ABSTRACT

Title of dissertation: ESSAYS ON FINANCIAL REGULATION IN MACROECONOMICS

Julien Yonathan Bengui, Doctor of Philosophy, 2012

Dissertation directed by: Professor Anton Korinek and Professor Enrique G. Mendoza
Department of Economics

This dissertation investigates two aspects of how to regulate the financial sector optimally in order to increase macroeconomic stability and mitigate the risk of future financial crises.

Chapter 1 analyzes the desirability of international coordination in financial regulation. It develops a two-country model of systemic liquidity risk-taking in which financial market imperfections provide a rationale for macro-prudential regulation. In the model, curbing liquidity risk-taking via regulation lowers the price of liquidity during financial crises and thereby reduces the costs associated with market incompleteness. But regulation also entails costs in the form of distortions to productive investment decisions. The discrepancy between the domestic dimension of the costs and the global dimension of the benefits of regulation generates free-riding incentives among regulators operating in different countries. The theory predicts that absent international coordination, national authorities are tempted to regulate their financial systems in a way that results in excessive illiquidity. It therefore speaks in favor of a stronger global coordination of banking regulation.

Chapter 2 analyzes the social optimality of private debt maturity choices. It studies debt maturity decisions in a dynamic macroeconomic model in which financial frictions give rise

to systemic risk in the form of amplification effects. Long-term liabilities provide insurance against shocks to the asset side of the balance sheet, but they come at an extra cost. The debt maturity structure therefore maps into an allocation of macroeconomic risk between lenders and leveraged borrowers, and fundamental shocks propagate more powerfully in the economy when the maturity is shorter. The market equilibrium is not constrained efficient as borrowers fail to internalize their contribution to systemic risk and take on too much short-term debt in a decentralized economy. The theory indicates that a tax on short-term debt – a form of macroprudential policy – leads to Pareto improvements and results in less volatile allocations and asset prices.

ESSAYS ON FINANCIAL REGULATION IN MACROECONOMICS

by

Julien Yonathan Bengui

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2012

Advisory Committee:
Professor Anton Korinek, Co-Chair
Professor Enrique G. Mendoza, Co-Chair
Professor Albert S. (Pete) Kyle
Professor John Shea
Professor Carlos A. Végh

Dedication

À ma grand-maman Rebecca, mes parents, Samy et Francine, mon frère Yann-Yves, et ma compagne Bich.

Acknowledgments

This dissertation could not have been written without the advice and encouragement of my dissertation advisors at the University of Maryland.

I am particularly grateful to Anton Korinek and Enrique Mendoza, the co-chairs of my dissertation committee, for their unflagging devotion, steady encouragement and endless support. Their exceptional dedication to advising and mentoring surpassed all my expectations. I thank Carlos Végh and John Shea for insightful comments on my work, unconditional support and generosity with their time. I also thank Pete Kyle for kindly agreeing to serve as an external advisor on my committee.

My doctoral research benefitted from support from several institutions. The Department of Economics and the Graduate School at the University of Maryland provided me with financial support, as did the Janggen-Pöhn-Stiftung and the Federal Reserve Bank of New York. The latter provided me with an outstanding research environment during the Summer preceding my job market. There, I owe a special thank to Rebecca Hellerstein and Tanju Yorulmazer for their kind support.

I also thank my classmates at the University of Maryland, in particular Sushant Acharya, Pablo Federico and Ha Nguyen, for their encouragement and friendship throughout my doctoral studies.

My parents, Samy and Francine, and my brother, Yann-Yves, provided me with unconditional love and support throughout my life. I am immensely grateful to them. I also warmly thank the Tran family for their continuous support. Above all, I thank my partner, Bich, for her endless love, care and understanding during my graduate school experience.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. MACRO-PRUDENTIAL POLICY COORDINATION AND GLOBAL REGULATORY SPILLOVERS

1.1 Introduction

The financial globalization process that started in the early 1980s was accompanied early on by attempts to harmonize banking regulation internationally. The original motivation for the first Basel accords (Basel I) was twofold: to ensure the stability of the global financial system and to eliminate distortions to competition arising from heterogenous regulatory regimes. Over time, national competitiveness concerns turned more dominant (Tarullo 2008), and they became the focus of the academic literature analyzing the international linkages relevant for banking regulation (see e.g. Acharya 2003, Dell Arricia and Marquez 2006). This focus on competitiveness issues raises the question of whether the goal of financial stability *in and of itself* requires an international coordination of banking regulation. This paper investigates this question in a model where agents' contribution to systemic risk calls for a *macro-prudential* approach to regulation.

We refer to liquidity as the aggregate amount of resources set aside to satisfy potential needs for funds. Liquidity has public goods properties during periods of market stress (Shin 2010). By limiting agents' exposure to liquidity risk, regulators can reduce the scarcity of liquidity during crises and alleviate credit crunches. But national regulators fail to adequately internalize the share of positive externalities associated with global liquidity that operates across borders. Consequently, individual countries try to free-ride on the foreign provision of liquidity. A lack of international coordination therefore results in insufficient macro-prudential regulation. This underprovision of ex-ante regulation leads to more severe and more costly financial crises ex-post.

The analysis is undertaken in the context of a two-country version of a model of liquidity demand, in the spirit of Holmstrom and Tirole (1998) and Caballero and Krishnamurthy (2001).[1] Ex-ante identical agents invest in risky long-term projects that may require an additional liquidity injection along the road. Liquidity shocks are imperfectly correlated across countries, implying opportunities for international risk-sharing. Because of moral hazard, cross-country insurance against these shocks is limited, but agents can set aside liquid resources ex-ante by investing in a short-term asset (i.e. self-insure), or alternatively, they can borrow ex-post on an international spot market up to some limit. In this environment, market incompleteness results in a constrained inefficiency of the competitive equilibrium. Agents fail to internalize that their collective investment choice affects the severity of a potential credit crunch, and they underinvest in short-term assets in equilibrium. Curbing agents' exposure to liquidity risk via prudential regulation can restore constrained efficiency. We compare three alternative allocation mechanisms: (a) the laissez-faire outcome (competitive equilibrium), (b) the constrained efficient outcome achieved by setting regulation cooperatively at a global level, and (c) the equilibrium of a policy game where regulation is chosen non-cooperatively by welfare maximizing national authorities.

[1] See also the related consumer liquidity demand models of Diamond and Dybvig (1983), Jacklin (1987), Bhattacharya and Gale (1987), Hellwig (1994), von Thadden (1999) and others.

The inefficiency of decentralized investment decisions in the model results from a pecuniary externality operating via the price of global liquidity, i.e. the international interest rate, in a crisis. This interest rate depends on the scarcity of liquidity. A marginal increase in the liquidity of the representative agent's ex-ante investment portfolio from its competitive equilibrium level lowers the interest rate and causes a redistribution of wealth from lenders to borrowers in a crisis. Due to the incompleteness of markets, borrowers value liquidity more highly than lenders ex-post. Since ex-ante, any agent could end up with a high or with a low valuation of liquidity, a marginally lower interest rate achieves a redistribution of resources from low valuation states of nature to high valuation states. Such a redistribution partially substitutes for missing risk markets and leads to a first order welfare gain. A global planner maximizing a representative agent's welfare would require agents to tilt their investment in favor of short-term assets, with the consequence of alleviating credit crunches when a crisis occurs. But while such regulation brings about an improvement in exchange efficiency, it distorts productive investment decisions away from their competitive level. In choosing the optimal extent of regulation, a global planner thus trades off an improvement in exchange efficiency with a deterioration in production efficiency in the world economy.

National planners, who set regulation non-cooperatively to maximize the welfare of a domestic representative agent, do perceive the dependence of the severity of a potential credit crunch upon ex-ante investment choices. But in contrast to a global planner, they attempt to shift surplus in favor of residents rather than restore constrained efficiency. In particular, national planners recognize that more domestic ex-ante liquidity hoarding (i.e. more domestic investment in short-term assets resulting from tighter regulation) would alleviate a foreign credit crunch by lowering the interest rate, which redistributes resources in favor of foreign residents in states of nature where foreigners with a high valuation of liquidity borrow from domestic lenders with a low valuation of liquidity. But national planners derive no benefits from alleviating credit crunches abroad. Since the pecuniary externality operates across borders, national planners do not internalize the full exchange efficiency benefits of regulation. At the same time, the production efficiency costs of regulation are incurred domestically. Consequently, national planners generally fall short of imposing the optimal extent of regulation. This underprovision of ex-ante regulation results in more severe and more costly financial crises ex-post, in the form of larger interest rate spikes and more forced liquidation of real investments. In fact, national planners' incentives to manipulate state-contingent terms of trade (i.e. interest rates) in favor of residents can cause the equilibrium of the non-cooperative regulation game to feature even less liquid and more risky investments than the laissez-faire benchmark. When this occurs, welfare is lower with uncoordinated regulation than under laissez-faire. In other words, uncoordinated regulation can be worse than no regulation at all.

The underprovision of regulation can be interpreted as a beggar-thy-neighbor outcome. Global liquidity mainly benefits distressed countries during crises, so it is attractive for a country to take advantage of foreign liquidity provision when one is distressed, but not to provide liquidity to distressed foreigners when one is intact. Viewed through the lens of the literature on international policy coordination (see e.g. Cooper 1985 and Persson and Tabellini 1995), the result can also be interpreted as arising from the attempt by national planners to make use of monopoly and monopsony power in the market for international liquidity during crises. Alternatively, the underprovision of regulation can be seen as resulting from a variation of the hold up problem (Grout 1984 and Tirole 1986). In the model, the returns from “investing” in a sound domestic financial system (via prudential regulation) within the trading relationship between the two countries exceed the returns outside the trading relationship. And once the investment is sunk (the regulation enacted), the investor (regulator) has to share the gross return with its trading partner (the other country). As in the classical hold up problem, this anticipated expropriation leads to underinvestment ex-ante.

The constrained inefficiency of competitive equilibria in incomplete markets economies is well known since the work of Hart (1975), Stiglitz (1982) and Geanakoplos and Polemarchakis (1986). So too is the suboptimality of uncoordinated macroeconomic policies since at least Johnson (1965) and Hamada (1976). The novelty of the present paper is to analyze in a common framework the interplay between distortions arising from market incompleteness and those resulting from openness and countries' monopoly and monopsony power in global markets. The analysis outlines the close link between the mechanics of policy incentives arising from these two kinds of distortions. A constrained global planner internalizes pecuniary externalities in much the same way as strategically acting governments do. But the former does so to improve exchange efficiency and reduce the cross-sectional wedges between marginal rates of substitutions caused by incomplete markets. In contrast, the latter use market power to shift surplus in their favor, generally at the cost of widening these wedges and reducing overall efficiency.

The model offers predictions about the international spillover effects of changes in regulation. Starting from the laissez-faire equilibrium, the introduction of a small regulation in one country increases risk-taking in the other country. The transmission channels work through the lowering of interest rates during crises brought about by the extra amount of liquidity set aside in the regulated country. The model also predicts that macro-prudential policies are strategic substitutes across countries, as a country's tightening of regulation, by increasing the amount of global liquidity, reduces the benefits of regulation for the other country. Finally the model delivers the result that, starting from the laissez-faire equilibrium, a unilateral adoption of liquidity regulation can be welfare reducing for the regulated country and welfare improving for the unregulated country.

The analysis is done in the context of a model where agents can borrow internationally when hit by liquidity shocks. We show that all results fully carry over to a setup where agents raise liquidity during crises by selling long-term assets rather than by borrowing. In that setup, the pecuniary externality works through an asset price and takes the form of cash-in-the-market pricing and fire-sale externalities (Allen and Gale 1998). The coordination problem between regulators can hence be alternatively interpreted as arising from a failure to commit mutually to supporting asset prices during crises.

At an abstract level, the regulation game analyzed in this paper corresponds to a liquidity demand model with two large agents. It is often argued that market power mitigates the harm caused by systemic externalities, because large agents partly internalize the effect of their actions on prices. We show here that this need not be the case. Whether market power attenuates or amplifies the distortions caused by market incompleteness crucially depends on the direction of the pecuniary externalities imposed on ex-post identical agents. Market power tends to attenuate market incompleteness distortions when these externalities are negative (like fire-sale externalities), but it tends to amplify distortions when the externalities are positive (like cash-in-the-market pricing externalities). In situations similar to our model, where agents impose both negative and positive externalities on ex-post identical agents, the effect of market power on market incompleteness distortions can potentially go either way.

Literature

The paper fits into a recent research agenda that motivates financial regulation from a second best perspective in incomplete markets environments. It is most closely related to the liquidity regulation approach of Allen and Gale (2004) and Farhi, Golosov, and Tsyvinski (2009), and to the sudden stop prevention analysis of Caballero and Krishnamurthy (2001, 2004) for emerging countries. As in these papers, the market failure calling for government intervention in the present paper originates from a pecuniary externality operating on a spot market for liquidity.[2] But our paper stands out from these by explicitly formulating a multi-country framework. The model structure is therefore closer to that of Castiglionesi, Feriozzi, and Lorenzoni (2010), whose main focus is on the positive implications of financial integration for the liquidity of banks' portfolios and the magnitude of interest rate spikes in crises. They briefly touch upon normative issues by solving numerically for the constrained efficient allocation, but ignore the potential coordination problem that arises in attempting to implement this allocation when regulation is set at a national level. In contrast, we consider the incentives faced by rationally acting national regulators, analytically characterize the equilibrium of the policy game and compare it with both the laissez-faire outcome and the constrained efficient allocation achieved by coordinating regulation globally.

In modeling the strategic interaction among national governments, this paper follows the game theoretic approach to macroeconomic policy coordination pioneered by Hamada

[2] Lorenzoni (2008) and Korinek (2011b) emphasize a related pecuniary externality working through asset prices and fire-sale spirals during financial crises. Bianchi (2011), Jeanne and Korinek (2011), Bianchi and Mendoza (2010) and Bengui (2011) investigate the quantitative relevance of these pecuniary externalities in infinite horizon macroeconomic models.

(1974, 1976, 1979).[3] Governments are assumed to set policies to maximize national welfare, while taking into account their power to affect international prices. By attempting to use market power to shift surplus in favor of domestic residents, governments generally end up in Pareto-inferior equilibria. Our results parallel those of studies where distortions induced by openness can overturn the direction of a desirable policy intervention, such as Corsetti and Pesenti (2001). In their model, monopoly distortions in production together with nominal rigidities make unexpected monetary expansions welfare improving (as in Blanchard and Kiyotaki 1987 and Ball and Romer 1989) when carried out simultaneously at home and abroad. But the same policy can be welfare reducing for a country acting in isolation because of adverse endogenous terms of trade movements. Similarly, in our model, financial market imperfections make prudential regulation unambiguously welfare improving when introduced jointly at home and abroad. But terms of trade movements working through the interest rate during crises and associated spillover effects can make the unilateral introduction of such a policy welfare reducing for a given country.

Our paper is also related to Acharya (2003), who studies the consequences of an international convergence of bank capital requirements in the presence of heterogenous national closure policies, and Dell Arricia and Marquez (2006), who analyze the incentives for national bank regulators to form a regulatory union. In both of those papers, regulatory spillovers operate through changes in the degree of competition faced by banks on international loan markets during tranquil times. In our model, this competition channel is absent and international spillovers only arise through pecuniary externalities operating in a global market

[3] Since Hamada's work, there has been a large literature on macroeconomic policy coordination and interdependence. See the reviews in Cooper (1985) and Persson and Tabellini (1995).

for liquidity during financial crises. Out paper therefore stands out in focussing on an international coordination motive directly linked to the financial stability objective of banking regulation.

The paper is structured as follows. The model is presented in section 1.2. Global and national regulations are analyzed in section 1.3. Section 1.4 works out the implications of the framework for the international spillovers of regulatory policies. Section 1.5 presents an alternative model in which the ex-post intermediation of funds occurs via an asset market rather than via a credit market. Section 1.6 concludes.

1.2 A two-country model of liquidity demand

This section presents the model in which the coordination problem between national regulators is analyzed. The environment is specified in section 1.2.1, and the competitive equilibrium is characterized in section 1.2.2.

1.2.1 Preferences, technology and markets

Agents, Time and Preferences The world economy is composed of two countries, indexed by $j \in \{A, B\}$, that are ex-ante identical with respect to preferences, endowments and technology. Each country is populated by a continuum of identical agents. Time lasts for three periods $t = 0, 1, 2$, and consumption takes place at date 2. Agents' preferences over date 2 consumption are represented by an increasing, concave and twice continuously differentiable utility function $u(\cdot)$.

Technology Each agent is born with an endowment of one unit of the consumption good at date 0, and decides how to allocate this endowment between investment in a risky and illiquid project k and investment in a safe storage technology ℓ. With probability $1-\alpha$, all projects in country j remain *intact*. An intact project does not require additional funds at date 1 and yields a date 2 return of $A > 1$. With probability α, a project becomes *distressed* and necessitates a renewed investment of one good per unit at date 1 for the project to yield a date 2 return of A. Each unit not shored up at date 1 yields a reduced date 2 return of $r < 1$. Distressed agents have the possibility to scale down investment at date 1. For an initial project of size k, a continuation scale θ results in a date 1 cost of θk and a date 2 return of $rk + \theta\Delta k$, where $\Delta \equiv A - r$. The storage technology yields one date $t+1$ good per date t unit invested, and can be accessed both at date 0 and date 1.

Uncertainty All the uncertainty is resolved at date 1. Liquidity shocks are imperfectly correlated across the two countries. The sample space is given by $\Omega = \{(i,i),(i,s),(s,i),(s,s)\}$, where in state (i,s) country A is intact and country B is distressed.The probability mass function $\pi : \Omega \to [0,1]$ assigns a probability to each state $\omega \in \Omega$.

Markets The imperfect correlation of liquidity shocks across countries creates opportunities for international risk-sharing. The assumption maintained throughout the paper, though, is that only non-state contingent bonds can be traded across borders. This can be motivated by the lack of verifiability of country-specific shocks.[4]

Agents can substitute for risk-sharing by borrowing and lending at date 1 on an inter-

[4] Because agents are ex-ante identical, non-contingent claims as well as claims contingent on the states (i,i) and (s,s) would not be traded at date 0 in equilibrium, so these markets can be abstracted from for the sake of simplicity.

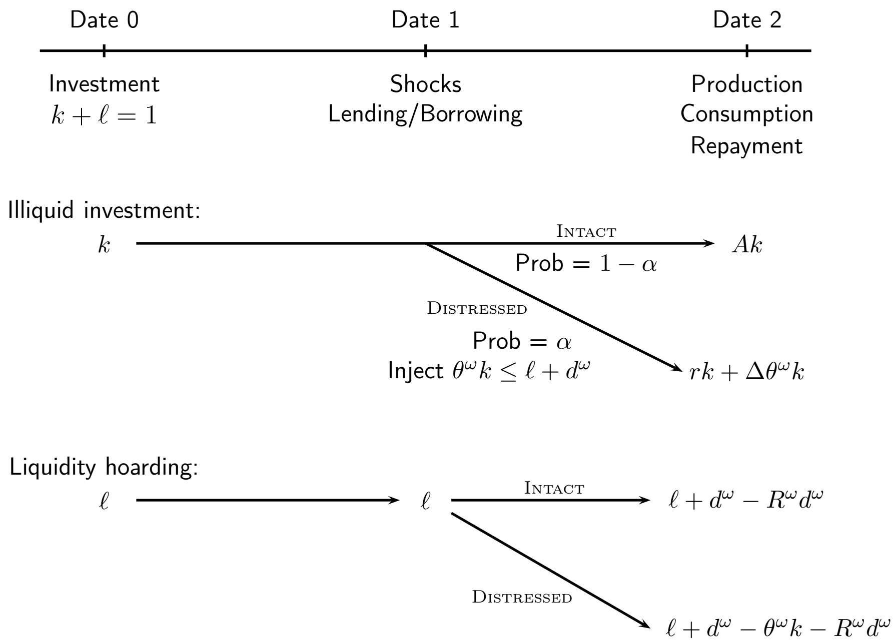

Fig. 1.1: Time line.

national spot market at interest rate R^ω. Distressed agents borrow d_j^ω, while intact agents lend $-d_j^\omega$. The size of loans on this market is constrained by distressed agents' ability to commit to repay their debt at date 2. We assume lenders can only repossess a fraction κ of distressed agents' date 2 output and that any excess production based on reinvestment at date 1 is neither observable nor verifiable. This implies that borrowing must satisfy the collateral constraint $R^\omega d_j^\omega \leq \kappa r k_j$. The time line is represented in Figure 1.1.

Assumptions on parameters

Assumption 1 (Yield of illiquid project)**.** *The yield on the illiquid project satisfies* $1 < A \leq \frac{3}{2}$ *and* $\Delta \equiv A - r > 1$.

The assumption that $A > 1$ ensures that the illiquid project has a higher yield than the liquid asset in normal times, albeit not excessively so. The assumption that $\Delta > 1$, on the other hand, captures the idea that distressed firms have a high marginal value of investment during crisis times. It implies that shoring up an additional unit of a distressed project, if feasible, is always socially desirable.

Assumption 2 (Probability of crisis)**.** *The probability α of a project becoming distressed satisfies $\underline{\alpha} < \alpha < \bar{\alpha}$, where $\underline{\alpha}$ is given by*

$$\underline{\alpha} = \frac{(A-1)u'(A)}{(A-1)u'(A) + (\Delta - r)u'(r)}$$

and $\bar{\alpha}$ is the smallest positive root of the quadratic equation

$$[(1-\alpha)^2+\rho\alpha(1-\alpha)](A-1)u'\Big(\frac{2A}{3}+\frac{1}{3}\Big)+(1-\rho)\alpha(1-\alpha)(A-\Delta)u'\Big(\frac{2A}{3}+\frac{\Delta}{3}\Big)+\alpha(r-\Delta)u'\Big(\frac{2r}{3}+\frac{\Delta}{3}\Big) = 0.$$

Assumption 2 captures the fact that financial crises are low probability events, but that they are likely enough to induce precautionary behavior in the form of some liquidity hoarding. $\alpha > \underline{\alpha}$ guarantees that crises are likely enough that agents find it optimal to hoard a positive amount of liquid assets, while $\alpha < \bar{\alpha}$ ensures that crisis are rare enough that when a crisis hits one of the two countries, the global aggregate amount of liquidity hoarded ex-ante is not sufficient to shore up all illiquid projects in the distressed country. Together, these two assumptions imply that there is partial liquidation in a crisis.

1.2.2 Competitive equilibrium

We start by considering equilibrium in the date 1 spot market for given date 0 decisions. We will then proceed backwards to solve for the competitive equilibrium and regulated equilibria at date 0.

Date 1 spot market equilibrium

The date 1 value of an intact agent in country j is given by

$$V_i^{\omega}(k_j, \ell_j) \equiv \max_{0 \leq -d_j^{\omega} \leq \ell_j} u\Big(Ak_j - d_j^{\omega}(R^{\omega} - 1) + \ell_j\Big). \tag{1.1}$$

The intact agent's date 2 consumption in (1.1) is given by the sum of the return on its illiquid project Ak_j, the return on the loan made on the date 1 spot market $-R^{\omega} d_j^{\omega}$, and the return on the funds invested at date 1 in the storage technology $\ell_j + d_j^{\omega}$. Without loss of generality, we assume that intact agents can only lend on the date 1 spot market. Their lending capacity is limited by their date 1 liquid resources ℓ_j.

The form of the objective in (1.1) implies a simple loan supply schedule for intact agents. For $R^{\omega} < 1$, intact agents do not want to lend at all. At $R^{\omega} = 1$, they are indifferent between lending any amount between 0 and ℓ_j. Finally, when $R^{\omega} > 1$ they are willing to lend all their available liquidity ℓ_j.

The date 1 value of a distressed agent in country j is given by

$$V_s^{\omega}(k_j, \ell_j) \equiv \max_{\theta_j^{\omega}, d_j^{\omega}} u\Big(rk_j + \Delta\theta_j^{\omega} k_j - \theta_j^{\omega} k_j + \ell_j - (R^{\omega} - 1)d_j^{\omega}\Big) \tag{1.2}$$

subject to

$$\begin{aligned} \theta_j^\omega k_j &\leq \ell_j + d_j^\omega & (1.3) \\ R^\omega d_j^\omega &\leq \kappa r k_j & (1.4) \\ \theta_j^\omega &\leq 1 & (1.5) \end{aligned}$$

A distressed agent's date 2 consumption in (1.2) is the sum of the return on its illiquid project $rk_j + \Delta\theta_j^\omega k_j$ and the return on the funds invested at date 1 in the storage technology $\ell_j + d_j^\omega - \theta_j^\omega k_j$, minus the debt repayment $R^\omega d_j^\omega$. (1.3) is the date 1 budget constraint indicating that reinvestment $\theta_j^\omega k_j$ needs to be covered by the sum of ex-ante liquidity hoarding ℓ_j and ex-post borrowing d_j^ω. (1.4) is a collateral constraint, and (1.5) indicates that investment cannot be scaled up at date 1.

Given the assumption that $\Delta > 1$ and the fact that θ_j^ω and d_j^ω enter additively in the expression of consumption, the loan demand and optimal continuation scale of distressed agents take simple forms. For $R^\omega < 1$, the agents hit their collateral constraint (1.4). For $1 \leq R^\omega < \Delta$, they borrow the minimum of the amount they need to salvage all their assets, $k_j - \ell_j$, and their borrowing limit, $\kappa r k_j / R^\omega$. At $R^\omega = \Delta$, they are indifferent between borrowing any amount between 0 and $\min\left\{k_j - \ell_j, \kappa r k_j/\Delta\right\}$. Finally, for $R^\omega > \Delta$, they do not want to borrow at all. The optimal continuation θ_j^ω is accordingly given by $\min\left\{1, \frac{\ell_j}{k_j} + \frac{\kappa r}{R^\omega}\right\}$ for $R^\omega < \Delta$, by any amount between $\frac{\ell_j}{k_j}$ and $\min\left\{1, \frac{\ell_j}{k_j} + \frac{\kappa r}{\Delta}\right\}$ when $R^\omega = \Delta$, and by $\min\left\{1, \frac{\ell_j}{k_j}\right\}$ for $R^\omega > \Delta$. The loan supply schedule of intact agents and the loan demand schedule of distressed agents are displayed in Figure 1.2. In the right panel, the loan demand curve is drawn for different values of κ, with the curves to the left associated with a lower κ.

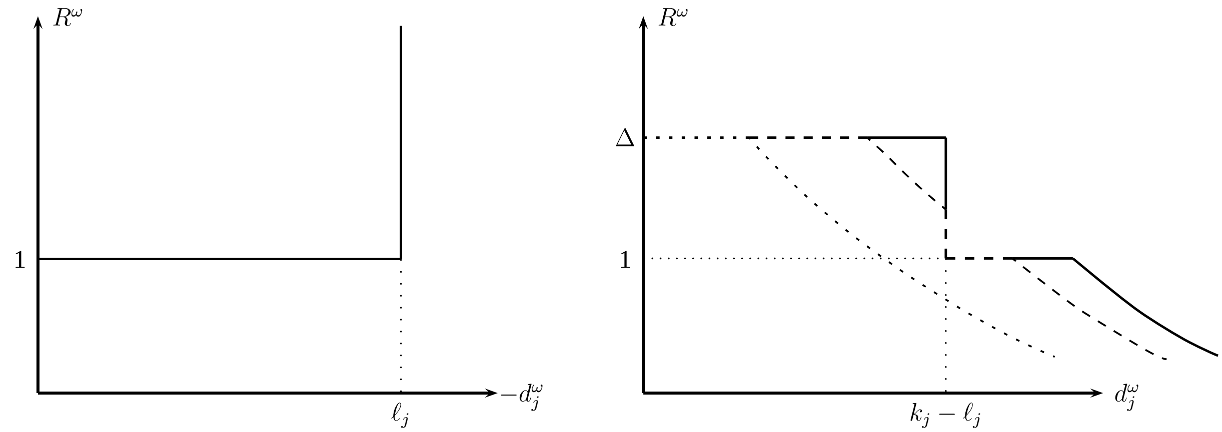

Fig. 1.2: Supply by intact (left) and demand by distressed (right) on date 1 spot market.

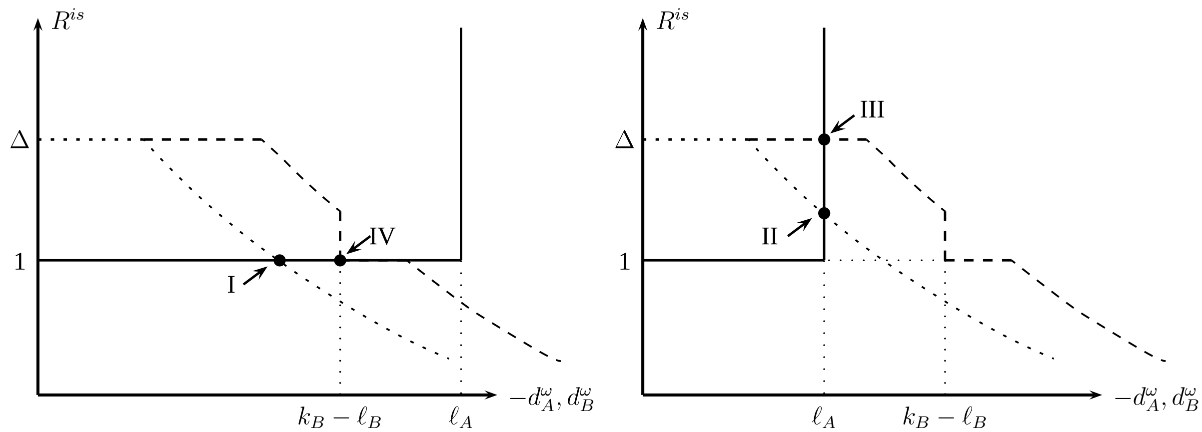

Fig. 1.3: Regions for spot market equilibrium in state (i, s).

Assumptions 1 and 2 guarantee that agents do not find it optimal to hoard more liquidity at date 0 than what is needed to shore up their own entire project were they to become distressed at date 1. The date 1 spot market equilibrium is therefore simply given by $R^\omega = 1$ and $d_A^\omega = d_B^\omega = 0$ in state (i, i), and by $R^\omega = \Delta$ and $d_A^\omega = d_B^\omega = 0$ in state (s, s).

In states of world where one country is intact and the other is distressed, i.e. in (i, s) and (s, i), the spot market equilibrium can a priori fall in four distinct regions, depending on where the loan demand and loan supply curves intersect, similarly to Caballero and Krishnamurthy (2001). These four types of equilibria are displayed in Figure 1.3.[5] As

[5] The equilibria are drawn for state (i, s), where country A is intact and country B is distressed, but

		Global liquidity	
		ample	*scarce*
Distressed country	*constrained*	region I	region II
	unconstrained	region IV	region III

Tab. 1.1: Regions for equilibrium in state (i, s) or (s, i).

represented in table 1.1, the regions can be categorized according to (a) whether global liquidity is ample or scarce, and (b) whether the distressed country's collateral constraint is slack or binds. When global liquidity is ample (regions I and IV), the equilibrium interest rate is low: $R^{is} = 1$. In region I, the distressed country is constrained, and there is partial liquidation ($\theta_B^{is} = (\kappa r k_B + \ell_B)/k_B < 1$). In region IV, the distressed country is unconstrained, and borrowing is high enough to allow for full continuation and avoid partial liquidation ($\theta_B^{is} = 1$). When, on the other hand, global liquidity is scarce (regions II and III), the equilibrium interest rate rises: $R^{is} > 1$. In these regions, all the global liquidity is used to shore up the distressed country's assets, but the aggregate shortage of date 1 resources results in partial liquidation ($\theta_B^{is} = (\ell_A + \ell_B)/k_B < 1$). In region II, the distressed country is constrained and can only pledge to offer to lenders a return $R^{is} = \kappa r k_B/\ell_A$, lower than the social marginal return Δ. In region III, on the other hand, the distressed country is unconstrained and the lender country can be compensated at the social marginal return of liquidity in consumption goods terms, $R^{is} = \Delta$. As is common in models with borrowing constraints (i.e. Bernanke and Gertler 1989, Kiyotaki and Moore 1997), a wedge between

equilibria in state (s, i) take identical forms, with the subscripts A and B interchanged.

the internal and external rate of return on investment arises in regions I and II.

The relevant region for equilibrium on the date 1 spot market in states (i, s) and (s, i) depends on date 0 choices in the two countries. The next section provides conditions on parameters under which date 0 choices in a competitive equilibrium lead to particular regions. The focus of the analysis will henceforth be on parameter configurations which lead the competitive equilibrium to be in region II.

Decentralized equilibrium

At date 0, an agent in country j takes the schedule of date 1 interest rates R^ω as given and solves

$$\max_{k_j, \ell_j} \sum_{\omega \in \Omega} \pi^\omega V^\omega(k_j, \ell_j) \tag{1.6}$$

subject to

$$k_j + \ell_j = 1, \tag{1.7}$$

where the date 1 value function $V^\omega(k_j, \ell_j)$ is equal to $V_i^\omega(k_j, \ell_j)$ if the agent is intact and to $V_s^\omega(k_j, \ell_j)$ if the agent is distressed. The first-order condition

$$\sum_{\omega \in \Omega} \pi^\omega \frac{\partial V^\omega(k_j, \ell_j)}{\partial k_j} = \sum_{\omega \in \Omega} \pi^\omega \frac{\partial V^\omega(k_j, \ell_j)}{\partial \ell_j} \tag{1.8}$$

together with the date 0 budget constraint (1.7) characterize the agent's optimal choice. A *competitive equilibrium* of the model consists of date 0 decisions $(k_j, \ell_j)_{j \in \{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega \in \Omega, j \in \{A,B\}}$ and prices $(R^\omega)_{\omega \in \Omega}$, such that (a) given prices, the decisions solve the problems in (1.6), (1.1) and (1.2); and (b) markets clear.[6] In what follows, the values of k and ℓ in a symmetric competitive equilibrium are denoted by k^{CE} and ℓ^{CE}.

Assumption 2 guarantees that the probability of a crisis is not large enough to produce a situation where the aggregate amount of liquidity set aside in a symmetric competitive equilibrium, $2\ell^{CE}$, is sufficient to avoid any liquidation in the states of the world where one country is intact and the other is distressed. In other words, assumption 2 ensures that $k^{CE} > 2/3$ in a symmetric competitive equilibrium. This implies that we can focus on situations in which only regions I, II or III in states (i, s) and (s, i) can arise in equilibrium.

To gain further insights into the properties of a competitive equilibrium of the model, it is useful to look at the agents' value function. In state ω, the value function is given by

$$V_i^\omega(k_j, \ell_j) = u\Big(Ak_j + R^\omega \ell_j\Big) \tag{1.9}$$

for an intact agent, and by

$$V_s^\omega(k_j, \ell_j) = u\Big(rk_j + (\Delta - R^\omega)\frac{\kappa r k_j}{R^\omega} + \Delta\ell_j\Big) \tag{1.10}$$

for a distressed agent. The terms in (1.9) are straightforward to interpret. For an intact agent, illiquid assets yield a return of A, while liquid assets yield a return of R^ω, with $1 \leq R^\omega \leq \Delta$. The marginal value of the illiquid asset in terms of date 2 consumption (rather than date 2 utility) is higher than that of the liquid asset. The terms in (1.10) are similarly straightforward. For a distressed agent, each unit of illiquid assets yields a baseline return of

[6] The continuation scale of an intact agent is by definition set to $\theta_j^\omega = 1$.

r, plus a net return of $\Delta - R^\omega$ on the $\kappa r / R^\omega$ of external financing raised against collateral, whereas a unit of liquid assets allows the continuation of one unit of illiquid assets, yielding a return of Δ. The marginal value of the liquid asset is higher than that of the illiquid asset.[7] Given concave utility, from the perspective of period 0 the illiquid asset is a bad hedge, while the liquid asset is a good hedge.

It turns out that the equilibrium can be further characterized as falling into one of the aforementioned three regions, depending on the tightness of financial constraints κ. There are thresholds $\underline{\kappa}$ and $\bar{\kappa}$, such that

- for $\kappa < \underline{\kappa}$, the symmetric competitive equilibrium leads to region I, i.e. $\frac{2}{3} < k^{CE} < \frac{1}{1+\kappa r}$,
- for $\underline{\kappa} \leq \kappa \leq \bar{\kappa}$, the symmetric competitive equilibrium leads to region II, i.e. $\frac{1}{1+\kappa r} \leq k^{CE} \leq \frac{\Delta}{\Delta + \kappa r}$,
- for $\kappa > \bar{\kappa}$, the symmetric competitive equilibrium leads to region III, i.e. $\frac{\Delta}{\Delta + \kappa r} < k^{CE} < 1$.

Hence, very tight financial constraints lead to region I, mildly tight constraints lead to region II, and loose constraints lead to region III. The interest rate in states (i, s) and (s, i) is pictured as a function of the tightness of financial constraints in Figure 1.4.

For the remainder of the paper, we focus on the case in which the symmetric competitive equilibrium leads to region II, via the following assumption.

Assumption 3 (Tightness of financial constraints)**.** *The tightness of financial constraints* κ *satisfies* $\underline{\kappa} < \kappa < \bar{\kappa}$.

[7] The marginal value of the illiquid asset for a distressed agent is the highest when $R^\omega = 1$, in which case it is given by $r + (\Delta - 1)\kappa r = [(1 - \kappa) + \kappa \Delta] r \leq \Delta r < \Delta$.

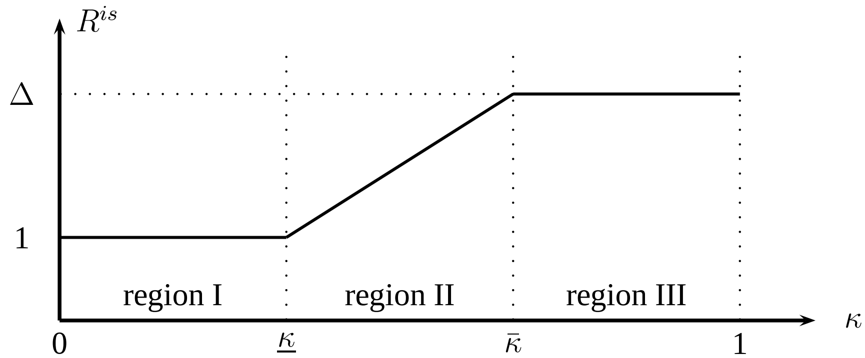

Fig. 1.4: Equilibrium interest rate in state (i, s) as a function of κ, in competitive equilibrium.

Assumption 3 guarantees that the symmetric competitive equilibrium leads to the interior of region II, i.e. that $\frac{1}{1+\kappa r} < k^{CE} < \frac{\Delta}{\Delta+\kappa r}$. The nature of the date 1 spot market equilibrium in region II provides a stylized description of actual liquidity crises in a globally integrated financial system in two key respects: (a) the aggregate shortage of liquidity results in a spike in the cost of borrowing, and (b) the pervasiveness of financial constraints causes a wedge between the internal and external marginal value of funds for distressed entities. Importantly, in this region, the price of liquidity (i.e. the interest rate) is a decreasing function of the amount of liquidity set aside ex-ante by lenders, and an increasing function of the amount of illiquid collateral owned by borrowers. Equilibrium in this region therefore captures the key intuition that the price of liquidity in crises decreases with the supply of it and increases with the demand for it. A lower ex-ante illiquid investment scale decreases the interest rate in crises, which benefits distressed borrowers more than it hurts intact lenders at the margin. Due to the incompleteness of markets, this pecuniary externality causes a market failure: the date 0 choices in a competitive equilibrium are not constrained efficient, as perturbating these allocations locally has first-order welfare effects via shifts in interest rates. The failure of the competitive equilibrium to be constrained efficient motivates the analysis of prudential regulation from a second best perspective.

1.3 Efficiency and planning problems

In assessing the welfare performance of competitive equilibria in incomplete markets economies, one is generally interested in whether the market system allocates resources efficiently given the set of markets operating (see Stiglitz 1982). This section analyzes alternative allocation mechanisms and compares their welfare properties with those of the decentralized equilibrium described in section 1.2.2. Sections 1.3.1 and 1.3.2 characterize the allocations resulting from global and national planners making investment decisions subject to the same set of enforcement and informational frictions as private agents. These allocation mechanisms are interpreted as regulated equilibria, as in Allen and Gale (2004).

1.3.1 Constrained global planner

Given the presumed failure of the first welfare theorem, it is natural to ask how a constrained social planner would want to regulate date 0 investment decisions. To this end, we start by considering a global planner who maximizes the sum of agents' expected utility in the two countries, makes date 0 decisions about k and ℓ instead of private agents in both countries, and lets the spot market operate competitively at date 1. Importantly, the planner is assumed to be subject to the same set of informational and enforcement constraints as the private sector.

Since agents are ex-ante homogenous, the planner is assumed to assign equal weights to agents in the two countries when making date 0 choices. The planner's date 1 value function $\tilde{V}^{\omega}(k_A, k_B, \ell_A, \ell_B)$ is given by the sum of the respective expressions in (1.9) and (1.10) in which the equilibrium interest rate has been substituted in. When the interest rate is not an explicit function of date 0 choices, as in states (i,i), (s,s), and $(i,s)/(s,i)$ in region I, III and IV, $\tilde{V}^{\omega}(k_A, k_B, \ell_A, \ell_B)$ coincides with the sum of the value functions perceived by the agents in the two countries. But when the interest rate depends explicitly on date 0 choices, as in states $(i,s)/(s,i)$ in region II, the planner's value functions are given by

$$\tilde{V}^{is}(k_A, k_B, \ell_A, \ell_B) = u\Big(Ak_A + \kappa r k_B\Big) + u\Big((1-\kappa) r k_B + \Delta(\ell_A + \ell_B)\Big), \quad (1.11)$$

$$\tilde{V}^{si}(k_A, k_B, \ell_A, \ell_B) = u\Big((1-\kappa) r k_A + \Delta(\ell_B + \ell_A)\Big) + u\Big(Ak_B + \kappa r k_A\Big). \quad (1.12)$$

Comparing the expressions in (1.11-1.12) with those in (1.9-1.10), it is apparent that the global planner's marginal valuation of the two assets does not coincide with the private marginal valuation in states (i,s) and (s,i). This result is formalized in the following lemma.

Lemma 1 (Differences in asset valuations between global planner and private agents)**.** *In region II, the global planner values*

1. *the intact country's liquid assets more highly than private agents in the state of nature where the other country is distressed, and*

2. *the distressed country's illiquid assets less highly than private agents in the state of nature where the other country is intact.*

Proof. See appendix. □

The two results in lemma 1 reflect the fact that the global planner internalizes the effect of the two countries' asset positions on the interest rate in crises, while private agents take this price as given. The undervaluation of liquid assets by private agents (part 1 of the lemma) can be traced back to two separate effects. First, the social return in terms of date 2 goods of a marginal unit of liquid assets in the hands of an intact agent is Δ, but because of binding financial constraints, intact agents only earn a marginal return of $R^{is} < \Delta$ in equilibrium. Second, given concave utility, distressed agents value date 2 resource more highly than intact agents, so the decrease in the interest rate brought about by an additional marginal unit of liquidity supply benefits the distressed borrower more than it hurts the intact lender. The overvaluation of illiquid assets by distressed agents (part 2 of the lemma) relies solely on this latter wealth redistributive effect. At the margin, a lower stock of illiquid assets for the distressed country reduces the amount of collateral available for loans, which reduces the demand for loans and therefore lowers the interest rate. This lowering of the interest rate transfers wealth from intact lenders to distressed borrowers, and results in a net gain in social welfare.

The differential asset valuation result emphasized in lemma 1 leads the planner to make date 0 investment choices that generally differ from those obtained in a competitive equilibrium. The global planner solves

$$\max_{(k_j,\ell_j)_{j\in\{A,B\}}} \sum_{\omega\in\Omega} \pi^{\omega}\tilde{V}^{\omega}(k_A, k_B, \ell_A, \ell_B) \tag{1.13}$$

subject to

$$k_j + \ell_j = 1 \qquad \text{for} \quad j \in \{A, B\}. \tag{1.14}$$

In other words, the planner makes date 0 choices while anticipating the effect of its decisions on the determination of the spot market equilibrium at date 1. A *globally regulated equilibrium* consists of date 0 decisions $(k_j, \ell_j)_{j\in\{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices, the private sector's decisions solve the problems in (1.1) and (1.2); (b) the global planner's decisions solve the problem in (1.13); and (c) markets clear. For future reference, the levels of k and ℓ chosen by a global planner in a symmetric optimal plan are denoted by $\tilde{k}$ and $\tilde{\ell}$.

How does $\tilde{k}$ relate to k^{CE}? As noted above, for $0 \le k < \frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r} < k \le 1$, the planner's objective coincides with the private agents' objectives, since in these regions (I, III and IV), the interest rate is not a function of date 0 choices locally. Furthermore, under assumptions 1, 2 and 3, the investment choice k^{CE} in a competitive equilibrium falls in the interior of the interval $[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}]$ while the private agents' objective is monotonically increasing over $\left[0, \frac{1}{1+\kappa r}\right)$ and monotonically decreasing over $\left(\frac{\Delta}{\Delta+\kappa r}, 1\right]$. Since over these latter two intervals, the planner's and private agents' objectives are the same, the planner's objective must be monotonically increasing over $\left[0, \frac{1}{1+\kappa r}\right)$ and monotonically decreasing over $\left(\frac{\Delta}{\Delta+\kappa r}, 1\right]$. The global planner's optimal investment choice $\tilde{k}$ therefore has to fall in the interval $[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}]$, or region II. $\tilde{k}$ thus necessarily satisfies

$$\sum_{\omega\in\Omega} \pi^\omega \frac{\partial \tilde{V}^\omega(\tilde{k}, \tilde{k}, 1-\tilde{k}, 1-\tilde{k})}{\partial k_j} - \sum_{\omega\in\Omega} \pi^\omega \frac{\partial \tilde{V}^\omega(\tilde{k}, \tilde{k}, 1-\tilde{k}, 1-\tilde{k})}{\partial \ell_j} \lesseqgtr 0 \qquad \text{for} \quad j \in \{A, B\}, \tag{1.15}$$

with "$\le$" if $\tilde{k} = \frac{1}{1+\kappa r}$, with "$=$" if $\frac{1}{1+\kappa r} < \tilde{k} < \frac{1}{1+\kappa r}$ and with "$\ge$" if $\tilde{k} = \frac{\Delta}{\Delta+\kappa r}$.

Proposition 1 (Excessive illiquidity in competitive equilibrium)**.** *A global planner chooses a more liquid and less risky investment portfolio than private agents in the competitive equi-*

librium, i.e. $\tilde{k} < k^{CE}$ *and* $\ell^{CE} < \tilde{\ell}$.

Proof. See appendix. □

Proposition 1 establishes the constrained inefficiency of decentralized investment decisions in the model, and provides a characterization of the constrained efficient allocation. At date 1, the global planner's and private agents' valuations of the liquid asset coincide in all states of nature, except in the one where agents lend to distressed foreigners, in which the planner's valuation is strictly higher. Similarly, the planner's and private agents' valuations of the illiquid asset coincide in all states, except in the one where agents borrow from intact foreigners, in which the planner's valuation is strictly lower. These wedges between the private and social valuations of the respective assets naturally lead the global planner to invest more in the liquid asset and less in the illiquid asset at date 0.

Since agents in both countries are ex-ante identical, the welfare metric is unambiguously given by a representative agent's expected utility at date 0. This criterion corresponds to the global planner's objective, re-scaled by 1/2. Since the planner's objective is strictly decreasing in k for $k \geq \tilde{k}$, welfare is strictly higher under global regulation than in the decentralized equilibrium.[8] As developed further in section 1.3.3, the global planner recognizes that a more liquid investment portfolio at date 0 brings about a redistribution of wealth from intact lenders to distressed borrowers in states (i, s) and (s, i).

Global regulation makes financial crises less severe in two respects. First, there is always less liquidation during a crisis with global regulation than in the laissez-faire case. In states of nature where one country is distressed and the other is intact, liquidation is given by

[8] The fact that the planner's objective is strictly decreasing in k for $\tilde{k} \leq k \leq \frac{\Delta}{\Delta+\kappa r}$ follows from the planner's second-order condition.

$1 - \theta^\omega = 1 - 2(1-k)/k$, while in the state where both countries are distressed, liquidation is given by $1 - \theta^\omega = 1 - (1-k)/k$. In both cases, $\tilde{k} < k^{CE}$ implies that there is less illiquid investment to shore up in a crisis and more available liquid resources to do so under global regulation than in a decentralized equilibrium. Second, global regulation results in less pronounced interest rate spikes when one country is hit and the other is not, since $R^{is} = R^{si} = \kappa r k/(1-k)$. The demand for funds is smaller, and the supply of funds is larger, resulting in a milder increase in the price of liquidity in a crisis.

1.3.2 Constrained national planners

In order to understand the source of tensions that can arise in an environment where regulations are set independently in each country, we now consider the case of national planners who make date 0 decisions in their respective countries and let the spot market operate competitively at date 1. The assumption that planners are subject to the same informational and enforcement frictions as private agents is maintained.

The national planners are assumed to maximize the expected utility of domestic agents when making date 0 choices. Country j planner's date 1 value function $\hat{V}^\omega(k_j, k_{-j}, \ell_j, \ell_{-j})$ is given by the expression in (1.9) or (1.10) in which the equilibrium interest rate has been substituted in. As was the case with the global planner, when the interest rate is not an explicit function of date 0 investment choices, the national planners' value function coincides with the private agents' value function[9]. But in states $(i,s)/(s,i)$ in region II, where the

[9] This is again the case in states (i,i), (s,s), and $(i,s)/(s,i)$ in regions I, III and IV

interest rate is a function of date 0 choices, the planners' value functions are given by

$$\hat{V}_i^{\omega}(k_j, k_{-j}, \ell_j, \ell_{-j}) = u\Big(Ak_j + \kappa r k_{-j}\Big), \tag{1.16}$$

$$\hat{V}_s^{\omega}(k_j, k_{-j}, \ell_j, \ell_{-j}) = u\Big((1-\kappa) r k_j + \Delta(\ell_{-j} + \ell_j)\Big). \tag{1.17}$$

A comparison of the expressions in (1.16-1.17) with those in (1.11-1.12) and (1.9-1.10) reveals that the national planner's valuation of the two assets coincides neither with the global planner's valuation, nor with the private valuation in states (i, s) and (s, i). These wedges in valuations are formalized in the following two lemmas.

Lemma 2 (Undervaluation of assets by national planners vs. global planner)**.** *A national planner values*

1. *its intact agents' liquid assets less highly than the global planner in states of nature where the other country is distressed, and*

2. *its distressed agents' illiquid assets less highly than the global planner in states of nature where the other country is intact.*

Proof. See appendix. □

The undervaluation of an intact agent's liquid assets by a national planner vs. the global planner (part 1 of the lemma) is a consequence of the public goods property of international liquidity in a crisis. When a country is distressed and the other one is intact, the marginal value of either country's liquidity holding accrues *entirely* to the distressed country. At the margin, the value of an additional unit of liquid assets for the intact country is thus literally zero. This somewhat surprising result follows from the fact that an extra unit of liquid assets decreases the interest rate at which the intact country is lending in a way that makes the total revenues from lending abroad insensitive to the intact country's holdings of liquid assets (at least locally):

$$\frac{\mathrm{d}(R^{is}\ell_A)}{\mathrm{d}\ell_A} = \underbrace{\frac{\kappa r k_B}{\ell_A}}_{R^{is}} - \ell_A \underbrace{\frac{\kappa r k_B}{\ell_A^2}}_{\partial R^{is}/\partial \ell_A} = 0.$$

The global planner, on the other hand, values liquid asset holdings by both countries equally and at their full social returns.

The undervaluation of distressed agents' illiquid assets by a national planner vs. the global planner (part 2 of the lemma) also reflects the fact that the distressed and intact countries share the social value of an additional unit of collateral, with the share depending on the degree of pledgeability κ. The distressed country's national planner only captures a share $1-\kappa$ of the marginal value of the illiquid asset in terms of date 2 goods, and thus naturally undervalues the illiquid asset.

The undervaluation results for national planners relative to the global planner also hold vis-a-vis private agents.

Lemma 3 (Undervaluation of assets by national planners vs private agents)**.** *A national planner values*

1. *its intact agents' liquid assets less highly than the agents themselves in states of nature where the other country is distressed, and*

2. *its distressed agents' illiquid assets less highly than the agents themselves in states of nature where the other country is intact.*

Proof. See appendix. □

The undervaluation of intact agents' liquid assets by national planners (part 1 of the lemma) reflects the fact that national planners internalize the drop in the interest rate caused by a marginally larger holding of liquid assets, while private agents take interest rates as given. The undervaluation of distressed agents' illiquid assets by national planner (part 2 of the lemma), on the other hand, follows directly from the fact that distressed private agents overvalue illiquid assets relative to the global planner, while national planners undervalue illiquid assets relative to the global planner.

Given the wedges in asset valuations between the national planners and the global planner, it is clear that regulations chosen at the national level will generally not coincide with the constrained optimal allocation. Country j's national planner solves

$$\max_{k_j,\ell_j} \sum_{\omega\in\Omega} \pi^\omega \hat{V}^\omega(k_j, k_{-j}, \ell_j, \ell_{-j}) \tag{1.18}$$

subject to (1.7). National planners make date 0 choices while anticipating the effect of their decisions on the determination of the spot market equilibrium at date 1, and taking the action of the other country's national planner as given. A *nationally regulated equilibrium* (NRE) consists of date 0 decisions $(k_j, \ell_j)_{j\in\{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices and date 0 decisions, the private sector's date 1 decisions solve the problems in (1.1) and (1.2); (b) given (k_{-j}, ℓ_{-j}), (k_j, ℓ_j) solves the problem in (1.18); and (c) markets clear. The levels of k and ℓ chosen by national planners in a symmetric nationally regulated equilibrium are denoted by $\hat{k}$ and $\hat{\ell}$.

How does $\hat{k}$ relate to k^{CE} and $\tilde{k}$? We observe that under symmetric choices, for $0 \leq k < \frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r} \leq k \leq 1$, the national planners' objectives coincide with both the private agents' and the global planner's objective since in these regions the interest rate does not depend on date 0 choices locally. An argument analogous to that used in section 1.3.1 implies that the national planners' investment choices in a symmetric nationally regulated equilibrium $\hat{k}$ have to fall in the interval $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$ (i.e. in region II). A necessary condition for a symmetric nationally regulated equilibrium is therefore given by

$$\sum_{\omega\in\Omega}\pi^{\omega}\frac{\partial\hat{V}^{\omega}(\hat{k},\hat{k},1-\hat{k},1-\hat{k})}{\partial k_j} - \sum_{\omega\in\Omega}\pi^{\omega}\frac{\partial\hat{V}^{\omega}(\hat{k},\hat{k},1-\hat{k},1-\hat{k})}{\partial \ell_j} \lesseqgtr 0 \qquad \text{for} \quad j \in \{A, B\}, \tag{1.19}$$

with "$\leq$" if $\hat{k} = \frac{1}{1+\kappa r}$, with "=" if $\frac{1}{1+\kappa r} < \hat{k} < \frac{1}{1+\kappa r}$ and with "$\geq$" if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$.

Proposition 2 (Excessive illiquidity of NRE relative to GRE)**.** *National planners choose a weakly less liquid and more risky investment portfolio than a global planner, i.e.* $\hat{\ell} \leq \tilde{\ell}$ *and* $\hat{k} \geq \tilde{k}$*. Furthermore, if* $\hat{k} > \frac{1}{1+\kappa r}$*, then national planners choose a strictly less liquid and more risky investment portfolio than a global planner, i.e.* $\hat{\ell} < \tilde{\ell}$ *and* $\hat{k} > \tilde{k}$*.*

Proof. See appendix. □

Lemma 2 had established that national planners (a) undervalue liquid assets when their country is intact and the foreign country is distressed, and (b) undervalue illiquid assets when their country is distressed and the foreign country is intact. Proposition 2 states that unless both equilibria result in a left corner solution (within region II) for the date 0 investment choice (i.e. when $\hat{k} = \tilde{k} = \frac{1}{1+\kappa r}$), the undervaluation of liquid assets by national planners dominates the undervaluation of illiquid assets, so that wedges in ex-post valuations of assets result in an excessive illiquidity of investment by national planners relative to the constrained efficient allocation. Failing to coordinate macro-prudential policy results in an insufficient amount of regulation. In other words, the public goods property of international liquidity during a crisis translates into a public goods property of prudential regulation. Since the global planner's objective is strictly decreasing in k for $k \geq \tilde{k}$, this insufficient amount of regulation results in a weakly lower welfare than in the constrained efficient allocation (strictly lower if $\hat{k} > \frac{1}{1+\kappa r}$). Insufficient provision of regulation also results in more liquidation and higher interest rates during financial crises.

Without imposing additional structure on the primitives, the relationship between $\hat{k}$ and k^{CE} is ambiguous. It is therefore not a priori clear whether national planners want to hoard more or less liquidity than private agents in a competitive equilibrium. It turns out that by putting more structure on preferences, one can obtain the result that national planners want to hoard less liquidity than private agents, as illustrated in the following proposition.

Proposition 3 (Excessive illiquidity of NRE relative to CE)**.** *When utility is logarithmic, national planners choose a less liquid and more risky investment portfolio than private agents in the competitive equilibrium, i.e.* $\hat{\ell} < \ell^{CE}$ *and* $\hat{k} > k^{CE}$*.*

Proof. See appendix. □

Proposition 3 illustrates that the national planners' extremely low valuation of liquidity in the state of nature where its country is intact but the foreign country is distressed can result in less liquidity and more risk-taking ex-ante than in the laissez-faire benchmark. This Proposition 3 can be generalized for the class of CRRA utility when the coefficient of relative risk aversion σ is not too large, i.e. when $\sigma < \bar{\sigma}$, for some $\bar{\sigma} > 1$. Intuitively, the relationship between $\hat{k}$ and k^{CE} depends upon two counteracting effects. By hoarding less liquidity than private agents in a competitive equilibrium, national planners benefit from a higher interest rate when their country is intact and lends abroad, but they suffer from a higher interest rate when their country is distressed and borrows from abroad. The interest rate is more sensitive to the lender's supply of liquidity than to borrower's collateral ($|\partial R^{is}/\partial \ell_A| > \partial R^{is}/\partial k_A$), but because of concave utility, goods are more valuable when a country is distressed and borrowing than when it is intact and lending. When risk-aversion is low, the utility benefits associated with the first effect dominate: monopoly rents in the market for liquidity (e.g. in state (i, s) for country A) are more important in utility terms than monopsony rents (in state (s, i)). National planners therefore find it optimal to set aside less liquidity than private agents. When risk-aversion is high, the utility costs associated with the second effect dominate because goods are a lot more valuable in the state where the country is distressed and borrowing. National planners then find it optimal to set aside more liquidity than private agents. In the case with low risk-aversion illustrated in Proposition 3, national planners find it optimal to reduce investment in the liquid asset *below* what would be chosen in the free market. This excessive illiquidity results in lower welfare, as measured by the ex-ante expected utility of a representative agent, than both the constrained efficient allocation (achieved by global regulation) *and* the decentralized equilibrium. It also results in more severe financial crises in the form of more liquidation and larger interest rate spikes.

The underprovision of liquidity in the absence of international coordination can be viewed as arising from the public goods property international liquidity in crises. Because global liquidity benefits mainly distressed countries during crises, it is attractive to take advantage of foreign liquidity provision when one is distressed, but not to provide liquidity to distressed foreigners when one is intact. Since hoarding liquidity ex-ante is costly in terms of forgone higher expected returns on long-term projects, in a non-cooperative equilibrium countries choose to contribute too little to the pool of international liquidity, and attempt to free-ride on the foreign contribution. This results in a form of beggar-thy-neighbor policy in the area of financial regulation.

Viewed through the lens of the literature on international policy coordination, the underprovision of regulation result can be interpreted as arising from the attempt by national regulators to make use of monopoly and monopsony power in the market for international liquidity during crises. Regulators recognize that less liquidity hoarding ex-ante results in liquidity supply being scarcer in the state of nature where their country will be lending to distressed foreigners. This scarcity is associated with a higher interest rate, and thus brings about a shift in surplus from foreign borrowers to domestic lenders. Regulators also recognize that less illiquid investment ex-ante results in liquidity demand being smaller in the state of nature where their country will be borrowing from intact foreigners. This smaller demand is associated with a lower interest rate, and thus to a shift in surplus from foreign lenders to domestic borrowers. Under the assumption of proposition 3, it turns out that the larger sensitivity of the interest rate to liquidity supply by intact agents than to liquidity demand by distressed agents results in an underinvestment in liquid assets ex-ante by national planners relative to the laissez-faire benchmark.

Finally, the underprovision of regulation result can be seen as arising from a variation of the *hold-up problem* (see Grout 1984 and Tirole 1986). The hold-up problem occurs "when part of the return on an agent's relationship-specific investment is ex-post expropriable by his trading partner" (Che and Sakovics 2008). In the game between national regulators, the allocation of ex-post surplus is achieved via a competitive spot market rather than via bargaining. But the fact is that this allocation results in the distressed country expropriating part of the return on the ex-ante investment in liquid assets by its trading partner (i.e. the intact country). As in the classical hold-up problem, this results in an underinvestment in liquid assets relative to the cooperative solution.

1.3.3 Exchange efficiency and production efficiency

This section offers a conceptual description of the trade-offs faced by a constrained planner (corresponding to the global planner) when choosing how to set regulations in the present model. In particular, it shows how the planner's decision to regulate can be cast into the management of wedges describing deviations from optimality conditions for exchange and production efficiency. This discussion clarifies why planners controlling only a subset of the economy (national planners) are doomed to miss the goal of regulation in such a framework.

It is well known that incomplete markets generally result in a failure of exchange efficiency. This failure takes the form of wedges between the marginal rates of substitution (MRS) between two goods across agents. In the model of section 1.2, the relevant MRS is the one between the consumption good in state (i, s) and the same consumption good in state (s, i):

$$MRS_j \equiv \frac{\pi^{is} u'(c_j^{is})}{\pi^{si} u'(c_j^{si})}$$

If markets were complete, agents in the two countries would trade securities contingent on these two states and the MRS would be equalized in a competitive equilibrium. When markets are incomplete, a wedge between the MRS, indicating a failure of exchange efficiency, generally persists in equilibrium. For an arbitrary symmetric date 0 investment choice $(k, 1-$

k), this exchange wedge is given by

$$\tau_e(k) \equiv 1 - \frac{MRS_A(k)}{MRS_B(k)}. \tag{1.20}$$

It can be shown that $\tau_e(k)$ is monotonically increasing over $[0,1]$, with $\tau_e(0) = 0$ and $\tau_e(1) = 1 - [u'(A)/u'(r)]^2$. In other words, the higher the investment in the illiquid project, the more serious the failure of exchange efficiency in this economy. Intuitively, more investment in illiquid projects leads to more risk, and since markets are incomplete, this higher risk has to be borne by agents. Under assumptions 1, 2 and 3, the symmetric competitive equilibrium falls in region II (i.e. $\frac{1}{1+\kappa r} < k^{CE} < \frac{\Delta}{\Delta+\kappa r}$), in which case the wedge is given by

$$\tau_e(k) = 1 - \left[\frac{u'\Big((A+\kappa r)k\Big)}{u'\Big([(1-\kappa)r - 2\Delta]k + 2\Delta\Big)} \right]^2.$$

Given that $\tau_e'(k) > 0$, by limiting the investment scale via regulation to $\tilde{k} < k^{CE}$, the global planner reduces the size of the wedge relative to the competitive equilibrium, and thereby reduces the severity of exchange inefficiency in the economy. Why doesn't the planner choose to reduce exchange inefficiency further by lowering the investment scale below $\tilde{k}$? The answer lies in the trade-off between exchange inefficiency and production inefficiency faced by the planner. The production wedge can be defined as

$$\tau_p(k) \equiv 1 - \frac{E[R_k^\omega(k)u'(c_j^\omega(k))]}{E[R_\ell^\omega(k)u'(c_j^\omega(k))]}, \tag{1.21}$$

where R_k^ω and R_ℓ^ω denote the returns on the illiquid and liquid assets, respectively, and $c^\omega(k)$

denotes a representative agent's equilibrium consumption in state ω, for a given symmetric investment choice of k at date 0.[10] The production wedge τ_p is zero at the competitive equilibrium investment decision k^{CE} and non-zero elsewhere. When lowering the investment scale down to $\tilde{k}$, a global planner trades off an improvement in exchange efficiency with a deterioration in production efficiency. Optimal regulation is then the result of the optimal management of these two wedges by the planner.

The above discussion makes it clear that national planners who have pricing power by definition *cannot* be expected to set regulation optimally in an environment where the motivation to regulate originates from a market incompleteness. National planners do not care about overall efficiency, but rather attempt to manipulate prices in a way that results in a shift in surplus in favor of the agents they represent. According to proposition 3, having them in charge of regulation can even result in a deterioration of both exchange efficiency - $\tau_e(k)$ is monotonically increasing so $\tau_e(k^{CE}) < \tau_e(\hat{k})$ - and production efficiency - $\tau_p(k)$ is zero only at k^{CE} - vis-a-vis the competitive equilibrium benchmark.

[10] For $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, the expectations in (1.21) are given by

$$\begin{aligned} E[R_k^\omega(k)u'(c_j^\omega(k))] &= \pi^{ii}Au'\Big((A-1)k+1\Big) + \pi^{is}Au'\Big((A+\kappa r)k\Big) \\ &\quad + \pi^{si}\Big[r + \Big(\Delta - \frac{\kappa rk}{1-k}\Big)\frac{\kappa r}{\frac{\kappa rk}{1-k}}\Big]u'\Big([(1-\kappa)r - 2\Delta]k + 2\Delta\Big) + \pi^{ss}ru'\Big((r-\Delta)k+\Delta\Big) \end{aligned}$$

and

$$E[R_\ell^\omega(k)u'(c_j^\omega(k))] = \pi^{ii}u'\Big((A-1)k+1\Big) + \pi^{is}\frac{\kappa rk}{1-k}u'\Big((A+\kappa r)k\Big) + \pi^{si}\Delta u'\Big([(1-\kappa)r-2\Delta]k+2\Delta\Big) + \pi^{ss}\Delta u'\Big((r-\Delta)k+\Delta\Big).$$

1.4 International spillovers

The model developed in section 1.2 can also be used to study the effects of changes in macro-prudential policies across borders. Section 1.4.1 analyzes the impact of changes in macro-prudential policy in one country on risk-taking by market participants abroad. Section 1.4.2 looks at how changes in regulation in one country affect the incentive to regulate in the other country. Section 1.4.3 studies the welfare effects of a unilateral introduction of regulation by one of the two countries.

1.4.1 Regulatory spillovers

How do market participants react to changes in macro-prudential policy abroad? To answer this question, we consider a version of the model of section 1.2 in which the date 0 choices are set exogenously in country B and made optimally by private agents in country A. A *competitive equilibrium with exogenous regulations abroad* consists of date 0 decisions (k_A, ℓ_A), date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices and country B regulations (k_B, ℓ_B), the decisions solve the problem in (1.6) for country A, and the problems in (1.1) and (1.2) for both countries; and (b) markets clear.

To understand the relevant transmission channels of prudential policy across countries, it is useful to denote the aggregate date 0 investment choices in country j by (K_j, L_j). While K_B and L_B are set exogenously by country B's regulator, K_A and L_A result from the optimal choices of private agents in country A. When taking decisions atomistically at date 0, private agents correctly forecast the function mapping aggregate investment choices into interest rates in the various states of nature at date 1, but they take aggregate investment decisions in both countries as given. In the neighborhood of the competitive equilibrium, the pricing functions are given by $R^{ii} = 1$, $R^{ss} = \Delta$,

$$R^{is} = \frac{\kappa r K_B}{L_A}, \qquad \text{and} \qquad R^{si} = \frac{\kappa r K_A}{L_B}. \tag{1.22}$$

Tighter regulations in country B are captured by a marginal decrease in K_B and a corresponding marginal increase in L_B. The following proposition establishes the direction of the effect of tighter regulations in country B on private agents' investment choice in country A.

Proposition 4 (Regulatory spillovers)**.** *In the neighborhood of a symmetric competitive equilibrium, tighter regulations in country B induce private agents in country A to choose a less liquid and more risky investment portfolio.*

Proof. See appendix. □

Tighter regulations in country B do not affect private agents' payoffs in country A in states (i, i) and (s, s). However, they result in lower interest rates in states (i, s) and (s, i). In state (i, s), the tighter regulations lead to a smaller demand for funds by country B's distressed agents, which pushes down the interest rate at which country A's intact agents lend. As can be seen from expression (1.1), this lower interest rate reduces the return on the liquid asset for country A's agents in that state. In state (s, i), the tighter regulations increase the supply of liquidity, which lowers the interest rate at which country A's distressed agents borrow. As is clear from expression (1.2), a lower interest rate in this state increases the return on the illiquid asset for country A's agents by increasing the wedge between the internal and external value of funds. Hence, tighter regulations, defined as increasingly proliquidity regulations, in country B decrease the return on the liquid asset and increase the return on the illiquid asset for private agents in country A. This naturally leads these agents to reallocate their investment portfolio towards more illiquid assets.

1.4.2 Spillovers in incentives to regulate

How are a regulator's incentives affected by a change in macro-prudential policy abroad? This section shows that the interaction between national regulators in the model of section 1.2 can be understood in terms of the strategic substitutability concept of Bulow, Geanakoplos, and Klemperer (1985). In light of the result in section 1.4.1 that tighter regulations abroad induce more risk-taking domestically, one could a priori expect that macro-prudential policies are strategic complements. In fact, the model delivers precisely the opposite result, as stated in the following proposition.

Proposition 5 (Strategic substitutabilities in national regulations)**.** *In the neighborhood of a symmetric competitive equilibrium, national regulations are strategic substitutes.*

Proof. See appendix. □

The intuition for this result is that in the neighborhood of the symmetric competitive equilibrium, the national planners' payoff functions only depend on the other country's investment choices in the states of nature where there is cross-border borrowing and lending, i.e. in states (i, s) and (s, i). As can be seen from (1.16) and (1.17), a marginal increase in the tightness of regulations in country $-j$, in the form of a marginal decrease in k_{-j} and a corresponding marginal increase in ℓ_{-j}, has the following effects:

- It decreases the interest rate payment $\kappa r k_{-j}$ which country j receives from country

$-j$ in the state of nature where intact agents in country j lend to distressed agents in country $-j$. This leads to an increase in the marginal value of the illiquid asset in that state, $Au'\Big(Ak_j + \kappa r k_{-j}\Big)$.

- It increases the loan size ℓ_{-j} which country j receives from country $-j$ in the state of nature where distressed agents in country j borrow from intact agents in country $-j$. This decreases the marginal value of investing in the liquid asset, $\Delta u'\Big((1-\kappa)r + \Delta(\ell_{-j}+\ell_j)\Big)$, more than it increases the marginal value of investing in the illiquid asset, $(1-\kappa)ru'\Big((1-\kappa)r + \Delta(\ell_{-j}+\ell_j)\Big)$.

The combination of these two effects makes the relative attractiveness of regulations that increase investment in liquid assets a decreasing function of the tightness of regulations in country $-j$. In other words, these two effects imply that the national regulators' actions are strategic substitutes.

1.4.3 Welfare effects of unilateral regulation

The analysis in section 1.3 implies that, starting from a competitive equilibrium, the introduction of a regulation simultaneously requiring agents in both countries to increase their holdings of liquid assets marginally is unambiguously welfare improving for all agents.[11] One might also be interested in the welfare effects of a unilateral introduction of regulation. In particular, does such an introduction of regulation increase welfare in the regulated country? Does it increase welfare in the unregulated country? These are relevant questions to the extent that their answer determines the incentives of a national regulator to move first in a

[11] This follows from the fact that the global planner's objective is strictly decreasing in k for $k \geq \tilde{k}$, while $k^{CE} > \tilde{k}$.

world where macro-prudential regulations are absent to start with. This section addresses these questions using the concept of a competitive equilibrium with exogenous regulation abroad, defined in section 1.4.1.

The focus is on the effects of the introduction of a small regulation requiring country B agents to increase their holding of liquid assets by $dL_B = -dK_B > 0$ relative to the symmetric competitive equilibrium level $\ell^{CE} = 1 - k^{CE}$. We consider in turn the welfare effects in the regulated country and in the unregulated country.

Welfare of regulated country

In a competitive equilibrium with exogenous regulations abroad, country A agents make their date 0 investment decisions taking prices and foreign regulation as given. In the neighborhood of the symmetric competitive equilibrium, the ex-ante welfare of agents in the regulated country (country B) is given by

$$
\begin{aligned}
W(L_B, \ell_A) &\equiv \sum_{\omega \in \Omega} \pi^{\omega} \hat{V}^{\omega}(1 - L_B, 1 - \ell_A, L_B, \ell_A) \qquad (1.23) \\
&= \pi^{ii} u\Big(A(1 - L_B) + L_B\Big) + \pi^{is} u\Big((1 - \kappa) r(1 - L_B) + \Delta(L_B + \ell_A)\Big) \\
&\quad + \pi^{si} u\Big(A(1 - L_B) + \kappa r(1 - \ell_A)\Big) + \pi^{ss} u\Big(r(1 - L_B) + \Delta L_B\Big).
\end{aligned}
$$

Starting from a symmetric competitive equilibrium, the marginal effect on country B's welfare of a tightening of regulation in country B is given by

$$
dW = \Big[\frac{\partial W}{\partial L_B} + \frac{\partial W}{\partial \ell_A} \frac{d\ell_A^{CE}}{dL_B}\Big] dL_B, \qquad (1.24)
$$

where the derivatives are evaluated at $(L_B, \ell_A) = (\ell^{CE}, \ell^{CE})$, and $dL_B > 0$. The first term in the brackets in (1.24) represents the direct effect of a change in regulation. The second term in the brackets reflects the indirect effect working through the spillovers operating in country A. Proposition 4 established that a tightening of regulation in country B induced a more illiquid date 0 portfolio choice in country A, i.e. that $\frac{d\ell_A^{CE}}{dL_B} < 0$. The partial derivatives of $W(L_B, \ell_A)$, evaluated at (ℓ^{CE}, ℓ^{CE}), are given by

$$\begin{aligned} \frac{\partial W}{\partial L_B} &= -\pi^{ii}(A-1)u'\Big(A-(A-1)\ell^{CE}\Big) - \pi^{is}[(1-\kappa)r-\Delta]u'\Big((1-\kappa)r-[(1-\kappa)r-2\Delta]\ell^{CE}\Big) \\ &\quad -\pi^{si}Au'\Big((A+\kappa r)(1-\ell^{CE})\Big) - \pi^{ss}(r-\Delta)u'\Big(r+(\Delta-r)\ell^{CE}\Big) \end{aligned} \tag{1.25}$$

and

$$\frac{\partial W}{\partial \ell_A} = \pi^{is}\Delta u'\Big((1-\kappa)r-[(1-\kappa)r-2\Delta]\ell^{CE}\Big) - \pi^{si}\kappa r u'\Big((A+\kappa r)(1-\ell^{CE})\Big). \tag{1.26}$$

Using the first-order condition of the symmetric competitive equilibrium, $g_{II}(k^{CE}) = 0$ ($g_{II}(k)$ is defined in (A.8)), $\frac{\partial W}{\partial L_B}$ can be written as

$$\frac{\partial W}{\partial L_B} = \pi^{is}\Delta\frac{\kappa r(1-k^{CE})}{\kappa r k^{CE}}u'\Big([(1-\kappa)r-2\Delta]k^{CE}+2\Delta\Big) - \pi^{si}\frac{\kappa r k^{CE}}{1-k^{CE}}u'\Big((A+\kappa r)k^{CE}\Big) \tag{1.27}$$

Without further restriction, it is not possible to determine the sign of $\frac{\partial W}{\partial L_B}$, so the direct effect of the tightening of regulation is ambiguous. The tightening of regulation pushes the interest rate down in the states (i, s) and (s, i) in which there is international borrowing/lending. In state (i, s), this lowering of the interest rate benefits country B's agents who can borrow more cheaply (first term in (1.27)). In state (s,i), it costs country B's agents who lend at a lower rate (second term in (1.27)). Whether the costs are smaller or larger than the benefits is a priori ambiguous. The direction of the indirect effect, however, is unambiguous. The proof of lemma 1 establishes that $(A+\kappa)k^{CE} > [(1-\kappa)r - 2\Delta]k^{CE} + 2\Delta$ and therefore the concavity of $u(\cdot)$ implies $\frac{\partial W}{\partial \ell_A} > 0$. The indirect effect $\frac{\partial W}{\partial \ell_A}\frac{d\ell_A^{CE}}{dL_B}$ in (1.24) is therefore strictly negative. For country B's agents, the losses from an increase in the costs of borrowing from abroad in state (i,s) (the first term in (1.26)) is only partially offset by the benefits of an increase in the interest rate payment from abroad in state (s,i) (the second term in (1.26)). A unilateral tightening of regulations in country B induces more risk-taking and illiquidity abroad, and this feeds back negatively into country B.

With further assumptions, the model implies the direct and the indirect effect work in the same direction. This results in an unambiguous overall effect of a tightening of regulation on the regulated country's welfare, as stated in the following proposition.

Proposition 6 (Beggar-thyself unilateral regulation)**.** *Absent initial regulation, when utility is logarithmic, the unilateral introduction of a (small) regulation is welfare reducing for the country introducing the regulation, i.e.* $\frac{dW}{dL_B} < 0$.

Proof. See appendix. □

Under log utility, the direct effect of a tightening of regulation on the regulated country's welfare is negative , for reasons similar to those underlying the result in proposition 3. In this case, losses from the extra costs of lending at a lower interest rate in state (s,i) are larger than the benefits from cheaper borrowing in state (i,s) for country B's agents.[12] While

[12] As before, log utility is merely a sufficient condition for direct effect of a change in regulation on the

introducing regulation simultaneously at home and abroad is welfare improving for both countries, a unilateral introduction of regulation can be welfare reducing for the regulated country, because of terms of trade effects and their associated spillovers. This result is reminiscent of Corsetti and Pesenti (2001), who find that an unexpected unilateral monetary expansion can be welfare reducing for a given country in an environment where, owing to monopolistic distortions in production and nominal rigidities, the same policy would be welfare improving if pursued simultaneously at home and abroad.

Welfare of unregulated country

In the neighborhood of the symmetric competitive equilibrium, the ex-ante welfare of agents in the unregulated country (country A) is given by

$$\begin{aligned}\Pi(\ell_A, L_B) &\equiv \sum_{\omega\in\Omega}\pi^{\omega}\hat{V}^{\omega}(1-\ell_A, 1-L_B, \ell_A, L_B) \\ &= \pi^{ii}u\Big(A(1-\ell_A)+\ell_A\Big)+\pi^{is}u\Big(A(1-\ell_A)+\kappa r(1-L_B)\Big) \\ &\quad +\pi^{si}u\Big((1-\kappa)r(1-\ell_A)+\Delta(\ell_A+L_B)\Big)+\pi^{ss}u\Big(r(1-\ell_A)+\Delta\ell_A\Big).\end{aligned} \tag{1.28}$$

Starting from a symmetric competitive equilibrium, the marginal effect on country A's welfare of a tightening of regulation in country B is given by

$$d\Pi = \Big[\frac{\partial\Pi}{\partial\ell_A}\frac{d\ell_A^{CE}}{dL_B}+\frac{\partial\Pi}{\partial L_B}\Big]dL_B, \tag{1.29}$$

regulated country's welfare to work in the same direction as the indirect effects. As for proposition 3, the proposition can be generalized for CRRA utility when $\sigma < \bar{\sigma}$, for some $\bar{\sigma} > 1$. The result that the overall effect is negative can be found to hold under less restrictive, albeit not easily characterizable, conditions.

where the derivatives are evaluated at $(\ell_A, L_B) = (\ell^{CE}, \ell^{CE})$. The overall effect of a tightening of regulation on the unregulated country's welfare is again given by the sum of an indirect effect (first term in (1.29)) and a direct effect (second term in (1.29)). The partial derivatives of $\Pi(\ell_A, L_B)$, evaluated at $(\ell_A, L_B) = (\ell^{CE}, \ell^{CE})$, are given by

$$\begin{aligned}\frac{\partial \Pi}{\partial \ell_A} &= -\pi^{ii}(A-1)u'\Big(A-(A-1)\ell^{CE}\Big) - \pi^{is}Au'\Big((A+\kappa r)(1-\ell^{CE})\Big) \\ &\quad -\pi^{si}[(1-\kappa)r-\Delta]u'\Big((1-\kappa)r-[(1-\kappa)r-2\Delta]\ell^{CE}\Big) - \pi^{ss}(r-\Delta)u'\Big(r+(\Delta-r)\ell^{CE}\Big) \\ &= \frac{\partial W}{\partial L_B},\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \Pi}{\partial L_B} &= -\pi^{is}\kappa r u'\Big((A+\kappa r)(1-\ell^{CE})\Big) + \pi^{is}\Delta u'\Big((1-\kappa)r-[(1-\kappa)r-2\Delta]\ell^{CE}\Big) \\ &= \frac{\partial W}{\partial \ell_A} < 0.\end{aligned}$$

The analysis is facilitated by the fact that locally, the effect a small change in a country's portfolio on its own welfare is identical for the regulated and for the unregulated country ($\frac{\partial \Pi}{\partial \ell_A} = \frac{\partial W}{\partial L_B}$), as is the effect of a small change in a country's portfolio on the other country's welfare ($\frac{\partial \Pi}{\partial L_B} = \frac{\partial W}{\partial \ell_A}$). The direct effect of a tightening of regulation on the unregulated country's welfare is therefore unambiguously positive, while the direction of the indirect effect is a priori ambiguous. Perhaps paradoxically, it is the unregulated agents' response to the introduction of regulation abroad that may make them worse off than in the absence of any regulation. Without this behavioral response to the introduction of regulation abroad, the unregulated country would necessarily be made better off by the decrease in risk-taking happening abroad. As before, on can make additional preference assumptions under which the direct and the indirect effects of the introduction of regulation work in the same direction. This results in an unambiguously positive overall effect of a tightening of regulation on the unregulated country's welfare.

Proposition 7 (Prosper-thy neighbor unilateral regulation)**.** *Absent initial regulation, when utility is logarithmic, the unilateral introduction of a (small) regulation is welfare improving for the unregulated country, i.e.* $\frac{d\Pi}{dL_B} > 0$*.*

Proof. See appendix. □

With logarithmic utility, the unregulated country's response to the introduction of regulation abroad contributes positively to its welfare. Agents in the unregulated country react by decreasing their investment in liquid assets and increasing their investment in illiquid assets. At the margin, the only impact on their welfare works through the marginal increase in the interest rate in states (i, s) and (s, i) resulting from this portfolio reallocation. The benefits from lending abroad at a higher rate in state (i, s) is larger than the cost of paying a higher rate of foreign loans in state (s, i). A unilateral regulation may therefore create *prosper thy-neighbor* effects via both the direct and indirect channels.

1.5 An asset market formulation

This section presents a variant of the model of section 1.2 in which the intermediation of funds during a crisis occurs via an asset market rather than via a credit market. Distressed agents cannot borrow at date 1, but they can sell some of their illiquid assets to intact agents in order to raise funds. When variables are appropriately relabeled, the date 1 equilibrium of this model is isomorphic to the date 1 equilibrium of the credit market model of section 1.2. All the results of sections 1.3.1 to 1.4 derived for the credit market model therefore also apply to the asset market model of the present section.

As in the baseline model of section 1.2, markets are incomplete. However, instead of being able to share risk indirectly by borrowing and lending on a credit market at date 1, agents are now able to buy and sell the illiquid asset on a spot market at a price q^ω. Intact agents buy x_j^ω, while distressed agents sell $-x_j^\omega$. We assume that sellers cannot sell more than a fraction η of their total capital holdings k_j. This amounts to assuming limited market liquidity for long-term projects, as in Kiyotaki and Moore (2008). We further assume that the long-term projects that are traded need to be shored up by distressed agents before being delivered to a buyer. Buyers of illiquid projects at date 1 therefore receive Ax_j^ω at date 2. All other assumptions of section 1.2.1 pertaining to preferences, technology and uncertainty are maintained.

At date 1, the value of an intact agent in country j is now given by

$$V_i^\omega(k_j, \ell_j) \equiv \max_{0 \leq x_j^\omega \leq \ell_j / q^\omega} u\Big(Ak_j + Ax_j^\omega + \ell_j - q^\omega x_j^\omega\Big) \tag{1.30}$$

The agent's date 2 consumption in (1.30) is given by the sum of the return on its initial holding of illiquid projects Ak_j, the return on the newly acquired illiquid projects Ax_j^ω and the return on the funds invested at date 1 in the storage technology $\ell_j - q^\omega x_j^\omega$. Without loss of generality, we assume that intact agents can only buy and not sell assets on the date 1 spot market. Their capacity to buy is limited by their date 1 liquid resources ℓ_j.

As in the model of section 1.2, the form of the objective in (1.30) leads to a simple asset demand schedule for intact agents. For $q^\omega < A$, intact agents exhaust their budget constraint and are willing to buy ℓ_j/q^ω units of the long-term asset. For $q^\omega = A$, they are indifferent between buying any amount between 0 and ℓ_j/A. Finally, for $q^\omega > A$, they do not want to buy any long-term assets. The intact agents' asset demand curve is therefore horizontal at $q^\omega = A$ and slopes downward for $q^\omega < A$, as shown in figure 1.5.

The date 1 value of a distressed agent in country j is given by

$$V_s^\omega(k_j, \ell_j) \equiv \max_{\theta_j^\omega, x_j^\omega} u\Big(r(1-\theta_j^\omega)k_j + A(\theta_j^\omega k_j + x_j^\omega) + \ell_j - q^\omega x_j^\omega - \theta_j^\omega k_j\Big) \tag{1.31}$$

subject to

$$\theta_j^\omega k_j \leq -q^\omega x_j^\omega + \ell_j, \tag{1.32}$$

$$-x_j^\omega \leq \eta k_j, \tag{1.33}$$

A distressed agent's date 2 consumption in (1.31) is given by the sum of the return on the long-term assets that were not shored up, $r(1-\theta_j^\omega)k_j$, the return on the long-term assets that were shored up but not sold $A(\theta_j^\omega k_j + x_j^\omega)$, and the return on the funds invested at date 1 in the storage technology $\ell_j - q^\omega x_j^\omega - \theta_j^\omega k_j$. (1.32) is the date 1 budget constraint stating that reinvestment $\theta_j^\omega k_j$ needs to be covered by the sum of ex-ante liquidity hoarding ℓ_j and proceeds of ex-post sale of assets $-q^\omega x_j^\omega$. (1.33) says that distressed agents cannot resell more than a fraction η of their initial long-term asset holdings k_j.

The form of the objective in (1.31) again yields a simple form for a distressed agent's asset supply schedule. For $q^\omega < A/\Delta$, an agent does not want to sell any assets, since the revenue from a sale is lower than the cost of shoring up the asset. At $q^\omega = A/\Delta$, an agent is indifferent between selling any amount it can. Finally, for $q^\omega > A/\Delta$, an agent wants to sell as much as possible. The distressed agents' asset supply curve is therefore horizontal at $q^\omega = A/\Delta$ and vertical at ηk_j for $q^\omega > A/\Delta$.

Asset market clearing requires $x_A^\omega + x_B^\omega = 0$. The equilibrium asset price necessarily satisfies $A/\Delta \leq q^\omega \leq A$. The equilibrium is simply given by $q^\omega = A/\Delta$ and $x_A^\omega = x_B^\omega = 0$ in state (i, i), and by $q^\omega = A$ and $x_A^\omega = x_B^\omega = 0$ in state (s, s). When country j is intact and country $-j$ is distressed (i.e. in states (i, s) and (s, i)), under the condition that $A/\Delta < \frac{\ell_j}{\eta k_{-j}} < A$, the equilibrium takes the form displayed in Figure 1.5, and equating the asset demand ℓ_j/q^ω with the asset supply ηk_{-j} yields an equilibrium asset price of

$$q^\omega = \frac{\ell_j}{\eta k_{-j}}. \tag{1.34}$$

The equilibrium features *cash-in-the-market pricing*, as in Allen and Gale (1998), in that the asset price depends positively on the amount of liquidity in the hands of the intact country's buyers. A marginal increase in the amount of liquidity set aside ex-ante by intact agents would push up the asset price by increasing the demand for the asset (shifting the downward sloping part of the demand curve to the right). The equilibrium also exhibits *fire sales* in that the asset price depends negatively on the amount of illiquid assets thrown on the market by the distressed country's sellers. A marginal decrease in the amount of ex-ante illiquid investment by distressed agents would push up the asset price by reducing the supply of the asset (shifting the vertical portion of the supply curve to the left). Naturally, such movements in the asset price have ex-ante welfare implications similar to movements in the

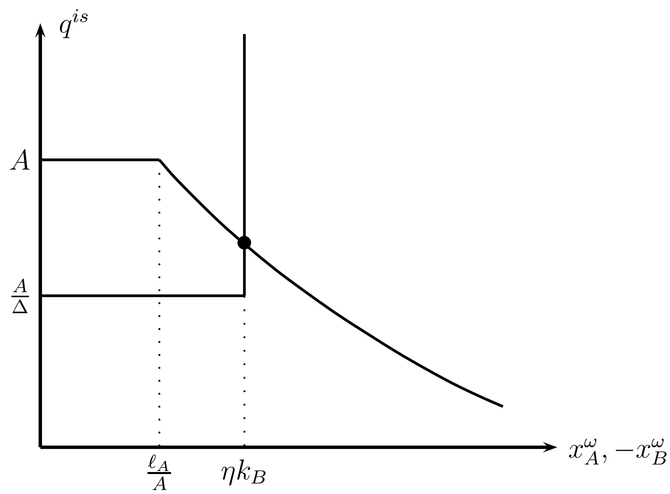

Fig. 1.5: Asset market equilibrium in state (i, s).

interest rate in the model of section 1.2.

In equilibrium, the date 1 value function of an intact agent is given by

$$V_i^\omega(k_j, \ell_j) = u\Big(Ak_j + \frac{A}{q^\omega}\ell_j\Big), \tag{1.35}$$

and that of a distressed agent is given by

$$V_s^\omega(k_j, \ell_j) = u\Big(r(1-\eta)k_j + \Delta(q^\omega - 1)\eta k_j + \Delta\ell_j\Big). \tag{1.36}$$

(1.35) indicates that for an intact agent, illiquid assets yield a return of A, while liquid assets, by allowing the purchase at price q^ω of assets that have an ultimate return of A, yield a return of A/q^ω. The terms in (1.36) can be interpreted similarly. For a distressed agent, each unit of liquid assets allows shoring up one unit of illiquid assets, yielding a return Δ, whereas each unit of illiquid assets yields a return of r for the share $1-\eta$ that cannot be sold off, and a return of $\Delta(q^\omega - 1)$ for the share η that can be sold off (an asset sale brings an extra $q^\omega - 1$ of date 1 liquidity, whose return is Δ).

A remarkable aspect of the asset market equilibrium is its complete isomorphism with the credit market equilibrium of the model of section 1.2. For $\eta = \frac{\kappa r}{A}$, the equilibrium of the asset market model corresponds exactly to the equilibrium of the credit market model, when prices are redefined according to $R^\omega = \frac{A}{q^\omega}$. When $\underline{\eta} < \eta < \bar{\eta}$, for $\underline{\eta} = \frac{\underline{\kappa} r}{A}$ and $\bar{\eta} = \frac{\bar{\kappa} r}{A}$, date 0 decisions in a symmetric competitive equilibrium result in the date 1 asset market equilibrium taking the form displayed in Figure 1.5. The results derived in sections 1.3.1 to 1.4 for the credit market model therefore automatically also apply to the asset market model of this section. This illustrates that the coordination problem between national regulatory authorities that is at the core of this paper does not rely on a particular specification of the market allowing distressed entities to raise funds during a crisis. In the asset market version of the model, global regulation calls for more investment in liquid assets and less investment in illiquid assets ex-ante, with the aim of supporting asset prices during financial crises. Marginally higher asset prices in a crisis redistribute wealth from intact buyers to distressed sellers, and thereby achieve a reduction in the cross-sectional wedges between marginal rates of substitution between goods in states (i, s) and (s, i). National planners do not aim at a reduction of these wedges, but rather try to shift surplus in favor of domestic residents, by pushing down the asset price in states where their residents are buying the asset and pushing it up in states where the residents are selling the asset.

1.6 Conclusion

This paper studies the international coordination problem inherent to the financial stability objective of banking regulation. It presents a model of systemic risk-taking where the stabilization benefits of macro-prudential regulation are global, but the costs of regulation stemming from distortions in production are incurred domestically. Absent coordination, this public goods problem naturally results in an underprovision of regulation.

Our findings have implications far beyond the framework of the particular model analyzed. They illustrate that when where pecuniary externalities operate across borders, there is a strong case for cooperation in policies whose underlying motivation is to correct such externalities. Hence, they imply that the case for international coordination extends to a large part of the growing research agenda that motivates financial crisis prevention policies from a second best perspective in incomplete markets environments (see reviews in Wagner 2009 and Korinek 2011a). The quantitative relevance of the implied coordination problem is likely to depend on the particular frictions and policies under scrutiny, and its assessment is an important area for future research.

At a more abstract level, the paper also shows that, contrary to common beliefs, market power can increase the magnitude of the distortions induced by systemic externalities. Whether market power attenuates or amplifies these distortions crucially depends on the direction of the relevant pecuniary externalities. The non-trivial interplay between distortions arising from market incompleteness and those due to non-competitive market structures is another fruitful avenue for future research.

2. SYSTEMIC RISK AND INEFFICIENT DEBT MATURITY

2.1 Introduction

The 2007-2009 global financial crisis has shown that liquidity problems, originally confined to a relatively small number of economic entities, can spread out rapidly. Through vicious spirals, liquidity shortages can lead to sudden losses of confidence in markets, causing massive asset price drops and cutbacks in bank lending. In the run-up to the crisis, highly leveraged entities, such as investment banks, hedge funds and off-balance-sheet vehicles, relied increasingly on very short-term liabilities to fund long-term assets. This trend is believed to have been a major factor behind the liquidity crunch that led to the unprecedented financial turmoil of 2008-2009 (Brunnermeier 2009). Was this widespread maturity mismatch just the efficient aggregate result of sound choices made by individually rational agents? Or was it in some sense *excessive*, in which case government intervention would have been warranted? This paper investigates this question by assessing within a quantitative theoretical framework the desirability of government policies that alter the debt maturity choice of leveraged economic agents.

In the wake of the recent crisis, academic economists, policymakers and observers have increasingly pushed for a broad reform of financial regulation.[1] Central to the proposed

[1] See, for example, The Group of Thirty (2009), Bank of England (2009) and Warwick Commission (2009).

reforms are macroprudential policies designed to limit behavior of market participants that tends to increase the whole financial system's vulnerability – so-called *systemic risk*. In addition to proposals to penalize high leverage and large institution sizes, most calls for new macroprudential regulations also suggest taxing large maturity mismatches.[2] But because of the general presumption that decentralized markets produce socially optimal outcomes through the "invisible hand," government interventions often need to be justified by the identification of a specific form of market failure. In our context, the market failure results from a "fire-sale externality" that causes excessive leverage and risk-taking by borrowers. Individual agents fail to internalize that by building up leverage and choosing a high risk exposure, they increase the likelihood of having to fire-sell assets beyond what would be socially desirable, thereby excessively depressing asset prices and tightening others' financing constraints in the event of adverse aggregate shocks.

This paper studies the debt maturity choice of leveraged agents in a formal framework where endogenous collateral constraints are a source of amplification of fundamental shocks. Long-term debt provides insurance against negative shocks to the value of assets held by leveraged borrowers, but it entails an extra cost over short-term debt because lenders need to be compensated for spending resources on enforcing long-term contracts. Borrowers choose their debt maturity by trading off the insurance benefits of long-term debt with its costs. But as they fail to internalize their contribution to systemic risk, they only consider the private insurance benefits of long-term debt and take on too little of it (i.e. they take on too much short-term debt) in a decentralized market equilibrium. In such an environment, where

[2] See Brunnermeier, Crocket, Goodhart, Persaud, and Shin (2009) and the Squam Lake Working Group on Financial Regulation (2009).

the stability of leveraged borrowers' net worth has "public goods" properties, government intervention in the form of a tax on short-term debt can lead to Pareto improvements and result in less volatile allocations and asset prices.

We consider a model with two sets of agents and introduce financial frictions, as those play a key role in the formal and informal analysis of systemic risk and macroprudential policy. The modeling framework builds on Kiyotaki and Moore (1997). Relatively patient agents (households) lend in equilibrium to less patient agents (entrepreneurs). Capital serves both as a factor of production and as collateral for loans. When the only type of claim that agents can trade is a one-period non-state-contingent bond, entrepreneurs are naturally more exposed to aggregate risk (productivity shocks) than households because of leverage. A negative shock disproportionately hurts the net worth of entrepreneurs, leading them to reduce borrowing and fire-sell assets. As in Kiyotaki and Moore (1997), fundamental shocks get amplified as these fire-sales lead to a further decline in asset prices and net worth, causing yet another round of deleveraging. By letting agents trade long-term bonds alongside the usual one-period non-state-contingent bonds, we allow for the possibility of better risk-sharing between households and entrepreneurs. Even though long-term bonds are a promise to non-state-contingent payments, their one-period return is state-contingent since the market price of the future payment stream generally depends on aggregate conditions, as in Angeletos (2002) and Buera and Nicolini (2004). Since the prices of both long-term bonds and physical assets are pro-cyclical, the issuance of long-term debt by borrowers (entrepreneurs) effectively shifts risk towards lenders (households). A longer debt maturity structure thus translates into a lower relative risk exposure of leveraged entrepreneurs. When adverse shocks hit, the value of the entrepreneurs' assets shrinks, but so does the value of their liabilities, which mitigates the effect on their net worth. By reducing the sensitivity of leveraged entrepreneurs' net worth's to fundamental shocks, a longer debt maturity structure also reduces the scope of financial amplification in the economy, resulting in less volatile allocations and asset prices.

In this environment where a shorter debt maturity maps into more volatile aggregate economic variables, we ask whether debt maturity choices made by individually rational agents result in socially efficient risk allocations. Short-term debt is cheaper than long-term debt, partly because in the model enforcing long-term contracts is costly. In choosing their debt maturity, entrepreneurs hence trade-off the private insurance benefits of long-term debt with the cost advantage of short-term debt. But since (1) lower net worth causes fire sales, (2) fire sales depress the price of capital, and (3) the price of capital matters for other entrepreneurs' borrowing capacity, the social insurance benefits of long-term debt outweigh its private benefits. As a result of this pecuniary externality in an incomplete market setting, entrepreneurs issue too much short-term debt and too little long-term debt in a decentralized equilibrium. We show that a constant tax on short-term debt can lead to Pareto improvements and less volatile aggregate economic variables by inducing entrepreneurs to rely on longer-term funding. In fact, in our model entrepreneurs are better off even when the proceeds of the tax are wasted in unproductive expenditures instead of being rebated lump-sum.

The paper is related to several strands of the literature. First, it relates to a broad theoretical literature in corporate finance and banking that analyzes debt maturity choice in partial equilibrium. For the most part, this literature attempts to rationalize the empirical prevalence of short-term debt in the financial and non-financial corporate sector. Flannery (1986) and Diamond (1991) argue that short-term debt issuance can act as a signaling device in frameworks with asymmetric information between borrowers and lenders. Diamond and

Dybvig (1983) rationalize demandable debt as an efficient mechanism to deal with depositors' exposure to liquidity shocks, while Calomiris and Kahn (1991) and Diamond and Rajan (2001) emphasize the incentive roles of short-term debt in environments with moral hazard. In contrast to this literature, the present paper stresses undesirable aspects of short-term debt and argues that too much of it may be issued in decentralized markets.

The second literature to which this paper relates is the macroeconomic literature on financial amplification or financial accelerator effects, which analyzes the role of financial frictions in general equilibrium. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) show that in the presence of financial frictions, endogenous variations in borrowers' net worth can lead to amplification of fundamental economic shocks. The formal modeling framework adopted in this paper shares several aspects of the quantitative theoretical implementations of these ideas by Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), Iacoviello (2005), Mendoza and Smith (2006) and others. On the normative side, Lorenzoni (2008) and Bianchi (2009) find that individual agents may overborrow as they do not internalize the tightening of financing conditions they impose on other agents through their subsequent deleveraging in the event of bad shocks. Korinek (2009) finds that atomistic agents in emerging markets may rely excessively on dollar debt, as they do not internalize the pressure put on the exchange rate through their cut in aggregate demand during financial crises, and Korinek (2011b) argues more generally that agents choose excessively risky financial structures in the presence of financial accelerator effects. This paper complements this body of research by showing that the debt maturity chosen by constrained borrowers in a decentralized equilibrium can be socially inefficient and can result in excessively volatile allocations and asset prices.

Finally, the paper also relates to a literature that analyzes the maturity structure of capital flows to emerging markets. Paralleling the results of the corporate finance papers mentioned above, Rodrick and Velasco (1999), Tirole (2003) and Jeanne (2009) argue that short-term debt can act as a disciplining device for opportunistic sovereign borrowers, although the latter recognize that under some circumstances, short-term debt accumulation by private agents can be socially excessive. Broner, Lorenzoni, and Schmukler (2008) explain emerging market governments' reliance on short-term debt by appealing to international lenders' risk aversion and fluctuations thereof. Like most of the corporate finance literature, the analysis in these papers is based on heavily stylized 3-period partial equilibrium models. In contrast, the present paper studies the positive and normative implications of debt maturity choices in a tractable infinite horizon dynamic stochastic general equilibrium framework with risk averse borrowers and lenders.

The environment is presented in Section 2.2 and the competitive equilibrium is defined and characterized in Section 2.3. Section 2.4 discusses the rationale for macroprudential policy. Section 2.5 presents the quantitative results and Section 2.6 concludes.

2.2 The model

We consider an environment, inspired by Kiyotaki and Moore (1997), with two sets of agents - households and entrepreneurs - and one source of (aggregate) risk. Both types of agents are risk-averse consumers and derive benefits from a physical asset. Entrepreneurs, who for modeling purposes are assumed to be less patient, borrow from households in equilibrium, and produce the consumption good out of the physical asset and labor using a constant returns to scale technology. Households supply labor and savings to the entrepreneurial sector, and use the physical asset for home production. Financial markets are both imperfect and incomplete. In addition to an enforcement friction that underlies a collateral constraint faced by borrowers, asset markets are exogenously assumed to be incomplete in that agents are only allowed to trade short-term and long-term non-state-contingent bonds. Short-term bonds are one-period non-state-contingent bonds, while long-term bonds are modeled as a perpetuity. Note that although the cash flows attached to a long-term bond are non-state-contingent, the one-period rate of return on this bond is state-contingent, as the price of long-term bonds generally varies with economic conditions. The presence of long-term bonds therefore creates risk-sharing opportunities in the economy by enabling agents to form bond portfolios with state-contingent returns. There are two goods: a consumption good and a capital good. The consumption good is perishable, while capital is in fixed supply and does not depreciate.

Households. There is a unit mass of identical infinitely-lived households in the economy. Each household maximizes expected lifetime utility, given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - G(L_t) \right]$$

where E_0 is the expectation operator conditional on period 0 information, β is a discount factor, $u(\cdot)$ is a constant-relative-risk-aversion (CRRA) period utility function for consumption, $G(\cdot)$ is an increasing and convex labor disutility function, C_t denotes period t consumption, and L_t denotes period labor supply. Households can invest in physical capital used to operate a home production technology. We assume that home production output received in period t depends on the stock of physical capital carried over into period $t+1$, K_{t+1}. This amounts to assuming that physical assets are traded *cum-dividend.* Households can also choose to invest in short-term bonds and long-term bonds. q_t^S denotes the price of the short-term (discount) bond. Similarly, q_t^L denotes the price of the long-term bond, which entitles its holder to payments of one unit of the consumption good in every future period until infinity. The return on holding short-term bonds between t and $t+1$ is known in t, while the one-period return on holding long-term bonds is state-dependent, because the $t+1$ market value of the remaining payment stream of a perpetuity depends on the state of the economy. A representative household chooses sequences of consumption, capital, short-term bonds and long-term bonds to maximize expected lifetime utility subject to the following period budget constraint:

$$C_t + q_t K_{t+1} + q_t^S B_{t+1}^S + q_t^L B_{t+1}^L = A_t F(K_{t+1}) + w_t L_t + q_t K_t + B_t^S + (1+q_t^L) B_t^L - \vartheta B_t^L,$$

where $F(\cdot)$ is an increasing and concave home production function, q_t is the price of capital, w_t is the wage rate, B_t^S is the household's holding of short-term bonds, B_t^L is its holding of long-term bonds, and ϑ represents a monitoring cost which long-term bond holders must incur each period to prevent borrowers from absconding with the remaining stream of payments due.[3]

[3] In our numerical solution, in order to preserve a meaningful bond portfolio choice even as the stochastic noise in the model approaches zero, we assume that this cost is of second-order, i.e. that $\vartheta \equiv (\epsilon_t^\vartheta)^2$, where ϵ_t^ϑ is a zero-mean i.i.d. random variable with variance σ_ϑ^2 of the same order of magnitude as the variance of shocks to TFP. The assumption that the monitoring cost is stochastic is made for technical reasons only and affects the equilibrium dynamics only at third- and higher orders. Because of the presence of portfolio choice, we solve the model by approximating the decision rules around a deterministic steady state to which the allocations and prices converge when the scale of the stochasticity become arbitrary small. If the monitoring cost was a constant independent of that scale, the bond portfolio choice would be trivial in the limit: as the insurance benefits of long-term debt depend on the scale of TFP shocks but the monitoring costs are kept fixed, for arbitrarily small TFP shocks, short-term contracts would strictly dominate long-term contracts. By making monitoring costs stochastic, we are able to maintain a non-trivial trade-off between short- and long-term debt, even as the scale of the stochastic noise approaches zero, through effectively stabilizing the

Further, as monitoring activities generally result in unproductive expenditures, we assume that the monitoring cost is a resource cost. Consequently, short-term debt contracts have a relative advantage over long-term debt contracts in that they avoid the need for unproductive monitoring activities. From both a private and a social perspective, this implies a trade-off between the insurance (risk-sharing) benefits of long-term debt and the cost advantage of short-term debt.[4] Section 2.3.2 offers a detailed discussion of individual agents' bond/debt maturity choice in the model.

Household behavior is characterized by the following four optimality conditions:

$$
\begin{aligned}
w_t &= \frac{G'(L_t)}{u'(C_t)}, && (2.1)\\
q_t u'(C_t) &= A_t F'(K_{t+1}) u'(C_t) + \beta E_t \left[q_{t+1} u'(C_{t+1}) \right], && (2.2)\\
u'(C_t) &= \beta \frac{1}{q_t^S} E_t \left[u'(C_{t+1}) \right], && (2.3)\\
u'(C_t) &= \beta E_t \left[\frac{1 + q_{t+1}^L}{q_t^L} u'(C_{t+1}) \right] - \beta E_t \left[\frac{\vartheta}{q_t^L} u'(C_{t+1}) \right]. && (2.4)
\end{aligned}
$$

(2.1) describes optimal labor supply, while (2.2), (2.3) and (2.4) are standard Euler equations characterizing the household's optimal holdings of the physical asset, short-term bonds and long-term bonds.

Entrepreneurs. There is a continuum of mass one of identical entrepreneurs with infinite horizon. Entrepreneurs consume and operate a technology which produces consumption

relationship between the costs and benefits of long-term debt.

[4] Kiyotaki and Moore (2003, 2005) consider a setup where borrowers need to pay a deadweight securitization cost to ensure the future liquidity (resellability) of long-term claims, putting forward the argument that multilateral commitment to repay debt is generally more demanding than bilateral commitment (the later being the only type of commitment relevant for short-term debt issuance). This securitization cost increases the cost of borrowing long-term. Our monitoring cost ϑ puts the burden of costly contract enforcement on the lender but achieves the same effect.

goods out of physical capital and labor inputs. To make entrepreneurs borrow in equilibrium, we assume that they discount the future more strongly than households. Each entrepreneur faces a collateral constraint which limits the value of his total debt to a multiple of the value of the capital he holds.[5] As developed below, this collateral constraint can be interpreted as an incentive compatibility constraint in an environment with enforcement frictions.[6] The lower discount factor ensures that entrepreneurs remain financially constrained in the long run. Each entrepreneur maximizes expected lifetime utility, given by:

$$E_0 \sum_{t=0}^{\infty} \gamma^t u(c_t),$$

where γ is a discount factor satisfying $\gamma < \beta$, and c_t denotes period t consumption. The entrepreneur's period budget constraint is given by

$$c_t + q_t k_{t+1} + q_t^S b_{t+1}^S + q_t^L b_{t+1}^L = A_t f(k_{t+t}, l_t) - w_t l_t + q_t k_t + b_t^S + (1 + q_t^L) b_t^L,$$

where k_t is the entrepreneur's holding of physical capital, l_t is the hired labor, b_t^S is his holding of short-term bonds, b_t^L is his holding of long-term bonds, and $f(\cdot)$ is a constant returns to scale production function.[7]

[5] Given a lower discount factor, in the absence of limits on their borrowing, entrepreneurs would accumulate debt to a point where their long-run consumption would converge towards zero.

[6] Note that, in principle, households are subject to the same enforcement friction as entrepreneurs and should therefore face the same collateral constraint. However, given their higher discount factor, households will turn out to be lenders and not borrowers in equilibrium. Imposing a borrowing constraint in their decision problem would thus be superfluous.

[7] We omit the monitoring cost in the budget constraint, because entrepreneurs will end up being issuers of long-term bonds in equilibrium ($b_t^L < 0$), and monitoring costs are borne by lenders (households).

Entrepreneurs also face a sequence of collateral constraints, given by

$$-q_t^S b_{t+1}^S - q_t^L b_{t+1}^L \leq \kappa q_t k_{t+1}. \tag{2.5}$$

This constraint limits the total value of outstanding debt to a fraction of the value of capital held by a borrower. It is akin to the collateral constraints in Aiyagari and Gertler (1999) and Kiyotaki and Moore (1997), and as in these papers it has the potential to generate financial amplification effects through the impact of asset price changes on agents' borrowing capacity. We interpret the constraint as arising from a limited enforcement problem, but refrain from modeling its micro-economic foundation explicitly.

A representative entrepreneur chooses sequences of consumption, capital, short-term bonds and long-term bonds to maximize expected life-time utility subject to a sequence of period budget constraints and collateral constraints. Optimal behavior by entrepreneurs is characterized by the following four conditions

$$
\begin{aligned}
A_t f_l(k_{t+1}, l_t) &= w_t & (2.6)\\
q_t u'(c_t) &= A_t f_k(k_{t+1}, l_t) u'(c_t) + \gamma E_t \left[q_{t+1} u'(c_{t+1}) \right] + \kappa \mu_t q_t & (2.7)\\
u'(c_t) &= \gamma \frac{1}{q_t^S} E_t \left[u'(c_{t+1}) \right] + \mu_t, & (2.8)\\
u'(c_t) &= \gamma E_t \left[\frac{1 + q_{t+1}^L}{q_t^L} u'(c_{t+1}) \right] + \mu_t, & (2.9)
\end{aligned}
$$

where μ_t is the non-negative multiplier on the collateral constraint, and by the complementary slackness condition

$$\mu_t \left(q_t^S b_{t+1}^S + q_t^L b_{t+1}^L + \kappa q_t k_{t+1} \right) = 0. \tag{2.10}$$

(2.6) describes entrepreneurs' labor demand, while (2.7), (2.8) and (2.9) are conventional Euler equations for capital, short-term bonds and long-term bonds. From (2.8) and (2.9) we see that the borrowing constraint can induce a wedge between the current marginal value of wealth and the discounted expected value of next period's marginal value of wealth. It is also apparent from (2.7) that when the collateral constraint binds, entrepreneurs value capital more highly as it helps relax the constraint.

Fundamentals. TFP is assumed to follow a first-order autoregressive process

$$\log(A_t) = \rho \log(A_{t-1}) + \epsilon_t^A,$$

where ϵ_t^A is an i.i.d. random variable with variance σ_A^2.[8]

2.3 Competitive Equilibrium

It is convenient to define the variables $B_t \equiv q_{t-1}^S B_t^S + q_{t-1}^L B_t^L$, $\Phi_t \equiv q_{t-1}^L B_t^L$, $b_t \equiv q_{t-1}^S b_t^S + q_{t-1}^L b_t^L$ and $\phi_t \equiv q_{t-1}^L b_t^L$. A competitive equilibrium of the model can then be defined by sequences of state-contingent allocations $\{c_t, C_t, l_t, L_t, b_{t+1}, B_{t+1}, \Phi_{t+1}, \phi_{t+1}, k_{t+1}, K_{t+1}\}_{t=0}^{\infty}$ and prices $\{w_t, q_t^S, q_t^L, q_t\}_{t=0}^{\infty}$ such that: (a) households maximize expected lifetime utility subject to their sequence of budget constraints, taking as given prices and initial conditions (B_0, Φ_0, K_0), (b) entrepreneurs maximize expected lifetime utility subject to their sequence of budget and collateral constraints, taking as given prices and initial conditions (b_0, ϕ_0, k_0),

[8] At a technical level and for the purpose of solving the model, a second source of uncertainty in the economy arises from monitoring costs. For the purpose of solving the model, we assume that ϵ_t^{ϑ} is an i.i.d. random variable with variance σ_{ϑ}^2. Given the retained formulation, up to a second-order of accuracy the realizations of this shock have no effect on the equilibrium dynamics of the model's variables. The presence of the monitoring cost (but not the realizations of ϵ_t^{ϑ}) is nonetheless a key determinant of debt/bond maturity choices.

and (c) the markets for labor, short-term bonds, long-term bonds and capital clear[9]:

$$
\begin{aligned}
L_t &= l_t, \\
b_{t+1} - \phi_{t+1} + B_{t+1} - \Phi_{t+1} &= 0, \\
\phi_{t+1} + \Phi_{t+1} &= 0, \\
k_{t+1} + K_{t+1} &= \bar{K},
\end{aligned}
$$

where $\bar{K}$ is the fixed supply of capital in the economy.

We quantitatively analyze the dynamics of the model in the neighborhood of a deterministic steady state to which competitive equilibrium allocations and prices converge when the scale of the stochastic noise in the model becomes arbitrarily small. Even though the maturity structure is not uniquely determined in the deterministic steady state, as short- and long-term bonds are perfect substitutes in the absence of stochastic shocks, the other "non-portfolio" variables are uniquely pinned down. By focusing our attention on the local dynamics of the model, we assume that the entrepreneur's collateral constraint always binds.[10]

2.3.1 Deterministic steady state

We consider a deterministic steady state in which long-term debt enforcement is costless ($\vartheta = 0$). From the household's Euler equations for short term and long term bonds, the bond prices are given by $q^S = \beta$ and $q^L = \beta/(1-\beta)$. The gross interest rates on these

[9] Goods market clearing then follows from Walras' law.

[10] In simulations, we find that the shadow price of this constraint remains positive in each of the 100,000 periods.

two bonds are thus equal and given by $r^S = r^L = \beta^{-1}$. Given that the two bonds have the same deterministic returns in the steady state, they are indistinguishable. This illustrates why the agents' debt portfolios are not uniquely pinned down in the absence of stochastic shocks and monitoring costs. From the entrepreneur's Euler equations (2.8) or (2.9), we have $\mu = (1-\gamma/\beta)u'(c) > 0$, meaning that the entrepreneurs are constrained in the deterministic steady state. Combining the two agents' Euler equations for capital, (2.2) and (2.7), we can write

$$AF'(K) = \frac{\beta(1-\beta)}{\beta(1-\beta)+(\beta-\kappa)(\beta-\gamma)} A f_k(k,l) \tag{2.11}$$

This expression illustrates that as long as $\kappa \neq \beta$, capital is allocated inefficiently in the deterministic steady-state. In the realistic case where $\kappa < \beta$, the marginal product of capital is higher in the entrepreneurial sector than in the household sector.

2.3.2 *Costs and benefits of long-term debt*

The central point of the paper is that debt maturity choices made by individually rational agents in an environment where financial frictions give rise to amplification effects are not necessarily efficient. In particular, in this model, debt contracts can have an excessively short maturity. It is thus worth clarifying the precise elements that affect the agents' debt maturity choices in the model.

Using the household's Euler equations, we can express the prices of capital and long-term bonds as

$$q_t = E_t \sum_{j=0}^{\infty} \frac{\beta^j A_{t+j} F'(K_{t+1+j}) u'(C_{t+j})}{u'(C_t)},$$

and

$$q_t^L = E_t \sum_{j=1}^{\infty} \frac{\beta^j u'(C_{t+j})}{u'(C_t)} (1-\vartheta).$$

From these expressions, it can be recognized that fluctuations in households' consumption resulting from aggregate shocks will be associated with co-movements in the prices of capital and long-term bonds due to fluctuations in the common stochastic discount factor. A positive TFP shock will lead to higher current household consumption (via higher wages and higher home production), lower current household marginal utility of consumption, and thus in equilibrium to higher prices of capital and long-term bonds.[11] At the same time, leveraged entrepreneurs who fund part of their capital holdings with debt are highly exposed to aggregate shocks: not only do they suffer from less productive capital when a bad shock hits, but they are also hurt by the asset price drop associated with the scarcity of current resources. Figure 2.1 represents stylized balance sheets of leveraged borrowers (entrepreneurs) in the cases with and without long-term debt. When only short-term debt is available, the market value of the debt is predetermined, while the value of the assets is state contingent. This results in a high sensitivity of entrepreneurs' net worth to aggregate shocks. When long-term debt is also available, the value of the debt is state-contingent, since the market price of the future payment stream attached to long-term debt depends on current conditions. In particular, the market value of the debt rises in good times and shrinks in bad times. Issuing long-term debt thus provides entrepreneurs with a hedge against fluctuations in the value of their assets. Equivalently, it allows entrepreneurs to pass on to households some of the risk to which they are naturally exposed.

[11] The strength of the price responses will depend, among other things, on how much households are exposed to aggregate shocks, but its direction will be unambiguous.

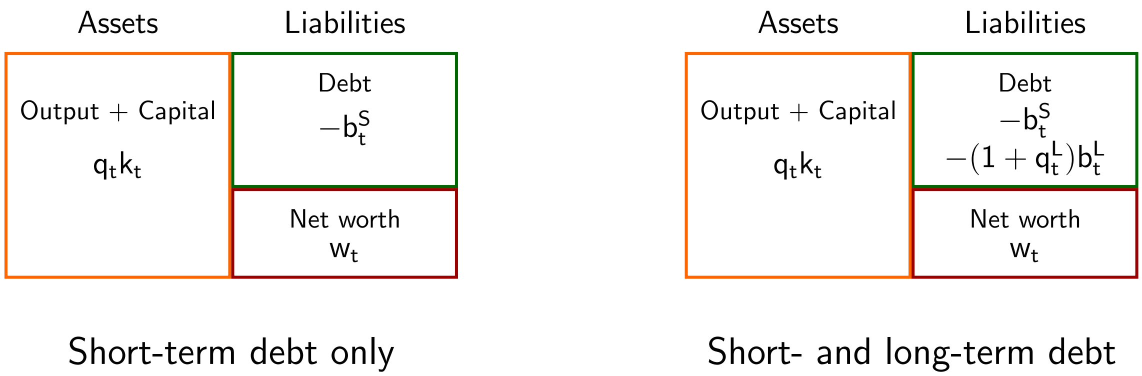

Fig. 2.1: Balance sheets of leveraged entrepreneur.

To which extent will private agents make use of the risk-sharing opportunity provided by long-term debt contracts? This will depend on the premium on long-term debt charged by lenders. In the absence of costly enforcement of long-term contracts (when $\vartheta = 0$), this premium will correspond to a pure term premium: since the return on long-term bonds is positively correlated with lenders' consumption in equilibrium, lenders will demand compensation for holding these bonds. Borrowers will then trade-off the insurance benefits of long-term debt with its extra cost, and choose a debt maturity such that those two are equalized. When the enforcement of long-term contracts is costly ($\vartheta > 0$), lenders will require compensation for holding long-term bonds beyond what can be attributed to a pure term premium. In effect, lenders will pass on the burden of costly enforcement to borrowers, since in equilibrium they need to be indifferent at the margin between saving in short- and long-term bonds. Faced with this higher cost of long-term debt, borrowers will generally choose debt maturity structures shorter than what they would choose in the absence of costly enforcement of long-term contracts. From a social perspective, short-term debt has an advantage in that it avoids the waste of resources in unproductive monitoring activities.

Our model thus captures in a reduced form the incentive benefits of short-term debt put forward by Calomiris and Kahn (1991) and others. A contribution of our paper is then to show that the market can fail to produce allocations that efficiently balance the trade-off between the risk-sharing benefits of long-term debt and the cost advantage of short-term debt.

2.3.3 Analytical results

It turns out that when enforcement of long-term debt is costless ($\vartheta = 0$), the availability of short- and long-term bonds results in competitive equilibrium allocations that achieve an efficient degree of risk-sharing. This is because in spite of borrowing constraints, short- and long-term bonds can provide agents with a very valuable hedge. In the absence of bonds with state-contingent returns, entrepreneurs are more exposed to risk than households because of their leveraged positions: they finance part of their capital holdings with debt and therefore tend to suffer more in the event of a negative TFP shock (which depresses the price of the capital). But given the state-contingent nature of the returns on long-term bonds, entrepreneurs can insure against fluctuations in the value of their capital by going short on long-term bonds, i.e. by issuing long-term debt, while going long on short-term bonds. Indeed, under some conditions, this opportunity results in a fully efficient allocation of risk between households and entrepreneurs, which leads to the following proposition.

Proposition 8. *When agents have log utility, TFP is serially uncorrelated and enforcement of long-term debt is costless (i.e. when $\vartheta = 0$), risk markets are effectively complete. As a result, aggregate risk is shared equally by households and entrepreneurs, and the wealth distribution is time-invariant. Furthermore, the entrepreneur's collateral constraint always* *binds, the capital allocation is fixed at its deterministic steady state value (i.e. fire sales never occur) and the economy displays no persistence.*

Proof. See appendix C. □

The intuition for the effective completeness of risk markets comes from the fact that when long-term debt enforcement is costless, under log utility and serially uncorrelated TFP, the capital and bond prices are all linear functions of the single state variable (TFP). It is therefore possible to construct a bond portfolio whose fluctuations in value match exactly the changes in the relative wealth distribution caused by fluctuations in TFP under asymmetric holdings of capital. The result is similar to the ones in Angeletos (2002) and Buera and Nicolini (2004), who find that a government can use non-contingent debt of different maturities to achieve complete markets Ramsey allocations. Our result requires a more stringent restriction on preferences than theirs, but puts less demand on asset markets: complete markets in these papers generally requires debt instruments of as many maturities as possible states of nature, while our result holds for an arbitrarily large number of states and just two maturities.

We view the equal sharing of aggregate risk as an efficient outcome, given identical risk tolerances and the CRRA property of the log utility function. The constant relative risk aversion makes the desired risk-exposure of agents independent of their wealth levels, so it is socially desirable to let households and entrepreneurs share aggregate risk equally, even though the former are wealthier than the later. A remarkable aspect of proposition 1 is that perfect risk sharing is achieved despite the presence of a collateral constraint, which a priori puts restrictions on the trade of the available financial claims.

The *no fire sales* outcome also represents a strong result. In the presence of short-term debt only, as in the generic model of Kiyotaki and Moore (1997), aggregate shocks relax or tighten leveraged borrowers' constraints, and result in them increasing or decreasing their capital holdings, thereby setting in motion a financial amplification mechanism. Under the conditions of proposition 1, aggregate shocks do indeed relax or tighten the collateral constraint of borrowers, and thus affect their demand for the physical asset, but happen to do so in exactly the same proportion as the wealth effect on the unconstrained agents' demand for the asset. In equilibrium, the asset price adjusts to induce agents to demand a constant amount of capital, and there is no transfer of assets between the constrained and unconstrained sectors. The price of capital appreciates following a good shock and depreciates following a bad shock, but the allocation of capital never deviates from its deterministic steady state value. The fact that fire sales never occur in equilibrium explains the efficiency of risk allocations. In a similar environment, Korinek (2011b) finds that risk allocations can fail to be constrained efficient even when agents have access to a full set of state-contingent assets. There, the inefficiency derives from the fact that agents undervalue wealth in states of nature where fire sales depress asset prices. Under less restrictive assumptions than the ones in proposition 1, fire sales will occur in our model and risk allocations will generally fail to be efficient.

But under the assumptions of proposition 1, the competitive equilibrium prices and allocations take particularly simple forms. Define the variable $Y_t \equiv [f(k_{t+1}, l_t) + F(K_{t+1})]$, such that aggregate output in period t is given by $Y_t A_t$. Capital and labor are always allocated as in the deterministic steady state: $k_{t+1} = k$, $K_{t+1} = K$, $l_t = l$ for all t. Households and entrepreneurs consume a constant fraction of aggregate output every period: $C_t =$

$(1-\omega)YA_t$ and $c_t = \omega YA_t$, where ω is related to the relative wealth positions.[12] The price of capital is $q_t = \frac{1}{1-\beta}F'(K)A_t$. Assuming normality of the innovations to log TFP, the prices of short-term bonds and long-term bonds are given by

$$q_t^S = \beta e^{\frac{\sigma_A^2}{2}} A_t, \qquad q_t^L = \frac{\beta}{1-\beta} e^{\frac{\sigma_A^2}{2}} A_t,$$

and the bond portfolio is given by

$$b^S = \frac{\kappa F'(K)k}{\beta^2 e^{\frac{\sigma_A^2}{2}}}, \qquad b^L = -\frac{\kappa F'(K)k}{\beta^2 e^{\frac{\sigma_A^2}{2}}},$$

with $B^S = -b^S$ and $B^L = -b^L$. The value of total bond holdings and long-term bond holdings are given by

$$b_{t+1} = -\frac{1}{1-\beta}\kappa F'(K)kA_t, \qquad \phi_{t+1} = -\frac{1}{1-\beta}\kappa\beta^{-1}F'(K)kA_t$$

The equilibrium debt maturity structure of entrepreneurs therefore consists of a fraction $\beta^{-1} > 1$ of long-term debt and a fraction $1-\beta^{-1} < 0$ of short-term debt. Despite being net borrowers, entrepreneurs have a long position in short-term bonds. The intuition for this result has to do with the nature of the risk-sharing problem that the market is trying to solve. The value of the asset side of leveraged entrepreneurs' balance sheet is state-contingent. Therefore, the market is looking for a bond portfolio whose realized one-period ahead return is also state-contingent in order to allow entrepreneurs to shift risk to the liability side of their balance sheet and pass it on to households. Yet long-term bonds have

[12] The value of ω is given in appendix C.

a non-state contingent component. This component corresponds to the first payment on the bond, whose value makes up a fraction $1-\beta^{-1}$ of the total value of the long-term bond. By borrowing long-term $\beta^{-1} > 1$ times their net debt and placing $1-\beta^{-1}$ in short-term bonds, entrepreneurs hold a debt portfolio whose realized one-period ahead return is entirely state-contingent. This particular debt portfolio is the only one that can achieve the socially desirable risk-allocation.

Finally, we observe that in the presence of aggregate risk the insurance provided by long-term debt is not free: there is a positive risk premium or *term premium* $\chi_t \equiv E_t[r^L_{t+1}] - r^S_{t+1}$ on long-term bonds in equilibrium, where $r^L_{t+1} \equiv (1+q^L_{t+1})/q^L_t$ and $r^S_{t+1} \equiv 1/q^S_t$ are the returns on long- and short-term bonds. The term premium is given by

$$\chi_t = \frac{e^{\frac{\sigma_A^2}{2}} - e^{-\frac{\sigma_A^2}{2}}}{A_t} > 0$$

This term premium is naturally an increasing function of the volatility of aggregate shocks σ_A, and it is countercyclical:

$$\frac{\partial \chi_t}{\partial \sigma_A} = \sigma_A \frac{e^{\frac{\sigma_A^2}{2}} + e^{-\frac{\sigma_A^2}{2}}}{A_t} > 0, \qquad \text{and} \qquad \frac{\partial \chi_t}{\partial A_t} = -\frac{e^{\frac{\sigma_A^2}{2}} - e^{-\frac{\sigma_A^2}{2}}}{A_t^2} < 0.$$

It is worth mentioning that the perfect risk-sharing result established under the assumptions of proposition 1 continues to hold in the more general case of CRRA utility and serially correlated TFP, but only up to a first-order of approximation. More precisely, as long as enforcement of long-term contracts is costless, households and entrepreneurs share risk equally up to a first-order, i.e. the policy functions for consumption take the following form

$$\hat{C}_t = \hat{A}_t + \eta_t^C + O(\epsilon^2),$$

$$\hat{c}_t = \hat{A}_t + \eta_t^c + O(\epsilon^2),$$

where $\hat{x}_t = \log(x_t/x)$, $O(\epsilon^2)$ represents terms of second- or higher order, and η_t^x are terms linear in the first-order components of endogenous state variables whose values are known as of $t-1$. In these cases, the long-run (zero-order) maturity choice of agents is efficient, and a constant tax on short- or long-term debt cannot lead to Pareto improvements.

2.4 Macroprudential policy

This section discusses the role of macroprudential policy in the model, and specifies its objectives and instruments.

2.4.1 Motivation

In the absence of costly enforcement of long-term debt contracts, the presence of short- and long-term bonds results in approximately effectively complete risk markets. In this case, equilibrium debt portfolio choices result in an allocation of risk that cannot be improved upon using a constant tax on short-term debt. However, when long-term debt is costly to enforce, debt portfolio choices are distorted and a wedge between the private and social benefits of the insurance provided by long-term debt leads to risk allocations that are not constrained efficient. Individual entrepreneurs fail to internalize that by issuing more long-term debt and less short-term debt, they reduce the volatility of the price of capital. This happens

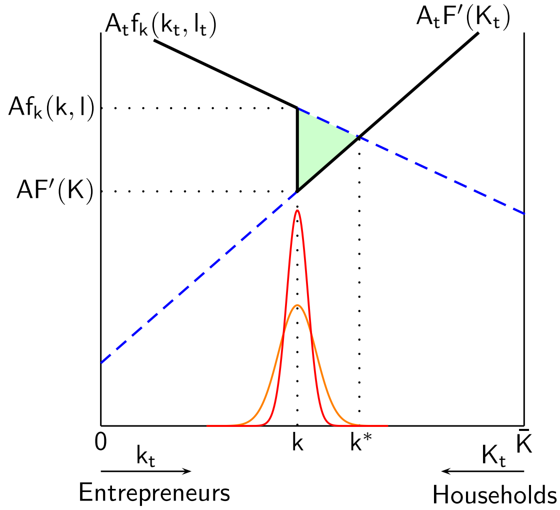

Fig. 2.2: Inefficient capital and risk allocations.

because the volatility of the asset price depends on the stability of entrepreneurs' net worth, as the collateral constraint makes the borrowing capacity of entrepreneurs a direct function of their wealth. Relying on long-term rather than short-term debt reduces the exposure of entrepreneurs' net worth to aggregate shocks, and therefore reduces the volatility of the price of capital. A more stable price of capital in turn leads to a more stable distribution of capital between the entrepreneurial and household sectors.

Figure 2.2 depicts the marginal product of capital in the two sectors. In the deterministic steady-state, capital is allocated inefficiently at k (the efficient capital allocation is at k^*) and the deadweight loss caused by the borrowing constraint corresponds to the area of the shaded triangle.[13] As the deadweight loss is a convex function of the deviation of the capital allocation from the efficient allocation, stable capital allocations are socially more desirable than volatile ones. The figure schematically represents two generic ergodic distributions of capital. The distribution with higher variance corresponds to a situation where entrepreneurs rely less on long-term debt and more on short-term debt relative to the distribution with smaller variance. Leveraged borrowers do not generally choose a so-

[13] The interpretation is identical to the one in Kiyotaki and Moore (1997).

cially efficient risk exposure because they fail to internalize their contribution to systemic risk. At the margin, relying more on long-term debt and less on short-term debt reduces the volatility of individual net worth. What borrowers fail to internalize, however, is that a lower volatility of individual net worth reduces the volatility of asset prices and thus lowers the volatility of other borrowers' net worth. Entrepreneurs perceive the private insurance benefits of long-term debt issuance, but they fail to recognize its wider social benefits arising from the relevance of the market price of capital for financial constraints. The market failure underlying this inefficiency result is a "fire-sale" externality similar to that emphasized by Lorenzoni (2008) and Korinek (2009, 2009b). It is a particular application of the general proposition of the constrained suboptimality of competitive equilibria in incomplete markets settings by Stiglitz (1982) and Geanakoplos and Polemarchakis (1986).

2.4.2 Welfare measures and policy instruments

Let $E_{CE}[u(C)-G(L)]$ and $E_{CE}[u(c)]$ denote the unconditional expected utilities of households and entrepreneurs under the ergodic distribution induced by a competitive equilibrium without government intervention. We consider a government that has the ability to impose a constant tax on entrepreneurs' issuance of short-term debt and to rebate the proceeds of this tax to entrepreneurs.

With macroprudential policy, the entrepreneur's problem is to maximize expected lifetime utility subject to a sequence of collateral constraints and a sequence of budget constraints given by

$$c_t + q_t k_{t+1} + q_t^S b_{t+1}^S + q_t^L b_{t+1}^L = A_t f(k_{t+1}, l_t) - w_t l_t + q_t k_t + (1+\tau^S) b_t^S + (1+q_t^L) b_t^L + T_t^E,$$

where τ^S is tax on short-term debt, and T_t^E is a transfer. The household's problem is unaffected. A competitive equilibrium with macroprudential policy is defined by sequences of state-contingent prices $\{w_t, q_t^S, q_t^L, q_t\}_{t=0}^{\infty}$, allocations $\{c_t, C_t, l_t, L_t, b_{t+1}, B_{t+1}, \Phi_{t+1}, \phi_{t+1}, k_{t+1}, K_{t+1}\}_{t=0}^{\infty}$, and policy instruments $(\tau^S, \{T_t^E\}_{t=0}^{\infty})$ such that: (a) households maximize expected lifetime utility subject to their sequence of budget constraints, taking as given prices, policies and initial conditions (B_0, Φ_0, K_0), (b) entrepreneurs maximize expected lifetime utility subject to their sequence of budget and collateral constraints, taking as given prices, policies and initial conditions (b_0, ϕ_0, k_0), (c) the markets for short-term bonds, long-term bonds and capital clear, and (d) the government runs a balanced budget:

$$T_t^E + \tau^S b_t^S = 0.$$

Let $E[u(C)-G(L)]$ and $E[u(c)]$ denote the unconditional expected utilities of households and entrepreneurs under the ergodic distribution induced by a competitive equilibrium with macroprudential policy. We assume that the government's objective in setting macroprudential policy is to maximize the unconditional expected utility of entrepreneurs subject to providing households with an unconditional expected utility at least as high as in a competitive equilibrium without government intervention, i.e. the government solves

$$\max_{\tau^S} E[u(c)] \qquad \text{s.t} \quad E[u(C) - G(L)] \geq E_{CE}[u(C) - G(L)].$$

2.5 Quantitative analysis

2.5.1 Solution

Standard perturbation methods for solving dynamic stochastic general equilibrium (DSGE) models are inappropriate to solve the model presented in this paper because of the presence of portfolio choice in an incomplete market setting.[14] Progress has recently been made in this area with the methods proposed by Devereux and Sutherland (2011, Devereux and Sutherland (2009), Tille and van Wincoop (2008) and Evans and Hnatkovska (2008) to produce approximate solutions for two-country DSGE models featuring portfolio choice under incomplete markets. Despite a collateral constraint and differences in discount factors, the structure of our two-agent model is remarkably similar to the two-country DSGE models for which these methods are designed. We thus use the approach of Devereux and Sutherland together with the "standard" algorithm of Schmitt-Grohe and Uribe (2004) to obtain a second-order accurate solution of our model. The general principle underlying Devereux and Sutherland's approach is due to Samuelson (1970) and states that in order to derive the solution for portfolio choice up to N-th order accuracy, the portfolio problem must be approximated up to the $N+2$-th order. Appendix D provides details on the model solution.

[14] The failure of standard perturbation methods in models with portfolio choice is easily understood. These methods usually approximate the model solution around the deterministic steady state. Portfolio choices, however, are not uniquely defined in the deterministic steady state, as assets (i.e. in our case short-term and long-term bonds) are perfect substitutes. Hence, the deterministic steady state does not deliver a natural approximation point. Further, a linearized solution features certainty equivalence, while portfolio choices explicitly depend on the risk characteristics of the available assets.

2.5.2 *Functional forms and calibration*

We adopt the following functional forms for utility and producton functions. Utility from consumption takes the standard CRRA form

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma},$$

disutility from labor is assumed to be given by

$$G(L) = \frac{L^{\zeta}}{\zeta},$$

entrepreneurial production is assumed to be Cobb-Douglas

$$f(k, l) = k^{\alpha_e} l^{1-\alpha_e},$$

and home production is given by

$$F(K) = \nu K^{\alpha_h}.$$

A and $\bar{K}$ are normalized to 1. The parameters β, σ, ζ, α_e are set to standard values from the business cycle literature: $\beta = 0.99$, $\sigma = 2$, $\zeta = 1.01$, $\alpha_e = 0.36$. Following Iacoviello (2005), we set the entrepreneur's discount factor γ to 0.98, which implies an entrepreneurial internal rate of return twice as big as the equilibrium real interest rate. We set κ to 0.3, which matches an entrepreneurial debt ratio (debt over assets) of 30%. Welch (2004) finds a mean debt ratio of 29.8% in a sample of over 60,000 large publicly traded firms in the period 1964-2000. The capital share in home produciton is set to $\alpha_h = 0.13$, as in Greenwood and

		Source/target
Steady-state productivity	$A = 1$	Normalization
Fixed capital stock	$\bar{K} = 1$	Normalization
Household's discount factor	$\beta = 0.99$	Standard DSGE value
Entrepreneur's discount factor	$\gamma = 0.98$	Internal rate of return 2x real rate
Coefficient of relative risk aversion	$\sigma = 2$	Standard DSGE value
Elasticity parameter of labor supply	$\zeta = 1.01$	Virtually flat labor supply
Share of collateralizable assets	$\kappa = 0.3$	Debt ratio = 30%, Welch (2004)
Capital share in entrepreneur's output	$\alpha_e = 0.36$	Standard DSGE value
Capital share in home production	$\alpha_h = 0.13$	Greenwood and Hercowitz (1991)
Shift parameter in home production	$\nu = 0.41$	Corporate share of productive capital = 75%
Monitoring cost on LT contracts	$\vartheta = 0.030$	Long-run debt duration = 3.9 years
TFP process	$\rho = 0.57$	Autocorrelation $\rho_Y = 0.84$ and
	$\sigma_A = 0.012$	standard deviation $\sigma_Y = 0.017$ of U.S. GDP

Tab. 2.1: Parameter values

Hercowitz (1991). The scale factor ν in home production is set to yield a steady state ratio of productive assets held by the corporate sector $k/(K + k)$ of 75%, in line with Flow of Funds data. The monitoring cost parameter θ is set 0.030, implying a long-run average maturity structure (duration) of 3.9 years (corresponding to a maturity structure with weights of 0.85 on short-term debt and 0.15 on long-term debt).[15] This seems consistent with the descriptive results in Barclay and Smith (1995), who study a large sample of non-financial corporations from 1974 to 1992.[16] Finally, we use simulations to set the parameters governing the TFP process, ρ and σ_A, at values that make the model match the cyclical time series properties of quartely U.S. real GDP for the period 1947-2007. In the data, we find an autocorrelation of GDP of 0.84 and standard deviation of 0.017, and setting the parameters to $\rho = 0.57$ and $\sigma_A = 0.012$ replicates these moments. Table 2.1 summarizes the calibration.

[15] The duration of the short-term bond is one quarter, while the duration of long-term bond is $1/(1-\beta) =$ 100 quarters.

[16] Standard data sources like COMPUSTAT only report the the percentage of debt that matures in more than one, two, three, four and five years. Uncovering a debt duration from this data is difficult, but a value of 3.9 years seems broadly consistent with the data. Given the uncertainty surrounding this parameter value, it will be worth making robustness checks with respect to it.

2.5.3 *Results*

Positive implications of debt maturity

Figure 2.3 illustrates the impulse responses of some of the model's variables to a 1% negative innovation to log TFP in the benchmark model with only short-term debt, as well as in the multiple maturities model under two different assumptions regarding the enforcement cost of long-term contracts (in the absence of macro-prudential policy). The responses in the benchmark case with only short-term debt are represented with dashed lines, while those in the model with multiple maturities are represented with solid lines. The light (red) line is the multiple maturities model with costless enforcement of long-term debt ($\vartheta = 0$) and the dark (black) line is the multiplied maturities model with costly enforcement with the monitoring cost calibrated to yield an equilibrium maturity structure with an average duration of 3.9 years. Several comments are in order. First, we observe that when enforcement of long-term contracts is costless, risk markets are approximately effectively complete and shocks do not cause first-order changes in the wealth distribution. Entrepreneurs and households share risk equally.[17] In this case, capital does not get reallocated much following the shock. Consequently, the path of aggregate output follows mainly the mean-reverting path of TFP. In contrast, when long-term contracts are costly to enforce, borrowers choose more short-term debt in their portfolios, and entrepreneurs suffer relatively more than households from a negative aggregate shock. This disproportionately large deterioration in their net worth tightens their collateral constraint and induces a reallocation of capital towards the unconstrained household sector: fire sales occur. The standard financial amplification mechanism

[17] This can be observed from the fact that their consumption responses are identical. Again, this holds only up to a first-order approximation.

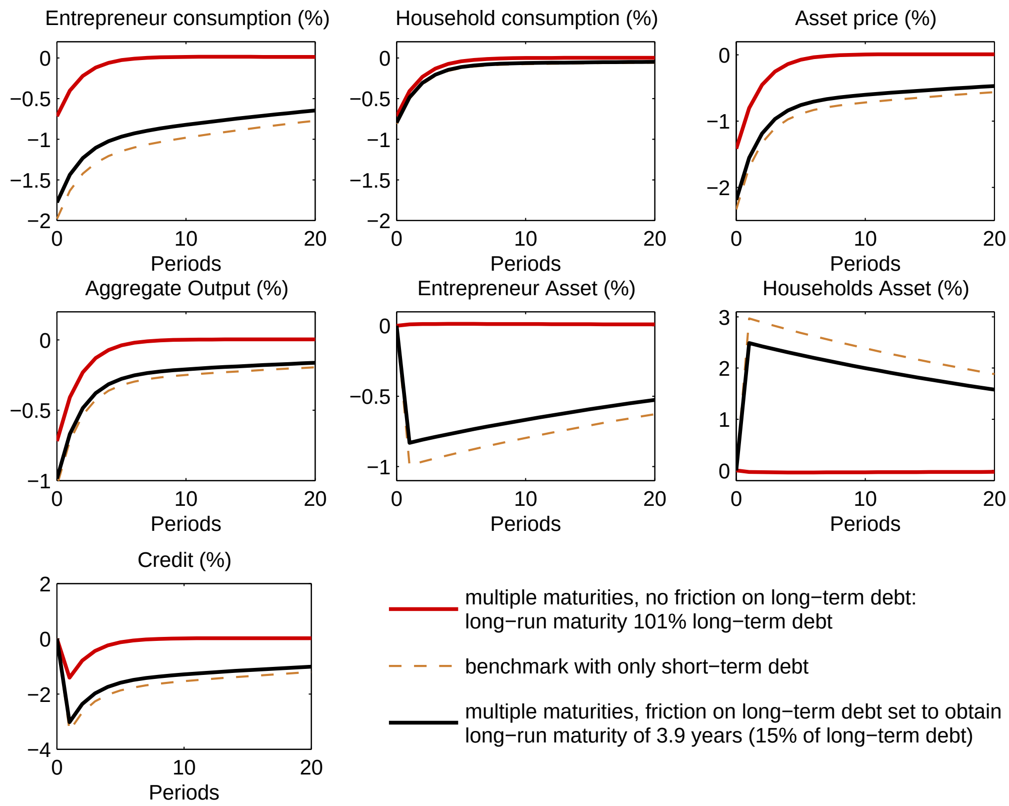

Fig. 2.3: Impulse responses to 1% negative TFP shock in benchmark model with only short-term debt, and with multiple maturities (with and without monitoring cost on long-term debt).

is at work here. The negative shock reduces the entrepreneurs' wealth, thereby reducing their ability to borrow. Facing tightening financial conditions, entrepreneurs fire-sell assets, causing further declines in the asset price (as households value the physical capital at a decreasing rate). This decline in the asset price reduces the entrepreneurs' wealth further, leading to yet another round of fire sales. After the shock and the fire sales, entrepreneurs slowly rebuild their capital holdings because it takes time for them to re-accumulate net worth and re-establish borrowing capacity at its pre-shock level. When long-term debt is costly to enforce, the high risk exposure of leveraged borrowers is a source of amplification and persistence of fundamental shocks, as in the benchmark case with only short-term debt.

Normative results

In order to analyze the normative implications of macroprudential policy, we compute welfare by calculating the unconditional expected utility of the households and entrepreneurs using a second-order accurate solution for consumption, and a second-order Taylor expansion of the period utility function. Hence, our welfare measures correctly reflect changes in both the unconditional mean and the unconditional variance of consumption and labor supply brought about by taxes on short-term debt.

In the competitive equilibrium without macroprudential policy, monitoring costs are such that the long-run debt maturity structure consists of 85% of short-term debt and 15% of long-term debt, resulting in an average duration of the debt portfolio of 3.9 years. A 100% short-term maturity structure would yield a duration of 1 quarter, while a 100% long-term maturity structure would correspond to a duration of 25 years (i.e. $1/(1-\beta) = 100$ quarters). By taxing entrepreneurs' issuance of short-term debt, a macroprudential authority can effectively lengthen the equilibrium long-run debt maturity structure. Figure 2.4 shows households' (upper panels) and entrepreneurs' (lower panels) welfare in equivalent consumption terms at different long-run equilibrium maturity structures induced by various levels of taxes on short-term debt. The figure illustrates that by using taxes to lenghten the maturity structure, the macroprudential authority does more than just shifting risk from entrepreneurs to households. Maturity structures longer than in the competitive equilibrium can reduce risk and increase welfare for both groups of agents. This result is explained by the positive spillovers that arise when an individual entrepreneur lenghtens his debt maturity. Such a lengthening leads to a more stable net worth, and therefore to fewer fire sales and more stable capital holdings at the individual level. This in turn leads to less volatile asset prices, which helps out other entrepreneurs, as well as to more stable wages, which benefits households. Those positive spill-over effects are not perceived by atomistic entrepreneurs in a decentralized equilibrium. In the language of financial markets, each individual entrepreneur fails to internalize the reduction in *systemic risk* that a lengthening of his own maturity structure would bring about.

On the other hand, figure 2.4 illustrates that the socially desirable maturity structure does not simply coincide with the maturity structure that would arise in the absence of costly enforcement of long-term contracts - a structure consisting of 101% of long-term debt. When long-term debt enforcement is costly, a longer maturity structure is beneficial for risk-sharing purposes, but it entails more resources spent on monitoring expenses. When the macroprudential authority uses taxes to induce agents to substitute long-term debt for short-term debt, it also causes more resources to be wasted in unproductive monitoring costs. The model hence captures what is widely seen as the trade-off faced by policy makers when trying to distort maturity choices away from their competitive equilibrium levels.

The optimal maturity structure in the model consists of 50% of short-term debt and 50% of long-term debt, or a debt duration of 12.5 years. This contrasts remarkably with the debt maturity structure of the competitive equilibrium without government intervention. The optimal constant tax is given by $\tau^S = 0.00013$, which represents an annual tax of 0.052% of the face value of short-term bonds. The optimal tax corresponds to about half of the term premium implied by the model. The fact that this tax is quantitatively small relates to the equity premium puzzle or its bond counterpart, the term premium puzzle: with CRRA utility, a realistic calibration of the aggregate shock process results in welfare costs of uncertainty

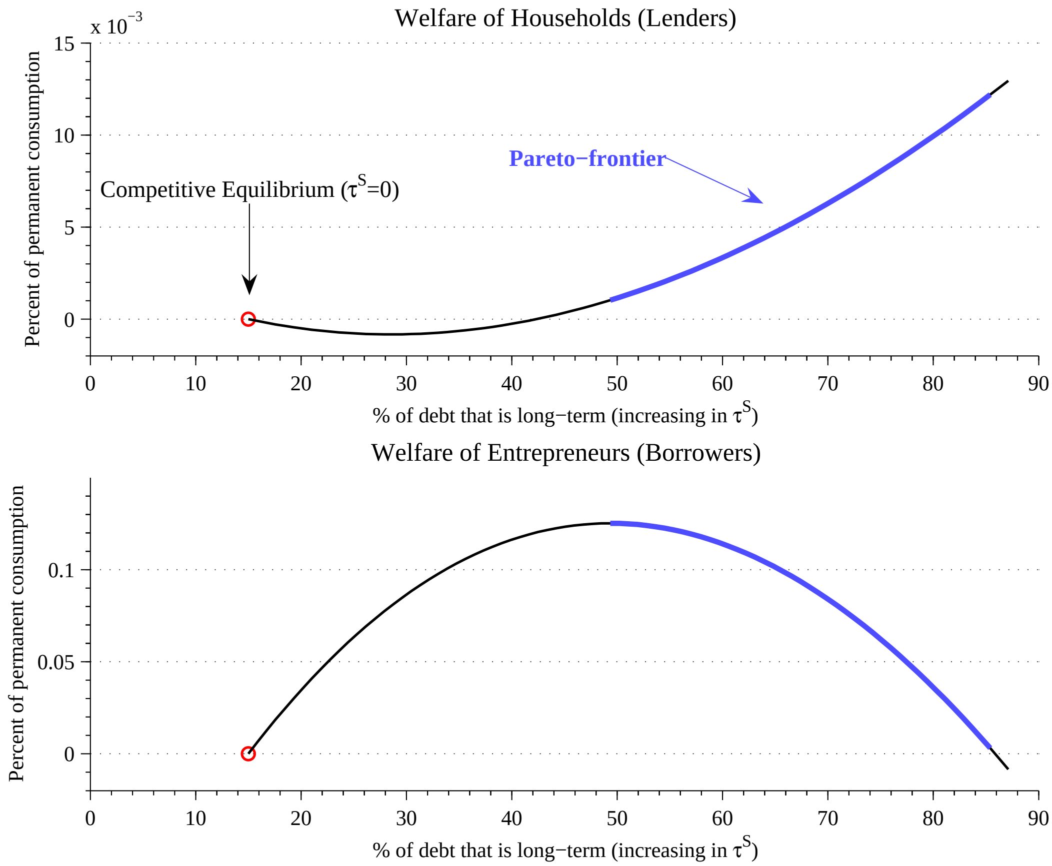

Fig. 2.4: Welfare and debt maturity structure: tax on short-term debt.

that are so low that model-based risk premia are unrealistically small. Accordingly, a tiny tax on short-term debt suffices to move the equilibrium maturity structure by considerable amounts.

To illustrate the implications of macroprudential policy for allocations and prices, figure 2.5 plots ergodic densities for some of the variables based on simulations for 100,000 periods of the model without government intervention and with the tax on short-term debt set at its optimal level. The numbers to the right of the densities in each panel are the variables' standard deviations din the cases without government intervention and with the optimal tax on short-term debt, respectively. The figure reveals that the tax on short-term debt

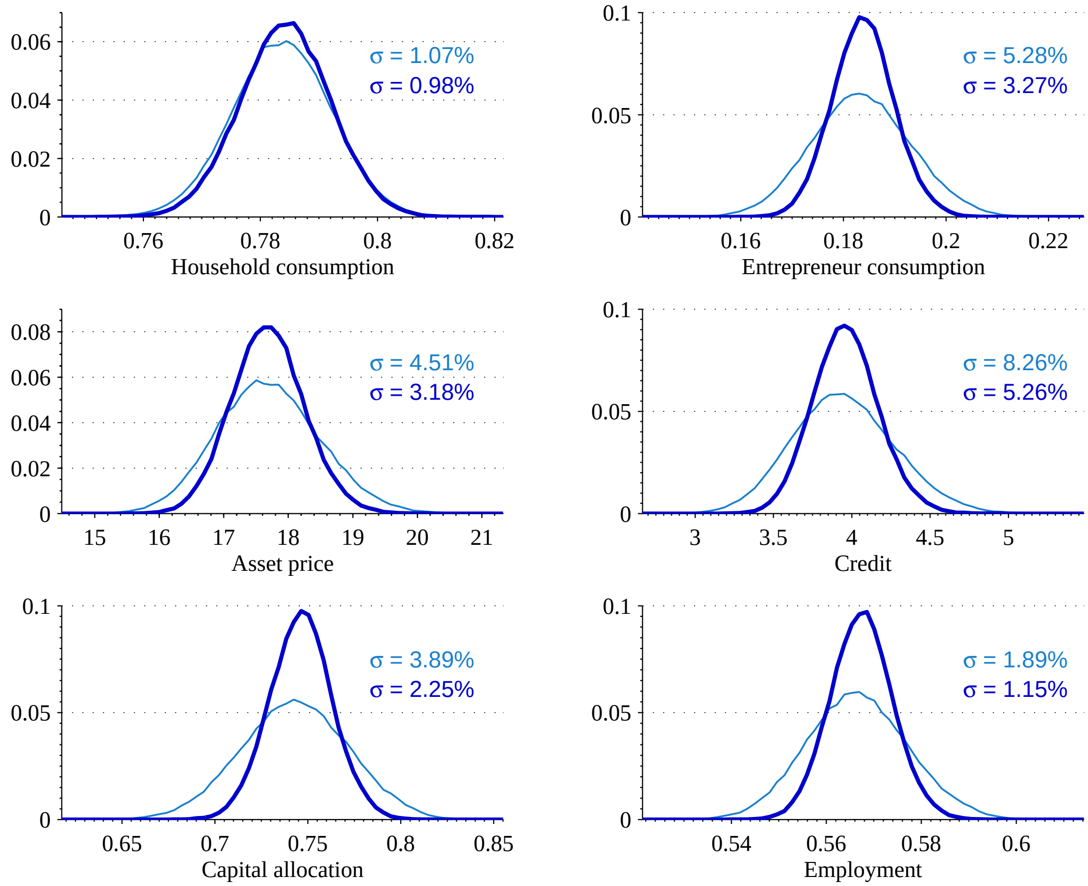

Fig. 2.5: Ergodic densities of the model's variables in the competitive equilibrium without macroprudential policy (thin line) and in the competitive equilibrium with macroprudential policy (socially optimal tax on short-term debt, thick line).

significantly reduces the volatility of entrepreneurial consumption, asset prices, credit, employment and the capital allocation across the two sectors, while it only marginally reduces consumption risk for households.

The consequences of a tax on short-term debt are further illustrated in figure 2.6, where the ergodic densities of output with and without the tax on short-term debt are displayed. The standard deviation of output is reduced by the tax from 1.73% to 1.31%, a 25% decrease. Moreover, when a crisis is defined as an output level more than one standard deviation below mean output in the laissez-faire case, we find that the tax on short-term debt reduces the long-run probability of a crisis by a factor of eight, lowering it from 2.33% to 0.28%. The model hence predicts that taxing short-term debt can bring about a quantitatively significant reduction in the incidence of crises.

As mentioned in section 2.4.1, less volatile factor allocations also contribute to reduce the deadweight loss caused by the collateral constraint. This effect can be seen from the 0.14% increase in long-run mean gross output brought about by the optimal tax on short-term debt. Interestingly, even when the additional monitoring costs caused by a lengthening of the maturity structure are subtracted from aggregate income, the long-run mean net output (available for consumption since there is no investment) is still 0.09% higher under the optimal tax on short-term debt than in the laissez-faire case. In the framework of the model, the reduction in macroeconomic volatility achieved by taxing short-term debt and inducing agents to rely on longer term financing does not come at the expense of lower mean output, even after netting out the increased resource loss associated with more monitoring activities.

2.6 Conclusion

This paper develops a quantitative general equilibrium model of the determinants of private agents' debt maturity. By featuring a number of practically relevant financial frictions, the model captures the largely agreed on trade-off between the insurance benefits of long-term debt and the borrowing cost benefits of short-term debt. The analysis indicates that when asset prices affect financing constraints in the economy, debt maturity choices made in decentralized markets are generally not constrained efficient. In particular, the results

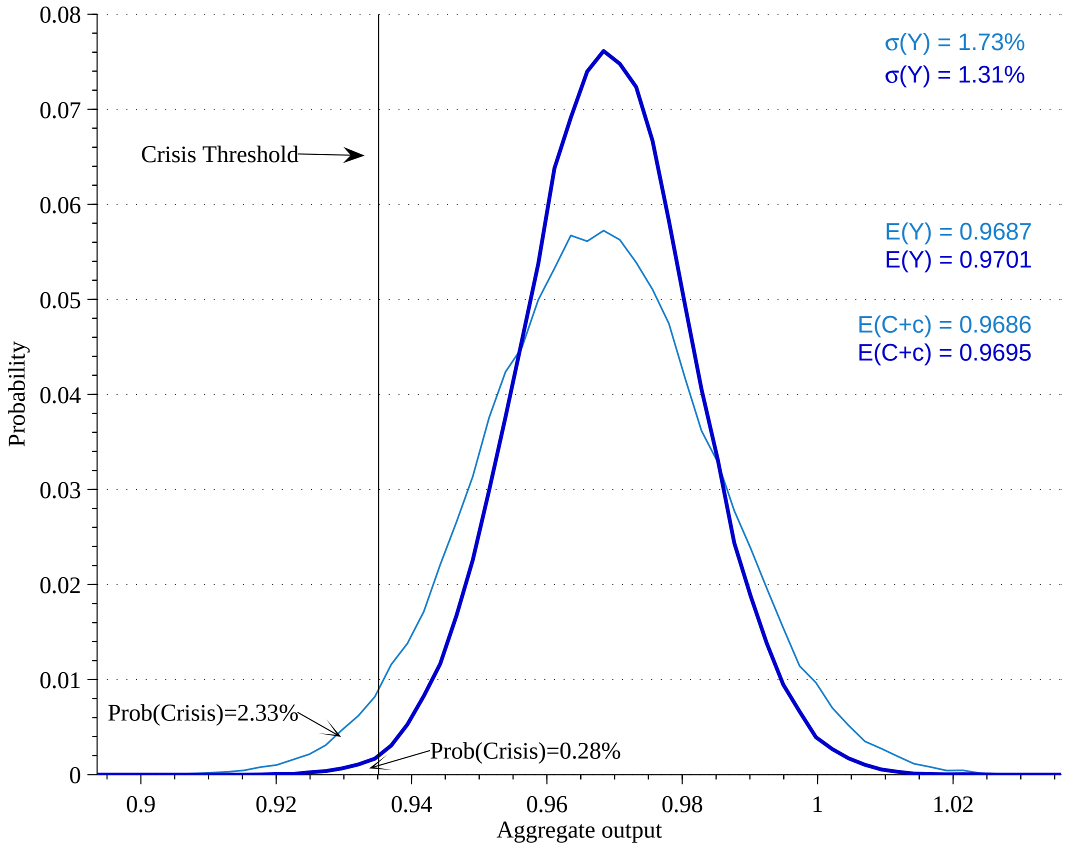

Fig. 2.6: Ergodic density of aggregate output in the competitive equilibrium without macroprudential policy (thin line) and in the competitive equilibrium with macroprudential policy (socially optimal tax on short-term debt, thick line).

suggest that individual borrowers issue too much short-term debt and too little long-term debt in a competitive equilibrium, as they fail to internalize the reduction in systemic risk brought about by a lengthening of their debt maturity structure. The paper also shows that a constant tax on short-term debt can lead to Pareto improvements and result in substantially less volatile allocations and asset prices. The analysis hence provides a theoretical foundation for a macroprudential policy framework that would penalize short-term debt.

APPENDIX

A. PROOFS (CHAPTER 1)

Proof of lemma 1

Without loss of generality, consider state (i, s). For part 1., differentiating (1.9) for $j = A$ and (1.11) with respect to ℓ_A yields

$$\frac{\partial V_i^{is}(k_A, \ell_A)}{\partial \ell_A} = R^{is} u'\Big(Ak_A + R^{is}\ell_A\Big), \tag{A.1}$$

and

$$\frac{\partial \tilde{V}_i^{is}(k_A, k_B, \ell_A, \ell_B)}{\partial \ell_A} = \Delta u'\Big((1-\kappa) r k_B + \Delta(\ell_A + \ell_B)\Big). \tag{A.2}$$

Now, observe that $\ell_A + \ell_B < k_A$ and $r < 1 < R^{is}$ imply that the following inequality holds

$$\Delta(\ell_A + \ell_B) + r(\ell_A - \ell_B) < \Delta k_A + 2R^{is}\ell_A.$$

Adding and subtracting r on the left-hand side, one obtains

$$\Delta(\ell_A + \ell_B) + rk_B - rk_A < \Delta k_A + 2R^{is}\ell_A.$$

Since $\Delta \equiv A - r$, this implies

$$\Delta \ell_B + (\Delta - R^{is})\ell_A + rk_B < Ak_A + R^{is}\ell_A. \tag{A.3}$$

Together with $R^{is} = \kappa r k_B/\ell_A < \Delta$ and the concavity of $u(\cdot)$, this implies that for given $(k_A, k_B, \ell_A, \ell_B)$ in region II, the expression in (A.2) is larger than the one in (A.1).

For part 2., differentiating (1.10) for $j = B$ and (1.12) with respect to k_B yields

$$\frac{\partial V_i^{is}(k_B, \ell_B, R^{is})}{\partial k_B} = [r + (\Delta - R^{is})\frac{\kappa r}{R^{is}}]u'\Big(rk_B + (\Delta - R^{is})\frac{\kappa r k_B}{R^{is}} + \Delta \ell_B\Big), \tag{A.4}$$

and

$$\frac{\partial \tilde{V}_i^{is}(k_A, k_B, \ell_A, \ell_B)}{\partial k_B} = (1-\kappa)ru'\Big((1-\kappa)rk_B + \Delta(\ell_A + \ell_B)\Big) + \kappa r u'\Big(Ak_A + \kappa r k_B\Big). \tag{A.5}$$

(A.3), $R^{is} = \kappa r k_B/\ell_A < \Delta$ and the concavity of $u(\cdot)$ imply that for given $(k_A, k_B, \ell_A, \ell_B)$ in region II, the expression in (A.5) is smaller than the one in (A.4).

Proof of lemma 2

For part 1., differentiating (1.16) with respect to ℓ_A yields

$$\frac{\partial \hat{V}_i^{id}(k_A, k_B, \ell_A, \ell_B)}{\partial \ell_A} = 0. \tag{A.6}$$

Clearly, for any $(k_A, k_B, \ell_A, \ell_B)$ in region II, the expression in (A.6) is smaller than the one in (A.2).

For part 2., differentiating (1.17) with respect to k_B yields

$$\frac{\partial \tilde{V}_i^{di}(k_A, k_B, \ell_A, \ell_B)}{\partial k_B} = (1-\kappa) r u'\Big((1-\kappa) r k_B + \Delta(\ell_A + \ell_B)\Big). \tag{A.7}$$

For any $(k_A, k_B, \ell_A, \ell_B)$ in region II, the expression in (A.7) is smaller than the one in (A.5).

Proof of lemma 3

For part 1., we simply observe that the expression in (A.6) is smaller than the one in (A.1). Similarly, for part 2., it is clear that the expression in (A.7) is smaller than the one in (A.4).

Proof of proposition 1

We start by defining the two functions characterizing a symmetric competitive equilibrium and a symmetric globally regulated equilibrium, respectively,

$$\begin{aligned}
g_{II}(k) \equiv\ & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big)+\pi^{is}\Big(A-\frac{\kappa rk}{1-k}\Big)u'\Big((A+\kappa r)k\Big) \qquad \text{(A.8)}\\
&+\pi^{si}\Big[r+\Big(\Delta-\frac{\kappa rk}{1-k}\Big)\frac{\kappa r}{\frac{\kappa rk}{1-k}}-\Delta\Big]u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)+\pi^{ss}(r-\Delta)u'\Big(rk+\Delta(1-k)\Big),
\end{aligned}$$

and

$$\begin{aligned}
\tilde{g}_{II}(k) \equiv\ & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big)+\pi^{is}\Big[Au'\Big((A+\kappa r)k\Big)-\Delta u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big] \qquad \text{(A.9)}\\
&+\pi^{si}\Big[(1-\kappa)ru'\Big((1-\kappa)rk+2\Delta(1-k)\Big)+\kappa ru'\Big((A+\kappa r)k\Big)-\Delta u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big]\\
&+\pi^{ss}(r-\Delta)u'\Big(rk+\Delta(1-k)\Big).
\end{aligned}$$

Under assumption 3, the value of k in a symmetric competitive equilibrium, k^{CE}, falls in the interior of region II, $k^{CE}\in\left(\frac{1}{1+\kappa r},\frac{\Delta}{\Delta+\kappa r}\right)$, and is implicitly given by $g_{II}(k^{CE})=0$. A necessary condition for a globally regulated equilibrium is that $\tilde{g}_{II}(\tilde{k})=0$ if $\frac{1}{1+\kappa r}<\tilde{k}<\frac{\Delta}{\Delta+\kappa r}$, $\tilde{g}_{II}(\tilde{k})\leq 0$ if $\tilde{k}=\frac{1}{1+\kappa r}$ and $\tilde{g}_{II}(\tilde{k})\geq 0$ if $\tilde{k}=\frac{\Delta}{\Delta+\kappa r}$. Note that if $g'_{II}(k)<0$, $\tilde{g}'_{II}(k)<0$ and $\tilde{g}_{II}(k)<g_{II}(k)$ over $\left[\frac{1}{1+\kappa r},\frac{\Delta}{\Delta+\kappa r}\right]$, then either $\tilde{k}\in\left[\frac{1}{1+\kappa r},k^{CE}\right)$ with $\tilde{g}_{II}(\tilde{k})=0$, or $\tilde{k}=\frac{1}{1+\kappa r}<k^{CE}$ with $\tilde{g}_{II}(\tilde{k})<0$. It follows that showing that $g'_{II}(k)<0$, $\tilde{g}'_{II}(k)<0$ and $\tilde{g}_{II}(k)<g_{II}(k)$ over $\left[\frac{1}{1+\kappa r},\frac{\Delta}{\Delta+\kappa r}\right]$ is sufficient to prove that $\tilde{k}<k^{CE}$.

The derivative of $g_{II}(\cdot)$ is given by:

$$\begin{aligned}
g_{II}'(k) \;\; \equiv \;\; & \pi^{ii}(A-1)^2 u''\Big((A-1)k+1\Big) \\
& +\pi^{is}\Big[-\frac{\kappa r(1-k)+\kappa rk}{(1-k)^2}u'\Big((A+\kappa r)k\Big)+\Big(A-\frac{\kappa rk}{1-k}\Big)(A+\kappa r)u''\Big((A+\kappa r)k\Big)\Big] \\
& +\pi^{si}\Big\{\Big[r+\Big(\Delta-\frac{\kappa rk}{1-k}\Big)\frac{\kappa r}{\frac{\kappa rk}{1-k}}-\Delta\Big]\Big((1-\kappa)r-2\Delta\Big)u''\Big((1-\kappa)rk+2\Delta(1-k)\Big) \\
& -(\Delta+\kappa r)u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big\} \\
& +\pi^{ss}(r-\Delta)^2 u''\Big(rk+\Delta(1-k)\Big).
\end{aligned}$$

Each single term of $g_{II}'(k)$ is strictly negative for $k \in \Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$, so $g_{II}'(k) < 0$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$.

Similarly, the derivative of $\tilde{g}_{II}(\cdot)$ is given by:

$$\begin{aligned}
\tilde{g}_{II}'(k) \;\; \equiv \;\; & \pi^{ii}(A-1)^2 u''\Big((A-1)k+1\Big) \\
& +\pi^{is}\Big[A(A+\kappa r)u''\Big((A+\kappa r)k\Big)-\Delta[(1-\kappa)r-2\Delta]u''\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big] \\
& +\pi^{si}\Big[[(1-\kappa)r-\Delta][(1-\kappa)r-2\Delta]u''\Big((1-\kappa)rk+2\Delta(1-k)\Big)+\kappa r(A+\kappa r)u''\Big((A+\kappa r)k\Big)\Big] \\
& +\pi^{ss}(r-\Delta)^2 u''\Big(rk+\Delta(1-k)\Big).
\end{aligned}$$

Every single term is strictly negative for $k \in \Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$, sot $\tilde{g}_{II}'(k) < 0$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$.

It remains to show that $\tilde{g}_{II}(k) < g_{II}(k)$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$. Defining $\tilde{\Phi}(k) \equiv \tilde{g}_{II}(k) -$

$g_{II}(k)$, we have

$$\begin{aligned}\tilde{\Phi}(k) &= -\pi^{is}\Big[\Delta u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)-\frac{\kappa rk}{1-k}u'\Big((A+\kappa r)k\Big)\Big]\\ &\quad -\pi^{is}\kappa r\Big[u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)-u'\Big((A+\kappa r)k\Big)\Big].\end{aligned}$$

Lemma 1 established that the terms in brackets are positive for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, implying that $\tilde{\Phi}(k) < 0$ for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$. It follows that $\tilde{k} < k^{CE}$, and therefore that $\tilde{\ell} > \ell^{CE}$.

Proof of proposition 2

We start by defining the function characterizing a symmetric nationally regulated equilibrium

$$\begin{aligned}\hat{g}_{II}(k) \equiv\ & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big)+\pi^{is}Au'\Big((A+\kappa r)k\Big) \\ & +\pi^{si}\Big[(1-\kappa)ru'\Big((1-\kappa)rk+2\Delta(1-k)\Big)-\Delta u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big] \\ & +\pi^{ss}(r-\Delta)u'\Big(rk+\Delta(1-k)\Big).\end{aligned} \tag{A.10}$$

Under assumption 3, the value of k in a symmetric competitive equilibrium, k^{CE}, falls in the interior of region II, $k^{CE} \in \left(\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right)$, and is implicitly given by $g_{II}(k^{CE}) = 0$ (with $g_{II}(\cdot)$ defined in A.8). A necessary condition for a globally regulated equilibrium is that $\tilde{g}_{II}(\tilde{k}) = 0$ (with $\tilde{g}_{II}(\cdot)$ defined in A.9) if $\frac{1}{1+\kappa r} < \tilde{k} < \frac{\Delta}{\Delta+\kappa r}$, $\tilde{g}_{II}(\tilde{k}) \leq 0$ if $\tilde{k} = \frac{1}{1+\kappa r}$ and $\tilde{g}_{II}(\tilde{k}) \geq 0$ if $\tilde{k} = \frac{\Delta}{\Delta+\kappa r}$. Similarly, a necessary condition for a nationally regulated equilibrium is that $\hat{g}_{II}(\hat{k}) = 0$ if $\frac{1}{1+r\kappa} < \hat{k} < \frac{\Delta}{\Delta+\kappa r}$, $\hat{g}_{II}(\hat{k}) \leq 0$ if $\hat{k} = \frac{1}{1+\kappa r}$ and $\hat{g}_{II}(\hat{k}) \geq 0$ if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$. Note that if $\hat{g}'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > \tilde{g}_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+\kappa r}\right]$, then we can be a priori in either of the following three situations

- $\hat{k} = \tilde{k} = \frac{1}{1+\kappa r}$, with $\hat{g}_{II}(\hat{k}) \leq 0$, $\tilde{g}_{II}(\tilde{k}) < 0$,

- $\tilde{k} = \frac{1}{1+\kappa r} < \hat{k}$, with $\hat{g}_{II}(\hat{k}) \geq 0$, $\tilde{g}_{II}(\tilde{k}) \leq 0$,

- $\frac{1}{1+\kappa r} < \tilde{k} < k^{CE}$ and $\tilde{k} < \hat{k}$ with $\hat{g}_{II}(\hat{k}) \geq 0$, $\tilde{g}_{II}(\tilde{k}) = 0$.

It follows that showing that $\hat{g}'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > \tilde{g}_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ is sufficient to prove that $\hat{k} \geq \tilde{k}$. Moreover, if in addition $\hat{k} > \frac{1}{1+\kappa r}$, then $\hat{k} > \tilde{k}$.

The fact that $\tilde{g}'_{II}(k)$ for $k \in \left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ has been established as part of proposition 1. The derivative of $\hat{g}_{II}(\cdot)$ is given by:

$$\begin{aligned}
\hat{g}'_{II}(k) \quad \equiv \quad & \pi^{ii}(A-1)^2 u''\Big((A-1)k+1\Big) + \pi^{is} A(A+\kappa r) u''\Big((A+\kappa r)k\Big) \\
& + \pi^{si}[(1-\kappa)r - \Delta][(1-\kappa)r - 2\Delta] u''\Big((1-\kappa)rk + 2\Delta(1-k)\Big) \\
& + \pi^{ss}(r-\Delta)^2 u''\Big(rk + \Delta(1-k)\Big).
\end{aligned}$$

Every single term is strictly negative for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, so $\hat{g}'_{II}(k) < 0$ over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$.

Now, defining $\check{\Phi}(k) \equiv \hat{g}_{II}(k) - \tilde{g}_{II}(k)$, we have

$$\check{\Phi}(k) = \pi^{is}\Big[\Delta u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) - \kappa r u'\Big((A+\kappa r)k\Big)\Big] > 0$$

The term in bracket is positive for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, so $\check{\Phi}(k) > 0$ for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$. It follows that

- $\hat{k} \geq \tilde{k}$ and $\hat{\ell} \leq \tilde{\ell}$,

- if $\hat{k} > \frac{1}{1+\kappa r}$ then $\hat{k} > \tilde{k}$ and $\hat{\ell} < \tilde{\ell}$.

Proof of proposition 3

Under assumption 3, $k^{CE} \in \left(\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right)$ and $g_{II}(k^{CE}) = 0$ (with $g_{II}(\cdot)$ defined in (A.8)). A necessary condition for a nationally regulated equilibrium is that $\hat{g}_{II}(\hat{k}) = 0$ (with $\hat{g}_{II}(\cdot)$ defined in (A.10)) if $\frac{1}{1+\kappa r} < \hat{k} < \frac{\Delta}{\Delta+\kappa r}$, $\hat{g}_{II}(\hat{k}) \leq 0$ if $\hat{k} = \frac{1}{1+\kappa r}$ and $\hat{g}_{II}(\hat{k}) \geq 0$ if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$. Note that if $g'_{II}(k) < 0$, $\hat{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > g_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$, then either $\hat{k} \in \left(k^{CE}, \frac{\Delta}{\Delta+\kappa r}\right]$ with $\hat{g}_{II}(\hat{k}) = 0$, or $\hat{k} = \frac{\Delta}{\Delta+\kappa r} > k^{CE}$ with $\hat{g}_{II}(\hat{k}) \geq 0$. The fact that $g'_{II}(k) < 0$ and $\hat{g}'_{II}(k) < 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ has already been established in the proofs of propositions 1 and 2. It follows that showing that $\hat{g}_{II}(k) > g_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ is sufficient to prove that $\hat{k} > k^{CE}$.

Defining $\hat{\Phi}(k) \equiv \hat{g}_{II}(k) - g_{II}(k)$, we have

$$\begin{aligned} \hat{\Phi}(k) &= \frac{\kappa r k}{1-k} u'\Big((A+\kappa r)k\Big) - \Delta \frac{\kappa r(1-k)}{\kappa r k} u'\Big((1-\kappa) r k + 2\Delta(1-k)\Big) \\ &= \frac{\kappa r}{(A+\kappa r)(1-k)} - \frac{\Delta(1-k)}{(1-\kappa) r k^2 + 2\Delta(1-k)k} \end{aligned} \tag{A.11}$$

We now show that under the logarithmic utility assumption, $\hat{\Phi}(k) > 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$.

We observe that $\hat{\Phi}(k)$ is strictly positive over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ if and only if the transformed function

$$\hat{\Phi}_T(k) \equiv [(A+\kappa r)(1-k)][(1-\kappa) r k^2 + 2\Delta(1-k)k]\hat{\Phi}(k)$$

is strictly positive over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. Conveniently, $\hat{\Phi}_T(k)$ is quadratic:

$$\hat{\Phi}_T(k) = (1-\kappa)\kappa r^2 k^2 + 2\Delta\kappa r(1-k)k - \Delta(A+\kappa r)(1-k)^2,$$

with a derivative given by

$$\hat{\Phi}'_T(k) = 2(1-\kappa)\kappa r^2 k + 2\Delta\kappa r(1-k) - 2\Delta\kappa r k + 2\Delta(A+\kappa r)(1-k).$$

When evaluated at the bounds $\frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r}$, both $\hat{\Phi}_T(k)$ and $\hat{\Phi}'_T(k)$ are strictly positive under assumption 1 that $A \leq \frac{3}{2}$:

$$\begin{aligned}
\hat{\Phi}_T(\frac{1}{1+r\kappa}) &= \frac{\kappa r^2}{(1+\kappa r)^2}\Big[(1-\kappa) + \Delta\kappa[2-(A+\kappa r)\Big] > 0 \\
\hat{\Phi}_T(\frac{\Delta}{\Delta+r\kappa}) &= \frac{\Delta\kappa r^2}{(\Delta+\kappa r)^2}\Big[\Delta(1-\kappa) + \Delta\kappa[2\Delta-(A+\kappa r)]\Big] > 0 \\
\hat{\Phi}'_T(\frac{1}{1+r\kappa}) &= \frac{2\kappa r}{1+\kappa r}\Big[(1-\kappa)r + \Delta[2\kappa r + (A-1)]\Big] > 0 \\
\hat{\Phi}'_T(\frac{\Delta}{\Delta+r\kappa}) &= \frac{2\Delta\kappa r}{\Delta+\kappa r}\Big[(1-\kappa)r + 2\kappa r + (A-\Delta)\Big] > 0
\end{aligned}$$

Due to linearity, $\hat{\Phi}'_T(\cdot)$ is strictly positive over $\Big[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\Big]$. This, together with the strict positiveness of $\hat{\Phi}_T(\frac{1}{1+r\kappa})$ and $\hat{\Phi}_T(\frac{\Delta}{\Delta+r\kappa})$, guarantees that $\hat{\Phi}_T(\cdot)$ is strictly positive over $\Big[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\Big]$. In turn, this implies that $\hat{\Phi}(\cdot)$ is strictly positive as well over $\Big[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\Big]$. It follows that $\hat{k} > k^{CE}$, and therefore $\hat{\ell} < \ell^{CE}$.

Proof of proposition 4

For regulations in country B sufficiently close to a competitive equilibrium (i.e. K_B in the neighborhood of k^{CE}), the date 0 investment decision of private agents in country A

satisfies $f(k_A^{CE}, K_B) = 0$, where

$$\begin{aligned} f(k, K_B) \;\equiv\;& \pi^{ii}(A-1)u'\Big((A-1)k+1\Big) + \pi^{is}\Big(A - \frac{\kappa r K_B}{1-k}\Big)u'\Big(Ak + \kappa r K_B\Big) \\ &+\pi^{si}\Big[r + \Big(\Delta - \frac{\kappa r k}{1-K_B}\Big)\frac{\kappa r}{\frac{\kappa r k}{1-K_B}} - \Delta\Big]u'\Big((1-\kappa)rk + \Delta(2-k-K_B)\Big) \\ &+\pi^{ss}(r-\Delta)u'\Big(rk + \Delta(1-k)\Big). \end{aligned} \tag{A.12}$$

The partial derivatives of $f(k, K_B)$ are given by

$$\begin{aligned} \frac{\partial f}{\partial k} \;=\;& \pi^{ii}(A-1)^2u''\Big((A-1)k+1\Big) \\ &+\pi^{is}\Big[-\frac{\kappa r K_B}{(1-k)^2}u'\Big(Ak+\kappa r K_B\Big) + \Big(A - \frac{\kappa r K_B}{1-k}\Big)Au''\Big(Ak+\kappa r K_B\Big)\Big] \\ &\pi^{si}\Big\{-\frac{\Delta(1-K_B)}{k^2}u'\Big((1-\kappa)rk + \Delta(2-k-K_B)\Big) \\ &+\Big[r + \Big(\Delta - \frac{\kappa r k}{1-K_B}\Big)\frac{\kappa r}{\frac{\kappa r k}{1-K_B}} - \Delta\Big][(1-\kappa)r - \Delta]u''\Big((1-\kappa)rk + \Delta(2-k-K_B)\Big)\Big\} \\ &+\pi^{ss}(r-\Delta)^2u''\Big(kr + \Delta(1-k)\Big), \end{aligned}$$

and

$$\begin{aligned} \frac{\partial f}{\partial K_B} \;=\;& \pi^{is}\Big[-\frac{\kappa r}{1-k}u'\Big(Ak+\kappa r K_B\Big) + \Big(A - \frac{\kappa r K_B}{1-k}\Big)\kappa r u''\Big(Ak+\kappa r K_B\Big)\Big] \\ &\pi^{si}\Big\{-\frac{\Delta}{k}u'\Big((1-\kappa)rk + \Delta(2-k-K_B)\Big) \\ &+\Big[r + \Big(\Delta - \frac{\kappa r k}{1-K_B}\Big)\frac{\kappa r}{\frac{\kappa r k}{1-K_B}} - \Delta\Big](-\Delta)u''\Big((1-\kappa)rk + \Delta(2-k-K_B)\Big)\Big\}. \end{aligned}$$

When evaluated at $(k, K_B) = (k^{CE}, k^{CE})$, each of the individual terms making up $\frac{\partial f}{\partial k}$ and $\frac{\partial f}{\partial K_B}$ are negative, and therefore $\frac{\partial f}{\partial k}$ and $\frac{\partial f}{\partial K_B}$ are negative. By the implicit function theorem

$$\frac{dk_A^{CE}}{dK_B} = -\frac{\frac{\partial f}{\partial K_B}}{\frac{\partial f}{\partial k}} < 0.$$

Hence k_A^{CE} is decreasing in K_B.

Proof of proposition 5

In the neighborhood of the symmetric competitive equilibrium, country A's national planner payoff can be written as

$$\begin{aligned}\Pi(k_A, k_B) &\equiv \sum_{\omega\in\Omega} \pi^{\omega} \hat{V}^{\omega}(k_A, k_B, 1-k_A, 1-k_B) \\ &= \pi^{ii} u\Big(Ak_A + (1-k_A)\Big) + \pi^{is} u\Big(Ak_A + \kappa r k_B\Big) + \pi^{si} u\Big((1-\kappa) r k_A + \Delta(1-k_A+1-k_B)\Big) \\ &\quad + \pi^{ss} u\Big(rk_A + \Delta(1-k_A)\Big)\end{aligned} \tag{A.13}$$

The cross derivative is given by

$$\frac{\partial^2 \Pi}{\partial k_A \partial k_B} = \pi^{is} \kappa r A u''\Big(Ak_A + \kappa r k_B\Big) + \pi^{si}(-\Delta)[(1-\kappa)r - \Delta] u''\Big((1-\kappa) r k_A + \Delta(1-k_A+1-k_B)\Big)$$

This cross derivative is unambiguously negative. It follows that the regulators' actions are strategic substitutes.

Proof of proposition 6

Note that $\frac{\partial W}{\partial L_B}$ can be written as

$$\begin{aligned}\frac{\partial W}{\partial L_B} &= -\pi^{is}\Big[\frac{\kappa r k^{CE}}{1-k^{CE}}u'\Big((A+\kappa r)k^{CE}\Big) - \Delta\frac{\kappa r(1-k^{CE})}{\kappa r k^{CE}}u'\Big([(1-\kappa)r-2\Delta]k^{CE}+2\Delta\Big)\Big] \\ &= -\pi^{is}\hat{\Phi}(k^{CE})\end{aligned}$$

where $\hat{\Phi}(k) \equiv \hat{g}_{II}(k) - g_{II}(k)$, for $g_{II}(\cdot)$ and $\hat{g}_{II}(\cdot)$ defined in (A.8) and (A.10), respectively. The proof of proposition 3 established that under the logarithmic utility assumption, $\hat{\Phi}(k)$ is strictly positive over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$. Since $k^{CE} \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, it follows that $\hat{\Phi}(k^{CE}) > 0$, and therefore that $\frac{\partial W}{\partial L_B} < 0$. Proposition 4 established that $\frac{d\ell_A^{CE}}{dL_B} < 0$, and it was argued in the text that $\frac{\partial W}{\partial \ell_A} > 0$. Hence the direct effect and the indirect effect of a change in country B's regulation on country B's welfare work in the same direction:

$$\frac{dW}{dL_B} = \underbrace{\frac{\partial W}{\partial L_B}}_{(-)} + \underbrace{\frac{\partial W}{\partial \ell_A}\frac{d\ell_A^{CE}}{dL_B}}_{(-)} < 0.$$

Proof of proposition 7

The proof of proposition 6 showed that under the logarithmic utility assumption, $\frac{\partial W}{\partial L_B} < 0$. Since $\frac{\partial \Pi}{\partial \ell_A} = \frac{\partial W}{\partial L_B}$, it follows that $\frac{\partial \Pi}{\partial \ell_A} < 0$. Proposition 4 established that $\frac{d\ell_A^{CE}}{dL_B} < 0$. Further, it was argued in the text that $\frac{\partial W}{\partial \ell_A} > 0$. Since $\frac{\partial \Pi}{\partial L_B} = \frac{\partial W}{\partial \ell_A}$, it follows that $\frac{\partial \Pi}{\partial L_B} > 0$. Hence the direct effect and the indirect effect of a change in country B's regulation on country A's welfare work in the same direction:

$$\frac{d\Pi}{dL_B} = \underbrace{\frac{\partial\Pi}{\partial\ell_A}\frac{d\ell_A^{CE}}{dL_B}}_{(+)} + \underbrace{\frac{\partial\Pi}{\partial L_B}}_{(+)} > 0.$$

B. DETERMINISTIC STEADY-STATE (CHAPTER 2)

This appendix gives details on the model's deterministic steady state.

Combining entrepreneur's optimal labor demand condition with household's optimal labor supply condition, we get

$$Af_l(k,l) = \frac{G'(l)}{u'(C)}. \tag{B.1}$$

From the households' first-order conditions, the steady-state asset prices are given by $q^S = \beta$, $q^L = \beta/(1-\beta)$ and $q = AF'(K)/(1-\beta)$. From the entrepreneurs' first-order conditions, the steady-state multiplier is given by $\mu = (1-\gamma/\beta)u'(c)$ and the asset prices must satisfy

$$q = Af_k(k,l)/[1-\gamma-\kappa(1-\gamma/\beta)].$$

Combining the two expressions for the asset price q, we obtain

$$AF'(K) = \frac{\beta(1-\beta)}{\beta(1-\beta)+(\beta-\kappa)(\beta-\gamma)}Af_k(k,l)$$

Now, given that the entrepreneur's collateral constraint binds, the market value of debt is given by

$$\beta B^S + \frac{\beta}{1-\beta}B^L = -\beta b^S - \frac{\beta}{1-\beta}b^L = \kappa q k$$

and therefore

$$B^S + \frac{1}{1-\beta}B^L = -b^S - \frac{1}{1-\beta}b^L = \beta^{-1}\kappa q k.$$

From the household's budget constraint, steady state household consumption C is given by

$$C = AF(K) + Af_l(k,l)l + \beta^{-1}\kappa AF'(K)k.$$

Hence, the steady state allocation of physical capital and labor supply are a solution to the two equations

$$F'(K) = \frac{\beta(1-\beta)}{\beta(1-\beta) + (\beta-\kappa)(\beta-\gamma)} f_k(k,l)$$

and

$$Af_l(k,l) = \frac{G'(l)}{u'\left(AF(K) + Af_l(k,l)l + \beta^{-1}\kappa AF'(K)k\right)}.$$

For further references, it is useful to define the household's beginning of period wealth as $Z_t \equiv q_t K_t + B_t^S + (1+q_t^L)B_t^L$ and the entrepreneur's beginning of period wealth as $z_t \equiv q_t k_t + b_t^S + (1+q_t^L)b_t^L$. Also, define aggregate wealth as $\Omega_t \equiv Z_t + z_t$. The steady state wealth positions are given by

$$\begin{aligned} Z &= qK + \beta^{-1}\kappa q k \\ z &= qk - \beta^{-1}\kappa q k. \end{aligned}$$

Hence, the relative wealth positions are given by

$$
\begin{aligned}
\omega &= \frac{z}{Z+z} \\
&= \frac{(1-\beta^{-1}\kappa)k}{K+k}
\end{aligned}
$$

It is also useful to define the entrepreneur's share of consumption in aggregate output as $\tilde{\omega} \equiv c_t/[Y_t A_t]$ and the according household's share of consumption in aggregate output as $(1-\tilde{\omega}) \equiv C_t/[Y_t A_t]$. In the steady state, these shares are given by

$$\tilde{\omega} = \frac{f(k,l) - f_l(k,l)l - \beta^{-1}\kappa F'(K)k}{F(K) + f(k,l)}$$

and

$$1 - \tilde{\omega} = \frac{F(K) + f_l(k,l)l + \beta^{-1}\kappa F'(K)k}{F(K) + f(k,l)}.$$

C. PROOFS (CHAPTER 2)

Proof of Proposition 1

We show here that under the assumptions of log utility and serially uncorrelated TFP, the costless enforcement model results in effectively complete risk markets, perfect risk sharing between entrepreneurs and households, and no persistence in allocations and asset prices. We also show that the entrepreneur's collateral constraint binds in every period and state. We proceed by first postulating allocations and prices, and then showing that given these prices, the allocations satisfy both agents' optimality conditions and constraints.

Assume $u(\cdot) = \ln(\cdot)$, $\rho = 0$ and $\vartheta = 0$. Consider initial values for capital and bond allocations that result in a distribution of wealth corresponding to the one prevailing in the deterministic steady state (see Appendix B), i.e. $Z_0 = (1-\omega)\Omega_0$ and $z_0 = \omega\Omega_0$, where $\Omega_0 \equiv Z_0 + z_0$.

Define $Y_t \equiv F(K_t) + f(k_t, l_t)$, denote by k and K the allocation of capital in the deterministic steady state and by l employment in the deterministic steady-state. We conjecture the following allocations: $k_t = k$, $K_t = K$, $l_t = 1$, $c_t = \tilde{\omega} Y A_t$ and $C_t = (1-\tilde{\omega}) Y A_t$ (see Appendix B for the value of $\tilde{\omega}$). It is straightforward to verify that with those allocations, both agents' optimality conditions for labor supply/demand, Euler equations for capital, short-term bonds and long-term bonds are satisfied when asset prices are given by

$$q_t = \frac{1}{1-\beta} F'(K) A_t, \qquad q_t^S = \beta A_t E_t[e^{-\epsilon_{t+1}^A}], \qquad q_t^L = \frac{\beta}{1-\beta} A_t E_t[e^{-\epsilon_{t+1}^A}],$$

and the wage is given by

$$w_t = A_t f_l(k, l),$$

provided that the entrepreneurs' collateral constraint binds as the multiplier on the constraint satisfies

$$\mu_t = (1 - \gamma/\beta)/(\omega Y A_t).$$

Also, conjecture that bond holdings are time invariant and given by

$$b^S = \frac{\kappa F'(K) k}{\beta^2 E_t[e^{-\epsilon_{t+1}^A}]}, \qquad b^L = -\frac{\kappa F'(K) k}{\beta^2 E_t[e^{-\epsilon_{t+1}^A}]},$$

and $B_S = -b_S$, $B_L = -b_L$.

It is straightforward to verify that the two agents budget constraints and the entrepreneurs' borrowing constraint hold with equality in every period and state. By showing that both agents' optimality conditions, budget constraints and borrowing constraints all hold with equality at the candidate allocations and prices, we have established that these constitute a competitive equilibrium.

The allocations are identical to the ones obtained in an environment with a full set of Arrow securities, where the household's and entrepreneur's budget constraints would be given by

$$
\begin{aligned}
C(s^t) + q(s^t)K(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)B(s_{t+1}, s^t) &= A(s^t)F(K(s^t)) + w(s^t)L(s^t) + q(s^t)K(s^{t-1}) + B(s^t) \\
c(s^t) + q(s^t)k(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}, s^t) &= A(s^t)f(k(s^t), l(s^t)) - w(s^t)l(s^t) + q(s^t)k(s^{t-1}) + b(s^t),
\end{aligned}
$$

and the entrepreneur's borrowing constraint would be

$$
-\sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}, s^t) \leq \kappa q(s^t)k(s^t).
$$

In these expressions, s_t has the usual interpretation of the state at time t, while s^t describes the entire history up to time t. $B(s_{t+1}, s^t)$ and $b(s_{t+1}, s^t)$ denote the holdings of recursive Arrow securities available for trade at time t, and $q(s_{t+1}, s^t)$ denotes the price of these securities.

D. MODEL SOLUTION UNDER INCOMPLETE MARKETS (CHAPTER 2)

This appendix gives some details on the solution of the model when markets are incomplete. The solution approach follows the one by Devereux and Sutherland (2009, 2011) and is adapted to be used together with the regular algorithm of Schmitt-Grohe and Uribe (2004) that computes first- and second-order approximations to the policy function in DSGE models without portfolio choices.

For future reference, we distinguish between portfolio variables and non-portfolio variables. The welfare calculations of Section 2.5 require a second-order accurate solution for households' consumption C_t and entrepreneurs' consumption c_t, which are both non-portfolio variables. As the model presented in this paper belongs to a general class of models in which second-order accurate solutions for non-portfolio variables only require a first-order accurate solution for portfolio variables, we only need to solve for the debt portfolio up to a first-order. Following Devereux and Sutherland (2009, 2011), we do this in steps. We first solve for the zero-order of the debt portfolio, and then use this zero-order portfolio to obtain a first-order accurate solution for non-portfolio variables. We then use this first-order solution to solve for the first-order portfolio. Finally, we use the first-order portfolio to obtain a second-order accurate solution for non-portfolio variables.

D.1 Zero-order portfolio and first-order non-portfolio variables

Solving for ϕ

Rewrite the model equilibrium conditions as

$$
\begin{aligned}
ce^{\hat{c}_t} + qe^{\hat{q}_t}ke^{\hat{k}_{t+1}} + (\hat{b}_{t+1} + b) &= \alpha_e A e^{\hat{A}_t} k^{\alpha_e} e^{\alpha_e \hat{k}_{t+1}} l^{1-\alpha_e} e^{(1-\alpha_e)\hat{l}_t} + qe^{\hat{q}_t}ke^{\hat{k}_t} \\
&\quad + r^S e^{\hat{r}_{St}}(\hat{b}_t + b) + \tau^S r^S e^{\hat{r}^S_t}(\hat{b}_t + b - \hat{\phi}_t - \phi) \\
&\quad + \beta^{-1}(e^{\hat{r}_{Lt}} - e^{\hat{r}_{St}})(\hat{\phi}_t + \phi) \\
Ae^{\hat{A}_t}k^{\alpha_e}e^{\alpha_e\hat{k}_{t+1}}l^{1-\alpha_e}e^{(1-\alpha_e)\hat{l}_t} + A\nu e^{\hat{A}_t}K^{\alpha_h}e^{\alpha_h\hat{k}_{t+1}} &= ce^{\hat{c}_t} + Ce^{\hat{C}_t} - \left(\epsilon_t^{\vartheta}\right)^2 \frac{1}{q^L e^{\hat{q}^L_{t-1}}}(\hat{\phi}_t + \phi) \\
(\hat{b}_{t+1} + b) + (\hat{B}_{t+1} + B) &= 0 \\
ke^{\hat{k}_{t+1}} + Ke^{\hat{K}_{t+1}} &= \bar{K} \\
-(\hat{b}_{t+1} + b) &= \kappa q e^{\hat{q}_t} k e^{\hat{k}_{t+1}} \\
\frac{qe^{\hat{q}_t} - Ae^{\hat{A}_t}\nu\alpha_h K^{\alpha_h-1}e^{(\alpha_h-1)\hat{K}_{t+1}}}{C^\sigma e^{\sigma\hat{C}_t}} &= \beta E_t\left[\frac{qe^{\hat{q}_{t+1}}}{C^\sigma e^{\sigma\hat{C}_{t+1}}}\right] \\
\frac{1}{C^\sigma e^{\sigma\hat{C}_t}} &= \beta r^S e^{\hat{r}_{St+1}} E_t\left[\frac{1}{C^\sigma e^{\sigma\hat{C}_{t+1}}}\right] \\
\frac{1}{C^\sigma e^{\sigma\hat{C}_t}} &= \beta E_t\left[r^L e^{\hat{r}_{Lt+1}}\frac{1}{C^\sigma e^{\sigma\hat{C}_{t+1}}}\right] - \beta E_t\left[\frac{\left(\epsilon_t^{\vartheta}\right)^2}{q^L e^{\hat{q}^L_t}}\frac{1}{C^\sigma e^{\sigma\hat{C}_{t+1}}}\right] \\
\frac{qe^{\hat{q}_t} - Ae^{\hat{A}_t}\alpha_e k^{\alpha_e-1}e^{(\alpha_e-1)\hat{k}_{t+1}}l^{1-\alpha_e}e^{(1-\alpha_e)\hat{l}_t}}{c^\sigma e^{\sigma\hat{c}_t}} &= \gamma E_t\left[\frac{qe^{\hat{q}_{t+1}}}{c^\sigma e^{\sigma\hat{c}_{t+1}}}\right] + \kappa\mu e^{\hat{\mu}_t} \\
\frac{1}{c^\sigma e^{\sigma\hat{c}_t}} &= \gamma(1+\tau^S) r^S e^{\hat{r}_{St+1}} E_t\left[\frac{1}{c^\sigma e^{\sigma\hat{c}_{t+1}}}\right] + \mu e^{\hat{\mu}_t} \\
\frac{1}{c^\sigma e^{\sigma\hat{c}_t}} &= \gamma E_t\left[r^L e^{\hat{r}_{Lt+1}}\frac{1}{c^\sigma e^{\sigma\hat{c}_{t+1}}}\right] + \mu e^{\hat{\mu}_t} \\
E_t\hat{A}_{t+1} &= \rho\hat{A}_t \\
r^L e^{\hat{r}^L_t} &= \frac{1 + q^L e^{\hat{q}^L_t}}{q^L e^{\hat{q}^L_{t-1}}} \\
Ae^{\hat{A}_t}(1-\alpha_e)k^{\alpha_e}e^{\alpha_e\hat{k}_{t+1}}l^{-\alpha_e}e^{-\alpha_e\hat{l}_t} &= l^{\zeta-1}e^{(\zeta-1)\hat{l}_t}C^\sigma e^{\sigma\hat{C}_t}
\end{aligned}
$$

where $\hat{x}_t \equiv \ln(x_t/x)$ for all variables except ϕ_t and b_t, for which $\hat{\phi}_t \equiv \phi_t - \phi$ and $\hat{b}_t \equiv b_t - b$. Taking first-order Taylor expansions of the above equations results in a linear system where $\hat{\phi}_t$ does not appear and ϕ appears only in the first equation

$$c\hat{c}_t + qk\hat{k}_{t+1} + \hat{b}_{t+1} = \text{Terms in } \hat{A}_t, \hat{k}_t, \hat{l}_t + r^S b\hat{r}_{St} + r^S\hat{b}_t + \beta^{-1}\phi(\hat{r}_{Lt} - \hat{r}_{St}) + O(\epsilon^2),$$

where $O(\epsilon^2)$ denotes terms second and higher orders. The first-order system cannot be simply solved as in a model without portfolio choice, because the zero-order portfolio ϕ is unknown. However, the first-order system can be solved conditional on some ϕ. From the first-order approximation of the households' two Euler equation for short- and long-term bonds, we notice that $E_t r_{Lt+1} = r_{St+1} + O(\epsilon^2)$. Hence, the excess return on the portfolio $\beta^{-1}\phi(\hat{r}_{Lt} - \hat{r}_{St})$ is a zero mean i.i.d. random variable up to a first-order of approximation. Devereux and Sutherland's approach consists in replacing this term by an exogenous zero mean i.i.d. random variable, solving the first-order approximation of the model, later on recognizing that this term is the excess return and using the first-order solution of the modified model together with a second-order approximation of the agents' Euler equations to solve for the zero-order portfolio. To operationalize this approach using the standard algorithm of Schmidt-Grohe and Uribe (SGU), we alternatively replace the excess return term in the exact entrepreneur's budget constraint by an exogenous variables $\hat{\xi}_t$, so that

$$\hat{\xi}_t \equiv \beta^{-1}(e^{\hat{r}_{Lt}} - e^{\hat{r}_{St}})(\hat{\phi}_t + \phi).$$

A first-order approximation of the entrepreneur's budget constraint then gives

$$c\hat{c}_t + qk\hat{k}_{t+1} + \hat{b}_{t+1} = \text{Terms in } \hat{A}_t, \hat{k}_t, \hat{l}_t + r^S b\hat{r}_{St} + r^S\hat{b}_t + \hat{\xi}_t + O(\epsilon^2),$$

while a first-order approximation of the definition of $\hat{\xi}_t$ yields

$$\hat{\xi}_t = \beta^{-1}\phi(\hat{r}_{Lt} - \hat{r}_{St}) + O(\epsilon^2).$$

We thus replace the excess return term by $\hat{\xi}_t$ in the entrepreneur's budget constraint and append $E_t\hat{\xi}_{t+1} = 0$ to the set of equilibrium conditions of the model. Denote by $\tilde{\epsilon}_t$ the vector of actual exogenous shocks in the model (so far $\tilde{\epsilon}_t = [\epsilon_t^A, \epsilon_t^\vartheta]'$). Also define $\epsilon_t \equiv [\epsilon_{0t}, \tilde{\epsilon}_t']'$. The first-order accurate solution of the modified model using the SGU algorithm is given by

$$\begin{aligned} y_t &= g_x x_t + O(\epsilon^2), \\ x_{t+1} &= h_x x_t + \eta\sigma\epsilon_{t+1} + O(\epsilon^2), \end{aligned}$$

where x_t is a vector of (exogenous and endogenous) state variables, y_t is a vector of control variables and g_x, h_x are coefficient matrices. The solution for the control variables can thus also be written as

$$y_t = g_x h_x x_{t-1} + g_x\eta\sigma\epsilon_t + O(\epsilon^2)$$

Defining $\Theta_1 \equiv g_x h_x$ and $\Theta_2 \equiv g_x\eta\sigma$, the solution for the control variables can be expressed as

$$y_t = \Theta_1 x_{t-1} + \Theta_2\epsilon_t + O(\epsilon^2)$$

In tensor notation and removing time subscripts, we have

$$[y']^i = [\Theta_1]^i_a[x]^a + [\Theta_2]^i_c[\epsilon']^c + O(\epsilon^2)$$

By extracting the appropriate rows of this solution, we can write

$$\begin{aligned}
(\sigma\hat{C}' - \sigma\hat{c}') &= [\Theta_1]^1_a[x]^a + [\Theta_2]^1_c[\epsilon']^c + O(\epsilon^2) \\
(\hat{r}'_L - \hat{r}'_S) &= [\Theta_2]^2_c[\epsilon']^c + O(\epsilon^2)
\end{aligned}$$

We now split ϵ into ϵ_0 and $\tilde{\epsilon}$ and define $[\Theta^0_2]^i \equiv [\Theta_2]^i_1$, $[\tilde{\Theta}_2]^i_c \equiv [\Theta_2]^i_{c+1}$. Hence, we can write the two expressions for $(\sigma\hat{C}' - \sigma\hat{c}')$ and $(\hat{r}'_L - \hat{r}'_S)$ as

$$(\sigma\hat{C}' - \sigma\hat{c}') = [\Theta_1]^1_a[x]^a + [\Theta^0_2]^1\epsilon'_0 + [\tilde{\Theta}_2]^1_c[\tilde{\epsilon}']^c + O(\epsilon^2) \tag{D.1}$$

$$(\hat{r}'_L - \hat{r}'_S) = [\Theta^0_2]^2\epsilon'_0 + [\tilde{\Theta}_2]^2_c[\tilde{\epsilon}']^c + O(\epsilon^2) \tag{D.2}$$

Now we use the fact that $\epsilon'_0 = \hat{\xi}' = \phi\beta^{-1}(\hat{r}'_L - \hat{r}'_S) + O(\epsilon^2)$ in (D.1) and (D.2) to get

$$(\sigma\hat{C}' - \sigma\hat{c}') = [\Theta_1]^1_a[x]^a + \left[\frac{\phi\beta^{-1}[\Theta^0_2]^1[\tilde{\Theta}_2]^2_c}{1 - \phi\beta^{-1}[\Theta^0_2]^2} + [\tilde{\Theta}_2]^1_c\right][\tilde{\epsilon}']^c + O(\epsilon^2) \tag{D.3}$$

$$(\hat{r}'_L - \hat{r}'_S) = \frac{[\tilde{\Theta}_2]^2_c}{1 - \phi\beta^{-1}[\Theta^0_2]^2}[\tilde{\epsilon}']^c + O(\epsilon^2) \tag{D.4}$$

Combining the second-order approximations to the household's and entrepreneur's portfolio equations yields:

$$E\left[\left(\sigma\hat{C}' - \sigma\hat{c}'\right)\left(\hat{r}'_L - \hat{r}'_S\right)\right] = -(1-\beta)\sigma^2_\vartheta + \tau^S + O(\epsilon^3)$$

Substituting the first-oder accurate expressions for $(\sigma\hat{C}' - \sigma\hat{c}')$ and $(\hat{r}'_L - \hat{r}'_S)$ into this portfolio equation leads to the following quadratic equation for the zero-order portfolio

$$m\phi^2 + n\phi + p = 0 + O(\epsilon^3),$$

where

$$\begin{aligned}
m &= [(1-\beta)\sigma^2_\vartheta - \tau^S]\beta^{-2}\left([\Theta^0_2]^2\right)^2 \\
n &= \beta^{-1}\left([\Theta^0_2]^1[\tilde{\Theta}_2]^2_c - [\Theta^0_2]^2[\tilde{\Theta}_2]^1_c\right)[\tilde{\Theta}_2]^2_d[\Sigma]^{cd} - 2[(1-\beta)\sigma^2_\vartheta - \tau^S]\beta^{-1}[\Theta^0_2]^2 \\
p &= [\tilde{\Theta}_2]^1_c[\tilde{\Theta}_2]^2_d[\Sigma]^{cd} + (1-\beta)\sigma^2_\vartheta - \tau^S
\end{aligned}$$

The two solutions are given by

$$\phi = \frac{-n \pm \sqrt{n^2 - 4mp}}{2m} + O(\epsilon)$$

Hence unless $[\Theta^0_2]^2 = 0$ (the excess return does not depend on the portfolio up to a first-order), the monitoring cost potentially introduces multiple equilibrium zero-order portfolios. In our case, only one of the two solutions of the quadratic equation is a valid equilibrium, but it is worth noting that this is not always the case. In the costless enforcement case where

$(1-\beta)\sigma_{\vartheta}^2 = 0$ and short-term debt is not taxed ($\tau^S = 0$) the zero-order portfolio is given by the solution of a linear equation:

$$\phi = \beta \frac{[\tilde{\Theta}_2]^1_c [\tilde{\Theta}_2]^2_d [\Sigma]^{cd}}{\left([\Theta_2^0]^2 [\tilde{\Theta}_2]^1_c - [\Theta_2^0]^1 [\tilde{\Theta}_2]^2_c\right) [\tilde{\Theta}_2]^2_d [\Sigma]^{cd}} + O(\epsilon),$$

which is analogous to the expression in Devereux and Sutherland (2011).

First-order solution for non-portfolio variables, once ϕ is known

Once ϕ has been computed, we can go back to the original model, replace ϕ_t by ϕ in the entrepreneur's budget constraint. Note that this is not a correct equilibrium condition, but just an artifice to solve the model using the SGU algorithm. Since linearizing this artificial equation gives the same expression as a linearization of the true budget constraint, as far as the first-order accuracy is concerned, we can solve the model using this modified budget constraint.

D.2 First-order portfolio and second-order non-portfolio variables

We are now interested in obtaining a first-order accurate solution for ϕ_t and a second-order accurate solution for non-portfolio variables.

First-order solution for ϕ

Taking a second order approximation to the entrepreneur's budget constraint yields

$$
\begin{aligned}
c\hat{c}_t + \frac{1}{2}c\hat{c}_t^2 + qk\hat{k}_{t+1} + \frac{1}{2}qk\hat{k}_{t+1}^2 + qk\hat{q}_t\hat{k}_{t+1} + \hat{b}_{t+1} \quad = \quad & \text{Terms in } \hat{A}_t, \hat{k}_t, \hat{l}_t + qk\hat{k}_t \\
& + \frac{1}{2}qk\hat{k}_t^2 + qk\hat{q}_t\hat{k}_t + \tau^S r^S(b-\phi) + \beta^{-1}b\hat{r}_{St} + \frac{1}{2}r^S b\hat{r}_{St}^2 \\
& + r^S\hat{b}_t + r^S\hat{r}_{St}\hat{b}_t + \phi\beta^{-1}(\hat{r}_{Lt} - \hat{r}_{St}) \\
& + \frac{1}{2}\phi\beta^{-1}(\hat{r}_{Lt}^2 - \hat{r}_{St}^2) + \beta^{-1}\hat{\phi}_t(\hat{r}_{Lt} - \hat{r}_{St}) + O(\epsilon^3)
\end{aligned}
$$

We now postulate that $\hat{\phi}_t$ is linear in the model's state variables:

$$
\hat{\phi}_t = [\psi]_k[x_{t-1}]^k + O(\epsilon^2)
$$

Following Devereux and Sutherland, we again replace the excess return term in the original entrepreneur's budget constraint by

$$
\hat{\xi}_t \equiv \beta^{-1}(e^{\hat{r}_{Lt}} - e^{\hat{r}_{St}})(\hat{\phi}_t + \phi).
$$

A second-order approximation of the modified entrepreneur's budget constraint gives

$$
\begin{aligned}
c\hat{c}_t + \frac{1}{2}c\hat{c}_t^2 + qk\hat{k}_{t+1} + \frac{1}{2}qk\hat{k}_{t+1}^2 + qk\hat{q}_t\hat{k}_{t+1} + \hat{b}_{t+1} \quad = \quad & \text{Terms in } \hat{A}_t, \hat{k}_t, \hat{l}_t + qk\hat{k}_t \\
& + \frac{1}{2}qk\hat{k}_t^2 + qk\hat{q}_t\hat{k}_t + \tau^S r^S(b-\phi) + r^S b\hat{r}_{St} + \frac{1}{2}r^S b\hat{r}_{St}^2 \\
& + r^S\hat{b}_t + r^S\hat{r}_{St}\hat{b}_t + \hat{\xi}_t + O(\epsilon^3)
\end{aligned}
$$

while a second-order approximation of the definition of $\hat{\xi}_t$ yields

$$\hat{\xi}_t = \phi\beta^{-1}(\hat{r}_{Lt} - \hat{r}_{St}) + \phi\beta^{-1}(\hat{r}_{Lt}^2 - \hat{r}_{St}^2) + \beta^{-1}\hat{\phi}_t(\hat{r}_{Lt} - \hat{r}_{St}) + O(\epsilon^3).$$

Now, given that $\hat{\phi}_t$ is a function of x_{t-1}, the term $\beta^{-1}\hat{\phi}_t(\hat{r}_{Lt} - \hat{r}_{St})$ satisfies

$$E_t[\beta^{-1}\hat{\phi}_{t+1}(\hat{r}_{Lt+1} - \hat{r}_{St+1})] = 0 + O(\epsilon^3),$$

i.e. the realized excess return on the time-varying element of the portfolio is a zero-mean i.i.d. random variable up to a second-order of accuracy. As we did for the first-order accurate model solution, we therefore initially treat this term as an exogenous zero mean i.i.d. random variable ϵ_{0t}. We thus use the budget constraint with $\hat{\xi}_t$ in place of the excess return term and append to the other model equilibrium conditions the following equation:

$$E_t\hat{\xi}_{t+1} = \phi\beta^{-1}(\hat{r}_{Lt} - \hat{r}_{St}) + \phi\beta^{-1}(\hat{r}_{Lt}^2 - \hat{r}_{St}^2).$$

The second-order accurate solution of the model using the SGU algorithm is given by

$$\begin{aligned}
[x']^i &= [h_x]^i_a[x]^a + \frac{1}{2}[h_{xx}]^i_{ab}[x]^a[x]^b + \frac{1}{2}[h_{\sigma\sigma}]^i\sigma^2 + \sigma[\eta]^i_c[\epsilon']^c + O(\epsilon^3),\\
[y]^i &= [g_x]^i_j[x]^j + \frac{1}{2}[g_{xx}]^i_{jk}[x]^j[x]^k + \frac{1}{2}[g_{\sigma\sigma}]^i\sigma^2 + O(\epsilon^3).
\end{aligned}$$

Forwarding the solution for control variables one period ahead, substituting the solution for contemporaneous state variables and distinguishing terms of first- and second-order for the state vector leads to

$$[y']^i = [D_0]^i + [D_1]^i_a([x^f]^a + [x^s]^a) + [D_2]^i_c[\epsilon']^c + [D_3]^i_{ab}[x]^a[x]^b + [D_4]^i_{cd}[\epsilon']^c[\epsilon']^d + [D_5]^i_{ac}[x]^a[\epsilon']^c + O(\epsilon^3),$$

where the arrays $[D_0]$, $[D_1]$, $[D_2]$, $[D_3]$, $[D_4]$, $[D_5]$ are some functions of the arrays $[h_x]$, $[h_{xx}]$, $[h_{\sigma\sigma}]$, $[g_x]$, $[g_{xx}]$ and $[g_{\sigma\sigma}]$. We can thus write the second-order accurate conditional solution for the difference in marginal utility $(\sigma\hat{C}' - \sigma\hat{c}')$ and the excess return $(\hat{r}'_L - \hat{r}'_S)$ as

$$\begin{aligned}
(\sigma\hat{C}' - \sigma\hat{c}') &= [D_0]^1 + [D_1]^1_a([x^f]^a + [x^s]^a) + [D_2]^1_c[\epsilon']^c + [D_3]^1_{ab}[x^f]^a[x^f]^b \\
&\quad + [D_4]^1_{cd}[\epsilon']^c[\epsilon']^d + [D_5]^1_{ac}[x^f]^a[\epsilon']^c + O(\epsilon^3) \\
(\hat{r}'_L - \hat{r}'_S) &= [D_0]^2 + [D_1]^2_a([x^f]^a + [x^s]^a) + [D_2]^2_c[\epsilon']^c + [D_3]^2_{ab}[x^f]^a[x^f]^b \\
&\quad + [D_4]^2_{cd}[\epsilon']^c[\epsilon']^d + [D_5]^2_{ac}[x^f]^a[\epsilon']^c + O(\epsilon^3)
\end{aligned}$$

Now, we again split ϵ into ϵ_0 and $\tilde{\epsilon}$, and define $[D_2^0]^i \equiv [D_2]^i_1$, $[\tilde{D}_2]^i_c \equiv [D_2]^i_{c+1}$, $[\tilde{D}_4]^i_{cd} \equiv [D_4]^i_{c+1,d+1}$ and $[\tilde{D}_5]^i_{ac} \equiv [D_5]^i_{a,c+1}$. Note that second-order terms that contain ϵ'_0 disappear from the second-order accurate expressions (i.e. are part of the $O(\epsilon^3)$ term) since ϵ_0 is actually itself a second-order term. Hence, we can write the two expressions for $(\sigma\hat{C}' - \sigma\hat{c}')$ and $(\hat{r}'_L - \hat{r}'_S)$ as

$$\begin{aligned}
(\sigma\hat{C}' - \sigma\hat{c}') &= [D_0]^1 + [D_1]^1_a([x^f]^a + [x^s]^a) + [D_2^0]^1\epsilon'_0 + [\tilde{D}_2]^1_c[\tilde{\epsilon}']^c + [D_3]^1_{ab}[x^f]^a[x^f]^b \\
&\quad + [\tilde{D}_4]^1_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + [\tilde{D}_5]^1_{ac}[x^f]^a[\tilde{\epsilon}']^c + O(\epsilon^3), \qquad \text{(D.1)} \\
(\hat{r}'_L - \hat{r}'_S) &= [D_0]^2 + [D_1]^2_a([x^f]^a + [x^s]^a) + [D_2^0]^2\epsilon'_0 + [\tilde{D}_2]^2_c[\tilde{\epsilon}']^c + [D_3]^2_{ab}[x^f]^a[x^f]^b \\
&\quad + [\tilde{D}_4]^2_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + [\tilde{D}_5]^2_{ac}[x^f]^a[\tilde{\epsilon}']^c + O(\epsilon^3) \qquad \text{(D.2)}
\end{aligned}$$

Since up to a first-order, the expected excess return is zero, we know that $[D_1]^2_a[x^f]^a = 0$. Further, since up to a second-order, the expected excess return is constant, we know that $[D_1]^2_a[x^s]^a$ and $[D_3]^2_{ab}[x^f]^a[x^f]^b$ are constants. Taking expectations on both sides of (D.2) implies that

$$[D_0]^2 = E[(\hat{r}'_L - \hat{r}'_S)] - [D_1]^2_a[x^s]^a - [D_3]^2_{ab}[x^f]^a[x^f]^b - [\tilde{D}_4]^2_{cd}[\Sigma]^{cd}$$

so that $(\hat{r}'_L - \hat{r}'_S)$ can be written as

$$(\hat{r}'_L - \hat{r}'_S) = E[(\hat{r}'_L - \hat{r}'_S)] - [\tilde{D}_4]^2_{cd}[\Sigma]^{cd} + [D^0_2]^2\epsilon'_0 + [\tilde{D}_2]^2_c[\tilde{\epsilon}']^c + [\tilde{D}_4]^2_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + [\tilde{D}_5]^2_{ac}[x^f]^a[\tilde{\epsilon}']^c + O(\epsilon^3)$$

We now recognize that ϵ'_0 is endogenous and given by

$$\epsilon'_0 = \beta^{-1}\hat{\phi}'(\hat{r}'_L - \hat{r}'_S) = \beta^{-1}[\psi]_k[\tilde{D}_2]^2_c[x^f]^k[\tilde{\epsilon}']^c$$

and use this to rewrite the second-order accurate expressions for $(\sigma\hat{C}' - \sigma\hat{c}')$ and $(\hat{r}'_L - \hat{r}'_S)$ as

$$\begin{aligned}
(\sigma\hat{C}' - \sigma\hat{c}') &= [D_0]^1 + [D_1]^1_a([x^f]^a + [x^s]^a) + [\tilde{D}_2]^1_c[\tilde{\epsilon}']^c + [D_3]^1_{ab}[x^f]^a[x^f]^b \\
&\quad + [\tilde{D}_4]^1_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + \left([\tilde{D}_5]^1_{kc} + [D^0_2]^1\beta^{-1}[\tilde{D}_2]^2_c[\psi]_k\right)[x^f]^k[\tilde{\epsilon}']^c + O(\epsilon^3), \\
(\hat{r}'_L - \hat{r}'_S) &= E[(\hat{r}'_L - \hat{r}'_S)] - [\tilde{D}_4]^2_{cd}[\Sigma]^{cd} + [\tilde{D}_2]^2_c[\tilde{\epsilon}']^c \\
&\quad + [\tilde{D}_4]^2_{cd}[\tilde{\epsilon}']^c[\tilde{\epsilon}']^d + \left([\tilde{D}_5]^2_{kc} + [D^0_2]^2\beta^{-1}[\tilde{D}_2]^2_c[\psi]_k\right)[x^f]^k[\tilde{\epsilon}']^c + O(\epsilon^3).
\end{aligned}$$

The first-order accurate solutions for $\hat{C}'$, $\hat{c}'$, $\hat{r}'_L$, $\hat{r}'_S$ and $\hat{q}_L$ are also useful:

$$
\begin{aligned}
\hat{C}' &= [D_1]^3_a[x^f]^a + [\tilde{D}_2]^3_c[\tilde{\epsilon}']^c + O(\epsilon^2),\\
\hat{c}' &= [D_1]^4_a[x^f]^a + [\tilde{D}_2]^4_c[\tilde{\epsilon}']^c + O(\epsilon^2),\\
\hat{r}'_L &= [D_1]^5_a[x^f]^a + [\tilde{D}_2]^5_c[\tilde{\epsilon}']^c + O(\epsilon^2),\\
\hat{r}'_S &= [D_1]^6_a[x^f]^a + [\tilde{D}_2]^6_c[\tilde{\epsilon}']^c + O(\epsilon^2),\\
\hat{q}_L &= [h_x]^4_a[x^f]^a + O(\epsilon^2),
\end{aligned}
$$

where we note that $[D_1]^5_a = [D_1]^6_a$.

Combining the third-order approximations to the household's and entrepreneur's portfolio equations yields:

$$
\begin{aligned}
&E\left[\left(\sigma\hat{C}' - \sigma\hat{c}'\right)(\hat{r}'_L - \hat{r}'_S) - \frac{1}{2}\left(\sigma^2(\hat{C}')^2 - \sigma^2(\hat{c}')^2\right)(\hat{r}'_L - \hat{r}'_S) + \frac{1}{2}\left(\sigma\hat{C}' - \sigma\hat{c}'\right)\left((\hat{r}'_L)^2 - (\hat{r}'_S)^2\right)\right]\\
&= -(1-\beta)\sigma^2_\vartheta + \tau^S + (1-\beta)\sigma^2_\vartheta\hat{q}_L + (1-\beta)\sigma^2_\vartheta\sigma E\hat{C}' + \tau^S\hat{r}'_S - \tau^S E\hat{c}' + O(\epsilon^4)
\end{aligned}
$$

We assume that all third moments of the vector of exogenous shocks $\tilde{\epsilon}'$ are zero. Substituting the expressions derived above for $(\sigma\hat{C}' - \sigma\hat{c}')$, $(\hat{r}_L - \hat{r}_S)$, $\hat{C}'$, $\hat{c}'$, $\hat{r}'_L$, $\hat{r}'_S$ and $\hat{q}_L$ into the terms of the third-order portfolio equation leads, after a number of manipulations and simplifications, to a zero-order accurate expression for the coefficients of the assumed linear decision rule for $\hat{\phi}_t$:

$$
\psi_a = -\beta\frac{[\tilde{D}_5]^1_{ac}[\tilde{D}_2]^2_d[\Sigma]^{cd} + [\tilde{D}_5]^2_{ad}[\tilde{D}_2]^1_c[\Sigma]^{cd} - (1-\beta)\sigma^2_\vartheta\left([D_1]^5_a + [h_x]^4_a\right)}{[D^0_2]^1[\tilde{D}_2]^2_c[\tilde{D}_2]^2_d[\Sigma]^{cd} - (1-\beta)\sigma^2_\vartheta[D^0_2]^2 + \tau^S[D^0_2]^2} + O(\epsilon)
$$

For $\sigma_\vartheta = \tau^S = 0$, this expressions collapses to the one in Devereux and Sutherland (2009).

Second-order solution for non-portfolio variables, once first-order ϕ_t is known

Once the ψ_a's have been computed, we can go back to the original model and replace ϕ_t by

$$\phi\beta^{-1}(\hat{r}_{Lt} - \hat{r}_{St}) + \frac{1}{2}\phi\beta^{-1}(\hat{r}_{Lt}^2 - \hat{r}_{St}^2) + \beta^{-1}\left([\psi]_k[x_{t-1}]^k\right)(\hat{r}_{Lt} - \hat{r}_{St})$$

in the entrepreneur's budget constraint, and replace ϕ_t by ϕ in the economy's resource constraint. Note that again, these are not a correct equilibrium condition, but just an artifice to solve the model using the SGU algorithm. Since taking a second-order approximation of the modified entrepreneur's budget constraint gives the same expression as a second-order approximation of the true budget constraint and taking a second-order approximation of the modified resource constraint gives the same expression as the second-order approximation of the true resource constraint, as far as second-order accuracy is concerned, we can solve the model using these modified constraints.

REFERENCES

Acharya, V. V. (2003). Is the international convergence of capital adequacy regulation desirable? *Journal of Finance 58*(6), 2745–2782.

Aiyagari, S. R. and M. Gertler (1999). Overreaction of asset prices in general equilibrium. *Review of Economic Dynamics 2*, 3–35.

Allen, F. and D. Gale (1998). Optimal financial crises. *Journal of Finance 53*(4), 1245–1284.

Allen, F. and D. Gale (2004). Financial intermediaries and markets. *Econometrica 72*(4), 1023–1061.

Angeletos, G.-M. (2002). Fiscal policy with noncontingent debt and the optimal maturity structure. *Quarterly Journal of Economics 117*(3), 1105–1131.

Ball, L. and D. Romer (1989). Are prices too sticky? *Quarterly Journal of Economics 104*, 507–524.

Bank of England (2009). The role of macroprudential policy. Discussion paper, Bank of England.

Barclay, M. and C. Smith (1995). The maturity structure of corporate debt. *Journal of Finance L*, 609–631.

Bengui, J. (2011). Systemic risk and inefficient debt maturity. Mimeo, University of Maryland.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctualtions. *American Economic Review 79*, 1–15.

Bernanke, B., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantatative business cycle framework. In J. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1, pp. 1341–1393. Elsevier Science B.V.

Bhattacharya, S. and D. Gale (1987). Preference shocks, liquidity and central bank policy. In W. P. Heller, R. M. Starr, and D. A. Starrett (Eds.), *New Approaches to Monetary Economics*. Cambridge University Press.

Bianchi, J. (2009). Overborrowing and systemic externalities in the business cycle. University of maryland working paper.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review*. forthcoming.

Bianchi, J. and E. G. Mendoza (2010). Overborrowing, financial crises and 'macroprudential' policy. Mimeo, University of Maryland.

Blanchard, O. J. and N. Kiyotaki (1987). Monopolistic competition and the effects of aggregate demand. *American Economic Review 77*, 647–666.

Broner, F., G. Lorenzoni, and S. Schmukler (2008). Why do emerging economies borrow short term. Mimeo.

Brunnermeier, M., A. Crocket, C. Goodhart, A. D. Persaud, and H. Shin (2009). The fundamental principles of financial regulation. Icmb-cepr geneva report on the world economy 11.

Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives 23*(1), 77–100.

Buera, F. and J. P. Nicolini (2004). Optimal maturity of government debt without state contingent bonds. *Journal of Monetary Economics 51*, 531–554.

Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy 93*(3), 488–511.

Caballero, R. J. and A. Krishnamurthy (2001). International and domestic collateral constraints in a model of emerging market crises. *Journal of Monetary Economics 48*(1), 513–548.

Caballero, R. J. and A. Krishnamurthy (2004). Smoothing sudden stops. *Journal of Economic Theory 116*(1), 104–127.

Calomiris, C. and C. Kahn (1991). The role of demandable debt in structuring optimal banking arrangements. *American Economic Review 81*(3), 497–513.

Carlstrom, C. T. and T. S. Fuerst (1997). Agency costs, net worth, and business fluctualtions: A computable general equilibrium analysis. *American Economic Review 87*, 893–910.

Castiglionesi, F., F. Feriozzi, and G. Lorenzoni (2010). Financial integration and liquidity crises. Mimeo, Princeton University.

Che, Y.-K. and J. Sakovics (2008). Hold-up problem. In S. N. Durlauf and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics* (Second ed.). Palgrave Macmillan.

Cooper, R. N. (1985). Economic interdependence and coordination of economic policies. In G. M. Grossman and K. Rogoff (Eds.), *Handbook of International Economics*, Volume 2. Elsevier.

Corsetti, G. and P. Pesenti (2001). Welfare and macroeconomic interdependence. *Quarterly Journal of Economics 116*(2), 421–445.

Dell Arricia, G. and R. Marquez (2006). Competition among regulators and credit market integration. *Journal of Financial Economics 79*, 401–430.

Devereux, M. and A. Sutherland (2009). Country portfolio dynamics. Working paper, UBC.

Devereux, M. and A. Sutherland (2011). Country portfolios in open economy macro models. *Journal of the European Economic Association 9*, 337–389.

Diamond, D. (1991). Debt maturity structure and liquidity risk. *Quarterly Journal of Economics 106*(3), 709–737.

Diamond, D. and P. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy 91*(3), 401–419.

Diamond, D. and R. Rajan (2001). Liquidity risk, liquidity creation and financial fragility: A theory of banking. *Journal of Political Economy 109*(2), 287–327.

Evans, M. D. D. and V. Hnatkovska (2008). A method for solving general equilibrium models with incomplete markets and many financial assets. Mimeo, UBC.

Farhi, E., M. Golosov, and A. Tsyvinski (2009). A theory of liquidity and regulation of financial intermediation. *Review of Economic Studies 76*(3), 973–992.

Flannery, M. (1986). Asymmetric information and risky debt maturity choice. *Journal of Finance 41*(1), 19–37.

Geanakoplos, J. and H. Polemarchakis (1986). Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete. In W. A. Barnett and K. J. Singleton (Eds.), *Uncertainty, Information and Communication, Essays in Honor of Kenneth J. Arrow*. Cambridge University Press.

Greenwood, J. and Z. Hercowitz (1991). The allocation of capital and time over the business cycle. *The Journal of Political Economy 99*, 1188–1214.

Grout, P. A. (1984). Investment and wages in the absence of binding contracts: a nash bargaining approach. *Econometrica 52*(2), 449–460.

Hamada, K. (1974). Alternative exchange rate systems and the interdependence of monetary policies. In R. Aliber (Ed.), *National Monetary Policies and the International Financial System*. University of Chicago Press.

Hamada, K. (1976). A strategic analysis of monetary interdependence. *Journal of Political Economy 84*, 677–700.

Hamada, K. (1979). Macroeconomic strategy and coordination under alternative exchange rates. In R. Dornbush and J. Frenkel (Eds.), *International Economic Policy*. Johns Hopkins University Press.

Hart, O. (1975). On the optimality of equilibrium when the market structure is incomplete. *Journal of Economic Theory 11*, 418–443.

Hellwig, M. (1994). Liquidity provision, banking, and the allocation of interest rate risk. *European Economic Review 38*, 1363–1389.

Holmstrom, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy 106*(1), 1–40.

Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review 95*, 739–764.

Jacklin, C. J. (1987). Demand deposits, trading restrictions, and risk sharing. In E. Prescott and N. Wallace (Eds.), *Contractual Arrangements for Intertemporal Trade*, pp. 26–47. University of Minnesota Press.

Jeanne, O. (2009). Debt maturity and the international financial architecture. *American Economic Review 99*(5), 2135–2148.

Jeanne, O. and A. Korinek (2011). Managing credit booms and busts: a pigouvian taxation approach. Mimeo, University of Maryland.

Johnson, H. G. (1965). An economic theory of protectionism, tariff bargaining, and the formation of custums union. *Journal of Political Economy 73*(3), 256–283.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*, 211–248.

Kiyotaki, N. and J. Moore (2003). Inside money and liquidity. Mimeo, London School of Economics.

Kiyotaki, N. and J. Moore (2005). Financial deepening. *Journal of the European Economic Association 3*(2–3), 701–713.

Kiyotaki, N. and J. Moore (2008). Liquidity, business cycles, and monetary policy. Mimeo, Princeton University.

Korinek, A. (2009). Excessive dollar borrowing in emerging markets: Balance sheet effects and macroeconomic externalities. University of maryland working paper.

Korinek, A. (2011a). The new economics of prudential capital controls. *IMF Economic Review*. forthcoming.

Korinek, A. (2011b). Systemic risk-taking: Amplification effects, externalities, and regulatory responses. Mimeo, University of Maryland.

Lorenzoni, G. (2008). Inefficient credit booms. *Review of Economic Studies 75*(3), 809–833.

Mendoza, E. G. and K. A. Smith (2006). Quantitative implications of a debt-deflation theory of sudden stops and asset prices. *Journal of International Economics 70*, 82–114.

Persson, T. and G. Tabellini (1995). Double-edged incentives: Institutions and policy coordination. In G. M. Grossman and K. Rogoff (Eds.), *Handbook of Devlopment Economics*, Volume 3. Elsevier.

Rodrick, D. and A. Velasco (1999). Short-term capital flows. In B. Pleskovic and J. E. Stiglitz (Eds.), *Annual World Bank Conference on Development Economics*. The World Bank.

Samuelson, P. (1970). The fundamental approximation theorem of portfolio analysis in terms of means, variance and higher moments. *Review of Economic Studies 37*, 537–542.

Schmitt-Grohe, S. and M. Uribe (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control 28*, 755–775.

Shin, H. S. (2010). *Risk and Liquidity*. Oxford University Press.

Squam Lake Working Group on Financial Regulation (2009). Reforming capial requirements for financial institutions. Working paper, Council on Foreign Relations.

Stiglitz, J. (1982). The inefficiency of stock market equilibrium. *Review of Economic Studies 49*, 241–261.

Tarullo, D. K. (2008). *Banking on Basel: The Future of International Financial Regulation*. Peterson Institute for International Economics.

The Group of Thirty (2009). Financial reform: a framework for financial stability. Technical report, Working Group on Financial Reform chaired by Paul Volker.

Tille, C. and E. van Wincoop (2008). International capital flows. Mimeo, University of Virginia.

Tirole, J. (1986). Investment and wages in the absence of binding contracts: a nash bargaining approach. *Journal of Political Economy 94*(2), 235–259.

Tirole, J. (2003). Inefficient foreign borrowing: A dual- and common-agency perspective. *American Economic Review 93*(5), 1678–1702.

von Thadden, E.-L. (1999). Liquidity creation through banks and markets: Multiple insurance and limited market access. *European Economic Review 43*, 991–1006.

Wagner, W. (2009). In the quest of systemic externalities: A review of the literature. *CESifo Economic Studies*. forthcoming.

Warwick Commission (2009). The warwick commission on international financial reform: In praise of unlevel playing fields. Technical report, The Warwick Commission.

Welch, I. (2004). Capital structure and stock returns. *The Journal of Political Economy 112*, 106–131.