged default set and the initial guess of the recovery rate, we compute the new short-term and long-term prices q_1^s and q_1^l according to the international investors' problem. If the new prices are sufficiently close to the old ones, stop iterating on prices and continue; otherwise update the price functions using an adaptive successive over-relaxation (ASOR) procedure that combines data from iterations i and $i-1$, as described in Mendoza and Smith (2002). Then repeat step IV and V using the new price functions and iterate until prices converge.

VII. Given the converged prices and value functions, solve the Nash bargaining problem for the new debt recovery rate α_1 by the Newton method. If the new recovery schedule is sufficiently close to the old one, we stop iterating; otherwise, we return to step IV.

To accelerate convergence, we follow Chow and Tsitsiklis (1991) in that we start with a coarse grid and solve the whole problem, then we refine the grid with more points (with linear interpolation to fill the unknown values), and resolve the whole problem.

Appendix B

Appendix to Chapter 2

Proof of Proposition 1. Suppose B is the proposer in the current period, then the equilibrium debt recovery rate is determined as follows

$$\alpha^{B^*}(y, b) = \operatorname{argmax} V_B^{prop}(y, b) \quad (\text{B.1})$$

$$s.t. \quad V_B^{prop}(\alpha^{B^*}(y, b)) \geq V_B^{pass}(\alpha^{B^*}(y, b))$$

$$V_L^{acpt}(\alpha^{B^*}(y, b)) \geq V_L^{rejt}(\alpha^{B^*}(y, b))$$

If no such debt recovery rate α^{B^*} exists, B chooses to pass.

So in equilibrium, if B chooses to propose, the debt recovery rate α^{B^*} ensures $V_B^{prop} \geq V_B^{pass}$ and $V_L^{acpt} \geq V_L^{rejt}$. Then what would L do if she is the proposer in the current period? In this case, there is at least one debt recovery rate that makes “proposing” better than “pass” for the lender himself. This debt recovery rate is just α^{B^*} , and the value of proposing is equal to V_L^{acpt} above and the value of pass is the same as V_L^{rejt} above. Given that $V_L^{acpt}(\alpha^{B^*}) \geq V_L^{rejt}(\alpha^{B^*})$, when the lender is the proposer, “proposing” would also be chosen.

If the proposer B chooses to wait, it means there is no α^{B^*} satisfying $V_B^{prop} \geq V_B^{pass}$ and $V_L^{acpt} \geq V_L^{rejt}$. In these circumstances, if the identity of proposer is changed to be the lender, then the lender cannot find a feasible debt recovery rate to settle the debt either. Therefore, the lender would also choose “pass”.

When the lender is the real proposer, we can show the same thing by the logic above. Thus, Proposition 1 holds.

Computation Algorithm

We discretize the spaces of asset holdings, defaulted debts and debt recovery rates using three 201-point grids. The limits of the asset space are set to ensure that they do not bind in equilibrium. The space of defaulted debts is set to be $[-2, 0]$, and the lower bound is never hit in simulations. The debt recovery rate has an upper bound of 1 and a lower bound of 0. The limits of endowment are set to be ten standard deviations above and below the mean output, which is normalized to 1. We then approximate the distribution of endowment shocks with a 51-point Markov chain using a quadrature based procedure (Hussey and Tauchen (1991)). Then we employ the following algorithm:

I. Guess initial payoffs for the sovereign and the debt holder. Our guess is $\Delta_{B0} = \Delta_{L0} = 0$.

II. Guess an initial price for bond. Our guess is the risk-free price: $q_0 = \frac{1}{1+r}$.

III. Guess an initial value for the value function. Our guess is 0.

IV. Given the initial guesses of Δ_{B0} , value function, and q_0 , we solve the country's optimization problem when it repays maturing debt. We use Bellman equation iteration to find the value function V_0^R and the optimal choices for assets. We then compute the value function for default, V_0^D . By comparing V_0^R and V_0^D , we are able to obtain the government's default decisions for all y and b . We also obtain the default set by finding the cut-off endowment shock at which $V_0^R = V_0^D$.

V. If the newly-obtained value function is close enough to the old one (we use

a numerical tolerance of 10^{-5}), then we proceed to the next step; otherwise, return to step IV using the new value function. Iterate the value function until convergence is obtained and then proceed to step VI.

VI. Given the converged default set and the initial guess of the lender's payoff Δ_{L0} , we compute the new price q_1 according to the international investors' problem. If the new price is sufficiently close to the old one, stop iterating on price and continue; otherwise update the price function, repeat step IV and V using the new price function and iterate until price converges.

VII. Given the converged price function and value function, solve the stochastic bargaining game as follows:

(i) When B is the proposer, we first calculate the value of "pass" by B , and then the value of rejecting (equivalent to waiting) by L . These values do not depend on the proposal made by B . We then start a grid search of the debt recovery rate from the lower bound and calculate the value of "proposing" corresponding to each debt recovery rate. Once we find the maximum debt recovery rate that ensures the value of proposing to be larger than, or equal to the value of pass, stop the grid search and record this maximum debt recovery rate. We then calculate the values of accepting by the lender for each debt recovery rate from the lower bound to the maximum one we have just recorded, and find the smallest one that makes accepting (at least weakly) better than rejecting. For some y and b , such debt recovery rate exists; for the others, no such debt recovery rate exists. Thus, we record the debt recovery rate in the former case and let $\alpha^B = 0$ in the latter case to denote the choice of pass. Based on the newly found borrower's choices of "proposing" and "pass"

and her debt recovery function, we update the borrower and the lender's payoffs.

(ii) When L is the proposer, we follow the same algorithm as in VII(i) except that we first find the minimum debt recovery rate the lender would propose and then do a grid search from this minimum debt recovery rate to the upper bound and find the maximum one in this set that ensures the borrower to accept the proposal. In this step, we can also find the “proposing/pass” choices of the lender and the equilibrium debt recovery function. We then update the payoff functions again. If these payoff functions are sufficiently close to the earlier ones, we stop iterating; otherwise, we return to step IV and redo the whole process using the new payoff functions until convergence is achieved.

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