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**Probability of Capture and
Rejection of Primary Multiple-
Access Interference in Spread-
Spectrum Networks**

by

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PROBABILITY OF CAPTURE AND REJECTION OF PRIMARY MULTIPLE-ACCESS INTERFERENCE IN SPREAD-SPECTRUM NETWORKS

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ABSTRACT

The probability of capture is evaluated for the situation in which several transmitters use the same spread-spectrum code to contend for the attention of a single receiver. The first stage in the capture mechanism is that of acquisition of capture; randomization of the arrival time has been proposed by Davis and Gronemayer to provide delay capture and the probability of this occurrence has been derived. We are concerned with the second stage, that of retaining capture in the presence of interference from the contending users. The probability of retaining capture is computed via accurate approximations and upper bounds for direct-sequence, frequency-hopped and hybrid spread-spectrum signaling formats and for different data modulation and demodulation schemes. The calculation of the overall probability of capture is carried out for spread-spectrum systems with and without forward-error-control; in the latter case Reed-Solomon codes, as well as binary convolutional codes, are considered. Finally, the capability of rejecting primary multiple-access interference in spread-spectrum radio networks is examined by computing the maximum number of users which can contend for the same receiver without causing the probability of capture to fall below some desirable level.

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I. INTRODUCTION

In networks which employ spread-spectrum signaling and receiver-oriented transmission policies the capture effect and the capability of rejecting primary multiple-access interference are important issues. However, except for the study of acquisition of power capture considered in [1]-[2] and of delay capture considered in [3], no other quantitative results are available in the literature. This paper is concerned with the presentation of quantitative results on the capture phenomena in spread-spectrum packet radio networks.

Capture phenomena characterizes the ability of a receiver to successfully receive a packet (with nonzero probability) even though part or all of the packet arrives at the receiver overlapped in time by other packets. The basic mechanism for capture is the ability of the receiver to synchronize with and lock on to one packet and subsequently reject other overlapping packets as noise [4]. We consider packet radio networks which employ receiver-oriented transmission policies. When a node wishes to communicate with another node it uses the spread-spectrum code assigned to the intended receiver for its transmission. In this way all the neighbors of a receiving node may be competing for its attention and since all will be using the same spread-spectrum code, collisions may occur, and no successful reception takes place.

More specifically, we focus on the problem of contention among many transmitters for a common receiver. When the spread-spectrum code that is employed does not repeat within a packet duration, the competing users' packets would be strongly correlated over each data symbol if they arrived at a receiver simultaneously, but would be pseudo-orthogonal if they arrived with a time offset. This property allows the first packet to be captured and successfully received while the others are rejected as noise (with some

probability). In general, there is a vulnerable period at the beginning of a packet, denoted by T_a and called *capture time* or *acquisition interval*, during which collision with the same portion of another packet results in a loss of both. The capture time may be as small as one dwell-time of a FH/SS system or as small as a few chip times in a DS/SS system [4]. This type of capture is known as *delay (or code) capture*.

The probability of capture is defined as

$$P_c(K) \equiv Pr(1 \text{ user successful} \mid K \text{ users contend}) ; \quad K \geq 2.$$

$P_c(K)$ can be decomposed into two parts; the probability of acquisition of capture, $P_a(K)$, and the probability of retaining capture, denoted by $P_r(K)$ (i.e., $P_c(K) = P_a(K) P_r(K)$). The probability of acquisition for the *delay capture* has been already derived by Gronemeyer and Davis [3]. By introducing a time of arrival randomization procedure, with parameter T_u , to eliminate discrimination as a function of range, $P_a(K)$ is given by:

$$P_a(K) = \begin{cases} 1 & K=1 \\ (1-Q)^K & K \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

where $Q \equiv \frac{T_a}{T_u}$, is called the *capture ratio* [3].

The probability of retaining capture pertains to the rejection of the interfering packets as noise, and therefore depends on the SS modulation scheme and correlation properties of the signature sequences or hopping patterns. We model the signature sequence (or hopping pattern) being used by all competing users, as a random SS code (e.g., random binary sequence for DS/SS and random hopping pattern for FH/SS). In the DS/SS and hybrid (FH-DS) SS cases, the exact calculation of these probabilities is intractable (because of partial autocorrelation functions of random signature waveform

involved), so we use the appropriate approximations which depend on the average signal-to-noise ratio; the techniques used are similar to those of [5]-[6] for pure coherent or noncoherent multiple-access systems (with secondary interference). In the FH/SS case, a tight upper bound on the probability of retaining capture is derived based on the methods of hits introduced in [7]. For hybrid (FH-DS) SS systems we use the techniques of [8]-[9]. Finally, the effect of several FEC coding schemes (e.g., Reed-Solomon codes, binary convolutional codes) on increasing the probability of capture is analyzed.

II. DIRECT-SEQUENCE SS SYSTEMS

The transmitted signal of the k -th user in a BPSK DS/SS system can be written as

$$s_k(t) = Ab_k(t)\psi(t)a(t)\cos(2\pi f_c t + \theta_k)$$

In this expression $A = \sqrt{2P}$ is the power constant of the transmitted signal, θ_k is the phase angle introduced by the modulator, and $\psi(t) = \psi(s)$ for $s = t \pmod{T_c}$ where

$\psi(s)$ is a chip waveform. The chip waveform has duration T_c and $\frac{1}{T_c} \int_0^{T_c} \psi^2(t) dt = 1$.

The k -th data signal $b_k(t)$ is a sequence of rectangular pulses of duration T . The j -th pulse has amplitude $b_j^{(k)} = b_k(t)$ for $jT \leq t < (j+1)T$ where $b_j^{(k)}$ takes values $+1$ or -1 . The information sequence is modeled as a sequence of mutually independent random variables taking values $+1$ or -1 with equal probability. The waveform $a(t)$ is a sequence of unit amplitude positive and negative rectangular pulses of duration T_c . The l -th code pulse has amplitude $a_l = a(t)$ for $lT_c \leq t < (l+1)T_c$. We assume that there are N code pulses in each data pulse ($T = NT_c$). The code sequence amplitudes are modeled as i.i.d random variables, taking positive and negative values with equal probability. We assume that there are L data bits per packet, and the signature sequence does not repeat within a packet length. In other words, the period of the sequence $\{a_l\}$, is greater than or equal to NL .

It is assumed that K nodes, sharing the same frequency band, contend for a common receiver in a time slot. The received waveform is then given by

$$r(t) = n(t) + \sum_{k=1}^K s_k(t - \tau_k)$$

where $n(t)$ is the additive white gaussian noise of (two-sided) spectral density $\frac{N_0}{2}$ and

τ_k denotes the time delay along the communication link between k -th transmitter and

the receiver. The time delays are modeled as uniformly distributed independent random variables over $[0, T_u]$.

Now, assume that the receiver acquires the capture of i -th transmitted signal, i.e., we assume that receiver is now in synchronization with $s_i(t)$. From receiver's point of view, we can then assume that $\tau_i = 0$. Since we are concerned with relative phase shifts modulo 2π , there is no loss in generality in assuming $\theta_i = 0$. The output of the correlation receiver during the m -th bit interval is (let $x(t) = a(t)\psi(t)$).

$$Z = \int_{mT}^{(m+1)T} r(t)x(t)\cos(2\pi f_c t) dt \quad (1)$$

After simplification, we can write Z as

$$Z = \frac{AT}{2} \left[b_m^{(i)} + \eta + \sum_{k \neq i} I_k(b^{(k)}, \hat{\tau}_k, \phi_k) \right] \quad (2)$$

where η is a normally distributed random variable with zero mean and variance $\frac{N_0}{A^2 T}$.

$I_k(\cdot, \cdot, \cdot)$ is the interference term due to the k -th node and is defined by

$$I_k(b^{(k)}, \hat{\tau}_k, \phi_k) = \frac{1}{T} \{ b_j^{(k)} R(\hat{\tau}_k) + b_{j+1}^{(k)} \hat{R}(\hat{\tau}_k) \} \cos \phi_k \quad (3)$$

where $R(\tau) = \int_0^\tau x(t)x(t-\tau)dt$ and $\hat{R}(\tau) = \int_\tau^T x(t-\tau)x(t)dt$ are the partial correlation functions of signature "waveform" [5], and $\phi_k = \theta_k - 2\pi f_c \tau_k$, are i.i.d random variables uniformly distributed over $[0, 2\pi]$. The random variable $\hat{\tau}_k$ is defined as the relative delay of the j -th bit of k -th signal with respect to the m -th bit of the i -th signal and is uniformly distributed over $[0, T]$. It is easy to see that the random variables, I_k , depend on the partial correlations of the signature sequence being used.

A. Performance evaluation.

We are concerned with the evaluation of probability of retaining capture, once the acquisition has been achieved. This is equivalent to the evaluation of the packet error probability, since the problem of retaining capture corresponds to the correct reception of the packet. The evaluation of the packet error probability in our model is complicated due to the fact that symbol errors within the packet are not independent and no tractable correlation among them exist. One crude approximation of this quantity is to evaluate the bit error probability at the output of the correlation receiver, and then invoke the assumption of independence of errors to write the packet error probability as

$$1 - P_r(K) = 1 - (1 - P_{ee}(K))^L \quad (4)$$

To evaluate the above quantity, we need the average probability of symbol (bit in case of binary signaling) error, P_{ee} . Examination of the random variable Z reveals that as a result of statistical dependency of interference terms (I_k 's), the exact calculation of this probability is intractable. Therefore, we use an approximation, treating the interference terms as additional noise, evaluate the average signal-to-noise ratio at the output of the correlation receiver, and define the conditional probability of error given that K users are contending by

$$P_{ee}(K) = Q[SNR(K)]. \quad (5)$$

where $Q(\cdot)$ is the complementary error function [5]. The signal to noise ratio is defined as

$$SNR(K) = \frac{AT/2}{\sqrt{Var(Z)}} \quad (6)$$

Here, we have assumed, without loss of generality, that $b_m^{(i)} = +1$. Evaluation of variance of Z leads to

$$\text{Var}(Z) = \frac{A^2 T^2}{4} \left[\frac{N_0}{A^2 T} + \sum_{k \neq i} E(I_k^2) \right] \quad (7)$$

This result follows from the fact that I_k 's are zero mean and uncorrelated.

Our task is to evaluate the mean-squared value of I_k 's. As is shown in Appendix, this expectation results in

$$E(I_k^2) = \frac{m_\psi}{2N} \left[2 + \left(1 - \frac{1}{N}\right) \Delta_\tau \right] \quad (8)$$

where, m_ψ is the waveshaping parameter ($m_\psi = \frac{1}{3}$ for rectangular waveform), and Δ_τ is the conditional probability of the interarrival time between the i -th and k -th node being less than a bit duration given that it is greater than the acquisition time.

Substituting the above in the expression for SNR gives

$$SNR(K) = \left[\left(\frac{2E_b}{N_0} \right)^{-1} + \frac{(K-1)m_\psi}{2N} \left\{ 2 + \left(1 - \frac{1}{N}\right) \Delta_\tau \right\} \right]^{-\frac{1}{2}} \quad (9)$$

For non-coherent MFSK DS/SS systems, the Gaussian approximation techniques of [6] results in an expression for the average symbol error probability :

$$P_{es}(K) = \sum_{m=1}^{M-1} \binom{M-1}{m} \frac{(-1)^{m+1}}{m+1} \exp \left\{ -\frac{m}{2(m+1)} \Lambda(N_0) \right\} \quad (10)$$

where $\Lambda(N_0)$ is given by

$$\Lambda(N_0) = \left[\left(\frac{2E_b \log_2 M}{N_0} \right)^{-1} + \frac{(K-1)m_\psi}{2MN'} \left(2 + \left(1 - \frac{1}{N'}\right) \Delta_\tau \right) \right]^{-1}$$

and $N' = N \log_2 M$, is the number of chips per M -ary symbol.

Using (4), we have evaluated the probability of retaining capture for BPSK and non-coherent MFSK DS/SS systems for different values of parameters of the system.

III. SLOW FREQUENCY-HOP SS SYSTEMS

This section is concerned with the capture performance of SFH/SS systems with binary or M-ary FSK modulation and noncoherent demodulation in a time-slotted packet radio network environment. The basic model for the FH/SS system that we consider is described in [7]. We assume that all nodes in the network use the same random memoryless hopping pattern. Here again, the model for acquisition of capture is that of [3]. To carry out the analysis, we assume that the capture interval, T_a , is equal to the dwell time of the FH system.

Now, given that the receiver has acquired capture of the first packet arriving at the front end of the receiver, all the other packets arriving later have differential delays of greater than T_a with the first packet. This means that the probability of a hit from an interfering packet is the same as the case of multi-user interference. Assuming that the number of frequency bins, q , is much greater than 1 ($q \gg 1$), the probability of a hit is given by [7]:

$$P_h \approx \frac{1}{q} \left(1 + \frac{\log_2 M}{N_b} \right) \quad (11)$$

where N_b is the number of data bits per dwell-time.

Given that K users simultaneously transmit in a given slot, the probability of a hit from k users ($0 \leq k \leq K-1$) is given by

$$P_h(k) = \binom{K-1}{k} P_h^k (1-P_h)^{K-1-k} ; 0 \leq k \leq K-1 \quad (12)$$

The conditional probability of error, $P_{e|h}(k)$, for MFSK modulation with non-coherent detection, given that k hits (assuming all hits are full hits) has occurred, is same as equation (10) with $\Lambda(N_0)$ given by

$$\Lambda(N_0) = \left[\left(\frac{2 E_b \log_2 M}{N_0} \right)^{-1} + k \frac{m_\psi}{M} \right]^{-1}$$

Therefore, the average probability of symbol error is given by

$$P_{es}(K) = \sum_{k=0}^{K-1} P_h(k) P_{e|h}(k) \quad (13)$$

Equation (13) is also resulted from Gaussian approximation techniques as described in detail in [9].

Notice that when the capture interval, T_a , is less than a dwell time, there is always hits on symbols of the captured packet. To eliminate this problem, we introduce a method to randomize the starting phase of the hopping pattern for each user independent of other users. The procedure is as follows:

Each user upon transmitting his packet, chooses a frequency among the available frequency bins with probability $\frac{1}{q}$. Assuming that the hopping pattern is " f_1, f_2, \dots, f_n "; if the j -th bin is selected, the hopping pattern for the packet is " $f_j, f_{j+1}, \dots, f_n, f_1, \dots, f_{j-1}$ ". This is the j -th starting phase for the hopping pattern. Now, consider user i has acquired the capture of receiver, and let user k be the interfering signal. Then with probability $\frac{1}{q}$, $f_1^{(i)} = f_1^{(k)}$ and therefore a hit occurs. It is also possible that $f_1^{(k)} = f_2^{(i)}$ and again a hit occurs. Other than these two cases, the probability of a hit is the same as the multi-user case in multiple access systems. Therefore, we can upperbound the probability of a hit as

$$\bar{P}_h \leq \frac{2}{q} + (1 - \frac{2}{q})P_h \quad (14)$$

where P_h is the probability of a hit in multi-user systems [7].

We observe that in the FH/SS system, errors are independent in every other hop. This means that by proper interleaving (i.e., depth of 2), we can achieve the

independence of errors in the packet and therefore, the probability of retaining capture can be written as

$$P_r(K) = (1 - P_{ee}(K))^L \quad (15)$$

IV. CODED SPREAD-SPECTRUM SYSTEMS

In this section, we consider the effect of FEC coding schemes on the capture performance of aforementioned spread-spectrum systems. The coding schemes most commonly employed are Reed-Solomon and binary convolutional codes.

The difficulty with the performance evaluation of coded systems again arises from the fact that symbol errors on the channel are not independent. This problem can be eliminated to certain degree in FH systems with proper interleaving and some conditions on the parameters of the system, as we elaborate on this point later. In case of DS/SS systems, there is no feasible method to justify the assumption of independent errors; hence we again make the crude approximation that symbol errors are independent.

A. Reed-Solomon coding for FH/SS Systems

In order to evaluate the packet error probability, first, we assume that each packet is made up of two RS codewords. Then we transmit the code symbols of each codeword in every other hop and hence achieve independence of errors within each codeword (We are implicitly assuming that each RS symbol is transmitted in one dwell-time). This way, the errors on the symbols of a codeword entering the decoder are independent. We declare a packet lost when any of the codewords is in error. Therefore, invoking the union bound, the packet error probability is upper bounded by

$$1 - P_r(K) \leq 2 P_{e,c} \quad (16)$$

where $P_{e,c}$ is the codeword error probability. Note that this bound is valid for $P_{e,c} \leq \frac{1}{2}$. To generalize this method, when the packet is made up of an even number of codewords (i.e., L is divisible by twice the codeword length n), the packet error probability can be upperbounded as

$$1 - P_r(K) \leq 1 - (1 - 2P_{e,c})^{L/2n} \quad (17)$$

We consider RS codes over $GF(M^m)$, where m M-ary symbols are in each RS symbol. When we use RS codes for error correction only, the probability of symbol error for the uncoded system, p_s , can be upper bounded by

$$p_s \leq 1 - (1 - P_h)^{K-1} (1 - P_{e,M})^m \quad (18)$$

where there are $K-1$ interfering signals, and $P_{e,M}$ is the probability of error for an M-ary FSK system with noncoherent demodulation in AWGN environment. Here the probability of a hit is given by

$$P_h = \frac{1}{q} \left(1 + \frac{m}{N_s} \right) \quad (19)$$

where N_s is the number of M-ary symbols in each dwell time ($N_s \geq 1$). When the capture time is less than the dwell-time we use the starting phase randomization method and evaluate the probability of a hit accordingly (i.e., use (14)).

In the case of Erasure/Error correction, the decoder erases a symbol if interference from others is present. Hence, the probability of an erasure is given by

$$\epsilon_s = 1 - (1 - P_h)^{K-1} \quad (20)$$

and the probability of symbol error is

$$p_s = (1 - \epsilon_s) [1 - (1 - P_{e,M})^m] \quad (21)$$

The codeword error probabilities are calculated, using the standard formulas, as given for example in [10], by proper substitution of p_s and/or ϵ_s .

B. Binary Convolutional Coding for FH/SS systems

We consider convolutional coding with non-coherent BFSK modulation for SFH/SS systems. As we mentioned earlier, in FH system, errors are independent in every other hop. Therefore, if we consider the code sequence of length L as two separate code

sequences of length $L/2$, and then transmit the code symbols of these subsequences alternately in each hop interval, the symbol errors at the input of Viterbi decoder for each subsequence are independent. When the symbol errors at the input to the decoder are independent, Pursley and Taiple [11] have shown that the packet error probability can be upper bounded as

$$1 - P_r(K) \leq 1 - (1 - P_u)^L \quad (22)$$

where P_u is the union bound on the first-event error probability [10]. P_u can also be replaced by the bound on the bit error probability, P_{eb} , which results in a looser bound on packet error probability. Now, applying this result to the two subsequences that make up the packet and use of union bound gives us the upper bound on the packet error probability as

$$1 - P_r(K) \leq 2[1 - (1 - P_{eb})^{L/2}] \quad (23)$$

A simple calculation shows that this bound is valid for all values of P_{eb} (P_u) satisfying

$$P_{eb} \leq 1 - \left(\frac{1}{4}\right)^{1/L}$$

The bit error probability at the output of the decoder is upper bounded by

$$P_{eb} \leq \sum_{j=d_{free}}^{\infty} w_j P_j \quad (24)$$

where w_j is the total information weight of all sequences which produce paths of weight j through the trellis, and P_j is the probability of the error event that the decoder chooses a path at distance j from the correct path. This can be evaluated as the error probability of a binary repetition code of length j [10]. In the calculation of P_j , we may use (13) or (18) (with $M=2$ and $m=1$), if there is no *side information*, and (20)-(21), when *side information is present*.

It is important to note that for the above arguments to be valid we must transmit each bit of the code sequence in separate hop intervals. This can be achieved by setting $N_b = 1$. For $N_b > 1$, the above bound will hold only as an approximation.

V. HYBRID (DS-SFH) SS SYSTEMS

For DS-SFH/SS employing random signature sequences, the probability of symbol error is evaluated via equation (13) [8]-[9]. In case of coherent hybrid systems, $P_{e|h}(k)$ is obtained via signal-to-noise approximation discussed in section II, and is given by

$$P_{e|h}(k) = Q_x[SNR(k+1)] \quad (25)$$

where $SNR(k)$ is given by (2.10). For non-coherent hybrid systems, $P_{e|h}(k)$ is given by (10) with $\Lambda(N_0)$ given by

$$\Lambda(N_0) = \left[\left(\frac{2E_b \log_2 M}{N_0} \right)^{-1} + \frac{k m_\psi}{2MN'} \left(2 + \left(1 - \frac{1}{N'} \right) \Delta_r \right) \right]^{-1} \quad (26)$$

where again $N' = N \log_2 M$, is the number of chips per M-ary symbol.

V. NUMERICAL RESULTS AND CONCLUSIONS

Probability of capture, $P_c(K)$, as a function of number of contending nodes, K , is evaluated for all the systems described above. In all the Figures that follow, the actual plots are discrete, but the discrete points are connected together via straight lines.

Figure 1 & 2, present the capture performance of DS/SS BPSK and non-coherent BFSK systems, respectively. As we increase N , the performance of the systems change from a threshold behavior to graceful degradation. The degradation due to non-coherent demodulation is apparent; for example, for the probability of capture fixed at 10^{-2} , the coherent system supports about 85 users with $N = 255$, while the non-coherent system supports up to 55 users for the same value of N . Figure 3 shows the performance of an uncoded SFH/SS system for the cases when $T_a < T_h$ and $T_a \geq T_h$. It is obvious that for SFH systems, the capture time should be greater than dwell interval, but the tradeoff is the wasting of a larger fraction of slot time for synchronization purposes. Figure 4 & 5 present the performance of RS coded SFH system for different values of code rate. Note the great improvement in performance when Erasure correction capability is employed. For example, at the capture probability of 10^{-2} , the (32,12) code supports about 60 users with side-information present, while with no side-information, this degrades to about 22 users. Of course, in most channel access protocols, for the purpose of robustness, it is assumed that no side information is present at the receiver. Figure 6 & 7 present the performance of a SFH system employing binary CC. For Figure 6. we have used EQ. (18) for the evaluation of p_s ($M=2$, and $m=1$). This bound degrades fast as the number of interfering nodes increases, and is insensitive to high signal-to-noise ratios. For Figure 8., we have used EQ. (13). In this case, we are overestimating the system's performance. Figure 8 shows the improvement achieved by use of P_u in

place of P_{eb} in EQ. (23). Numerical results of [12] has been used in evaluation of P_u . In Figure 9.-10., we present the performance of SFH system employing CC's with different rates and constraint lengths. As we mentioned at the end of section IV, the case of $N_b > 1$ (i.e., Fig. 9) serves as an approximation, while with $N_b = 1$, we have a valid bound on the capture performance of the system.

Finally, in Table 1., we present the maximum number of users accomodated by a Hybrid DS-SFH system when the probability of capture is fixed at 10^{-3} . For a fixed bandwidth-expansion factor of $N q = 700$, the addition of modest amount of DS code sequence greatly improves the performance of purely FH system.

APPENDIX

We are interested in the evaluation of mean-squared value of random variable I_k , defined in eq. (3). Given the signature sequence, the expectation reduces to

$$E\left\{I_k^2 \mid \{a_l\}\right\} = \frac{1}{T} \int_0^T \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{4} \left(\sum_{\underline{b}^{(k)}} I_k \right)^2 d\phi d\tau \quad (\text{A1})$$

where $\underline{b}^{(k)} = (b_j^{(k)}, b_{j+1}^{(k)})$. Simplifying the above, we get

$$E\left\{I_k^2 \mid \{a_l\}\right\} = \frac{1}{2T^3} \int_0^T \left(R^2(\tau) + \hat{R}^2(\tau) \right) d\tau \quad (\text{A2})$$

Now, referring to Fig. A1, we can observe that when the difference between arrival times of the i -th packet and k -th one is less than the bit duration, T , the interference due to the k -th signal is caused by the m -th and $(m-1)$ -th bit of the k -th node. This means that $b_m^{(i)}$ and $b_m^{(k)}$ are coded with the same segment of the signature sequence.

In this case we have

$$\begin{aligned} R(\tau) &= \left(\sum_{i=0}^l a_i a_{i+N-l-1} \right) R_\psi(\tau - lT_c) + \left(\sum_{i=0}^{l-1} a_i a_{i+N-l} \right) \hat{R}_\psi(\tau - lT_c) \\ \hat{R}(\tau) &= \left(\sum_{i=0}^{N-l-1} a_{i+l} a_i \right) \hat{R}_\psi(\tau - lT_c) + \left(\sum_{i=0}^{N-l-2} a_{i+l+1} a_i \right) R_\psi(\tau - lT_c) \end{aligned} \quad (\text{A3})$$

where l is the integer part of τ/T_c , and R_ψ and \hat{R}_ψ are the partial autocorrelation functions for the chip waveform; that is, for $0 \leq s \leq T_c$

$$\hat{R}_\psi(s) = \int_s^{T_c} \psi(t) \psi(t-s) dt$$

and $R_\psi(s) = \hat{R}_\psi(T_c - s)$.

On the other hand, when the interarrival time is greater than T , all the bits involved are coded with different windows of the signature sequence (see Fig. A2). In this case we have

$$R(\tau) = \left[\sum_{i=0}^l a_i a_{i+N-l-1} \right] R_\psi(\tau - lT_c) + \left[\sum_{i=0}^{l-1} a_i a_{i+N-l} \right] \hat{R}_\psi(\tau - lT_c) \quad (\text{A4})$$

$$\hat{R}(\tau) = \left[\sum_{i=0}^{N-l-1} a_{i+l} a_i \right] \hat{R}_\psi(\tau - lT_c) + \left[\sum_{i=0}^{N-l-2} a_{i+l+1} a_i \right] R_\psi(\tau - lT_c)$$

Substituting (A3)-(A4) in (A2), taking the expectation with respect to the signature sequence and using the fact that

$$\int_0^T R^2(\tau) d\tau = \sum_{l=0}^{N-1} \int_{lT_c}^{(l+1)T_c} R^2(\tau) d\tau \quad (\text{A5})$$

we obtain

$$E\{I_k^2\} = \frac{m_\psi}{2N^3} \left\{ 1 + (N^2 + N - 2)\Delta_\tau + (2N - 1)(1 - \Delta_\tau) + 2N(N - 1) \right\} \quad (\text{A6})$$

Simplifying the above we obtain

$$E\{I_k^2\} = \frac{m_\psi}{2N} \left[2 + \left(1 - \frac{1}{N}\right) \Delta_\tau \right] \quad (\text{A7})$$

where $m_\psi = \frac{1}{T_c^3} \int_0^{T_c} R_\psi^2(s) ds = \frac{1}{T_c^3} \int_0^{T_c} \hat{R}_\psi^2(s) ds$ ($m_\psi = \frac{1}{3}$ for rectangular waveform).

Δ_τ is the conditional probability of the interarrival time between the i -th and k -th node being less than a bit duration given that it is greater than the acquisition time; i.e.,

$$\Delta_\tau = P(\tau_k - \tau_i \leq T \mid \tau_k - \tau_i > T_a; \text{ for all } k \neq i)$$

Using the results of [3], It can be shown that Δ_τ is given by

$$\Delta_\tau = \frac{Q^K - R^K + K(R - Q)}{Q^K + (1 - Q)K - 1} \quad ; \quad K \geq 2 \quad (\text{A8})$$

where $R = \frac{T}{T_u}$, $Q = \frac{T_a}{T_u}$, and $T_a < T \leq T_u$.

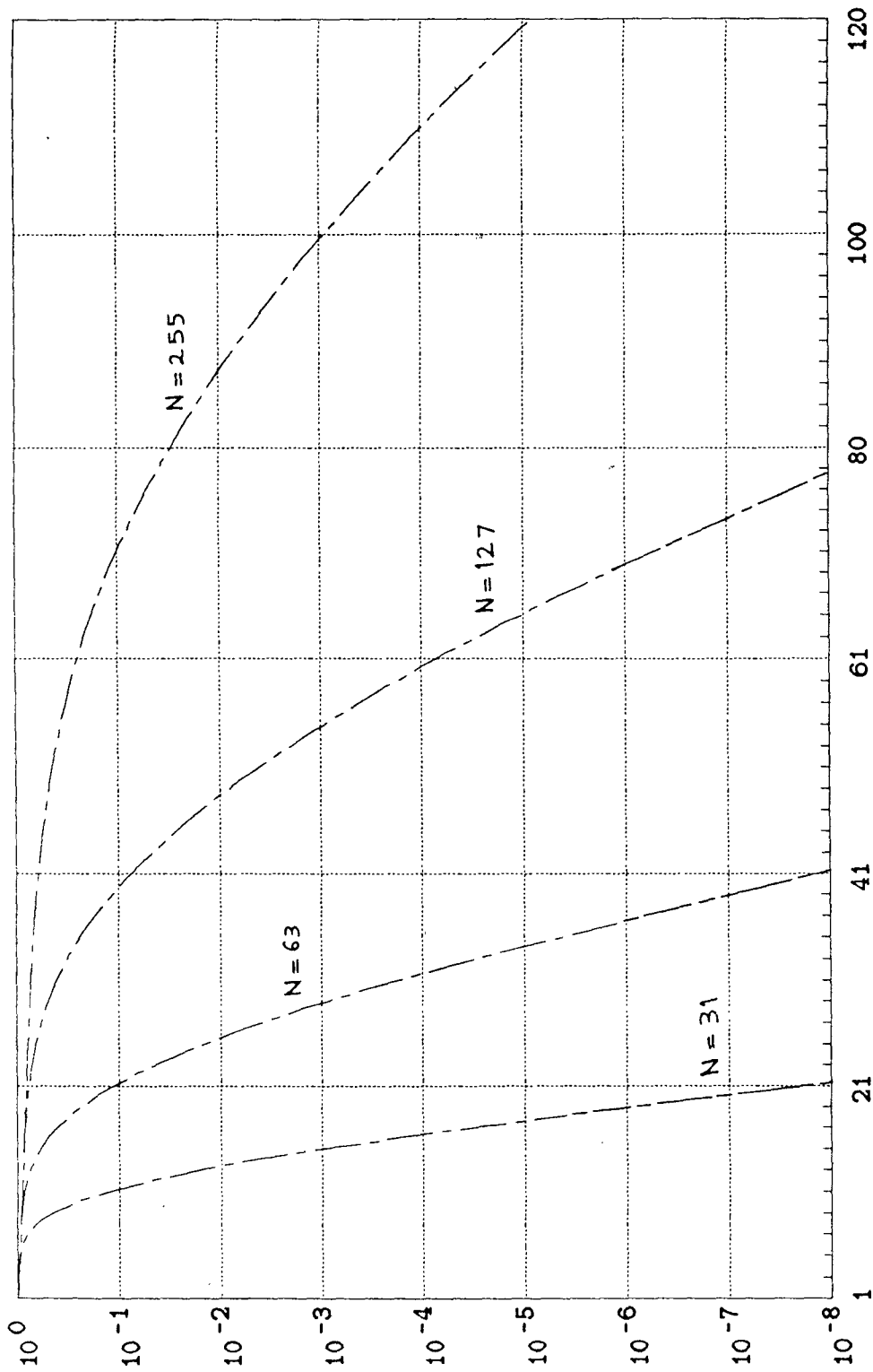


Figure 1. $P_c(K)$ vs. K for DS/SS system with coherent BPSK modulation

$$\left(\frac{E_b}{N_0} = 16 \text{ dB}, L = 1000, Q = 0.01 \right)$$

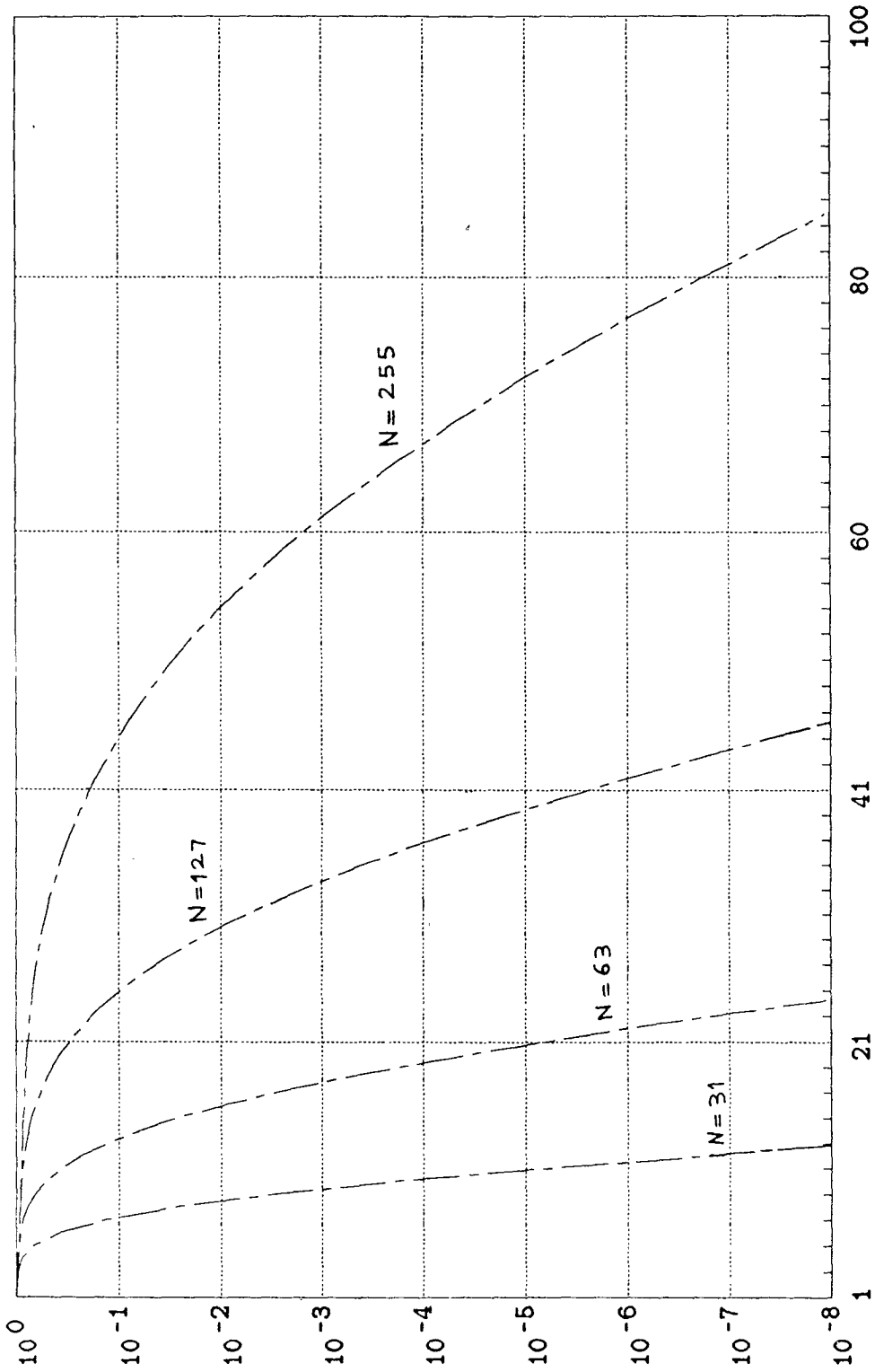


Figure 2. $P_c(K)$ vs. K for DS/SS system with non-coherent BFSK modulation

$$\left(\frac{E_b}{N_0} = 16 \text{ dB}, L = 1000, Q = 0.01 \right)$$

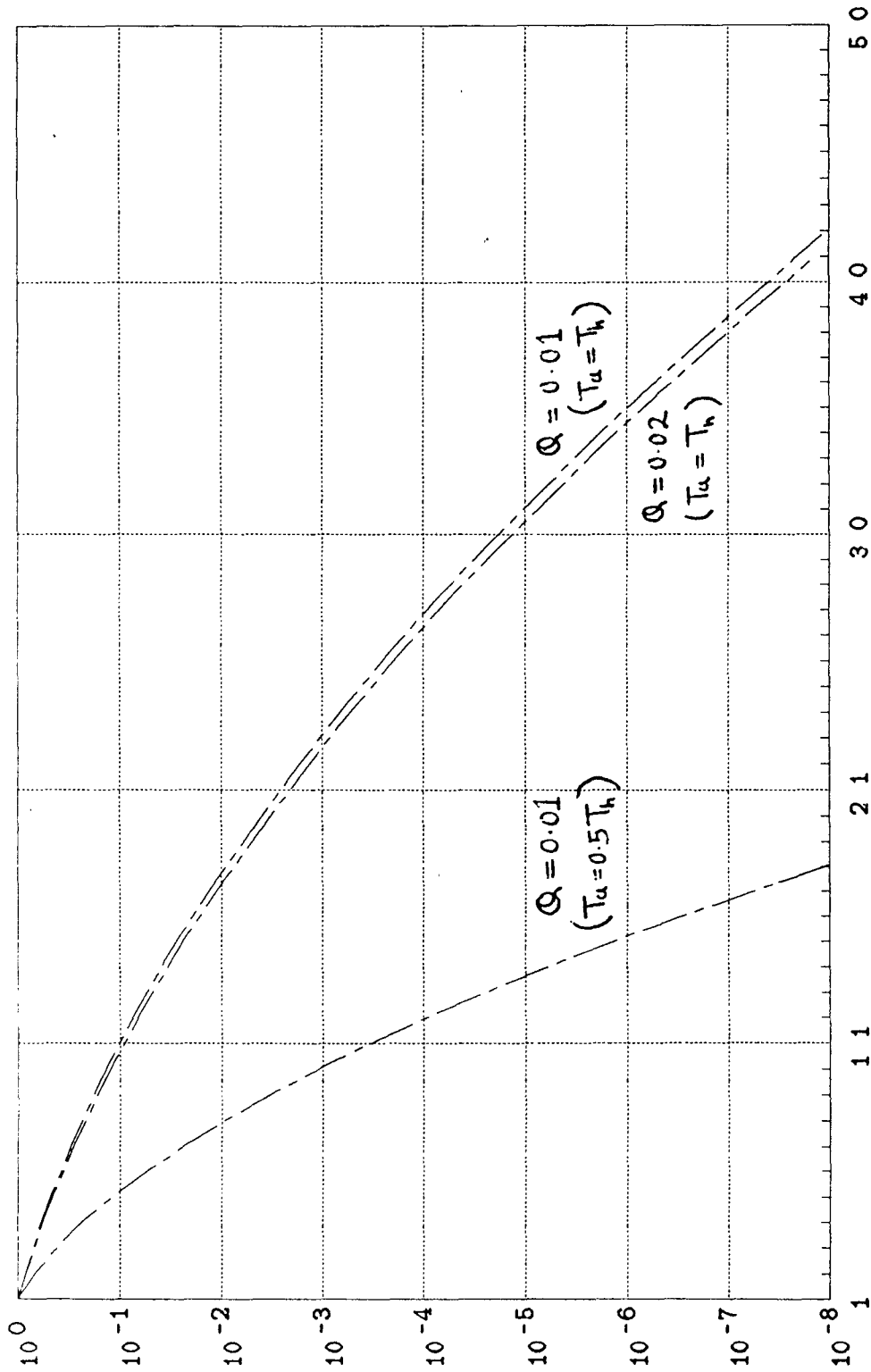


Figure 3. $P_e(K)$ vs. K for SFH/SS with non-coherent 8-ary FSK modulation

$$\left(\frac{E_b}{N_0} = 16 \text{ dB}, L = 1000, Q = 0.01, q = 100, N_b = 10 \right)$$

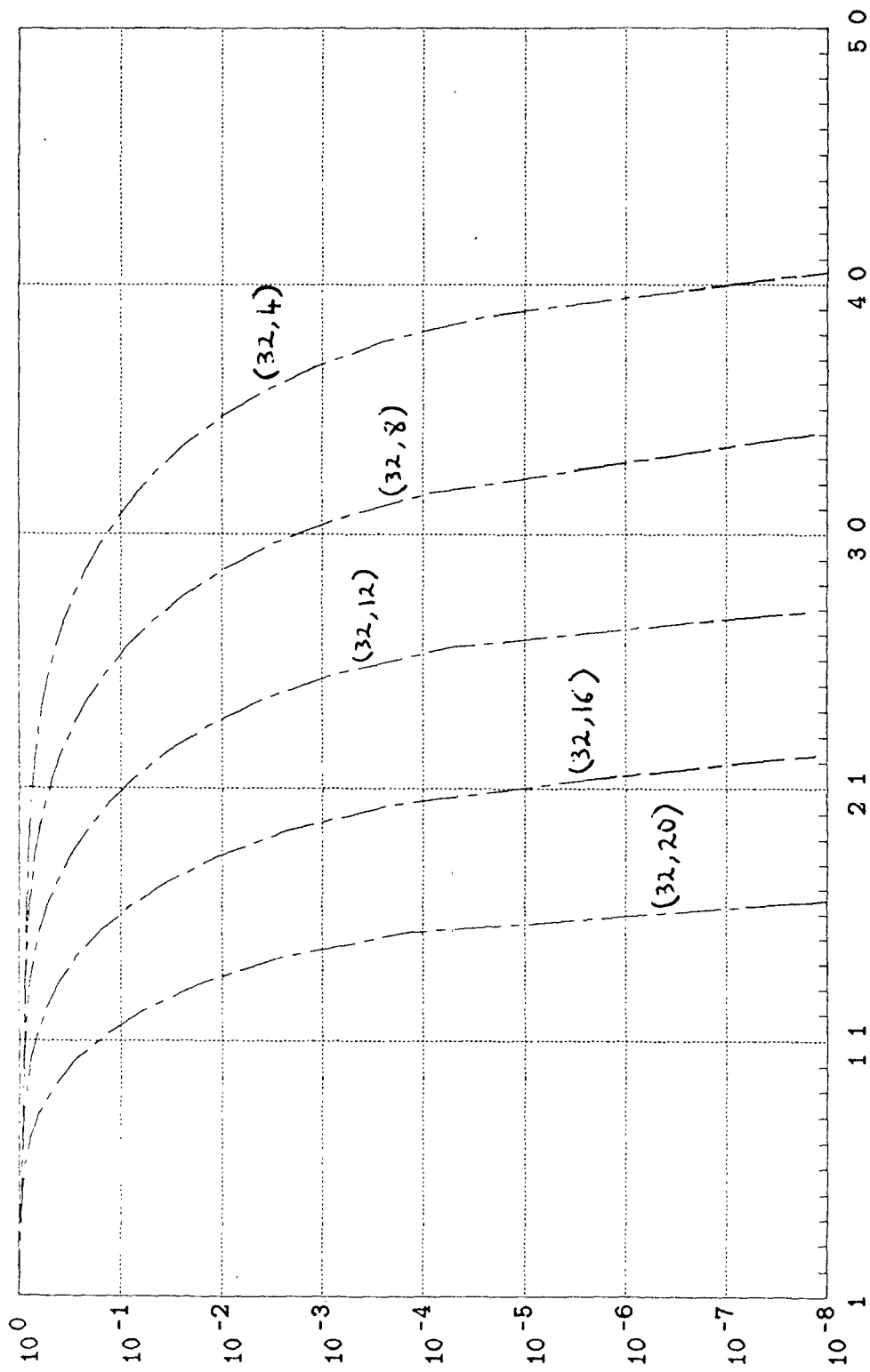


Figure 4. $P_e(K)$ vs. K for SFH/SS with non-coherent BFSK modulation and Reed-Solomon coding with Error correction decoding

$$\left(\frac{E_b}{N_0} = 16 \text{ dB}, L = 320, Q = 0.01, q = 100, N_b = 10, m = 5 \right)$$

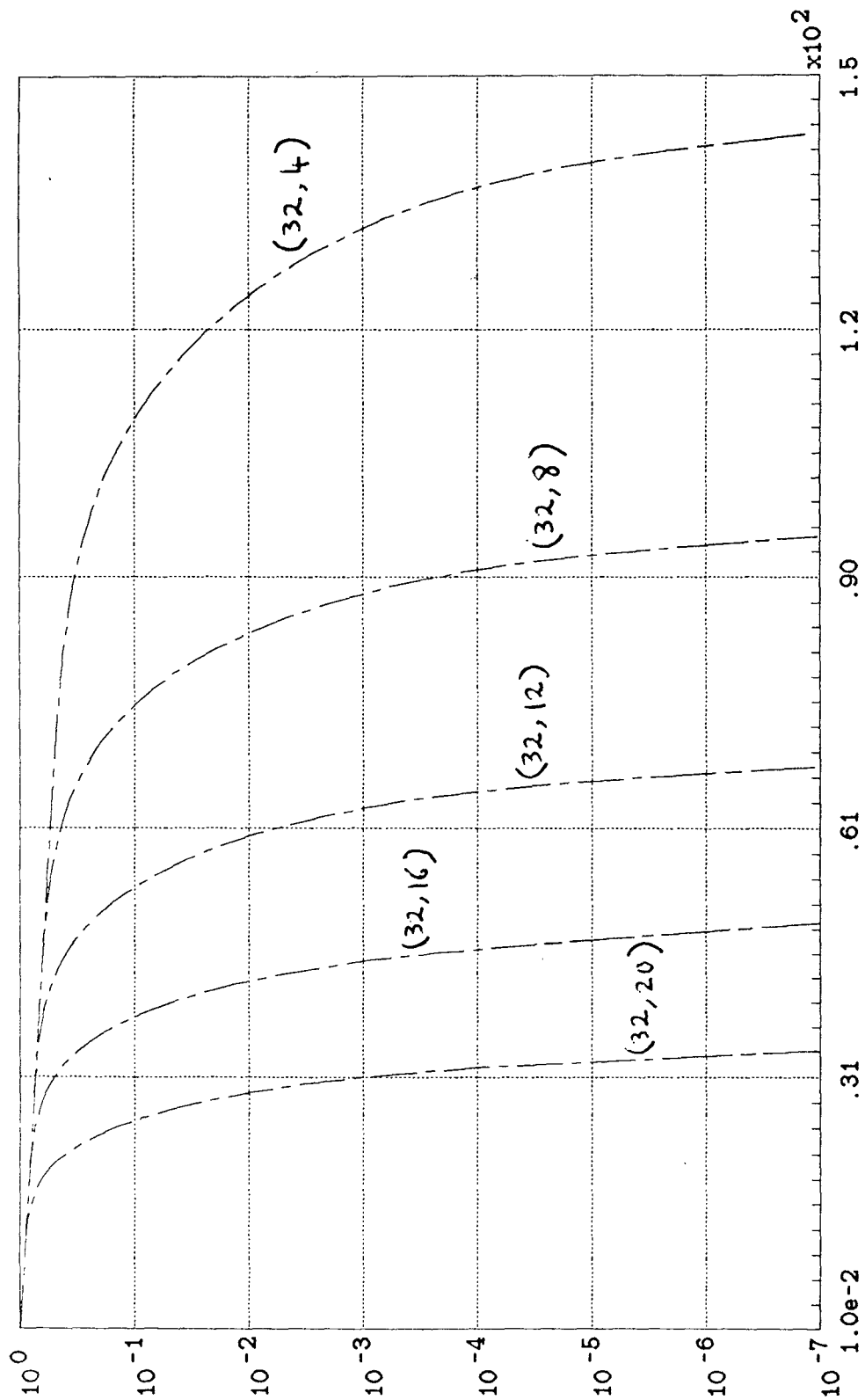


Figure 5. $P_c(K)$ vs. K for SFH/SS with non-coherent BFSK modulation

and Reed-Solomon coding with Erasure/Error correction decoding

$$\left(\frac{E_b}{N_0} = 16 \text{ dB}, L = 320, Q = 0.01, q = 100, N_b = 10, m = 5 \right)$$

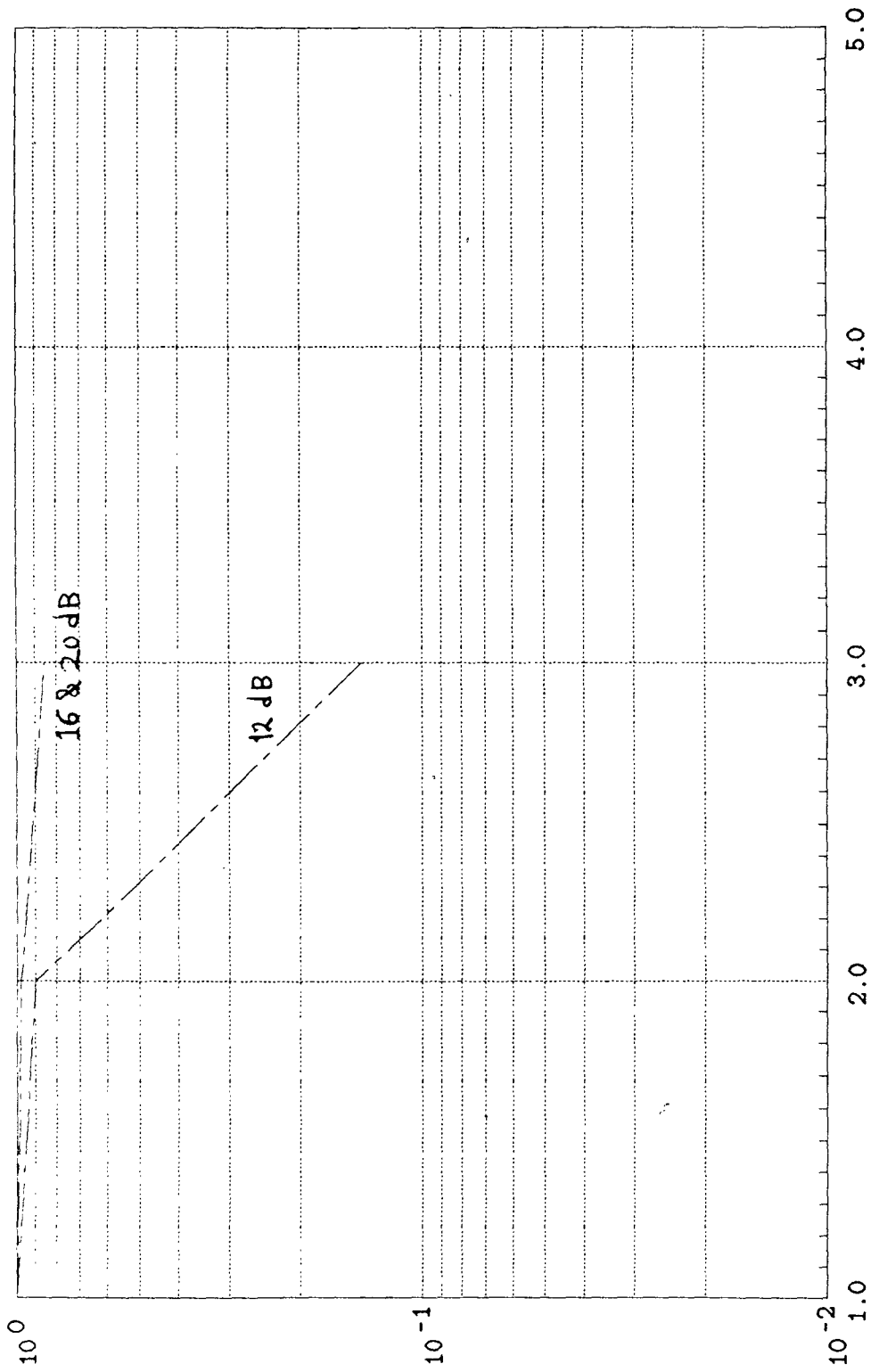


Figure 6. $P_c(K)$ vs. K for SFH/SS with non-coherent BFSK modulation

and convolutional coding ($R_c = 1/2, K_c = 7$).

($L = 1000, N_b = 10, q = 100$, hard bound on p_s)

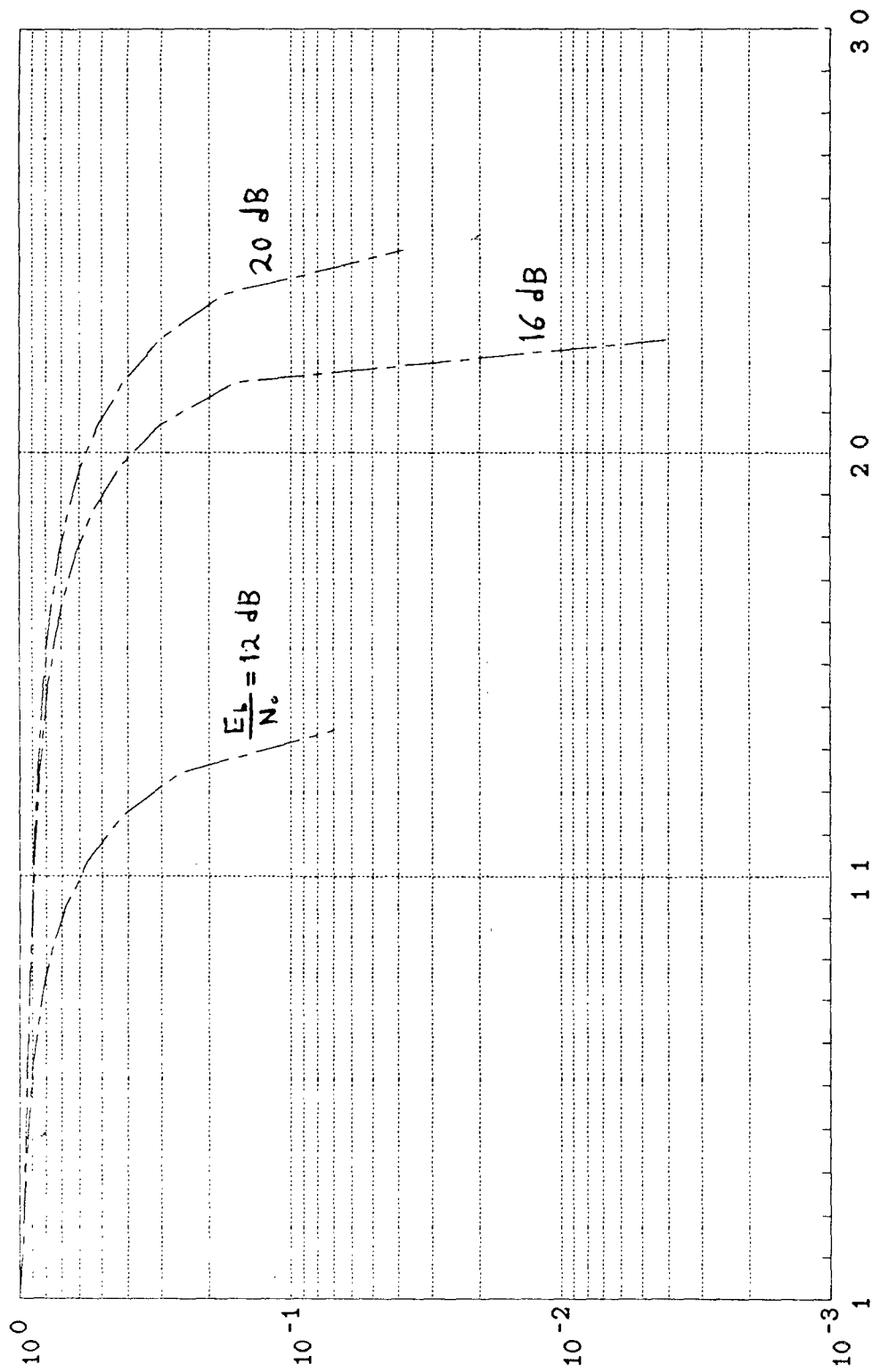


Figure 7. $P_c(K)$ vs. K for SFH/SS with non-coherent BFSK modulation and convolutional coding ($R_c = 1/2$, $K_c = 7$).

($L = 1000$, $N_b = 10$, $q = 100$, Gauss. approx. on p_s)

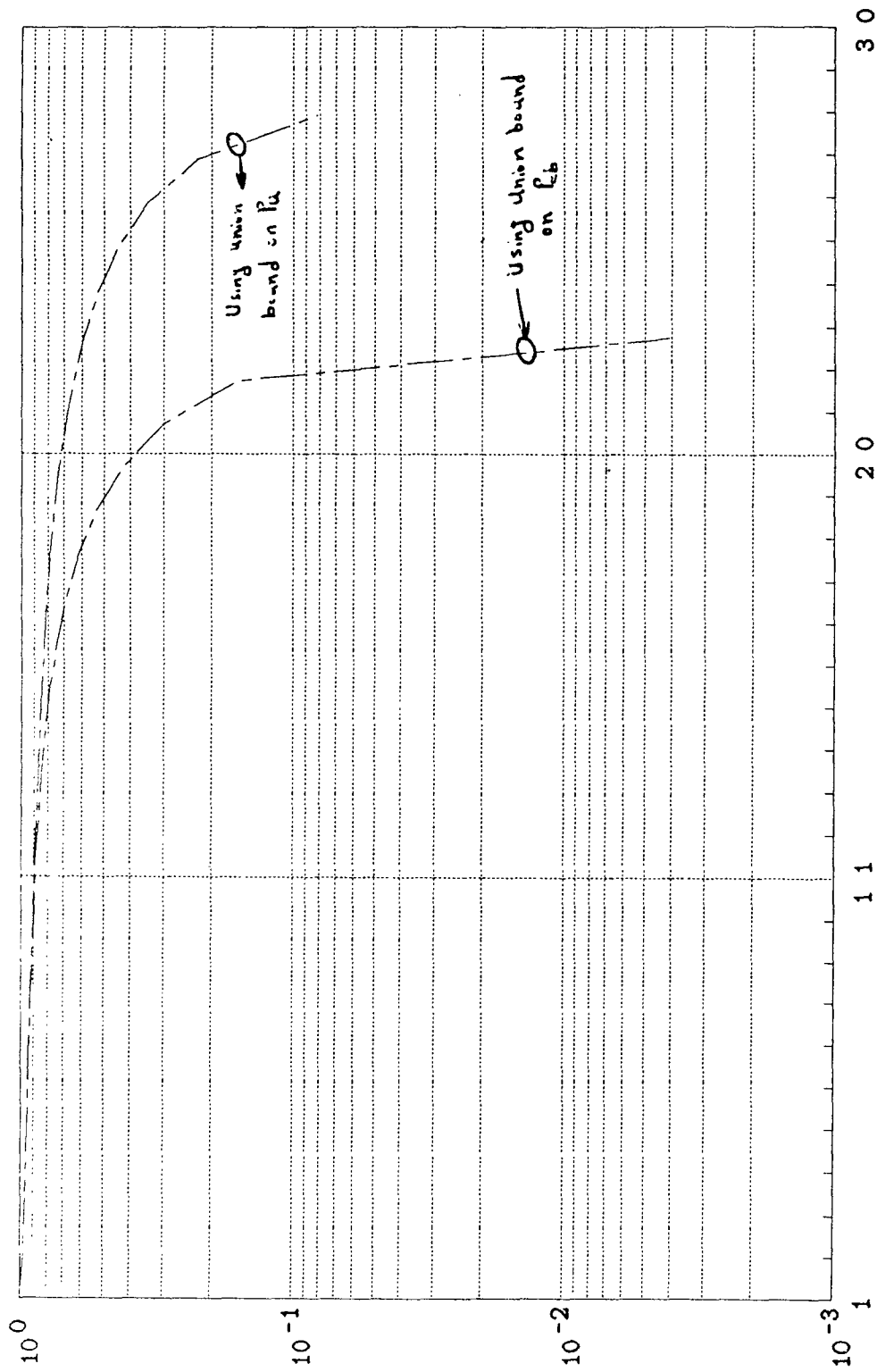


Figure 8. $P_e(K)$ vs. K for SFH/SS with non-coherent BFSK modulation

and convolutional coding ($R_c = 1/2, K_e = 7$).

($L = 1000, N_t = 10, q = 100$, Gauss. approx. on p_s)

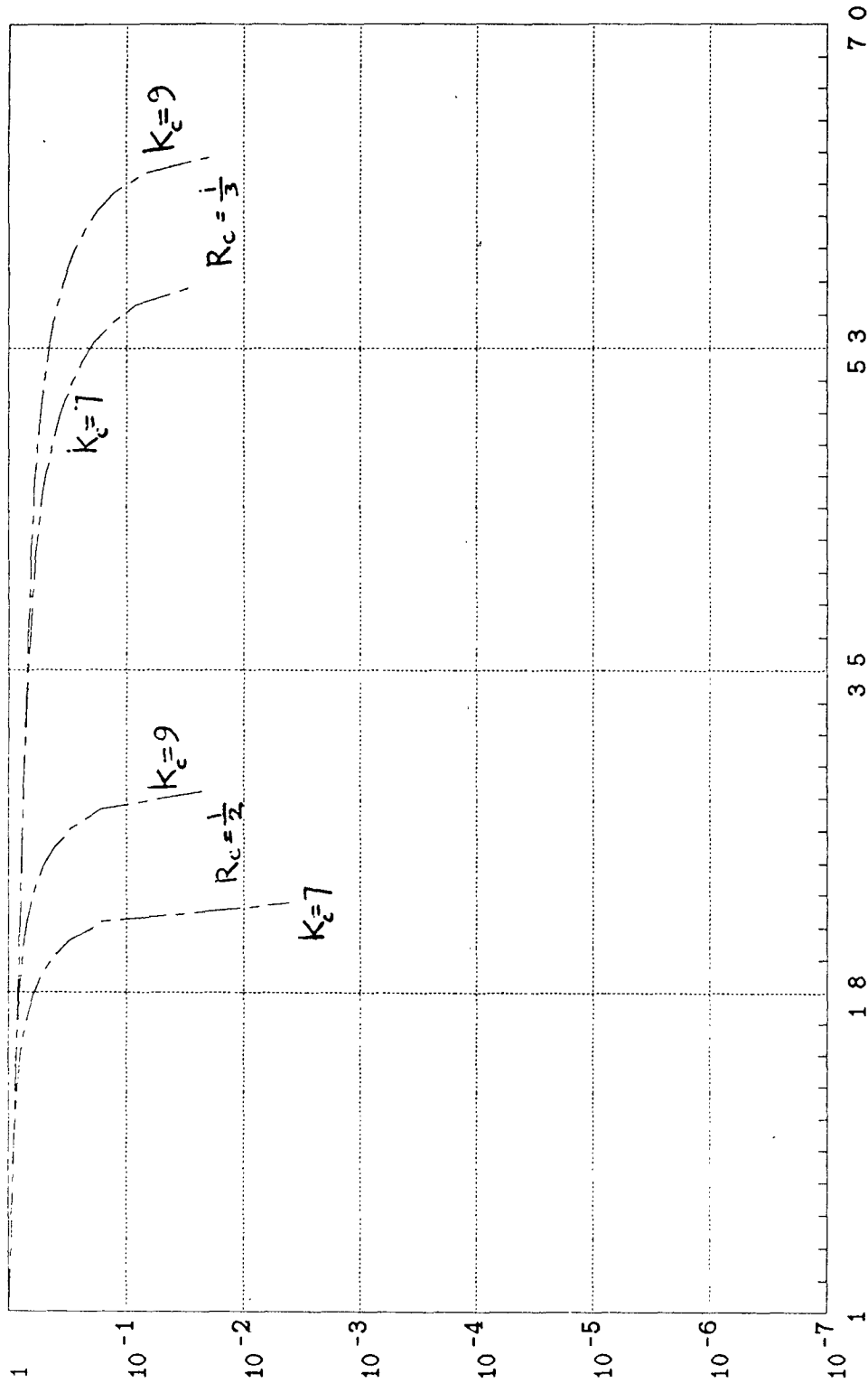


Figure 9. $P_e(K)$ vs. K for SFH/SS with non-coherent BFSK modulation and convolutional coding with different constraint length and rates.

$$\left(\frac{E_b}{N_0}\right) = 16 \text{ dB}, L = 1000, N_b = 10, q = 100, \text{Guass. approx. on } p_s$$

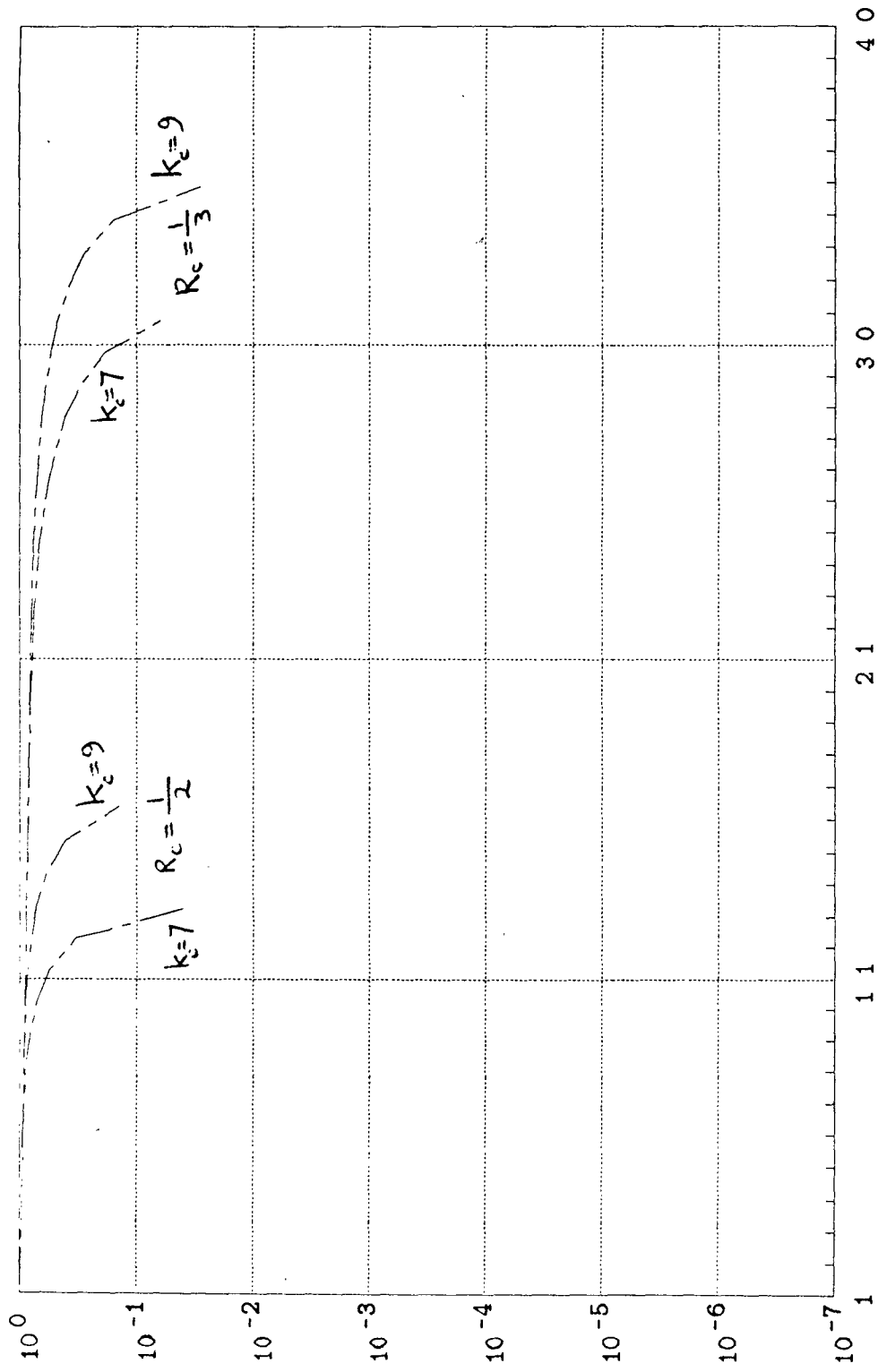


Figure 10. $P_e(K)$ vs. K for SFH/SS with non-coherent BFSK modulation and convolutional coding with different constraint length and rates.

$$\left(\frac{E_b}{N_0} = 16 \text{ dB}, L = 1000, N_b = 1, q = 100, \text{Guass. approx. on } p_s \right)$$

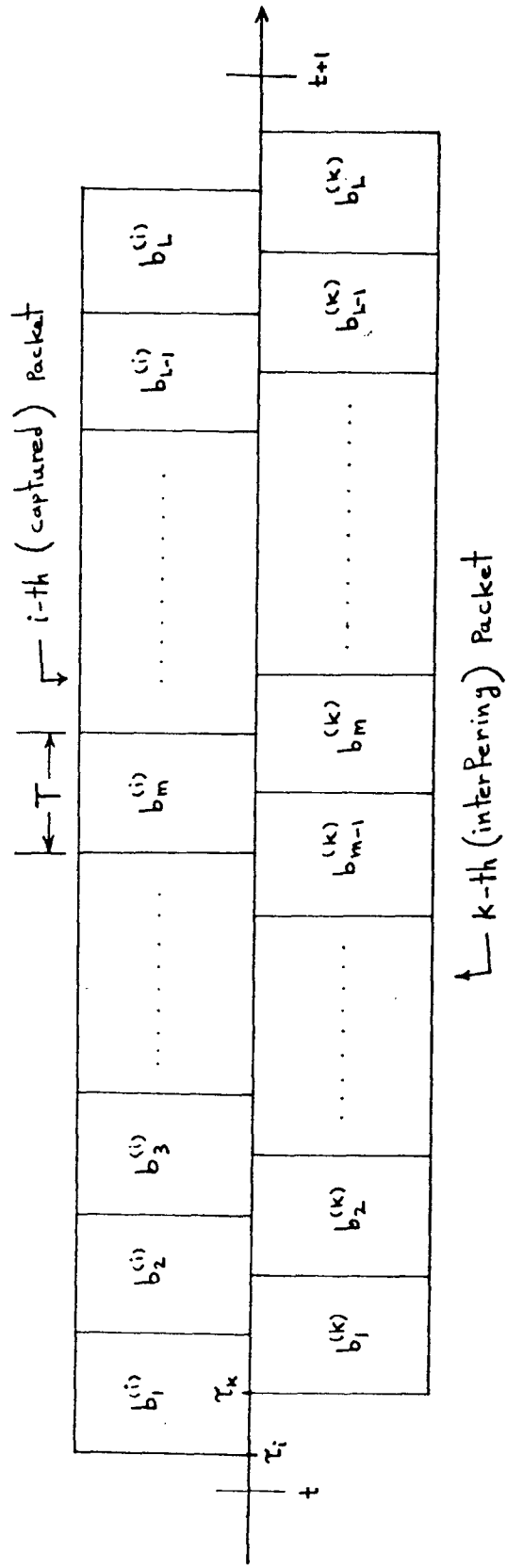


Figure A1. The effect of interfering packet on the captured packet for $\tau_k - \tau_i < T$.

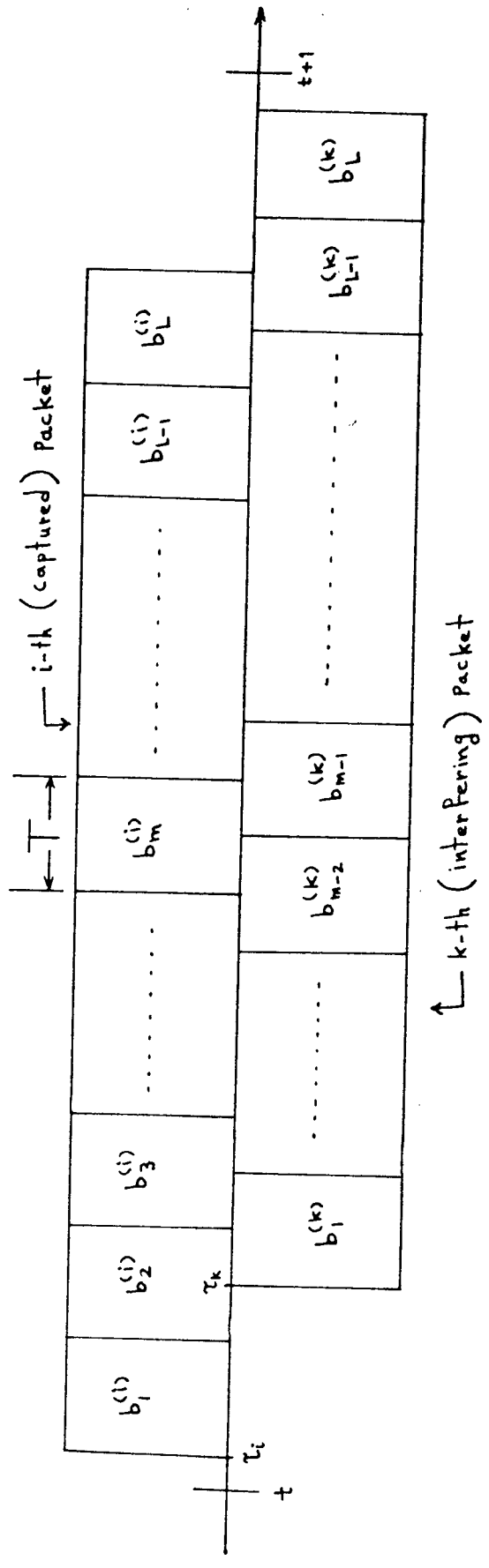


Figure A2. The effect of interfering packet on the captured packet for $\tau_k - \tau_i \geq T$.

Table 1. Maximum number of users accommodated by a Hybrid DS-SFH system with non-coherent binary FSK modulation ($\frac{E_b}{N_0} = 12dB$, $Q = 0.01$, $L = 1000$, $N_b = 1$, $P_c = 10^{-3}$).

N	q	K^*
1	700	17
14	50	44
70	10	51
700	1	78

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