ABSTRACT

Title of Dissertation: ESSAYS ON ECONOMIC POLICY AND FIRM DYNAMICS
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This dissertation examines the impact of economic policies on aggregate economy by analyzing their effects on firms' behavior. It employs theoretical and quantitative macroeconomic models to explore how these policies affect social welfare.

In Chapter 1, I study the second-best optimal carbon taxes when negative externalities from carbon emissions coexist with another inefficiency, specifically, the misallocation of production inputs across heterogeneous firms. This research holds relevance because governments may possess policy tools to address climate change, yet lack other means to alleviate additional inefficiencies. The motivation for this research stems from two prominent empirical facts. First, there is enormous heterogeneity in emission intensity across firms, even within narrowly defined 4-digit industries. Second, there is dispersion in the marginal products of production inputs, such as capital and labor, for firms within the industry, which is interpreted as evidence of misallocation of production inputs. Using a theoretical model, I show that when firms with lower emission intensity exhibit higher marginal products of production inputs, a carbon tax

yields a double dividend: 1) it reduces carbon emissions; 2) it enhances allocative efficiency by reallocating resources to more distorted firms. Using firm-level data, I show that firms with lower emission intensity indeed have higher marginal products of capital and labor. Based on the empirical evidence, I develop a quantitative firm dynamics model that incorporates carbon emissions, emission externalities, adjustment costs, and financial frictions. In a calibrated version of this model, the optimal carbon tax is three times higher than in a counterfactual economy in which there is no relation between emission intensity and marginal products. Furthermore, I find that a policy directly targeting adjustment costs and financial frictions, if it exists, can simultaneously reduce carbon emissions and boost output, ultimately surpassing a carbon tax in increasing overall welfare.

In Chapter 2 (co-authored with Thomas Drechsel), we explore the optimal macroprudential policy when firms face earnings-based borrowing constraints. Conventional wisdom in the literature suggests that when agents face asset-based collateral constraints—where the amount of debt is limited by the value of their asset holdings—they tend to over-borrow compared to the socially efficient level of debt. In this case, optimal policy aims to reduce debt positions through taxes. The reason is that agents do not internalize the effects of their debt choices on asset prices. However, recent empirical evidence shows that firms primarily borrow against their earnings rather than their assets. We show that agents over-save (and under-borrow) relative to the social optimum, as they do not internalize changes in wages, which in turn affect firms' earnings. This is the opposite conclusion to the previous literature. A numerical model exercise demonstrates that incorrectly rolling out a tax policy derived under the assumption of asset-based constraints in an economy where firms actually borrow based on earnings leads to a consumption equivalent welfare loss of up to 2.55%. Thus, we argue that optimal macroprudential policy critically depends on the specific form of financial constraints.

In Chapter 3, I investigate how the 2020 Small Business Reorganization Act, a corporate bankruptcy reform in the U.S. designed to reduce debt reorganization costs for small businesses, affects the aggregate economy. Under current U.S. law, businesses have two bankruptcy options: Chapter 7 liquidation and Chapter 11 reorganization. In Chapter 7, an insolvent company sells all of its assets, repays existing debts, and exits the market. In contrast, Chapter 11 is designed to rehabilitate efficient but financially distressed businesses. However, legal scholars have long argued that Chapter 11 is too costly for small businesses, causing productive but insolvent firms to choose liquidation, which could be potentially harmful to the economy. Using a general equilibrium model with bankruptcy decisions of firms, I evaluate the Small Business Reorganization Act. The main contribution to the literature is that I calibrate and estimate the model parameters using novel data encompassing the universe of bankrupt firms in the U.S., whereas existing literature primarily relies on data from bankrupt publicly listed large firms. I find that the bankruptcy reform has small but positive impact on aggregate welfare, while output and productivity decrease. A lower Chapter 11 cost helps distressed firms to reorganize, but also prompts firms that would not declare bankruptcy absent the reform to reorganize. Despite this unintended consequence, welfare of the economy improves.

ESSAYS ON ECONOMIC POLICY AND FIRM DYNAMICS

by

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Chapter 1: Optimal Carbon Taxes and Misallocation across Heterogeneous Firms

1.1 Introduction

Greenhouse gas emissions have emerged as a pressing global challenge, prompting the need for comprehensive carbon management policies. When the externality from carbon emissions is the only inefficiency in an economy, the Pigouvian principle applies: the optimal carbon tax rate is equal to the marginal damage of carbon emissions. However, a carbon externality does not exist in isolation when there are other frictions in the economy. For example, it could be the case that firms with lower carbon emissions are more financially constrained than firms with higher emissions. Thus, it is crucial to understand how carbon policies interact with other inefficiencies in designing optimal carbon management policies.

This paper investigates the optimal carbon tax in an environment where firms are heterogeneous in emission intensity, coupled with the presence of production factor misallocation among these firms. When firms have different emission intensities, a carbon tax reallocates resources towards relatively cleaner firms. The extent to which this resource reallocation enhances allocative efficiency hinges on whether these cleaner firms yield higher marginal products of production factors in comparison to their more emission-intensive counterparts. If cleaner firms have higher marginal products in the absence of a carbon tax, reallocating resources to these cleaner firms increases aggregate productivity. Consequently, the introduction of a carbon tax would yield additional advantages for a social planner by mitigating the pre-existing factor misallocation.

The starting point of my analysis is a simple theoretical model, building on Hsieh and Klenow (2009), in which I formalize this intuition. I theoretically show that the correlation between emission intensity and marginal products, where the dispersion of marginal products arises from firm-level distortions, is a key statistic in determining the optimal carbon tax. When a positive correlation exists between emission intensity and marginal products, meaning that cleaner firms exhibit lower marginal products, implementing a carbon tax that directs resources towards these cleaner firms can lead to a further reduction in their marginal products. Consequently, this contributes to an increase in the dispersion of marginal products and exacerbates the existing misallocation. When the correlation between emission intensity and marginal products is negative, a carbon tax decreases the dispersion in marginal products and alleviates the prevailing misallocation. Hence, the optimal carbon tax rate would be higher in situations where a negative correlation between emission intensity and marginal products exists, compared to cases with a positive correlation. This happens as a social planner capitalizes on the advantage of addressing the prevailing misallocation.

I then show that in firm-level data cleaner firms tend to face higher distortions, reflected by higher marginal products. I use the merged Compustat-Worldscope data to estimate firm-level emission intensity and marginal products and incorporate industry-year indicators to account for potential industry-specific effects.[1] By doing so, I isolate the cross-sectional variation among firms within the same industry in the relation between emission intensity and marginal products. I find that cleaner firms have higher marginal revenue product of capital (MRPK), marginal

[1]The dispersion in emission intensity within 4-digit industries is sizable. Within 4-digit industries, a firm in the 90th percentile of the distribution of emission intensity emits greenhouse gases around 10 times more per unit of sales than firms in the 10th percentile.

revenue product of labor (MRPL), and revenue productivity (TFPR), with these variables serving as measures of distortions.[2] In addition, I find that firms with higher levels of productivity tend to exhibit lower emission intensities. This is particularly noteworthy, as previous literature suggests that productive firms have higher marginal products (Blackwood et al. (2021)). This finding contributes to explaining the negative correlation between emission intensity and marginal products.

Based on my theoretical and empirical insights, I build a novel quantitative model of firm dynamics. I expand a standard firm dynamics framework, as outlined in Khan and Thomas (2013a), by including environmental externalities as described by Golosov et al. (2014). The model features externalities stemming from carbon emissions, while also encompassing various standard distortions such as investment uncertainties, adjustment costs, and financial constraints. These distortions lead to productive firms having a higher marginal product of capital. Given that productive firms have lower emission intensities, as supported by empirical findings, firms with lower emission intensities tend to have a higher marginal product of capital. Therefore, the elasticity of emission intensity in relation to productivity, coupled with the degree to which productive firms have a higher marginal product of capital, jointly determine the relationship between emission intensity and marginal products.

I calibrate the model to capture key aspects related to carbon emissions, investment dynamics, and firm entry and exit. Specifically, my model successfully replicates the carbon damage to GDP ratio at the current temperature, as well as the essential moments related to the carbon cycle. These moments are pivotal in determining the optimal carbon tax level, as observed in a

[2] See Hsieh and Klenow (2009) and Hopenhayn (2014a) for an extensive discussion of variables that represent distortions.

standard carbon model like Golosov et al. (2014). As I extend the standard model to incorporate heterogeneity among firms in terms of carbon emissions, a new parameter emerges, governing the relationship between productivity and emission intensity across firms. I discipline this parameter using the relative standard deviation of emission intensity and productivity from microdata. Regarding investment and firm dynamics, my model effectively reproduces the correlation between productivity and marginal products, even though this correlation is not a direct target of the calibration process. In sum, the calibrated version of the model aptly replicates the negative relation between emission intensity and marginal products.

Using the calibrated model as a quantitative laboratory, I conduct four experiments. First, I compute the optimal carbon tax rate in a baseline economy, which amounts to $7.3 per ton of greenhouse gases emissions. This is equivalent to additional 5.4 percent of corporate income tax rate for the average firm. The calculation of this optimal tax rate necessitates a simulation of the transition from a non-carbon-tax economy to a carbon-tax environment. As this is computationally demanding, the Sequence-Space Jacobian method is employed to compute the transition path, following the approach outlined by Auclert et al. (2021).

Second, I explore the importance of the correlation between emission intensity and distortions by evaluating the optimal carbon tax in an environment where this correlation is zero. The analysis reveals that the optimal carbon tax diminishes to one-third of the baseline value, in alignment with the intuitive argument presented in the simplified model.

Third, I directly eliminate underlying distortions such as adjustment costs and financial frictions, which hamper the efficient allocation toward cleaner/productive firms. Even as total output rises, the elimination of distortions leads to a reduction in total carbon emissions. This result is driven by the substantial reallocation of resources toward cleaner firms, which are relatively under-producing in a distorted world. When contrasted with a carbon-tax scenario achieving equivalent carbon reduction, this exercise demonstrates that the removal of distortions results in a 9 percent higher welfare than that of the carbon-tax economy.

Fourth, I only remove financial frictions, since adjustment costs are usually understood as a feature of technology that policymakers cannot easily change. Both total carbon emissions and output increase, while the aggregate emission intensity decreases. This demonstrates that easing financial frictions results in the reallocation of resources towards cleaner firms. However, this effect is not strong enough to offset the overall increase in total output, primarily due to the presence of high adjustment costs.

This paper delivers two broader policy implications that could challenge conventional wisdom. First, a carbon tax might be more desirable for developing countries as opposed to developed ones. Often, policymakers in developing countries question the necessity of bearing the cost of carbon emissions, attributing much of global warming to developed counterparts. However, due to higher distortions, productive firms in developing countries tend to be smaller (Bartelsman et al. (2013a)). The findings of this paper suggest that a carbon tax might have a higher unintended benefit in alleviating existing misallocation within developing economies. Second, alternative policies geared towards directly mitigating the underlying distortions can be useful as a means of curbing carbon emissions. Sometimes, policymakers contend that price-based carbon policies, such as carbon taxes or cap-and-trade programs, face challenges in securing legislative approval.[3] As an alternative approach, the paper posits that policies directly targeting the underlying distortions could serve as effective tools in addressing climate change.

[3] See "Remarks by Heather Boushey on How President Biden's Invest in America Agenda has Laid the Foundation for Decades of Strong, Stable, and Sustained, Equitable Growth," May 31, 2023, Peterson Institute for International Economics.

The rest of the chapter proceeds as follows. First, I review the existing related literature in Section 1.2. In Section 1.3, I develop a simple model of heterogeneous firms to elucidate the main intuition. I empirically investigate a key statistic, derived from the simple model, using firm-level data in Section 1.4. I outline a quantitative firm dynamics model that incorporates externalities from carbon emissions, as well as heterogeneity in emission intensity and distortions in Section 1.5. Then I demonstrate how the quantitative model is calibrated to data in Section 1.6. I calculate the optimal carbon taxes in the quantitative model and conducts counterfactual analyses in Section 1.7. In Section 2.5, I present the conclusion.

1.2 Literature Review

This paper makes contributions to four areas of research. First, it contributes to the literature on second-best environmental policy in a distorted economy. Buchanan (1969) argues that the Pigouvian tax, which internalizes environmental damages, may be excessive in cases of insufficient competition, as concentrated industries are already producing below the socially optimal level. Goulder (1995) and Bovenberg (1999) study the optimal environmental tax when there are other distortionary taxes in play, such as capital and labor income taxes. In such cases, a government can recycle revenues from environmental taxes to mitigate the impact of these existing distortionary taxes. This is commonly referred to as the double-dividend hypothesis of environmental taxes, wherein such taxes not only reduce emissions (the first dividend) but also ameliorate inefficiencies arising from the distortionary tax system (the second dividend) simultaneously. I contribute to this literature by showing that misallocation across heterogeneous firms can be an additional source of inefficiencies, potentially yielding another dividend from environmental taxes.

Second, this paper advances the existing literature on climate change and environmental policies under firm heterogeneity. Lyubich et al. (2018) show substantial heterogeneity in emission intensity among manufacturing plants within the same industry, based on U.S. Census data. Berthold et al. (2023) investigate how a carbon pricing shock, identified using Känzig (2023)'s methodology, affects equity prices at the firm level. They find that relatively dirtier firms within a sector experience a larger decline in equity prices in response to an increase in carbon price. Caggese et al. (2023) quantify how climate change affects misallocation and aggregate productivity, by leveraging a general equilibrium structural model and grid-cell level temperature data. To my knowledge, Qi et al. (2021) is the closest in spirit to my paper. They extend a model based on Hsieh and Klenow (2009), demonstrating that correlated distortions lead to simultaneous reductions in output and increases in water pollution. However, my paper diverges from theirs in three key aspects. First, I focus on carbon emissions, while they focus on industrial water pollution.[4] Second, I incorporate externalities arising from carbon emissions, enabling a comprehensive normative analysis of carbon taxes. Lastly, in my quantitative model, I endogenize the wedges outlined in Hsieh and Klenow (2009) using investment uncertainty, adjustment costs, and financial frictions. This allows me to explore the precise role of specific distortions in influencing aggregate carbon emissions.

Third, this paper contributes to the literature on misallocation across heterogeneous firms. Numerous studies have investigated the sources of this dispersion in TFPR, examining whether these sources indeed indicate misallocation in terms of overall welfare. They also quantify the

[4]This distinction is potentially important because abatement technology can vary for different environmental objects. For instance, addressing carbon emissions may necessitate the use of carbon capture and storage (CCS) technology, which might not be cost-effective (Martin (2011)), whereas industrial pollution can often be more readily mitigated through end-of-pipe treatment.

extent to which each source contributes to misallocation.[5] Beyond quantification, this literature has evolved to explore how existing misallocation interacts with changing environments or policies.[6] My contribution to this literature involves examining how carbon taxes impact misallocation in the presence of heterogeneity in emission intensity and pre-existing misallocation that interacts with the emission heterogeneity. I formally show that the connection between emission intensity and distortions plays a crucial role in determining the level of optimal carbon taxes in this context.[7]

Lastly, this paper is related to the existing literature on optimal policy in a heterogeneous agent framework. Nuño and Moll (2018) derive constrained efficient allocation by considering the cross-sectional distribution as a control variable. Similarly, Ottonello and Winberry (2023) characterize constrained efficient allocation in the presence of non-rivalry of ideas and financial frictions. González et al. (2022) focus on the optimal monetary policy when firms are heterogeneous, while Dávila and Schaab (2023) explore the optimal monetary policy in the presence of heterogeneous households. Both papers formulate a Ramsey planner's problem, with the latter solving it using the Sequence-Space Jacobian method. My paper also employs the Sequence-Space Jacobian method, leveraging it to derive a transition path multiple times in order to determine an optimal policy choice, in my setting that of a carbon tax.

[5]Among many examples, see Asker et al. (2014) and David and Venkateswaran (2019) for adjustment costs; Buera et al. (2011) and Midrigan and Xu (2014a) for financial frictions; Dhingra and Morrow (2019) and Peters (2020) for markups; and David et al. (2016) for information frictions.

[6]Among others, see Gopinath et al. (2017) for a decrease in real interest rate on misallocation; González et al. (2022) and Baqaee et al. (2023a) for monetary policy; Andreasen et al. (2023) for capital control; Baqaee et al. (2023b) for market size.

[7]Restuccia and Rogerson (2008) demonstrate that the correlated distortion is much more critical in decreasing aggregate productivity. Hopenhayn (2014b) suggests that the conventional wisdom, which holds that an increase in the correlation between fundamentals and distortions decreases allocative efficiency, is not necessarily correct. My paper shows that the correlated distortion remains critical in investigating how changes in policies interact with existing misallocation.

1.3 Main intuition from a simple theoretical model

This section formalizes the main insights of this paper in a simple theoretical model. I demonstrate that the correlation between emission intensity and market distortions is a key metric for examining the impact of a carbon tax on allocative efficiency. This model provides a natural guideline for the empirical analysis presented in Section 1.4. It will be generalized in Section 1.5.

1.3.1 Model environment

My starting point is a model with heterogeneous firms in which externalities from carbon emissions exist, building upon Hsieh and Klenow (2009). In addition to differing in their productivity and output distortions, I assume that firms have different emissions per output. Firms produce homogeneous goods with a single factor and operate in a perfectly competitive market. The production function is given by a decreasing returns to scale function of firm productivity z and factor f.[8] In addition, total carbon emissions E are damaging to firms, and this carbon damage is modeled as aggregate productivity loss, following Golosov et al. (2014). Notably, firms do not account for the impact of their production choices on this overall carbon damage, thus leading to externalities arising from carbon emissions. The production function for firm i is

$$y_i = exp(-\gamma E) z_i f_i^{\alpha}, \quad \alpha < 1, \tag{1.1}$$

[8]In Hsieh and Klenow (2009), firms operate under the assumption of a production technology with constant returns to scale and a market structure characterized by monopolistic competition. Provided the profit function exhibits concavity, there will be a nondegenerate distribution of firms, and the fundamental logic of a simple model in this paper will remain unchanged.

where γ is the parameter that governs the degree of carbon damage.

I denote distortions that increase the marginal products of factor as τ. Following the *indirect* approach in the misallocation literature, distortions are represented as reduced-form output taxes that hinder the equalization of marginal products, as discussed by Restuccia and Rogerson (2017). The firm-level distortions aim to account for various frictions and distortions that could potentially generate dispersion in marginal products. In Section 1.5, I am going to employ a *direct* approach where I can generate dispersion in marginal products from structural frictions and distortions, such as uncertainties in investment, adjustment costs, and financial frictions.

Firms have different emission intensities $m_i = \frac{e_i}{y_i}$, where e_i are firm-level carbon emissions. I assume that firms take emission intensities as given, so carbon emissions e_i are generated as a by-product of their production. Firms could have different emission intensities at least for two reasons, as outlined by Shapiro and Walker (2018). First, firms may adopt different levels of abatement technologies. Second, differences in productivity can lead to varying emissions per unit of output, as long as carbon emissions are tied to input factors. In this simple model, I do not delve into the specific underlying causes for these differences in emission intensities. Instead, I focus on examining the implications of such heterogeneity. Additionally, firms are subject to a uniform per-emission carbon tax, denoted as τ_c.

A representative household and the government play passive roles. The household owns firms, supplies factors, and consumes. I assume that the total factor supply, $\bar{F}$, remains constant, and the household's utility solely depends on their consumption. This assumption is deliberately chosen to focus exclusively on the role of factor reallocation in determining the optimal carbon taxes. The government collects carbon taxes and output taxes (distortions) and then redistributes a lump-sum rebate to the household. As a result, carbon taxes and distortions do not directly affect the resource constraint:

$$C = Y, \tag{1.2}$$

where C is the consumption of the representative household and Y is the aggregate output.

1.3.2 Social welfare

The social planner aims to increase the consumption of the representative household by adjusting the carbon tax τ_c. Consumption of the representative household is linked to the production choices of firms. To show this, I define aggregate productivity TFP and represent it using aggregation:

$$TFP \equiv \frac{Y}{\bar{F}^{\alpha}} = exp(-\gamma E) \times \frac{\int z_i^{\frac{1}{1-\alpha}} (f_i/y_i)^{\frac{\alpha}{1-\alpha}} di}{[\int z_i^{\frac{1}{1-\alpha}} (f_i/y_i)^{\frac{1}{1-\alpha}} di]^{\alpha}}. \tag{1.3}$$

Since $C = Y = TFP \times \bar{F}^{\alpha}$ holds, and the factor supply is fixed, analyzing how the aggregate productivity TFP is affected by carbon taxes is enough to convey the welfare consequences of these taxes.

1.3.3 Firm problem

Given the model environment, firm i's optimization problem is

$$\max_{f_i}(1-\tau_i)y_i - pf_i - \tau_c e_i = (\underbrace{1-\tau_i-\tau_c m_i}_{\equiv \xi_i})y_i - pf_i \tag{1.4}$$

$$s.t.$$

$$y_i = exp(-\gamma E)z_i f_i^{\alpha},$$

where p is the factor price and ξ_i are the net distortions after carbon taxes. The first-order condition illustrates how marginal products (MP_i) have a one-to-one mapping to the after-carbon taxes distortions ξ_i:

$$MP_i \equiv \frac{\partial y_i}{\partial f_i} = \alpha exp(-\gamma E)z_i f_i^{\alpha-1} = \alpha\frac{y_i}{f_i} = \frac{p}{1-\tau_i-\tau_c m_i} = \frac{p}{\xi_i}. \tag{1.5}$$

By combining (1.3) and (1.5), aggregate productivity is represented as follows:

$$TFP = \underbrace{exp(-\gamma E)}_{\text{carbon damages}} \times \underbrace{\frac{\int z_i^{\frac{1}{1-\alpha}} \xi_i^{\frac{\alpha}{1-\alpha}} di}{[\int z_i^{\frac{1}{1-\alpha}} \xi_i^{\frac{1}{1-\alpha}} di]^{\alpha}}}_{\text{allocative efficiency}}. \tag{1.6}$$

Aggregate productivity TFP is higher when there are fewer carbon emissions and a higher degree of allocative efficiency. Assuming a joint log-normal distribution of (z_i, ξ_i), I express the factor misallocation term as a direct function of the dispersion in marginal products:

$$log(TFP) = -\gamma E + log\left(\int z_i^{\frac{1}{1-\alpha}} di\right)^{1-\alpha} - \frac{\alpha}{2(1-\alpha)}\sigma^2_{logMP_i},$$

where $\sigma^2_{logMP_i}$ is the dispersion in (log) marginal products.

1.3.4 Misallocation and the optimal carbon tax

When the carbon tax τ_c is zero, a firm with higher distortions τ_i experiences higher marginal products. In this case, the dispersion in marginal products only depends on the dispersion in distortions. However, when the government imposes a positive carbon tax, a firm with higher distortions does not necessarily have higher marginal products. If a firm with higher τ_i also has a lower m_i, it could result in lower marginal products because its 'effective' costs of carbon emissions $\tau_c m_i$ are lower. Consequently, depending on the relationship between distortions and emission intensity, carbon taxes could either increase or decrease the dispersion in marginal products.

Proposition 1 *Suppose* $E(m_i) = 0$. *(i) If the correlation between emission intensities and distortions is positive, i.e.,* $\rho = \rho(m_i, \tau_i) > 0$*, a carbon tax increases the dispersion in marginal products. (ii) If* $\rho < 0$*, a modest increase in carbon tax decreases the dispersion in marginal products, however, a sufficiently high carbon tax increases the dispersion in marginal products, i.e.,*

$$\frac{d\sigma^2_{logMP_i}}{d\tau_c} = \begin{cases} \geq 0, \textit{if } \rho \geq 0 \textit{ or } (\rho < 0, \tau_c \geq \frac{\rho\sigma_{\tau_i}}{\sigma_{m_i}}) \\ < 0, \textit{if } (\rho < 0, \tau_c < \frac{\rho\sigma_{\tau_i}}{\sigma_{m_i}}). \end{cases}$$

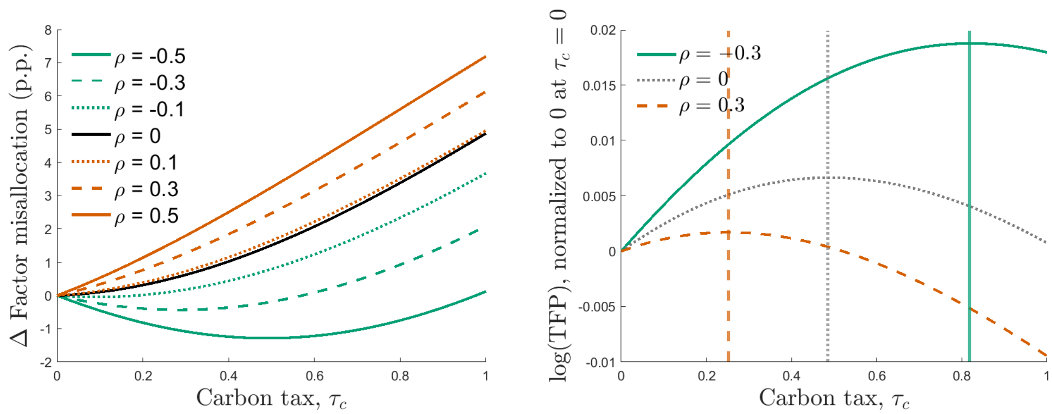
Proof. See Appendix A.1.1. ■

Proposition 1 shows the importance of the correlation between emission intensities and distortions in understanding how carbon taxes impact allocative efficiency. When there is a positive correlation, meaning cleaner firms have lower marginal products, implementing a carbon tax that directs resources towards these cleaner firms can lead to a further reduction in their marginal products. Consequently, this exacerbates the existing misallocation by increasing the dispersion of marginal products. A similar intuition applies when the correlation between emission intensity and marginal products is negative, particularly at a modest level of carbon tax. In this case, a carbon tax decreases the dispersion in marginal products and alleviates the existing misallocation. However, when the carbon tax is set excessively high, further increases in the carbon tax can make misallocation worse. This is because carbon taxes, combined with heterogeneous emission intensities, work as a source of the dispersion in marginal products. The correlation between emission intensity and distortions is a structural parameter in this simple model, but I endogeneize the relation in the quantitative model in Section 1.5.

1.3.5 Numerical illustration

Figure 1.1 shows how the reallocation of factors due to carbon taxes interacts with existing misallocation. The left panel demonstrates the impact of carbon taxes on allocative efficiency for different levels of correlation. When the correlation is positive, any level of carbon taxes exacerbates misallocation. However, in cases of a negative correlation, a modest increase in carbon taxes mitigates misallocation, while further increases deepen it. The right panel describes how carbon taxes affect aggregate productivity, the welfare measure in this model. As a result of

Figure 1.1: Factor misallocation and the optimal carbon taxes



(a) Misallocation (b) Optimal carbon taxes

Notes. I simulate 10,000 firms with different levels of productivity, distortions, and emission intensity, but with varying degrees of correlation between emission intensity and distortions, denoted as ρ. For each level of the ρ and carbon taxes, I calculate the degree of misallocation as $(1 - \text{allocative efficiency})$, and $log(TFP)$ using Equation (1.6). I plot the degree of misallocation and $log(TFP)$ relative to the value when the carbon tax is zero for each value of ρ. In Panel (b), the vertical lines represent carbon taxes that maximize $log(TFP)$. Parameter values: $\alpha = 0.8$, $\gamma = 0.005$, $\mu_{logz} = 0$, $\sigma_{logz} = 0.2$, $\mu_\tau = 0$, $\sigma_\tau = 0.2$, $\mu_m = 0$, $\sigma_m = 0.2$, and $\bar{F} = 10$.

the allocative efficiency improvements stemming from carbon taxes, the optimal level of carbon taxes is higher when the correlation between emission intensities and distortions is negative.[9]

It is important to note that there is a welfare gain from imposing carbon taxes, even when the correlation is positive and misallocation worsens. This is because firms do not internalize carbon damages in their production decisions. Carbon taxes make them internalize these carbon damages.

1.4 Empirical evidence on emissions and distortions

This section examines the relationship between emission intensities and distortions, leveraging firm-level data. The empirical analysis reveals that firms with lower emissions per output tend to

[9]In Appendix A.1.2, I demonstrate the impact of carbon taxes on the degree of misallocation and TFP when $E(m_i) > 0$. While carbon taxes may lead to an increase in misallocation in scenarios where the correlation between emission intensity and distortions is negative, this increase is less severe compared to cases with a positive correlation. Consequently, the optimal level of carbon tax remains higher for negative correlation cases.

exhibit higher marginal products, resulting in a negative correlation between emission intensities and distortions.

1.4.1 Description of data

I construct a firm-level panel dataset by merging three sources: firms' financial information from Compustat North America Fundamentals and carbon emissions data from Thomson Reuters Worldscope and Bloomberg. Compustat offers detailed accounting information for publicly traded companies, allowing me to compute the marginal products of firms. Worldscope and Bloomberg provide data on greenhouse gas emissions at the firm-level, measured in metric tons of CO_2 equivalent.[10] I use scope 1 carbon emissions, emissions from sources that a firm controls directly, as a measure of carbon emissions. I do not consider scope 2 and scope 3 carbon emissions to prevent any potential double-counting. My empirical analysis covers firms from 2002 to 2018, specifically focusing on non-financial companies incorporated in the U.S.

In the merged sample, the total carbon emissions for the year 2018 amount to approximately 2.1 gigatonnes of CO_2 equivalent, representing about 62.5% of the total emissions from the electric power and industrial sectors in the U.S. (EPA (2022)). Figure 1.2a provides a breakdown of carbon emissions by industry within the merged sample. It reveals that non-electricity generating sectors, such as manufacturing, transportation, and mining, contribute to roughly half of the total emissions. This industry distribution emphasizes the equal significance of reducing direct carbon emissions in these non electricity generating sectors, alongside efforts in the electricity generation sector. It is important to note that these carbon emissions are scope 1 emissions. This means that

[10] I primarily rely on Worldscope for carbon emissions data as it covers a larger number of firms. In cases where a firm lacks carbon emissions data in Worldscope, I turn to Bloomberg for this information.

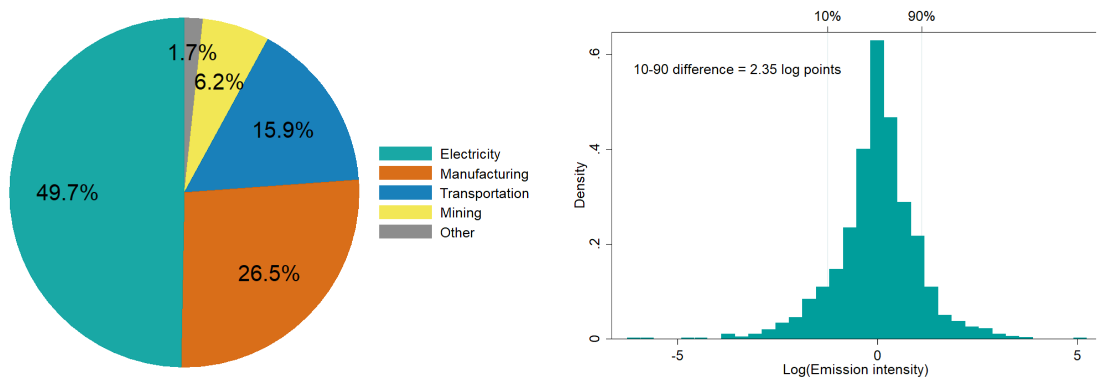
even if the electricity generation sectors were to fully transition to green energy sources, the emissions from these non-electricity generating sectors would remain unchanged. Furthermore, this industry composition is not unique to my merged sample; I observed a similar distribution of carbon emissions across industries in the U.S. EPA GHGRP dataset, which gathers plant-level carbon emissions data. This comparison is detailed in Appendix A.3.

Figure 1.2b presents the within-industry distribution of emission intensity. An important observation is that the dispersion in emission intensity within industries is large, echoing the findings of Lyubich et al. (2018) for manufacturing plants in the U.S. A firm in the 90th percentile of the distribution of emission intensity has a 2.35 higher log point of emission intensity than firms in the 10th percentile, implying that firms in the 90th percentile emit around 10 times more per unit of sales than firms in the 10th percentile. Moreover, the standard deviation of the logarithm of emission intensity in the raw sample is 2.65. When I compute the same statistics for residuals generated from a regression that controls for 4-digit industry-year dummies, the value is 1.01. This means that within-industry variations in emission intensity account for almost 40% of the variations in emission intensity.

1.4.2 Measurement and empirical specifications

The key variables in this empirical analysis are emission intensities and distortions. Emission intensities are measured as the ratio between carbon emissions to output, with the latter deflated using industry-specific output deflators. This is expressed as $m_i = \frac{e_i}{y_i}$, where m_i represents emission intensities, e_i are carbon emissions, y_i denotes real output, and i stands for a firm. The output is computed as real sales adjusted by the change in inventories.

Figure 1.2: Distribution of carbon emissions across and within industry in Year 2018



(a) Carbon emissions across industry (b) Emission intensity within industry

Notes. Panel (a) illustrates the distribution of carbon emissions across various industries. For Panel (b), I regress the firm-level logarithm of emission intensity on 4-digit industry-year dummies, which account for any variation in emission intensity across industries, and then take the residuals. Subsequently, I create a histogram of the residuals.

To measure distortions, I adopt the standard approach commonly employed in the misallocation literature. I employ three metrics: the marginal revenue product of capital, the marginal revenue product of labor, and revenue productivity. The MRPK is defined as the ratio of revenue to capital, $MRPK_i = y_i/k_i$.[11] Similarly, the MRPL is calculated as revenue per work, $MRPL_i = y_i/n_i$. TFPR is the geometric mean of MRPK and MRPL, where the weights determined by industry-specific cost shares (α_j), given by $TFPR_i = MRPK_i^{\alpha_j} MRPL_i^{1-\alpha_j}$. These cost shares are derived from industry-specific revenue elasticities, which are estimated using the control function approach pioneered by Olley and Pakes (1996). Furthermore, I gauge a firm's fundamentals, which encompass firm-specific demand and productivity. I will use the terms "fundamentals" and "productivity" interchangeably throughout this paper. These fundamentals are derived as the residuals from the production function estimation employed to calculate the industry-specific

[11] In fact, the marginal revenue product of capital is *proportional* to the revenue-capital ratio, as demonstrated by Hsieh and Klenow (2009). However, in my analysis, I use the revenue-capital ratio y_i/k_i as a proxy for $MRPK_i$. This choice is justified as I control for industry dummies to investigate the within-industry relationship between emission intensities and distortions in my empirical specifications. Thus, using y_i/k_i as a measure for $MRPK_i$ is innocuous.

revenue elasticities.[12] Productivity will be employed in an auxiliary empirical specification, which I will elaborate on below.

I present two empirical specifications. First, I examine the relationship between emission intensities and the distortion measures, concentrating specifically on the relationship within industries. This is achieved by incorporating 4-digit industry-year dummies as control variables. One rationale for focusing on within-industry relationships is the challenge in empirically discerning whether dispersions in marginal products across industries truly signify distortions. Firms in different industries are more likely to have fundamentally different production technologies, meaning that dispersion in marginal products could also capture differences in production technologies, not just distortions. Additionally, I take the logarithm of all variables to address unit-related concerns.

In the second empirical specification, I examine the relationship between productivity levels and emission intensities. This analysis aims to shed light on the connection between emission intensities and distortions. Drawing on Census manufacturing data, Blackwood et al. (2021) demonstrate a positive correlation of approximately of 0.7 between productivity and TFPR. This suggests that firms with higher productivity tend to experience greater distortions. If a firm with higher productivity also exhibits lower emission intensity, in conjunction with the findings of Blackwood et al. (2021), it could potentially support a negative correlation between emission intensities and distortions. To explore this relationship, I incorporate industry-year dummies, or two-way fixed effects, accounting for firm and year fixed effects. Additionally, I control for size and age. Size is measured as the logarithm of total assets, while age is calculated as the number of years since a firm's incorporation, utilizing information sourced from Jay

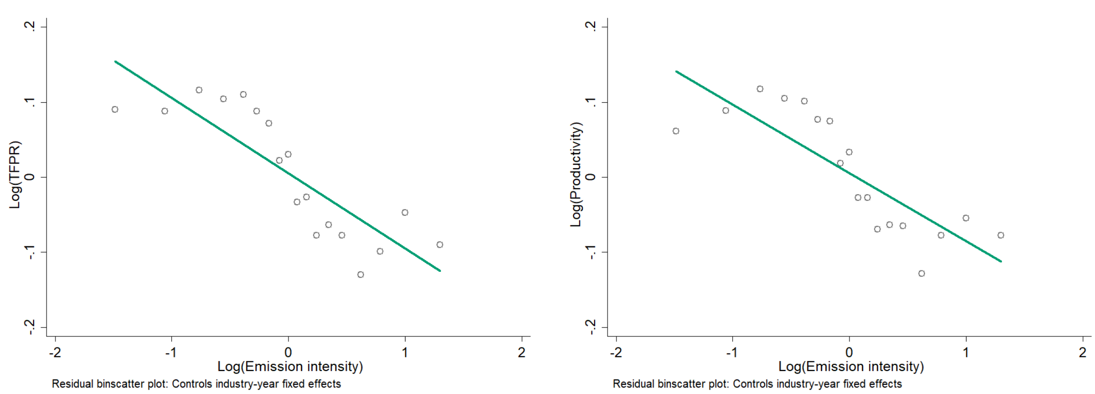
[12] Blackwood et al. (2021) provide an in-depth discussion regarding the distinction between TFPR and productivity. Although both measures pertain to revenue productivity, they rely on different elasticities. TFPR is calculated using cost shares, whereas productivity is based on revenue elasticities. It is emphasized that TFPR captures distortions, while revenue productivity, estimated with revenue elasticities, reflects a firm's productivity.

Ritter's website. Similarly, I take the logarithm of both productivity and emission intensities.

1.4.3 Empirical findings

Figure 1.3 provides binscatter plots as a preliminary view of the cross-sectional relationship between emission intensities, TFPR, and productivity. The left panel illustrates the connection between emission intensities and TFPR, while the right panel displays the relationship between emission intensities and productivity. To derive these plots, I first regress the logarithms of emission intensities, TFPR, and productivity on industry-year dummies, and then extract the residuals. These residuals are used to create bins based on the level of the logarithm of emission intensities. I then calculate the average of the residuals for each variable and plot them. Therefore, these binscatter plots specifically represent relationships within industries across firms.

Figure 1.3: Binscatter plots for TFPR and productivity with emission intensity



(a) TFPR (b) Productivity

Notes. I regress the logarithms of emission intensities, TFPR, and productivity on industry-year dummies and then extract the residuals. The industries are categorized at the SIC 4-digit level. Subsequently, I divide the residuals into bins based on the level of log(emission intensities). For each bin, I calculate the average values of log(emission intensities), log(TFPR), and log(productivity). These average values are then plotted along with the corresponding fitted linear lines.

Both panels, on the left and the right, indicate a negative relationship between emission intensities and both TFPR and productivity. In simpler terms, companies with lower emissions tend to face higher distortions and have higher productivity. Next, I will present the formal empirical results from the regression analysis. Initially, I will illustrate the empirical relationship between emission intensities and distortions, followed by the relationship between productivity and emission intensities.

Fact 1. Firms with lower emission intensities tend to face higher distortions.

Table 1.3 presents the empirical findings regarding the relationship between emission intensities and distortions. I control for 4-digit industry-year dummies to ensure that the results are primarily estimated by cross-sectional variation among firms within the same industries. Columns 1, 2, and 3 display the regression results when distortions are measured as MRPK, MRPL, and TFPR, respectively. Regardless of how distortions are measured, firms with lower emissions intensities tend to encounter higher levels of distortions. To put it in perspective, based on the TFPR result, a one-standard deviation reduction in emission intensity is associated with a 22% increase in TFPR.[13]

Table 1.1: Emission intensities and measures of distortions across firms

	(1) log(MRPK)	(2) log(MRPL)	(3) log(TFPR)
log(emissions/sales)	-0.152***	-0.075***	-0.094***
	(0.023)	(0.022)	(0.019)
Adj. R^2	0.827	0.734	0.650
Ind x Year FE			
N	2,848	2,820	2,820

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The table provides the results of empirical analysis, where I conduct regressions of distortions measures (MRPK, MRPL, and TFPR) on emission intensities. This analysis incorporates controls for 4 digit industry-year dummies and focuses on firms within non-financial sectors. Standard errors, which are presented in parentheses, are clustered at both the firm and year levels.

[13]If a firm exhibits a higher TFPR compared to other firms, primarily due to a higher markup, there might be a spurious correlation between emission intensity (defined using sales) and TFPR. Table A.1 presents regression results of emission intensity against distortion measures when emission intensity is defined by the costs of goods sold (COGS). The main results hold for this alternative definition of emission intensity as well.

Table 1.2, provides the within-industry relationship between emission intensities and distortions separately for different sectors. Specifically, my focus is on the mining, manufacturing, transportation, and electricity-generating sectors, as they collectively account for approximately 99% of the total carbon emissions and are the top four sectors in this regard. I use TFPR as a measure for distortions, although MRPK and MRPL yield similar findings. I maintain control for 4-digit SIC industry-year dummies, recognizing that these top four sectors encompass a broader scope than a typical 4-digit SIC industry classification. The estimates reveal that firms with lower emission intensities in the mining, manufacturing, and transportation sectors tend to face higher distortions. However, this pattern does not hold for firms within the electricity-generating sectors. Drawing on insights from the simplified theoretical model, one could conjecture that resource reallocation by imposing carbon taxes might alleviate existing misallocation in the mining, manufacturing, and transportation sectors. Nevertheless, its impact may be less significant for the electricity-generating sectors.

Table 1.2: Emission intensities and the measures of distortions by industries

log(TFPR)	Mining	Manufacturing	Transportation	Electricity
log(emissions/sales)	-0.156**	-0.110***	-0.364***	-0.014
	(0.070)	(0.026)	(0.058)	(0.028)
Adj. R^2	0.456	0.562	0.751	0.069
Ind x Year FE				
N	221	1,490	254	335

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The table provides the results of empirical analysis, where I conduct regressions of a distortions measure (TFPR) on emission intensities. This analysis incorporates controls for 4-digit SIC industry-year dummies and conducts separate analyses for firms within the mining, manufacturing, transportation, and electricity-generating sectors. Standard errors, which are presented in parentheses, are clustered at both the firm and year levels.

Fact 2. Firms with higher productivity tend to have lower emission intensities.

Table 1.3 displays the findings about the connection between firms' productivity and emission intensities. I explore this relationship employing various fixed effects and additional controls. For

Table 1.3: Emission intensities and productivity

log(emissions/sales)	(1)	(2)	(3)	(4)	(5)	(6)
log(productivity)	-0.750***	-0.643***	-0.638***	-0.880***	-0.624***	-0.653***
	(0.155)	(0.089)	(0.107)	(0.171)	(0.095)	(0.114)
Size				0.137**	-0.076	0.053
				(0.051)	(0.071)	(0.108)
Age				0.001	0.000	0.000
				(0.002)	(.)	(.)
Adj. R^2	0.825	0.978	0.981	0.827	0.979	0.981
Year FE						
Ind x Year FE						
Firm FE						
N	2,820	3,791	2,673	2,820	3,791	2,673

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The table provides the results of empirical analysis, where I conduct regressions of emission intensities on firms' productivity. Column (1) and (4) control industry-year dummies. In Column (2) and (5), I conduct two-way fixed effects regressions, accounting for both firm and year effects. Column (3) and (6) involve firm-fixed effects regressions, with additional control for industry-year dummies. Column (4) to (6) also include controls for firm size, measured by log(total assets), and firm age. Standard errors, which are presented in parentheses, are clustered at both the firm and year levels.

instance, column (1) and (4) control industry-year dummies, while I introduce firm-fixed effects in other specifications. Moreover, in columns (4) to (6), I include controls for size and age. Across all specifications, there is a consistent finding: firms with higher productivity tend to exhibit lower emission intensities.[14] These regression results align with the observations made in the binscatter plot. Notably, the size and age do not seem to play a substantial role in explaining emission intensities.[15]

In Table 1.4, I also investigate the relationship between productivity and emission intensities for different sectors. Echoing the findings in Table 1.3, firms with higher productivity tend to have lower emission intensities across all sectors, while this relationship is not statistically significant for firms in electricity-generating sectors. This could be the case if the variation in emission intensity within electricity-generating sectors is more influenced by the choice of fuel—such

[14] Shapiro and Walker (2018) show a negative correlation between non-carbon pollutions, such as NO_x, per real sales and total factor productivity at the firm level. However, their analysis does not control industry dummies.

[15] Table A.3 displays the regression results of emission intensity against productivity, using the COGS-based definition of emission intensity. These findings align with those presented in Table 1.3.

as coal, oil, natural gas, nuclear, and renewable energy sources—used by power plants, rather than the efficiency of electricity production. In contrast, firms in the mining, manufacturing, and transportation sectors may face greater challenges in using nuclear and renewable energy to operate their boilers, furnaces, ovens, and blast furnaces, so production efficiency might be more important in determining emission intensity for them.

Table 1.4: Emission intensities and productivity by industries

log(emissions/sales)	Mining	Manufacturing	Transportation	Electricity
log(productivity)	-0.791***	-1.221***	-1.349***	-0.474
	(0.247)	(0.249)	(0.181)	(0.835)
Size	0.028	0.165***	0.099	-0.096
	(0.090)	(0.054)	(0.065)	(0.290)
Age	0.001	0.001	0.007**	-0.009
	(0.003)	(0.003)	(0.003)	(0.021)
Adj. R^2	0.405	0.647	0.960	-0.037
Ind x Year FE				
N	221	1,490	254	335

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The table provides the results of empirical analysis, where I conduct regressions of emission intensities on firms' productivity. This analysis incorporates controls for 4-digit SIC industry-year dummies and conducts separate analyses for firms within the mining, manufacturing, transportation, and electricity-generating sectors. I also include controls for firm size, measured by log(total assets), and firm age. Standard errors, which are presented in parentheses, are clustered at both the firm and year levels.

1.5 A quantitative model with externalities and distortions

This section introduces a quantitative firm dynamics model, which generalizes the simple model discussed in Section 1.3. I endogenize frictions and distortions that impede the equalization of marginal products using investment uncertainty, adjustment costs, and financial frictions. Moreover, I incorporate endogenous capital accumulation. There are three types of economic agents: firms (comprising both incumbents and entrants), a representative household, and a government.

1.5.1 Firms

1.5.1.1 Incumbent firms

Incumbent firms operate within a perfectly competitive market. These firms differ in terms of productivity (z), capital (k), and debt (b). Productivity follows an exogenous process, evolving with a probability distribution $P(z'|z)$, while capital and debt are determined endogenously. Firms produce homogeneous goods using capital and labor (n), employing a production technology characterized by decreasing returns to scale. They incur fixed operating costs denoted as c_f. I include the concept of carbon damage, akin to the approach taken by Golosov et al. (2014), which is represented as the reduction in aggregate productivity due to carbon stock in the atmosphere. Incumbent firms consider this carbon damage as a given, thus there are externalities stemming from carbon emissions. The production technology is described by the following function:

$$y = exp(-\gamma_d S) z k^{\alpha} n^{\nu}, \quad \alpha + \nu < 1, \tag{1.7}$$

where S represents the aggregate carbon stock, and γ_d denotes the carbon damage parameter.

Firms generate emissions as a by-product of production, following the models of Copeland and Taylor (1994) and Shapiro and Walker (2018). Building on the empirical findings in Section 1.4, I assume that emissions (e) per unit of composite inputs decrease as productivity increases:

$$\frac{e}{k^{\alpha} n^{\nu}} = z^{-\eta}, \quad \eta > 0, \tag{1.8}$$

where η is a parameter that determines the sensitivity of emission intensity to productivity.

Equation (1.8) indicates that emissions per unit of output decline with increasing productivity, i.e., $\frac{e}{y} \propto z^{-(1+\eta)}$. While I adopt the 'emissions as a by-product of production' approach, this can be justified with a production function where non-energy inputs (capital and labor) and energy are perfect complements:

$$y = exp(-\gamma_d S)z \min\{k^\alpha n^\nu, z^\eta e\}, \tag{1.9}$$

where e represents energy, and using one unit of energy results in one unit of emission. The optimal behavior under the production function (1.9) implies $k^\alpha n^\nu = z^\eta e$. Hassler et al. (2012) find that energy and non-energy inputs have near-zero substitutability in the short-run.[16] Therefore, Equation (1.8) is a sensible assumption. Lastly, for each unit of emissions, firms face a carbon tax τ_c.

Firms in this model own and adjust their capital stock through investment. When making investments, firms know the expected future productivity but lack information about the actual realization of productivity given their present state. They also encounter capital adjustment costs, denoted as $\Phi(k, k')$, which encompass both convex and non-convex components. Lastly, firms face financial frictions, modeled as borrowing constraints (both asset-based and earnings-based) and restrictions on equity issuance, defined by the conditions:

$$b' \leq \max(\theta_k k, \theta_\pi \pi), \tag{1.10}$$

$$d \geq 0, \tag{1.11}$$

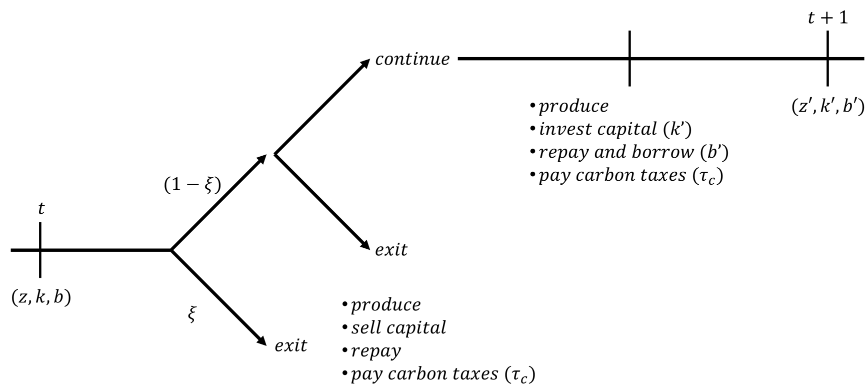
[16]They also find evidence of substitution between energy and non-energy inputs over the medium term. This is interpreted as an indication of directed technological change at the aggregate level.

where π represents profits after carbon taxes, θ_k and θ_π dictate the tightness of asset-based and earnings-based borrowing constraints, respectively. The term d denotes dividends.

Investment uncertainty, adjustment costs, and borrowing constraints all contribute to the dispersion in marginal products of capital, as shown by Gopinath et al. (2017). In Section 1.6.2, I will explain how these frictions and distortions create a positive correlation between productivity and TFPR, which in turn explains the negative correlation between emission intensity and TFPR, in combination with Equation (1.8).

Figure 1.4 provides a visual representation of the decision timeline for incumbent firms. At the start of each period, firms begin with idiosyncratic states (z, k, b) and confront aggregate variables including carbon stock, interest rates (r), and wages (w), which they treat as given. I denote these aggregate variables as $G = \{S, r, w\}$. They are also subject to exogenous exit shocks, which occur with a probability represented by ξ. After surviving these exit shocks, they make an optimal decision regarding whether to continue their operations or opt for an exit. Firms that choose to exit, whether due to exogenous shocks or endogenous decisions, engage in activities such as production, payment of carbon taxes, sale of undepreciated capital, and repayment of existing debt before leaving the market. In contrast, firms that weather the exit shocks and choose to continue operations engage in production, pay carbon taxes, make capital investments, repay existing debt, and borrow to finance these investments.

Figure 1.4: Timing for incumbent firms



Given these assumptions about incumbent firms, their beginning-of-the-period value function $V(z, k, b; G)$ is determined as follows:

$$V(z, k, b; G) = \xi V_x(z, k, b; G) + (1 - \xi) \max(V_x(z, k, b; G), V_c(z, k, b; G)), \tag{1.12}$$

where $V_x(z, k, b; G)$ and $V_c(z, k, b; G)$ represent value functions for exiting and continuing firms, respectively. For exiting firms, the value function is

$$V_x(z, k, b; G) = \max_n \{\pi + (1 - \delta)k - \Phi(k, 0) - b\}, \tag{1.13}$$

where $\pi = y - wn - c_f - \tau_c e$. When $V_x(z, k, b; G) > V_c(z, k, b; G)$ holds, incumbent firms optimally choose to exit.

The continuing firms choose labor, next period's capital and debt to maximize the discounted sum of dividends:

$$V_c(z,k,b;G) = \max_{n,k',b'} \left\{ d + \frac{1}{1+r} E_{z'|z} V(z',k',b';G') \right\} \tag{1.14}$$

subject to

$$d = \pi - (k' - (1-\delta)k) - \Phi(k,k') - b + \frac{1}{1+r} b' \geq 0$$

$$b' \leq \max(\theta_k k, \theta_\pi \pi).$$

1.5.1.2 Entrants

I adopt the approach of Clementi and Palazzo (2016) to model the problems faced by potential entrants. In every period, there is a constant mass M of potential entrants, each receiving a signal $q \sim Q(q)$ regarding their initial productivity. The initial productivity draws z' follow the same conditional probability distribution for the evolution of incumbents' productivity, denoted by $P(z'|q)$. If they decide to enter, they start their operations next period with initial capital k_0 and no debt. The value of a potential entrant with signal q, $V_e(q;G)$, is described as follows:

$$V_e(q;G) = -k_0 + \frac{1}{1+r} E_{z'|q} V(z', k_0, 0; G'). \tag{1.15}$$

Potential entrants choose to enter if $V_e(q) \geq 0$ holds. Here, since I assume $P(\cdot|q)$ is decreasing in q, $V_e(q;G)$ is strictly increasing in q. Thus, there exists a unique $\hat{q}$ such that $V_e(\hat{q};G) = 0$.

1.5.2 Representative household and government

The representative household's problem is standard. They make decisions on consumption (C), purchase bonds (B'), and hold shares in firms ($\{S_i'\}$) in order to maximize their lifetime utility. Additionally, they receive lump-sum transfers (T) from the government. The household supplies labor inelastically, with the labor supply denoted as $\bar{N}$. The recursive representation of their problem is described as follows:

$$V_H(B, \{S_i\}; G) = \max_{C, B', \{S_i\}} \{u(C) + \beta V_H(B', \{S_i'\}; G')\} \tag{1.16}$$

subject to

$$C + \frac{1}{1+r} B' + \int p_i S_i' di = w\bar{N} + \int (p_i + d_i) S_i di + B + T,$$

where p_i represents the stock price of firm i. The standard Euler equation illustrates the relationship between the interest rate and the marginal rate of substitutions across periods:

$$\frac{1}{1+r} = \frac{\beta u'(C')}{u'(C)}. \tag{1.17}$$

The government's role is also standard. They simply collect carbon taxes and provide lump-sum transfers to households to ensure balanced budgets:

$$T = \tau_c \int e_i di. \tag{1.18}$$

1.5.3 Evolution of carbon stock

Similar to Golosov et al. (2014), the law of the change in atmospheric carbon stock is determined by past carbon emissions and follows a linear relationship:

$$S' = (1 - \delta_c)S + \varphi E, \quad E = \int e_i di, \tag{1.19}$$

where the carbon stock decays at a geometric rate δ_c and a fraction of $(1-\varphi)$ of carbon emissions leaves the atmosphere immediately. This depreciation structure reflects the natural process by which the biosphere, land, and surface ocean absorb carbon stock.[17]

1.5.4 Evolution of the distribution of firms

The evolution of the distribution of firms, denoted as Γ, can be described as follows:

$$\Gamma(z', k', b') = (1 - \xi) \int_{(z,k,b)} \mathbb{1}_{[k'=k^*(z,k,b), b'=b^*(z,k,b), \text{no exit}]} dP(z'|z) d\Gamma(z, k, b) \tag{1.20}$$
$$+ M \int_{q \geq \hat{q}} \mathbb{1}_{[k'=k_0, b'=0]} dP(z'|q) dQ(q),$$

where $k^*(z, k, b)$ and $b^*(z, k, b)$ represent the optimal capital and debt choices for a firm with idiosyncratic state (z, k, b). A stationary distribution, denoted as Γ^*, is a fixed point of (1.20).

[17]In Golosov et al. (2014)'s model, a more general carbon depreciation structure is assumed: $S_t = \bar{S} + \sum_{s=0}^{t+T} (1 - d_s) E_{t-s}$, where $\bar{S}$ is the pre-industrial carbon stock and $(1 - d_s) = \varphi_P + (1 - \varphi_P)\varphi(1 - \delta_c)^s$. φ_P represents the proportion of carbon emissions that remain in the atmosphere permanently. I assume $\varphi_P = 0$ in order to compute a steady state. Without this assumption, the carbon stock would grow uncontrollably, leading to a convergence of aggregate productivity (net of carbon damages) towards zero, making a steady state ill-defined. See Nakov and Thomas (2023) for a similar discussion.

1.5.5 Equilibrium

I define the equilibrium for this economy; both in steady state, and along a perfect foresight transition path. A perfect foresight transition path is necessary to compute the optimal carbon taxes.

Steady state equilibrium A steady state equilibrium consists of (i) a policy vector $\phi(z,k,b) = \{n, k', b'\}$, value functions $V(z,k,b), V_x(z,k,b)$, and $V_c(z,k,b)$, (ii) a stationary distribution $\Gamma^*(z,k,b)$ and the entry cut-off $\hat{q}$, (iii) household consumption C^*, bond purchase B'^*, stocks $\{S_i^*\}$, and (iv) a wage w^*, an interest rate r^*, and carbon concentration S^*, such that:

1. For given S^*, r^*, and w^*, $\phi(z,k,b), V(z,k,b), V_x(z,k,b)$, and $V_c(z,k,b)$ solve the firm problem (1.12), (1.13), and (1.14).

2. The entry cut-off $\hat{q}$ satisfies, $V_e(\hat{q}) = 0$.

3. The equilibrium steady state carbon stock should be consistent with the carbon cycle: $\delta_c S^* = \int_{(z,k,b)} e(z,k,b) d\Gamma^*(z,k,b)$.

4. Household Euler equation: $r^* = \frac{1}{\beta} - 1$.

5. Market clearing conditions for labor, bonds, stocks, and goods market hold.

Perfect foresight transition path I consider a perfect foresight transition path from an economy with a zero carbon tax to one with positive carbon taxes. The equilibrium along this transition path is defined analogously to the steady state equilibrium.

1.6 Mapping the model to data

In this section, I calibrate the quantitative model and assess whether the calibrated model accurately reproduces key non-targeted data moments. Prior to delving into the calibration process, I elucidate the assumptions pertaining to the time frame, exogenous processes, adjustment costs, and household preferences. First, I assume that one period corresponds to a year. Second, the exogenous productivity process follows a standard AR(1) process:

$$log(z') = \rho_z log(z) + \epsilon_z, \;\; \epsilon_z \sim N(0, \sigma_\epsilon^2). \tag{1.21}$$

The entrant's signal q is drawn from the ergodic distribution of the productivity process. With regards to capital adjustment costs, I accommodate both convex and non-convex forms:

$$\Phi(k, k') = F\mathbb{1}_{k'-(1-\delta)k\neq 0} + \frac{\gamma}{2}\left(\frac{k' - (1-\delta)k}{k}\right)^2 k, \tag{1.22}$$

where F represents the fixed cost of capital adjustment costs, and γ denotes the parameter governing the convex adjustment costs. Finally, I assume a linear utility function for household preferences, $u(C) = C$, which ensures that interest rates remain constant both in steady state equilibrium and along a perfect foresight transition path.

1.6.1 Calibration

The baseline calibration matches investment behavior and carbon emissions at both the micro level and the aggregate evolution of carbon stock. The parameterization proceeds in two steps. Initially, I set a subset of parameters exogenously. Subsequently, I determine the remaining parameters to correspond with specific data moments. However, I will clarify these steps in three distinct categories: standard parameters, carbon parameters, and firm dynamics parameters. Parameters listed on rows shaded in green are calibrated internally.

Table 1.5 outlines the standard parameters utilized in this model. First, I set the household discount factor, β, at 0.96 to yield an annual interest rate of 4%, which is a standard value in firm dynamics literature. It is important to note that in climate economics literature, lower interest rates are commonly used. For instance, Stern (2007) employs a rate of 0.1%, while Nordhaus (2008) uses 1.5% for discounting future values. This implies that future utility holds relatively higher importance in welfare computations. Consequently, with lower interest rates, the socially optimal carbon taxes tend to be higher, as a planner places less emphasis on current economic costs and greater weight on the future benefits of emissions reduction through carbon taxes. The returns to scale $\alpha+\nu$ are set at 0.85, in line with Kaymak and Schott (2019). The labor coefficient in production, ν, is set at 0.56, which is equivalent to 0.85 multiplied by 2/3, consequently implying a capital coefficient (α) of 0.29. Additionally, the annual depreciation rate (δ) is set at 10%.

Table 1.5: Standard parameters

Parameter	**Description**	**Value**
β	Discount factor	0.96
α	Capital coefficient	0.29
ν	Labor coefficient	0.56
δ	Depreciation rate	0.10

In Table 1.6, I present the parameter values associated with carbon emissions. First, I assume that carbon emissions do not immediately dissipate from the atmosphere, $\varphi = 1$, based on my assumption that a period in the model corresponds to a year. Second, based on IPCC (2007), approximately half of a CO2 pulse is removed from the atmosphere after a span of 30 years. To reflect this, I set the carbon depreciation parameter (δ_c) at 0.02. Moving forward, according to Lyubich et al. (2018), the within-industry standard deviation of the logarithm of emission intensity is 2.47 times greater than that of the logarithm of productivity. Derived from Equation (1.8), this relationship can be expressed as:

$$\sigma_{log(\frac{e}{y})} = (1+\eta)\sigma_{log(z)}. \tag{1.23}$$

Therefore, I set the elasticity (η) of emission intensity with respect to productivity at 1.47. Lastly, I internally calibrate the carbon damage parameter (γ_d) to match the established carbon damage-to-GDP ratio from climate economics literature. The DICE-2023 model by Barrage and Nordhaus (2023) employs the following relationship to depict the connection between temperature changes since 1765 and carbon damages per GDP (Ω):

$$\Omega = 0.003467 \times (\Delta T)^2. \tag{1.24}$$

As of 2020, the temperature has risen by $1.25°C$ since 1765. According to Equation (1.24), this corresponds to a carbon damage per GDP of approximately 0.5%. To internally match this, I set the carbon damage parameter γ_d at 7.45×10^{-5}.

Table 1.7 provides parameter values related to the firm dynamics block. To begin, the persistence of the idiosyncratic productivity process is set at $\rho_z = 0.66$, and the standard deviation of productivity shocks is $\sigma_\epsilon = 0.12$, following Khan and Thomas (2013a). Second, parameters

Table 1.6: Carbon parameters

Parameter	Description	Value
$1-\varphi$	Immediate carbon depreciation	0
δ_c	Geometric carbon depreciation	0.02
η	Emission elasticity	1.47
γ_d	Carbon damage	7.45×10^{-5}

Notes. Parameters listed on rows shaded in green are calibrated internally.

related to financial frictions are drawn directly from studies focusing on micro-level data. Kermani and Ma (2020) show that the average liquidation recovery rate of PPE falls between 0.25 and 0.35, depending on whether industry-level or firm-level statistics are considered. As the liquidation recovery rate significantly impacts the tightness of asset-based constraints (as indicated by Lian and Ma (2021)), I set θ_k at 0.30, which stands as the median value between 0.25 and 0.35. Similarly, Drechsel (2023a) examines loan-level contract data, revealing an average debt-to-EBITDA ratio of 4.6.[18] Consequently, I establish θ_π as 4.6. Lastly, the mass of entrants is calibrated to normalize the equilibrium wage in steady state at 1.

The remaining parameters $(c_f, F, \gamma, \xi, k_0)$ are jointly calibrated to match salient moments related to firm dynamics and investment heterogeneity. First, the proportion of firms subject to earnings-based constraints is employed to calibrate the fixed operating cost parameter, c_f. As the operating cost increases, earnings decrease, subsequently leading to a decrease in the going-concern value of the firm. Consequently, firms are less likely to face earnings-based constraints. In the U.S., Lian and Ma (2021) report that 80% of firms face earnings-based borrowing constraints. Second, I use both the inaction rate (which represents the proportion of firms with an absolute investment rate lower than 1%) and the average investment rate to

[18] Drechsel and Kim (2022a) investigate implications of earnings-based borrowing constraints on macroprudential policy.

Table 1.7: Firm dynamics parameters

Parameter	Description	Value
ρ_z	Persistence of TFP	0.66
σ_ϵ	SD of innovations to TFP	0.12
θ_k	Borrowing limit (asset)	0.30
θ_π	Borrowing limit (earnings)	4.60
M	Mass of entrants	0.70
c_f	Fixed operating cost	0.07
F	Fixed adjustment cost	2.95×10^{-5}
γ	Convex adjustment cost	0.23
ξ	Exit shocks	0.02
k_0	Entrants' capital	0.15

Notes. Parameters listed on rows shaded in green are calibrated internally.

determine the adjustment costs parameters (F, γ). I obtain the values of the inaction rate (8%) and the average investment rate (12%) from Cooper and Haltiwanger (2006a). Third, the exit rate of establishments is used to identify the exogenous exit rate, ξ. I calculate the average exit rate for establishments as 9.8% for the period spanning from 2002 to 2018, using the Business Dynamics Statistics (BDS). Lastly, the relative number of employees of entrants compared to the average establishments is employed to determine the size of entrants' capital, k_0. I compute this relative size using data from the BDS, and on average, the size of entrants is found to be 31.4% of the average establishments.

Table 1.8 shows that the calibrated model reasonably matches the targeted moments. Specifically, it closely matches the proportion of firms facing earnings-based constraints, the inaction rate, the exit rate, the relative size of entrants, and the carbon damage to GDP. However, it slightly over-predicts the average investment rate.

Table 1.8: Target moments: model vs. data

Moment	Description	Data	Model
$\mathbb{E}[\mathbb{1}_{EBC}]$	Share of EBC firms	0.80	0.84
$\mathbb{E}[\mathbb{1}_{\lvert i/k\rvert<0.01}]$	Inaction rate	0.08	0.08
$\mathbb{E}[i/k]$	The average investment rate	0.12	0.17
$\mathbb{E}[\mathbb{1}_{Exit}]$	Exit rate	0.10	0.09
$n_0/\bar{n}$	The relative size of entrants	0.31	0.30
$1-exp(-\gamma_d S)$	Carbon damage to GDP	5.0×10^{-3}	5.0×10^{-3}

1.6.2 Non-targeted moments

As outlined in Section 1.3, the key statistic in determining the optimal carbon taxes is the relation between emission intensities and distortions. I investigated this moment empirically using firm-level data, as detailed in Section 1.4. However, I do not directly target the empirical findings for consistency of my calibration. My other targeted moments stem from Census microdata, which feature a distinct distribution of firms compared to Compustat data.[19]

Given my assumption of a direct negative relationship between productivity and emission intensity, which is supported by the empirical results in Table 1.3, it is crucial to examine the connection between productivity and distortions. Following Hopenhayn (2014a), TFPR, representing distortions in a model with perfectly competitive firms, is calculated as:

$$\begin{aligned} log(TFPR) &= log(y) - \frac{\alpha}{\alpha+\nu}log(k) - \frac{\nu}{\alpha+\nu}log(n) \\ &= \frac{\alpha}{\alpha+\nu}log(y/k) + \frac{\nu}{\alpha+\nu}log(y/l), \end{aligned} \tag{1.25}$$

where y/k and y/l represent the marginal products of capital (MRK) and labor (MPL) under a homogeneous production function. As MPL is always equalized in the absence of carbon taxes,

[19] Ottonello and Winberry (2020) follow a similar approach.

any dispersion in TFPR solely stems from dispersion in MPK.

Various factors in my quantitative model, including investment uncertainty, adjustment costs, and financial frictions, contribute to a positive correlation between productivity and MPK (or TFPR). First, due to investment uncertainty, firms commit to capital investment without knowing the precise productivity level for the next period. Consequently, even initially identical firms that invest the same amount of capital may yield different outcomes post-investment. In the subsequent period, a firm experiencing a high realization of productivity produces output using a lower level of capital than it would have utilized if it had prior knowledge of its productivity. This implies that a firm with a high realization of productivity possesses a relatively higher marginal product of capital.

Second, adjustment costs and borrowing constraints can also lead productive firms to have higher MPK. For instance, if we compare firms with the same amount of capital and debt but different levels of productivity (as is the case with new entrants), those with higher productivity typically exhibit a greater demand for investment. Due to the presence of adjustment costs and financial frictions, these highly productive firms may face limitations in their growth, leading them to have a higher MPK compared to firms with lower productivity. However, in a steady state, productive firms tend to hold larger amounts of capital and earn higher profits. These productive firms may encounter less restrictive borrowing limits. Thus, determining which of these forces dominates is a quantitative question.

To gain a clearer understanding of which forces exert a stronger influence, I undertake a simulation of the steady state economy of the calibrated quantitative model. Following this, I conduct two regression analyses. First, I regress log(TFPR) on log(z). Second, I regress log(TFPR) on the lag of log(z), denoted as log(z_{-1}). The second specification provides insight

Table 1.9: Relationship between distortions and ex-ante and ex-post productivity

	(1) log(TFPR)	(2) log(TFPR)
$\log(z)$	0.717***	
	(0.001)	
$\log(z_{-1})$		0.422***
		(0.001)
R^2	0.752	0.270
N	258,425	258,425

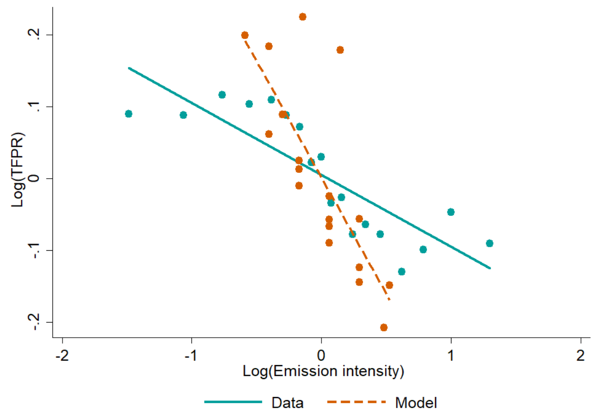
Notes. I conduct a simulation of the economy utilizing policy functions within the steady state of the calibrated model. In this simulation, I consider a pool of 3000 potential entrants. I generate a dataset covering a span of 150 years, but discard the initial 134 years in order to align with the sample period duration outlined in Section 1.4. To calculate TFPR, I employ Equation (1.25).

into how adjustment costs and financial frictions, while controlling for investment uncertainty, contribute to the relationship between productivity and TFPR by considering ex-ante productivity.

Table 1.9 indicates that, although controlling for investment uncertainty leads to a reduction in the regression coefficient and R^2, a notable and statistically significant positive relationship between productivity and TFPR persists. This relationship is attributed to the influence of adjustment costs and financial frictions.

Figure 1.5 shows a binscatter plot illustrating the connection between emission intensities and TFPR, comparing data to the model. The green dots and line represent the relationship between emission intensity (matching Figure 1.3a) and TFPR derived from the firm-level data. Meanwhile, the red dots and line show the relationship generated by my quantitative model. This figure demonstrates that my model reproduces the negative correlation between emission intensity and TFPR. The regression coefficient of log(TFPR) to log(emission intensity) was approximately -0.10 (as shown in Column (3) of Table 1.1). However, in my model, this regression implies a value of -0.31 for the same relationship. Furthermore, my model also aligns well with the correlation between log(productivity) and log(TFPR), which is calculated using Census

Figure 1.5: Relationship between emission intensity and TFPR: model vs. data



Notes. The binscatter plot generated from data, which is described with green dots and fitted line is the same with Figure 1.3a. The orange dots and fitted line represent the model counterpart.

microdata by Blackwood et al. (2021). In the Census data, this correlation ranges between 0.71 and 0.74, depending on the method used for estimating the production function. In my model, this correlation is 0.60.

The difference between the coefficients obtained from the model and those from the regression of log(TFPR) to log(emission intensity) in the Compustat data warrants examination. Several potential explanations for this discrepancy can be considered. First, it might be important to acknowledge the possibility of classic measurement errors in emission intensity in data. Greenhouse gas emissions reported by firms to data vendors like Worldscope and Bloomberg are estimations, and variations in estimation methods among firms can introduce errors. This could result in a downward bias in the regression coefficient in the data due to attenuation bias.

Second, when calibrating my parameters related to firm dynamics, I rely on empirical moments from Census microdata rather than moments from the Compustat-Worldscope data used in the empirical analysis. If there are disparities in the regression coefficient between Compustat and Census data, the utilization of different datasets could be a contributing factor. While I cannot directly compute the regression coefficient for the Census sample, there is additional information that sheds light on this hypothesis. For instance, the ratio between the standard deviation of log(emission intensity) and that of log(productivity) – which I used to calibrate the parameter η – is informative. In the Census sample, this ratio is 2.47, whereas in my Compustat-Worldscope sample, it is 3.17. This suggests that emission intensity is relatively more dispersed in the Compustat-Worldscope sample, which could account for the lower regression coefficient in the Compustat data.

Lastly, there could be a more structural issue within my quantitative model. I assume a one-to-one mapping between productivity and emission intensity, an assumption grounded in empirical findings in Section 1.4. However, there may exist alternative mechanisms that contribute to the dispersion in emission intensity. For instance, Lanteri and Rampini (2023) propose a theoretical model suggesting that financially constrained firms are more likely to employ capital with higher emissions due to the relatively higher cost of cleaner capital. This implies a positive correlation between emission intensity and distortions in production. If I incorporate this channel into the model, I anticipate that the regression coefficient of log(TFPR) to log(emission intensity) would be less negative.

1.7 Optimal carbon taxes and counterfactual analyses

In this section, I compute optimal carbon taxes and conduct three counterfactual analyses using my calibrated quantitative firm dynamics model.

1.7.1 The optimal carbon tax

I compute the optimal carbon tax with the following steps. First, for a given carbon tax level τ_c, I compute the steady state equilibrium. Second, I generate a perfect foresight transition path of consumption from the baseline economy, where carbon taxes are zero, to an economy with the carbon tax level τ_c. I employ the Sequence-Space Jacobian method, developed by Auclert et al. (2021), which allows for fast and accurate computation of this transition path. Third, I calculate the lifetime utility of the representative household over this transition path. I iterate these steps for different carbon tax levels $\tau_c \in [0, \bar{\tau}_c]$.[20] Consequently, I am able to generate a curve representing social welfare over different carbon tax levels, enabling me to identify the carbon tax level that maximizes welfare.

Two aspects of my procedure warrant clarification. First, I compute the simple optimal carbon tax level, rather than computing an optimal path of Ramsey policy or constrained efficient allocation.[21] Second, I consider the whole transition path because a carbon tax that maximizes steady state consumption might be misleading.[22] Researchers often use steady state consumption as a measure of welfare for policy evaluation due to the computational challenges of solving for transition paths in a heterogeneous firm model. I address this computational challenge by employing the Sequence-Space Jacobian method. Figure A.3 illustrates that using steady-state

[20] The upper bound of carbon tax $\bar{\tau}_c$ is set loosely enough to make sure that the optimal carbon tax for maximizing welfare is lower than this upper bound.

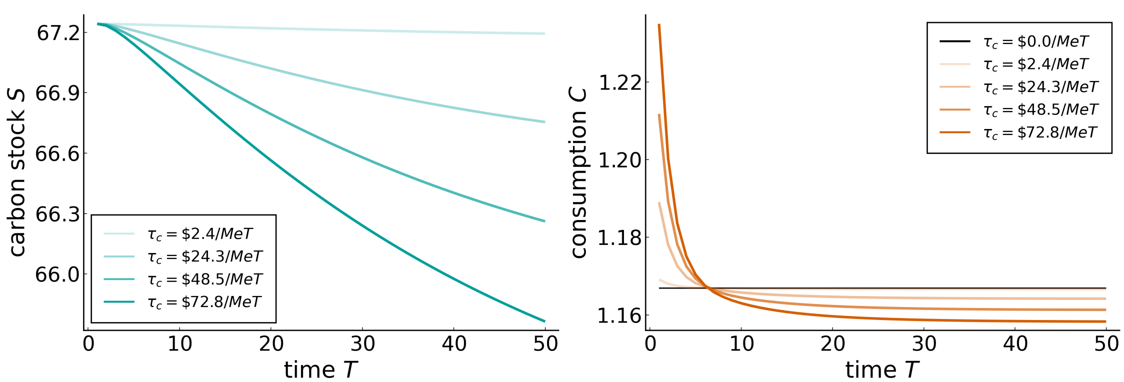
[21] The full path of Ramsey carbon tax would be difficult to characterize as the government should find the optimal path of carbon taxes when a state of the economy is characterized by a distribution of firms, which is an infinite-dimensional object. However, a recent development from the literature on optimal policy in a heterogeneous agent model could make this work doable. Among others, see Nuño and Moll (2018), González et al. (2022), and Ottonello and Winberry (2023). I leave applying their methodology to characterize the optimal path of carbon taxes for future work.

[22] In a neoclassical growth model, it is important to distinguish between the Golden Rule capital level, which maximizes steady state consumption, and the level of steady state capital chosen by an agent who maximizes lifetime utility in an efficient economy (referred to as the Golden Rule versus the modified Golden Rule). Mukoyama (2013) discusses this point in the context of unemployment insurance policy.

consumption as a welfare measure results in a zero optimal carbon tax. This outcome is not reasonable in a model that incorporates externalities stemming from carbon emissions.

Figure 1.6 provides an illustration of the economy's carbon stock and consumption change along a transition path for various levels of carbon taxes. As anticipated, higher carbon taxes lead to a more significant reduction in the carbon stock. Regarding consumption, the imposition of carbon taxes initially triggers increases due to smaller capital investment, eventually converging towards a lower steady state compared to that with zero carbon tax. Since I assume perfect complementarity between emissions and a composite input of capital and labor, carbon taxes that penalize carbon emissions also lead to a reduction in steady state capital. Consequently, steady state consumption decreases. This figure underscores why it can be misleading to rely solely on steady state consumption as a measure of welfare and the transition needs to be taken into account.

Figure 1.6: Carbon stock and consumption over a perfect foresight transition path



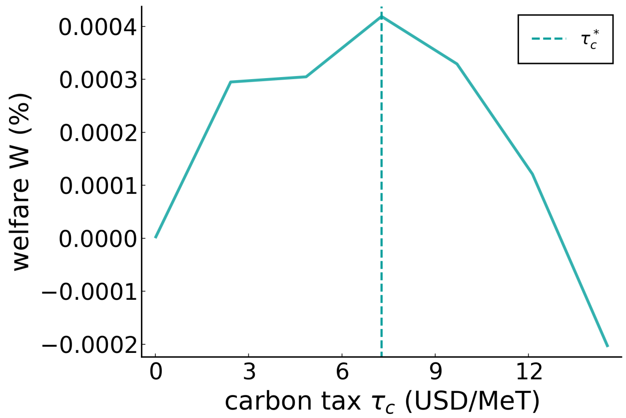
(a) Carbon stock

(b) Consumption

Notes. Panel (a) depicts the evolution of carbon stock, while panel (b) illustrates consumption along a perfect foresight transition path for different levels of carbon taxes. Darker shades of green and red indicate higher carbon tax levels. In both panels, the x-axis represents the transition time.

Figure 1.7 presents social welfare, quantified as the lifetime utility of the representative household along a perfect foresight transition path, for different levels of carbon taxes. The

Figure 1.7: Welfare curve over carbon taxes



Notes. The x-axis represents the different levels of carbon taxes. The y-axis measures the difference in welfare, which is the lifetime utility of the representative household over a transition path, compared to the case with zero carbon tax. This difference is expressed in units of consumption equivalent welfare.

carbon tax that maximizes social welfare is $7.3 per metric ton of CO2. This translates to an additional 5.4 percent of the corporate income tax rate for the average firm. In my Compustat-Worldscope sample, the average ratio of carbon emissions to taxable income is 0.0075 metric tons of CO2 per dollar. This implies that, for every dollar of taxable income, the average firm needs to allocate an extra 0.054 dollars towards the carbon tax (calculated as 0.0075×7.3). For high emitters, such as firms in the 90th percentile of the distribution for the ratio of carbon emissions to taxable income, the optimal carbon tax is equivalent to an additional 9.5 percent (computed as 0.013×7.3) of the corporate income tax rate.

The optimal carbon tax is relatively low compared to estimates from other studies. Nordhaus (2008) and Stern (2007) suggest a tax of 30 and 250 dollar per ton of coal, respectively, which are equivalent to 15 and 127 dollar per metric tons of CO2.[23] Golosov et al. (2014) propose even higher carbon taxes which range between 29 and 252 dollar per metric tons of CO2, where I use the same conversion factor for changing the tax on coal to CO2. One potential reason for this

[23]According to the U.S. EPA Greenhouse Gas Equivalencies Calculator, 1 ton of coal emits 1.968 metric tons of CO2.

disparity could be the different assumptions regarding the discount rate. I assume a 4% interest rate, whereas these other papers operate with interest rates between 0.1% and 1.5%. Stern (2007) sets a rate of 0.1 percent per year by adding a "moral" concern for future generations. In models where the social planner places greater emphasis on the future, as is the case in those studies, it is more likely for higher carbon taxes to be imposed. The fact that it is difficult to determine an appropriate value of discount rates necessitates caution when interpreting the social costs of carbon in a literal sense.

The welfare gain resulting from the optimal carbon tax is also relatively modest in my model. One reason is the perfect complementarity assumed between emissions and capital/labor. Within this framework, reducing emissions requires firms to directly curtail their production, incurring significant costs. Golosov et al. (2014) assume a Cobb-Douglas production function involving capital, labor, and energy, implying that energy can be substituted by capital and labor, potentially leading to relatively lower economic costs and therefore a higher welfare gain under the optimal carbon tax.

1.7.2 Counterfactual analysis

I conduct three counterfactual analyses to gain a deeper understanding of the quantitative model and to explore alternative policies that can effectively address climate change. The quantitative model maps the relation between emission intensity and marginal products to structural parameters, such as the elasticity of emission intensity with respect to productivity η. This feature enables me to conduct more economically interpretable counterfactual analyses. In addition, I specify the sources of frictions and distortions in the quantitative model, which was not the case in the simple model in Section 1.3. This allows me to separately investigate the role of each distortion in shaping carbon emissions.

Counterfactual 1: Zero correlation between emission intensity and distortions

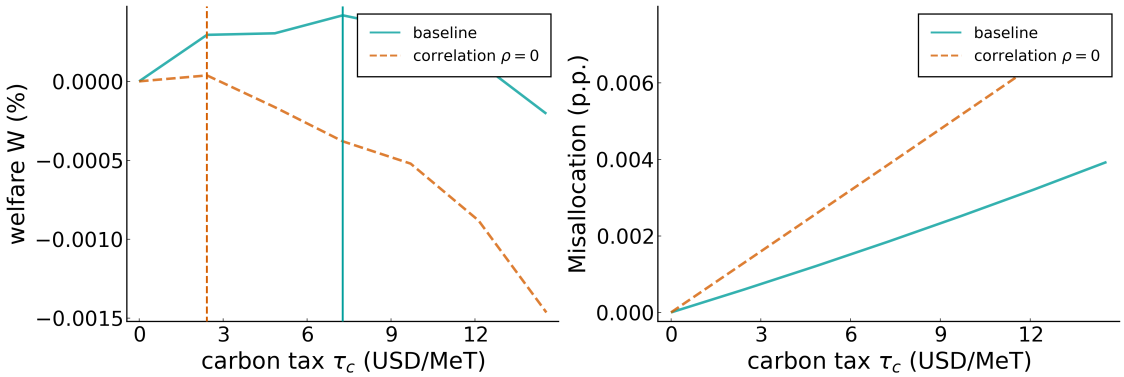
In this first counterfactual, I eliminate the correlation between emission intensity and distortions by setting $(1+\eta) = 0$ and compute the optimal carbon tax. Setting $(1+\eta) = 0$ implies that firms have exactly the same emission intensity and that the correlation between emission intensity and marginal products is zero. However, the dispersion in marginal products still exists as frictions and distortions are not affected by the homogeneity in emission intensity. Thus, I compute the optimal carbon tax where firms are homogeneous in emission intensity while they face different degrees of distortions. I anticipate that the optimal carbon tax rate in this counterfactual analysis will be lower than the baseline. This is because a uniform carbon tax treats all firms equally, which means there is no advantageous reallocation toward firms with higher marginal products.

Figure 1.8 illustrates the optimal carbon tax rate and the change in misallocation in response to carbon taxes for both the baseline calibration and the case where $(1 + \eta)$ is set to zero. Counterfactual optimal carbon tax rate is \$2.4 per metric ton of CO_2, which is one-third of the optimal carbon tax rate under the baseline calibration, where the correlation is negative. Examining how steady-state misallocation reacts to carbon taxes, we observe that, compared to the baseline economy, misallocation worsens when the correlation is zero.

Counterfactual 2: Remove both adjustment costs and financial frictions

In the second counterfactual, I eliminate the non-environmental distortions, including adjustment costs and financial frictions, which contribute to the dispersion in marginal products. Specifically, I set adjustment cost parameters F and γ to zero, and the borrowing constraints parameters

Figure 1.8: Welfare and misallocation when the correlation between emissions and distortions is zero



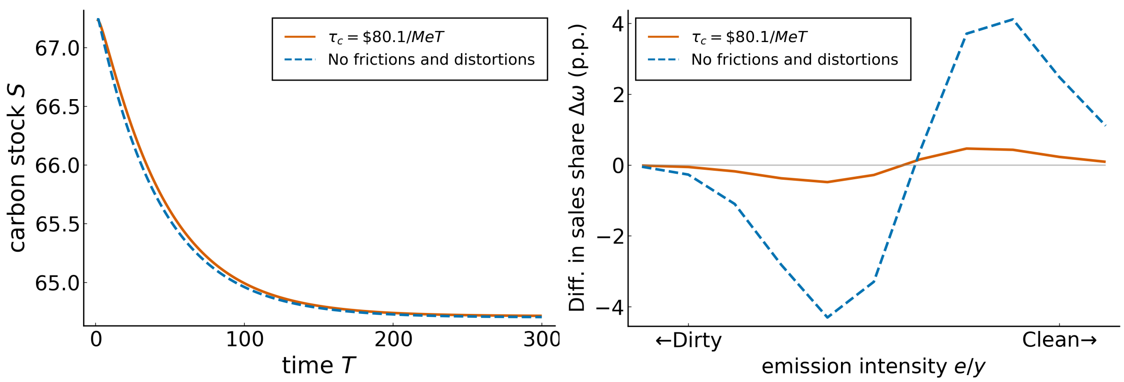
(a) Welfare (b) Misallocation

Notes. The x-axis represents the different levels of carbon taxes. In panel (a), the y-axis measures the difference in welfare, which is the lifetime utility of the representative household over a transition path, compared to the case with zero carbon tax. This difference is expressed in units of consumption equivalent welfare. In panel (b), the y-axis represents the difference in the extent of factor misallocation relative to the case with zero carbon tax. The solid lines correspond to the baseline case, while the dashed lines pertain to the case with zero correlation between emission intensity and distortions.

θ_k and θ_π to infinity. I study this scenario to inform a policymaker who might be constrained to alternative policies to carbon taxes, for example due to political constraints. By directly mitigating the underlying distortions that divert production factors from relatively cleaner firms, resources could be redirected back to these cleaner firms, resulting in an overall reduction in aggregate carbon emissions.

Figure 1.9 illustrates the effects of removing adjustment costs and financial frictions on carbon emissions. In Panel (a), we observe a reduction in carbon emissions as a result of eliminating these distortions. To achieve the same level of emission reduction, I would need to implement a carbon tax of \$80.1 per metric ton of CO_2, which is over 10 times the optimal carbon tax. Panel (b) examines the steady state share of sales for different levels of emission intensity in both the frictionless economy and the economy subject to a carbon tax of \$80.1 per metric ton of CO2. This is compared to the baseline economy, where there is no carbon tax but there are still frictions and distortions in place. The panel demonstrates that in both scenarios, the market share of firms with higher emission intensity decreases, while that of cleaner firms expands, compared to the baseline economy. Notably, when I eliminate adjustment costs and financial frictions, the shift towards cleaner firms is substantial. In contrast, this shift is relatively modest when I implement the large carbon tax. This suggests that the reduction in carbon emissions is primarily driven by the reallocation towards cleaner firms in a frictionless economy. In the scenario with the large carbon tax, it is more likely that the decrease in aggregate production is the main driver of carbon reduction.

Figure 1.9: Eliminate both adjustment costs and financial friction: carbon emissions

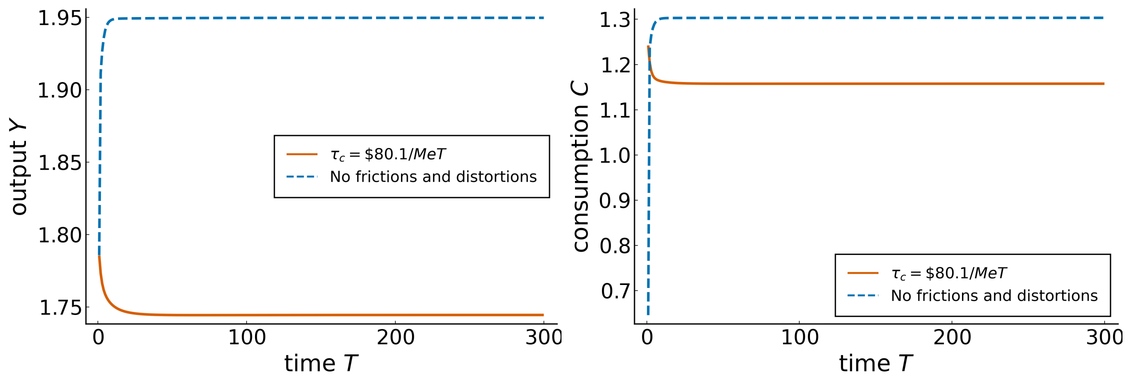


(a) Carbon stock (b) Differences in sales share

Notes. Panel (a): The x-axis denotes the transition time. The blue dashed line illustrates the transition paths towards a frictionless economy with without adjustment costs and financial frictions, while the orange solid line represents transition paths when I impose a \$80.1 per metric ton of CO_2 carbon tax, achieving an equivalent reduction in carbon emissions as the transition towards the frictionless economy; Panel (b): The x-axis represents emission intensity, while the y-axis shows the steady-state sales share relative to the baseline economy, presented in percentage points. The blue dashed line indicates the relative sales shares in the steady state of the frictionless economy. The orange solid line represents the relative sales shares in the steady state of the economy with a \$80.1 per metric ton of CO2 carbon tax. The gray line indicates zero.

Figure 1.10 shows the transition paths for output and consumption to an economy without adjustment costs and financial frictions, and to an economy with a \$80.1 per metric ton of CO_2 carbon tax. As anticipated, the elimination of frictions and distortions leads to an increase in output, while the imposition of a relatively high carbon tax suppresses output. In terms of consumption, despite an initial dip, the higher consumption in the frictionless economy is sustained for a much longer period. Comparing the welfare levels across these two consumption

Figure 1.10: Eliminate both adjustment costs and financial friction: output and consumption



(a) Output (b) Consumption

Notes. The x-axis denotes the transition time. The blue dashed lines illustrate the transition paths towards a frictionless economy without adjustment costs and financial frictions, while the orange solid lines represent transition paths when I impose a \$80.1 per metric ton of CO_2 carbon tax, achieving an equivalent reduction in carbon emissions as the transition towards the frictionless economy.

transition paths, I find a 9.05% increase in welfare, measured in terms of consumption equivalence, in the frictionless economy compared to the one with the \$80.1 per metric ton of CO_2 carbon tax.

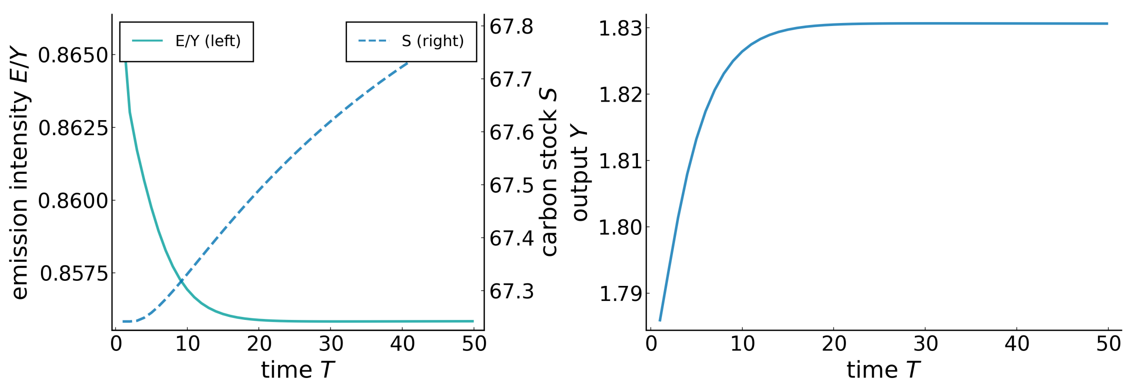
Observing the transition paths of consumption from two different policy approaches—eliminating friction and distortions, and imposing a large carbon tax—potentially raises an interesting point related to distributing the costs of combating carbon emissions across generations. Although I do not directly model this inter-generational aspect, it is worth noting that while the amount of carbon reduction is the same for both cases, there is an early consumption dip in the transition towards the frictionless economy, whereas there is an initial increase in consumption for the large carbon tax. This could imply that a society achieves higher welfare in a transition towards the frictionless economy, at the expense of current generations. Therefore, it would be interesting to consider an overlapping generations (OLG) structure to carefully analyze the welfare implications over different generations. This is an avenue I will explore in future work.[24]

[24]Fried et al. (2018) argue that recycling revenue through lump-sum rebates could lead to higher welfare compared to when a government recycles revenue to reduce distortionary taxes when they consider the transitional dynamics of living agents. However, the conventional wisdom holds true when the analysis focuses on steady state welfare.

Counterfactual 3: Remove only financial frictions

Adjustment costs are akin to technological constraints, so allowing a planner to remove them might be a stark assumption. In this counterfactual, I solely eliminate financial frictions by setting θ_k and θ_π to infinity. Figure 1.11 illustrates the transition paths for the aggregate emission intensity as well as the carbon stock and total output to the economy without financial frictions. In Panel (a), we observe an increase in carbon emissions accompanied by a decrease in aggregate emission intensity. Meanwhile, Panel (b) shows a rise in total output over this transition. This suggests that removing financial frictions reallocates resources towards cleaner firms. However, this reallocation is not potent enough to outweigh the increase in total output. Consequently, aggregate carbon emissions rise.

Figure 1.11: Remove only financial frictions: carbon emissions and output



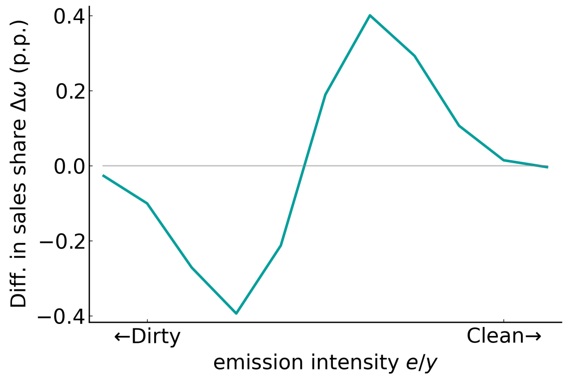
(a) Emission intensity and carbon stock

(b) Output

Notes. The x-axis denotes the transition time. In panel (a), the blue dashed line illustrates the transition paths of carbon stock, while the green solid line represents the transition paths of aggregate emission intensity, which is defined as the ratio of aggregate carbon emissions to aggregate output. The left y-axis and the right y-axis are for aggregate emission intensity and carbon stock, respectively.

Figure 1.12 directly illustrates the pattern of reallocation towards cleaner firms. It is evident that the sales share of cleaner firms expands while that of dirtier firms shrinks. Removing financial frictions certainly aids in reallocating resources towards cleaner firms. However, the extent of this reallocation is much smaller compared to the case when both adjustment costs

Figure 1.12: Remove only financial frictions



(a) Differences in sales share

Notes. The x-axis represents emission intensity, while the y-axis shows the steady-state sales share relative to the baseline economy, presented in percentage points. The green solid line indicates the relative sales shares in the steady state of the economy without financial frictions. The gray line indicates zero.

and financial frictions are eliminated. When only financial frictions are removed, the sales share of cleaner firms increases by approximately 0.4 percentage points, while that of dirtier firms decreases by a similar margin compared to the baseline economy. On the other hand, when both adjustment costs and financial frictions are eliminated, the change in the relative sales share ranges between -6 and 6 percentage points. This underscores that adjustment costs play a pivotal role in impeding the reallocation towards cleaner firms in my calibrated model.

1.8 Conclusion

This paper investigates the optimal carbon taxes in an environment where there is enormous within-industry heterogeneity in emission intensity and distortions. This exploration unfolds through theoretical, empirical, and quantitative lenses. From the simple theoretical model, I highlight the pivotal role played by the correlation between emission intensity and distortions. Specifically, carbon taxes that redirect resources towards cleaner firms have the potential to enhance allocative efficiency, particularly for those cleaner firms previously burdened by higher distortions. Utilizing firm-level data, I demonstrate that this correlation tends to be negative, indicating that carbon taxes are more inclined to reduce existing misallocation. Subsequently, through the development of a quantitative model integrating firm dynamics, carbon externalities, and firm-level frictions and distortions, I calculate the optimal carbon taxes, \$7.3 per metric tons of CO2, which is equivalent to additional 5.4 percent of corporate income tax rate. I conduct three counterfactual analyses. These exercises reveal that in cases where cleaner firms have been subject to greater distortion, policies directly targeting underlying distortions can serve as effective tools in combating climate change by reallocating resources towards cleaner firms.

Chapter 2: Macroprudential Policy with Earnings-Based Borrowing Constraints

2.1 Introduction

Should financial markets be regulated? If so, why and how? A large literature studies how the presence of borrowing constraints affects optimal regulatory policy (e.g. Dávila and Korinek, 2018, Bianchi and Mendoza, 2018). Most of this literature focuses on asset-based collateral constraints, which tie credit access to the resale value of an asset, such as a building or machine. The price of the asset can be a source of a pecuniary externality. Households or firms do not realize that their choices move asset prices in equilibrium, which in turn affects borrowing limits in the economy. A common conclusion is that agents borrow more than a social planner would prescribe. Optimal macroprudential policy therefore aims to limit debt, for example by imposing taxes on borrowing.

Meanwhile, a growing branch of research studies macroeconomic models with earnings-based borrowing constraints (e.g. Drechsel, 2023b). These constraints link firms' ability to obtain funds to their earnings, usually measured before interest, taxes, depreciation and amortization (EBITDA). Although earnings-based constraints are more prevalent for US corporations than asset-based constraints (Lian and Ma, 2020), there is still a limited understanding of how macroprudential policy should be conducted in their presence.[1,2]

The contribution of this paper is to advance our understanding of the normative consequences of earnings-based borrowing constraints in a theoretical framework. We provide analytical proofs under minimal assumptions, as well as a numerical analysis in a more general model. We contrast our insights with borrowing constraints that are commonly studied in the existing literature, for both closed and open economies.

Our findings are the following. First, in a simple closed economy setting we show how an earnings-based borrowing constraint leads to 'over-saving' and 'under-borrowing' from a welfare point of view. The intuition is that when saving increases (borrowing decreases) in the current period, saver (borrower) net worth will be higher next period. Under relevant economic conditions, which our analysis examines closely, such an increase in net worth leads real wages to rise next period. A higher real wage means higher costs and lower earnings for firms, which through the earnings-based borrowing constraint allows for less credit. However, when agents save or borrow today, they do not take into account this negative impact of their decisions today on the future borrowing limit through wages. Therefore agents save a larger (borrow a smaller) amount in the current period than what a social planner would implement as a constrained efficient allocation.

Second, this result is the opposite to what holds under asset-based borrowing constraints, which we analyze in our setting for comparison. In essence, in an earnings-based credit constraint an *input price* (through the wage bill) enters with a negative sign, while in an asset-based constraint an *asset price* (through the value of capital) enters with a positive sign. When future wages and

[1] There are a few exceptions, that is, normative analyses in which earnings do play some role in credit constraints, e.g. Bianchi (2016). We explain the differences to these formulations of financial constraints.

[2] We define macroprudential policy as regulatory policy that eliminates pecuniary externalities through *ex-ante* taxes. This includes policies that, if optimal, support borrowing through negative taxes (subsidies).

capital prices respond with the same sign to current saving and borrowing decisions, then the directions of the pecuniary externalities are the opposite for the two constraints.

Third, we compare earnings-based borrowing constraints to income-based borrowing constraints in a small open economy (SOE) setting with tradable and nontradable goods. With an income-based constraint, the external debt position of an economy is limited by its total income. As the wage bill is a payment from domestic producers to domestic employees, the wage does not affect total income and the relative price of nontradable goods is the only price that gives rise to a pecuniary externality. In contrast, an earnings-based constraint in the same economy determines borrowing capacity based on operating profits of producers, so both the price of nontradable goods and the wage give rise to pecuniary externalities. We show that prices of nontradable goods and wages respond with the same sign to current saving and borrowing decisions but enter with opposite signs in the earnings-based constraint. In consequence, there is an under-borrowing force through wages on top of a the over-borrowing force through nontradable goods prices that the literature has pointed out in this class of models.[3]

Finally, we study a numerical application in a general model with a wider array of economic channels. This includes additional externalities that work through redistribution, which are generally difficult to sign (Dávila and Korinek, 2018), but can be important in the context of collateral constraints (Lanteri and Rampini, 2021). In our main experiment, a planner calculates optimal taxes assuming that the economy features asset-based borrowing constraints. In an equally calibrated economy where firms actually borrow based on earnings, we impose these 'incorrect' taxes. We find that they lead to large welfare losses. For example, relative to imposing

[3]We also examine working capital constraints (Bianchi, 2016; Bianchi and Mendoza, 2010; Bocola and Lorenzoni, 2023; Jermann and Quadrini, 2012). We find that when firms need to pre-finance wages and also face earnings-based limits on credit, the pecuniary externality through wages is magnified.

the optimal policy, the wrongly designed tax policy leads to a loss of up to 2.55% in aggregate consumption. In light of comparable magnitudes in the literature, this is very sizable effect. Our findings make clear that optimal macroprudential policy critically depends on the specific form of financial constraints.

Our work contributes to two strands of research. The first strand studies pecuniary externalities with financial constraints.[4] Our approach is similar to Dávila and Korinek (2018) but considers a labor market and examines additional types of constraints. The introduction of a labor market provides new challenges in signing externalities, and a contribution of this paper is to determine relevant model restrictions. Our insight that higher wages tighten financial constraints is complementary to the mechanism in Bianchi (2016), where firms face working capital and equity constraints, and do not internalize that when they hire workers, wages increase, which in turn tightens equity constraints.[5] A few other studies consider income-based rather than asset-based credit constraints in normative analysis, for example Bianchi (2011) where tradable and nontradable income restrict the economy's external debt position. We contrast our results with the ones arising under those constraints. Benigno et al. (2013) and Schmitt-Grohé and Uribe (2020) also note the possibility of under-borrowing, but through channels different from ours. In Benigno et al. (2013), when the planner can use an ex-post stabilization tool, the constrained efficient allocation features more borrowing than the decentralized equilibrium. In Schmitt-Grohé and Uribe (2020) under-

[4]Important contributions include Mendoza (2006, 2010), Lorenzoni (2008), Jeanne and Korinek (2010), Korinek (2011), Bianchi (2011), Benigno et al. (2013), Bianchi (2016), Bianchi and Mendoza (2018). A related line of research studies *aggregate demand externalities* (Farhi and Werning, 2016; Schmitt-Grohé and Uribe, 2016). These do not work through financial constraints, but through the combination of nominal rigidities and other constraints, such as a fixed exchange rate. Wolf (2020) studies pecuniary externalities that arise from wage rigidities independently of financial constraints and aggregate demand channels.

[5]The pecuniary externality in Bianchi (2016) works through higher *labor demand* having a negative effect on other firms' dividend constraints. In our framework, the pecuniary externality arises from firms' current borrowing exerting a positive effect on future credit limits through *labor supply*.

borrowing is a result of precautionary savings in the face of self-fulling crises. Fazio (2021) proposes a framework with earnings-based constraints to study a credit crunch at the zero lower bound (ZLB) on interest rates. What distinguishes our paper from all of the above is that we compare a variety of credit constraints and systematically study the different policy implications. Another aspect that differentiates our paper is that we examine pecuniary externalities in a general labor market structure, with an explicit analysis of both labor demand and labor supply effects. Bianchi and Mendoza (2010), Bianchi (2016), Fazio (2021) and Bocola and Lorenzoni (2023) all focus on preferences without wealth effects on labor supply, while our setting features a more general labor supply specification. Finally, a related paper is Ottonello, Perez, and Varraso (2022) which focuses on the timing of collateral constraints and shows that policy conclusions can change depending on whether current or future prices of collateral affect credit access. Instead of timing, we focus on different variables entering borrowing constraints.

The second strand of research highlights the distinction between asset-based constraints and earnings-based constraints. Drechsel (2023b) studies how earnings-based borrowing constraints affect the transmission of macroeconomic shocks. Lian and Ma (2020) show that 80% of U.S. corporate debt is earnings-based. Caglio, Darst, and Kalemli-Özcan (2021b) show that earnings-based constraints are also prevalent for private small and medium-sized companies.[6] None of these papers consider normative implications.

This chapter is organized as follows. Section 2.2 provides the intuition behind pecuniary externalities with earnings-based borrowing constraints in a simple setting. Section 2.3 compares these insights with asset-based constraints and income-based constraints in SOEs. Section 3.5 presents the more general model. We provide more formal proofs for our earlier results, and

[6] di Giovanni et al. (2022) provide evidence for Spain and Camara and Sangiacomo (2022) for Argentina.

carry out the numerical policy experiments. Section 2.5 concludes.

2.2 Intuition for pecuniary externalities with earnings-based constraints

This section presents a simple two-period model in which borrowers face an earnings-based borrowing constraint as formulated in Drechsel (2023b). In this model, we derive our main theoretical intuition. We explain how pecuniary externalities will arise through the borrowing constraint from the way wages respond to agents' past financial decisions. We do so under different assumptions about preferences and the labor market structure.

2.2.1 Model setup

There are two time periods $t = 1, 2$. The economy is closed and populated by unit measures of borrowers and lenders, denoted by superscript $i \in \{b, l\}$. Agents have perfect foresight. Agent type i derives utility from consumption c_t^i in both periods and disutility from supplying labor ℓ_s^i at wage w in $t = 1$. Both agents are risk-neutral in $t = 2$. We examine different cases for risk aversion in $t = 1$. The borrower has access to a Cobb-Douglas production technology that uses labor ℓ_d and capital K as inputs in $t = 1$, and capital only in $t = 2$. The capital stock is fixed and owned by the borrower. The lender does not produce, but is endowed with resources e_t^l. Agent i can trade a risk-free bond x_2^i between the two periods at price m, where positive values of x indicate saving, negative values borrowing. The borrower faces the following earnings-based borrowing constraint:

$$-x_2^b \leq \phi_\pi(K^\alpha \ell_d^{1-\alpha} - w\ell_d) \tag{2.1}$$

where α is the capital share in production and $\phi_\pi > 0$ is a parameter that governs the tightness of the constraint. The difference between sales $K^\alpha \ell_d^{1-\alpha}$ and input costs $w\ell_d$ defines earnings (EBITDA) and restricts debt access (Drechsel, 2023b). Agent i holds an initial asset position x_1^i. This position results from choices in period $t = 0$ which we do not model explicitly, but which as we will describe below will be relevant in driving pecuniary externality. Taken together, the maximization problem of the borrower is

$$\max \left(\frac{(c_1^b)^{1-\gamma}}{1-\gamma} - \frac{(\ell_s^b)^{1+\psi}}{1+\psi} + \beta c_2^b \right) \tag{2.2}$$

subject to (2.1) and

$$c_1^b + m x_2^b \leq K^\alpha \ell_d^{1-\alpha} - w\ell_d + x_1^b + w\ell_s^b \tag{2.3}$$

$$c_2^b \leq A_2 K + x_2^b \tag{2.4}$$

γ and ψ are the risk aversion and Frisch elasticity parameters. The lender's problem is

$$\max \left(\frac{(c_1^l)^{1-\gamma}}{1-\gamma} - \frac{(\ell_s^l)^{1+\psi}}{1+\psi} + \beta c_2^l \right) \tag{2.5}$$

subject to

$$c_1^l + m x_2^l \leq e_1^l + w\ell_s^l + x_1^l \tag{2.6}$$

$$c_2^l \leq e_2^l + x_2^l \tag{2.7}$$

The setting nest the special cases in which agents are risk neutral ($\gamma = 0$) and in which only lenders supply labor ($\ell_s^b = 0$). We analyze these cases below.

2.2.2 Decentralized equilibrium

We solve the maximization problems of borrowers and lenders. The aggregate states of the model in $t = 1$ are denoted $X \equiv (X_1^b, X_1^l)$, and we characterize a symmetric equilibrium in which $x_1^i = X_1^i$ and the borrowing constraint binds. Combining labor market clearing $\ell_d = \ell_s^l + \ell_s^b$ with optimal choices gives

$$\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} K = \left(\frac{\beta w}{m}\right)^{\frac{1}{\psi}} + h(w, m, x_1^b) \tag{2.8}$$

where the labor supply function of the borrower $h(w, m, x_1^b)$ depends positively on w, negatively on m and x_1^b.[7] Bond market clearing $-x_2^b = x_2^l$ implies that

$$\alpha\phi_\pi \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} K = \frac{1}{m}\left(e_1^l + w\left(\frac{\beta w}{m}\right)^{\frac{1}{\psi}} + x_1^l - \left(\frac{m}{\beta}\right)^{\frac{1}{\gamma}}\right) \tag{2.9}$$

Condition (2.8) and (2.9) allow us to write the equilibrium wage and bond price as a function of the aggregate states X_1^b and X_1^l:

$$w = L(m, X_1^b) \tag{2.10}$$

$$m = B(w, X_1^l) \tag{2.11}$$

[7] The labor supply function $h(w, m, x_1^b)$ is implicitly defined by the borrower's optimality conditions $(l_s^b)^\psi = w(c_1^b)^{-\gamma}$ and $c_1^b = \alpha(1 + m\phi_\pi)(\frac{1-\alpha}{w})^{\frac{1-\alpha}{\alpha}} K + w l_s^b + x_1^b$.

where $\partial L/\partial m > 0$, $\partial L/\partial X_1^b > 0$, $\partial B/\partial w > 0$ and $\partial B/\partial X_1^l > 0$. (2.10) and (2.11) characterize the decentralized equilibrium in $t = 1$ in two price schedules for (w, m).

2.2.3 Sufficient statistics approach to pecuniary externalities

Agent i's initial asset position x_1^i results from past saving and borrowing decisions that are not explicitly modeled in this section. We study how wages change with the aggregate initial asset positions X, by determining the sign of $\partial w/\partial X$ in the equilibrium described by (2.10) and (2.11). These wage changes in turn affect the earnings-based borrowing constraint (2.1) because higher wages reduce earnings, all else equal. As wage changes in $t = 1$ and their effect on the constraint are not internalized by agents in $t = 0$, their past saving and borrowing decisions are not generally optimal when the borrowing constraint binds. Examining the sign of $\partial w/\partial X$ therefore provides the intuition for the direction in which pecuniary externalities in the earnings-based borrowing constraint result from saving and borrowing decisions in $t = 0$. In the more general model in Section 3.5, the decisions in $t = 0$ are explicitly modeled, a social planner problem is introduced, and the direction of the pecuniary externalities are proven formally.

In examining the direction of price responses to aggregate states we follow the "sufficient statistics" approach of Dávila and Korinek (2018) [henceforth 'DK18']. Similar to them, we sign the pecuniary externalities that result from past saving and borrowing decisions affecting the borrowing constraint in the current period. There are other externalities, in particular those that result from investment rather than saving and borrowing decisions and those that affect redistribution of resources across agents. It is challenging to sign these externalities in general, a result that DK18 refer to as "anything goes."

2.2.4 Equilibrium wage responses to past saving and borrowing decisions

To determine the sign of $\partial w / \partial X$, we examine the following three cases of our setting:

(i) lenders are risk neutral, borrowers are risk averse; only lenders supply labor

(ii) lenders and borrowers are risk averse; only lenders supply labor

(iii) lenders and borrowers are risk averse; all agents supply labor

Distinguishing between risk neutrality and risk aversion has two implications. First, with risk neutrality the interest rate in this economy is constant. Second, with $\gamma = 0$ in lenders' preferences, there is no wealth effect on labor supply.[8] Distinguishing which agents supply labor to begin with is relevant, because with earnings-based borrowing constraints the borrower is typically thought of as a firm. Therefore restricting the borrower to demanding labor and the lender to supplying it is a natural assumption. Making these distinctions about the setting helps us clarify the economic conditions under which the relevant pecuniary externalities will arise.

Case (i) Risk-neutrality of lenders implies that (2.11) becomes $m = \beta$ and the bond price does not depend on aggregate states. When borrowers do not supply labor, the second term of the right hand side of (2.8) disappears, and (2.10) simplifies to $w = L(m)$, so the wage also does not depend on aggregate states. Past saving and borrowing decisions do not move prices, so that $\partial w / \partial X = 0$ and no pecuniary externality operates through the earnings-based constraint. Agents' financial decisions in $t = 0$ will be constrained efficient.

Case (ii) The bond price schedule (2.11) is now a function of X_1^l, while wages depend on X_1^l only through m in (2.8). Lenders' decisions in $t = 0$ shift the bond price schedule, thereby

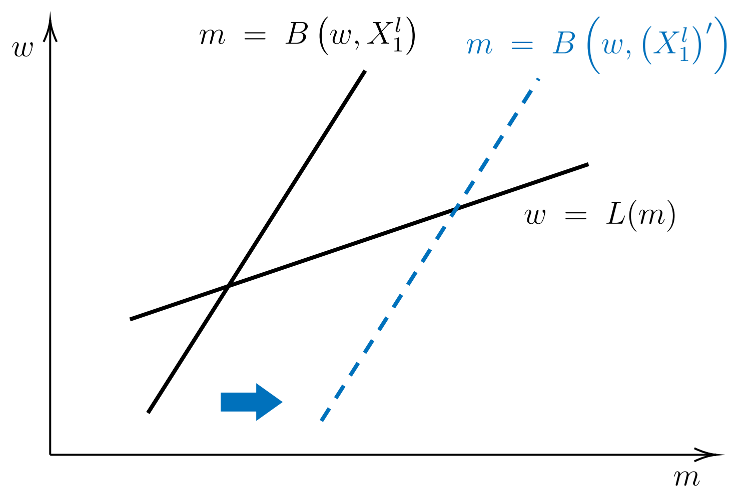
[8] To see this, note that the case $\left(c_1^l - \frac{(\ell_s^l)^{1+\psi}}{1+\psi}\right)$ represents Greenwood-Hercowitz-Huffman preferences.

affect equilibrium wages, so that $\partial w/\partial X \neq 0$. As lenders do not internalize this effect on the borrowing constraint, their $t = 0$ saving decision is not constrained efficient. To examine the direction of the pecuniary externality, note the following condition:

$$\frac{\partial w}{\partial X_1^l} \geq 0 \Leftrightarrow \frac{\partial B^{-1}}{\partial m} > \frac{\partial L}{\partial m} \tag{2.12}$$

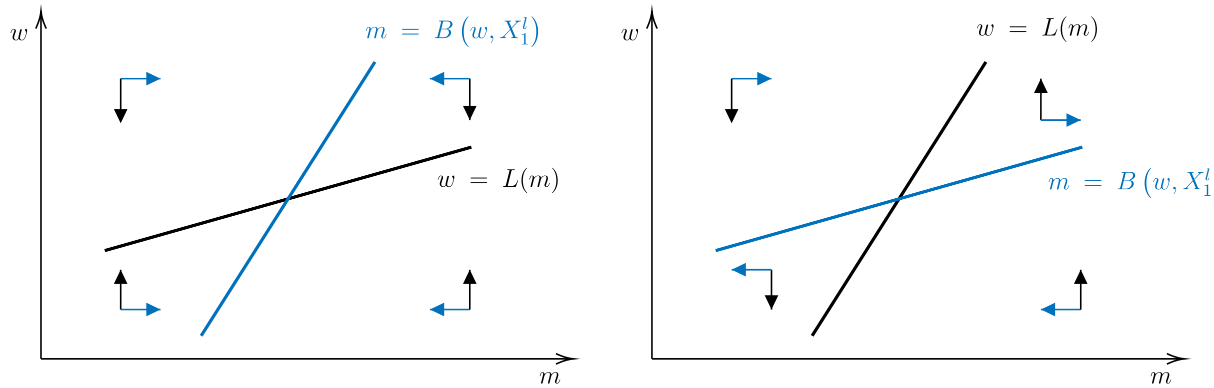
If the slope of B is steeper than the slope of L, higher lender net worth increases wages. In consequence, more saving by lenders in $t = 0$ tightens the earnings-based constraint by raising wages in $t = 1$. Figure 2.1 examines the equilibrium under condition (2.12).

Figure 2.1: Wage changes in response to past financial decisions – Case (ii)



On what grounds may the failure of condition (2.12), where $\frac{\partial B^{-1}}{\partial m} < \frac{\partial L}{\partial m}$, be ruled out? Figure 2.2 illustrates that in this case the equilibrium is unstable. The left panel presents a phase diagram corresponding to Figure 2.1, while the right panel shows a phase diagram when (2.12) does not hold. The equilibrium in the right panel is an unstable saddle point while the equilibrium in the left panel is fully stable. Thus, based on stability considerations, we argue that (2.12) is an appealing restriction. Further below, we show that this argument has an analogy under asset-based collateral constraints.

Figure 2.2: Equilibria with phase diagram under different conditions



It is possible to provide a sufficient condition on the model's parameters that ensures that (2.12) is satisfied: if $1 + \frac{\psi}{\alpha} > \frac{1-\alpha}{\alpha}$, then $\frac{\partial w}{\partial X_1^l} \geq 0$ holds. Conditional on the capital share of production, there needs to be a sufficiently strong labor supply elasticity for more lender net worth to raise wages. The Online Appendix provides the formal derivation of this sufficient condition. This derivation also makes clear that condition (2.12) generally depends on other model primitives, in particular the risk aversion γ. The condition $1 + \frac{\psi}{\alpha} > \frac{1-\alpha}{\alpha}$ therefore is not necessary, but sufficient. In the more general model below, we explore the calibration of the key parameters α and ψ.

Case (iii) When borrowers also supply labor, the wage schedule becomes a function of X_1^b.[9] Now both lenders' and borrowers' decisions in $t = 0$ affect the earnings-based constraint in $t = 1$ through equilibrium wages, and their decisions is thus not constrained efficient. The

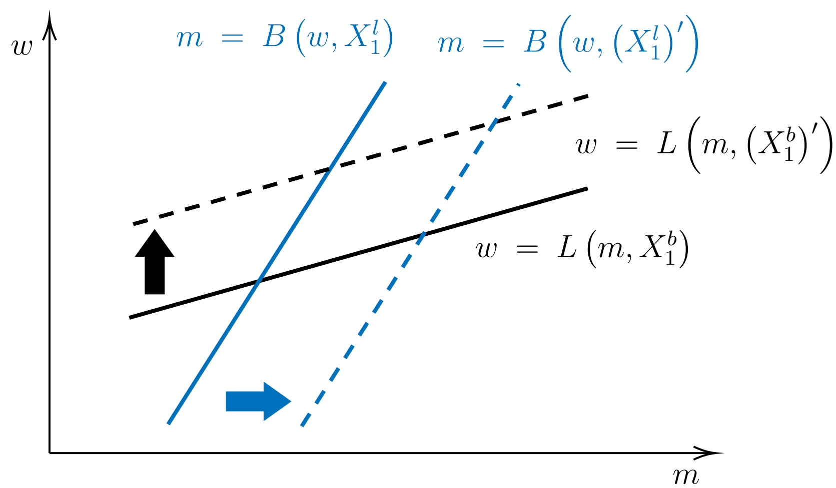
[9] This would not be true in the absence of wealth effect on borrowers' labor supply, in which case the borrowers' decisions would be constrained efficient. We omit this intermediate case, because the logic is similar to Case (i) for lenders' labor supply without wealth effects.

relevant condition generalizes to two derivatives in a similar fashion:

$$\frac{\partial w}{\partial X_1^l}, \frac{\partial w}{\partial X_1^b} \geq 0 \Leftrightarrow \frac{\partial B^{-1}}{\partial m} > \frac{\partial L}{\partial m} \tag{2.13}$$

Figure 2.3 examines the equilibrium under condition (2.13) graphically.

Figure 2.3: Wage changes in response to past financial decisions – Case (iii)



Similar to Case (ii), condition (2.13) can be supported based on stability arguments. It is not possible to derive a simple parametric sufficiency condition as in Case (ii), but it is again evident that the strength of labor supply is important as X_1^b enters (2.8) through $h(\cdot)$.

Labor demand vs. labor supply In our setting, inefficiencies in the decisions of lenders and borrowers in $t = 0$ arise from changes in labor supply in $t = 1$. To see this, note that in Case (ii) X_1^l enters in (2.11) because of wealth effects on lenders' labor supply and in Case (iii) X_1^b enters in (2.10) because of wealth effects on borrowers' labor supply. In both cases, labor demand is pinned down from optimal behavior within the period based on the predetermined capital stock, as the agents can always choose labor demand that maximizes their unconstrained objective as well as their borrowing capacity. Labor demand choices are thus not affected by changes in borrower net worth. Without labor supply reacting to changes in net worth, the allocation under an earnings-based borrowing limit would not exhibit constraint externalities through saving and borrowing choices. Providing this reasoning for signing pecuniary externalities with labor demand and labor supply is a central insight of our analysis, and makes our mechanism distinct from that in Bianchi (2016) and Bianchi and Mendoza (2010). Further below, we show that in interaction with working capital constraints, labor demand does give rise to additional pecuniary externalities with earnings-based borrowing constraints, similar to these papers.

2.2.5 Over-saving and under-borrowing effects with earnings-based constraints

The above analysis makes clear that when we consider stable equilibria in $t = 1$ with risk aversion, agents' decisions might not be constrained efficient.[10] In Cases (ii) and (iii), lenders in $t = 0$ will not internalize that saving more raises wages in $t = 1$ which in turn tightens the earnings-based constraint. From the point of view of a social planner, they thus over-save relative to the optimal allocation. In Case (iii) borrower decisions are not constrained efficient. Borrowers in $t = 0$ will not internalize that borrowing more lowers wages in $t = 1$ which in turn relaxes the earnings-based constraint. From the point of view of a social planner, they thus under-borrow relative to the optimal allocation.

We make the $t = 0$ choices as well as the planner problem explicit in a more general formulation of the model in Section 3.5. In that section, we formally prove the over-saving (under-borrowing) result by deriving the planner's optimal taxes/subsidies on borrowing. We

[10] Our analysis of the SOE setting in Section 2.3 makes clear that also with a fixed interest rate (risk neutral lenders) pecuniary externalities can arise.

show that the results on over-saving and under-borrowing hold in a more general setting as long as $\partial w/\partial X > 0$. Before generalizing the setting, we contrast the insights above with common formulations of financial constraints in the literature.

2.3 Comparison with constraints commonly studied in the literature

This section compares the implications of earnings-based borrowing constraints to those from common formulations of borrowing constraints in the macroprudential policy literature. We first focus on asset-based collateral constraints in the same setting as above. We then consider a small open economy environment and study income constraints on the economy's external debt position.

2.3.1 Over-borrowing effects with asset-based constraints

Suppose that capital is still in inelastic supply but can be traded at price q. The borrower faces the following commonly studied asset-based collateral constraint:

$$-x_2^b \leq \phi_k q k \tag{2.14}$$

where k is the capital choice and $0 < \phi_k < 1$ governs the tightness of the constraint. To demonstrate that the typical over-borrowing result holds in our setting, it is enough focus on the simplest setting with risk-neutral lenders and labor supply coming from lenders only (Case (i)). The borrower's problem becomes

$$\max \left(\frac{(c_1^b)^{1-\gamma}}{1-\gamma} + \beta c_2^b \right) \tag{2.15}$$

subject to (2.14) instead of (2.1) and subject to

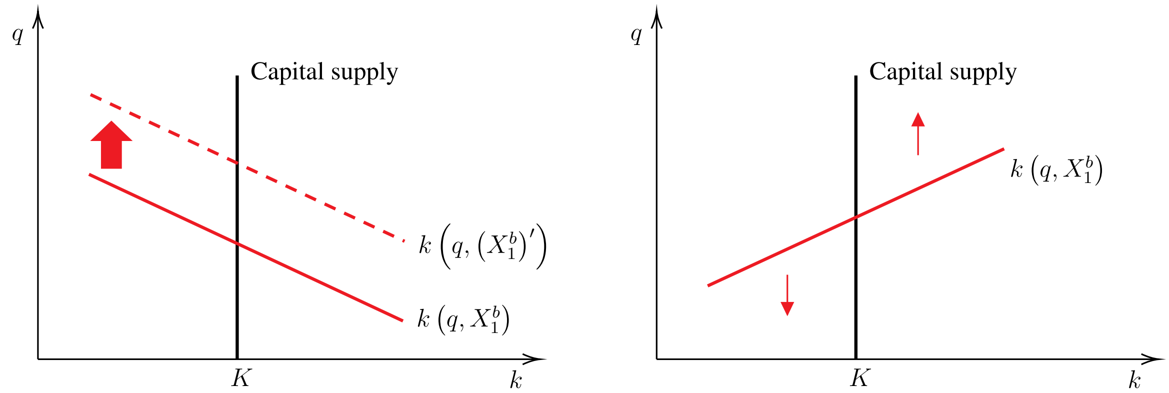
$$c_1^b + m x_2^b + qk \leq (k^\alpha \ell_d^{1-\alpha} - w\ell_d) + x_1^b + qK \tag{2.16}$$

$$c_2^b \leq A_2 k + x_2^b \tag{2.17}$$

The lender problem remains the same as in Section 2.2. We can derive a capital demand function in period $t = 1$ that depends on X_1^b, and may be upward-sloping or downward sloping.[11] Figure 2.4 shows both cases. When $\frac{\partial q}{\partial X_1^b} < 0$ (right panel), the equilibrium is not stable. Therefore $\frac{\partial q}{\partial X_1^b} \geq 0$ (left panel) is a sensible restriction. Indeed it typically features in the literature on asset-based constraints. Dávila and Korinek (2018) also show that failure of $\frac{\partial q}{\partial X_1^b} \geq 0$ leads to multiplicity and unstable equilibria. Economically, an increase in resources, holding the amount of available capital in the economy fixed, will increase the capital demand and thus put upward pressure on its price. When $\frac{\partial q}{\partial X_1^b} \geq 0$, borrowers in $t = 0$ will not internalize that saving more, and thus reducing net worth next period, will reduce capital prices in $t = 1$ and therefore tighten borrowing constraints. In consequence, they over-borrow relative to what a social planner would prescribe.

[11]Formally, solving for the $t = 1$ capital choice k as a function of q, X_1^b and predetermined capital supply gives the following relation: $k = \frac{1}{q(1-\beta\theta_k)} \left[\alpha (\frac{1-\alpha}{w})^{\frac{1-\alpha}{\alpha}} K + X_1^b + qK - \left(\frac{1}{1-\beta\theta_k} (\frac{1}{q}\beta A_2 - \beta\theta_k) \right)^{-\frac{1}{\gamma}} \right]$.

Figure 2.4: Capital price changes in response to past financial decisions



The contrast between the earnings-based and the asset-based constraint makes clear that when w and q respond with the same sign to current asset positions, then the directions of the pecuniary externalities coming from past saving and borrowing decisions are the opposite, as w and q enter with opposite sign in each constraint. Agents over-save and under-borrow with earnings-based constraints, but over-borrow with asset-based constraints. We reach the opposite conclusion from much of the previous literature on macroprudential policy with financial constraints.[12]

2.3.2 Earnings-based vs. income-based constraints in small open economies

We now consider an SOE version of the two-period model in Section 2.2. A representative household consumes tradable goods, which are the numéraire, and nontradable goods according to a CES aggregator $c = [\theta(c^T)^\rho + (1-\theta)(c^N)^\rho]^{\frac{1}{\rho}}$. The household receives an endowment of nontradable goods y^N and produces tradable goods with a Cobb-Douglas production technology

[12]Our over-saving and under-borrowing results require wealth effects on labor supply. In the absence of those effects, the normative conclusions with an earnings-based constraint would still be different from the typical over-borrowing result with an asset-based constraint, as they would imply constrained efficiency.

$y^T = zK^{\alpha}\ell^{1-\alpha}$.[13] The economy has access to a one-period bond on international markets. It is denominated in units of tradables and its exogenously fixed price is m. More details on the SOE setup are provided in the Online Appendix.

We now define income-based and earnings-based borrowing constraints and highlight the prices that enter in each constraint. Income-based constraints limit the amount that the economy can borrow externally by a fraction of *total* current income, the sum of profits, endowments, and wages, as for example in Bianchi (2011) and Benigno et al. (2013):

$$-x_2 \leq \phi_I((y_1^T - w\ell^d) + p_1 y_1^N + w\ell^s) = \phi_I(y_1^T + p_1 y_1^N), \tag{2.18}$$

where p_1 is the relative price of nontradable goods, and ℓ^d and ℓ^s are labor demand and supply. The key price in the income-based constraint is p_1, so $\partial p_1/\partial X$ determines the direction of the pecuniary externality. In Bianchi (2011), $\partial p_1/\partial X > 0$ and agents over-borrow under income-based borrowing constraints as they do not internalize that their debt positions shrink borrowing capacity through a lower p_1.

In contrast, earnings-based constraints are determined by a multiple of the EBITDA of *firms* rather than the *total* income of the economy. In the SOE economy this gives

$$-x_2 \leq \phi_\pi(y_1^T - w\ell^d + p_1 y_1^N). \tag{2.19}$$

because tradable firm earnings are $(y_1^T - w\ell^d)$ and nontradable firm earnings are $p_1 y_1^N$ (nontradable

[13] In the Online Appendix, we also consider the case with tradable goods being an endowment and production of nontradable goods. We reach similar conclusions in that alternative setting.

sector firms produce an endowment with zero costs). The key prices for the earnings-based constraints are the price of nontradable goods p_1 and wage w, so $\partial p_1/\partial X$ and $\partial w/\partial X$ are the relevant sufficient statistics. (p_1, w) are determined by

$$p_1 = \frac{1-\theta}{\theta}\left(\frac{(1+\alpha m\phi_\pi)y_1^T + X}{y_1^N} + m\phi_\pi p_1\right)^{1-\rho} \tag{2.20}$$

$$\left(\frac{(1-\alpha)z_1}{w}\right)^{\frac{1}{\alpha}} K = \ell^{s^*}, \tag{2.21}$$

where ℓ^{s^*} is the optimal labor supply which depends on preferences. In theory, $\partial p_1/\partial X$ can be either positive or negative depending on parameter values.[14] We focus our analysis on the case $\partial p_1/\partial X > 0$ as we want to contrast it with the standard over-borrowing result.

We consider two cases regarding labor supply: (i) labor supply is exogenously fixed; (ii) labor supply is endogenously determined.

Case (i) As ℓ^{s^*} is fixed, the equilibrium wage does not change with aggregate net worth i.e. $\frac{\partial w}{\partial X} = 0$. A pecuniary externality emerges only through the price of nontradable goods. With $\partial p_1/\partial X > 0$, the standard over-borrowing results hold.

Case (ii) In the SOE setting with endogenous labor supply, $sign(\partial p_1/\partial X) = sign(\partial w/\partial X)$. We show this formally in the Online Appendix. As we focus on $\partial p_1/\partial X > 0$, it is also the case that $\partial w/\partial X > 0$. Based on the arguments in Section 2.2, $\partial w/\partial X > 0$ leads to an under-borrowing force with earnings-based constraints. Thus, there is both an over-borrowing mechanism, which goes through the relative price of nontradable goods, and under-borrowing

[14] Schmitt-Grohé and Uribe (2020) show that $\partial p_1/\partial X$ can have either sign depending on parameter values, and that the equilibrium is unique with $\partial p_1/\partial X > 0$ under the calibration of Bianchi (2011). In other cases, the model features multiple equilibria and $\partial p_1/\partial X < 0$.

mechanism, which operates through wages.[15]

We conclude that in SOEs with earnings-based borrowing constraints, there is an under-borrowing force that features alongside the over-borrowing force present in income-based constraints. Which force dominates the other is a quantitative and empirical question, which we leave as an avenue for future research. Its answer depends on whether debt positions of SOEs are taken by households, firms or governments, as these agents might feature differential constraints. With income-based constraints the literature has taken a natural starting point, as they link to the total income across all of these agents. If, however, external borrowing is primarily done by firms subject to earnings-based constraints, then the contribution of the under-borrowing force could be first-order.

2.3.2.1 Discussion: working capital constraints

Firms sometimes pre-finance production inputs before revenues are collected. If the access to such *working capital*, in addition to other debt, is limited by an earnings-based constraint, this enhances the strength of the externality that operates through wages. To see this, suppose a firm takes the intertemporal position x_2^b as above, and in addition pre-finances a fraction ψ of its wage bill with an intraperiod working capital loan $x_{wc} = -\psi w \ell_d$. Such a setup is chosen, for example, by Bianchi and Mendoza (2010) and Bocola and Lorenzoni (2023). When we add working capital to our framework, an earnings-based constraint on *total borrowing* takes the form

$$-(x_2^b - \psi w \ell_d) \leq \phi_\pi (K^\alpha \ell_d^{1-\alpha} - w \ell_d) \tag{2.22}$$

[15] It could be interesting to study relative output price variation as a source of pecuniary externalities also in a closed economy setting with earnings-based constraints. Fazio (2021) explores this possibility in an environment with a manufacturing and a service sector, where manufacturing producers face a credit constraint that depends on their earnings.

which can be rearranged to

$$-x_2^b \leq \phi_\pi K^\alpha \ell_d^{1-\alpha} - (\phi_\pi + \psi) w \ell_d \tag{2.23}$$

which corresponds to (2.1), with the only difference that the parameter multiplying the wage bill is $(\phi_\pi + \psi) > \phi_\pi$. The presence of working capital thus strengthens the externality in the earnings-based constraint, leading to a more pronounced under-borrowing effect.[16]

Recall from above that in our framework without working capital there are no inefficiencies that operate through labor demand. This changes with a working capital constraint, as lower labor demand eases the working capital constraint. In this case, higher borrower net worth from past saving and borrowing decisions increases the equilibrium wage through higher labor demand. Thus, the under-borrowing effects from earnings-based constraints are magnified with working capital through both a higher parameter in front of the wage bill and an additional labor demand channel. Interestingly, in models such as Bianchi and Mendoza (2010) and Bocola and Lorenzoni (2023) agents have GHH preferences, so constraint externalities operate exclusively through labor demand.

2.4 General setting, formal proofs and numerical application

This section generalizes the model of Section 2.2 to feature three periods and capital investment. All agents are risk averse, produce and supply labor. The model is close to DK18, but with a labor market and different credit constraints. In this setting, we formally prove the

[16] To see this formally, in the proof of Proposition 2 in Section 3.5 a larger parameter multiplying the wage increases $\frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta}$ and thus drives $C_{N^i}^{b,\theta}$ more negative.

direction of the pecuniary externalities for which we developed the intuition above. We also carry out numerical model experiments.

2.4.1 Generalized model

There are three time periods $t = 0, 1, 2$. The state of nature is realized at date $t = 1$ and is denoted by $\theta \in \Theta$. Agent type $i \in \{b, l\}$ has a time separable utility function

$$U^i = E_0 \left[\sum_{t=0}^{2} \beta^t u^i(c_t^i, \ell_{st}^i) \right] \tag{2.24}$$

where $u^i(\cdot, \cdot)$ is strictly increasing and weakly concave in consumption, strictly decreasing and weakly convex in labor, and $u^i(c_0^i, \ell_{s0}^i) = u^i(c_0^i)$. There are consumption goods and capital goods. $e_t^{i,\theta}$ is the endowment of consumption goods agent i receives at date $t = 1, 2$ given state θ. Time-0 endowments are denoted by e_0^i. At date $t = 0$, agents can invest $h^i(k_1^i)$ units of consumption good to produce k_1^i units of date-1 capital goods.[17] The functions $h^i(\cdot)$ are increasing and convex and satisfy $h^i(0) = 0$. k_1^i can be used for the production of consumption goods in period $t = 1$ and be carried over for production in period $t = 2$. $k_2^{i,\theta}$ denotes the amount of capital that agent i carries from date 1 to 2. Capital fully depreciates after date 2. To produce consumption goods in $t \geq 1$, agent i employs both capital and labor to produce $F^i(k_t^{i,\theta}, \ell_{dt}^{i,\theta})$ units of the consumption good. $\ell_{dt}^{i,\theta}$ is labor demanded by agent i at date t. The production functions $F^i(\cdot, \cdot)$ are strictly increasing and weakly concave in each argument and satisfy $F^i(0, 0) = 0$.

At date $t = 0$, agents trade state-contingent assets that pay 1 unit of the consumption good in period $t = 1$ and state θ. $x_1^{i,\theta}$ denotes the date-0 state-θ purchases by agent i and m_1^θ is the

[17]Note that $k_1^{i,\theta} = k_1^i$ since it is chosen in $t = 0$, thus not conditional on the state of nature θ.

corresponding asset price, taken as given by the agent. Agent i spends $\int_{\theta\in\Theta} m_1^\theta x_1^{i,\theta} d\theta$ in total on these securities. Without further uncertainty between $t = 1$ and $t = 2$, agents trade non-contingent one-period bonds $x_2^{i,\theta}$ at time $t = 1$ at price m_2^θ. There is a competitive labor market. Wages at date $t \geq 1$ and state θ are denoted by w_t^θ. There is also a market to trade capital at a price q^θ at date 1 after production has taken place. There is no trading of capital at date 2. The budget constraints of agent $i \in \{b, l\}$ are

$$c_0^i + h^i(k_1^i) + \int_{\theta\in\Theta} m_1^\theta x_1^{i,\theta} d\theta = e_0^i \tag{2.25}$$

$$c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta}, \quad \forall\theta \tag{2.26}$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta}, \quad \forall\theta \tag{2.27}$$

where $\Delta k_2^{i,\theta} \equiv k_2^{i,\theta} - k_1^i$. Recall that the state θ materializes in $t = 1$ so choices in $t \geq 1$ are made conditional on the realized state of nature. There are constraints on the holdings of securities between periods $t = 0$ and $t = 1$, as well as between periods $t = 1$ and $t = 2$. At date $t = 0$, borrowers' holdings of $x_1^b = \{x_1^{b,\theta}\}_{\theta\in\Omega}$ are subject to a constraint

$$\Phi_1^b(x_1^b, k_1^b) \geq 0 \tag{2.28}$$

At date $t = 1$, borrowers' holdings of $x_2^{b,\theta}$ are subject to a state-dependent constraint

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0, \ \forall\theta \tag{2.29}$$

We assume $\Phi_1^l(\cdot) = \Phi_2^{l,\theta}(\cdot) = 0$, that is, lenders are financially unconstrained.

2.4.1.1 Decentralized equilibrium

A decentralized equilibrium consists of asset allocations $\{x_1^{i,\theta}, x_2^{i,\theta}\}_{i\in\{b,l\},\theta\in\Theta}$, real allocations $\{c_0^i, c_1^{i,\theta}, c_2^{i,\theta}, k_1^i, k_2^{i,\theta}, \ell_{d1}^{i,\theta}, \ell_{d2}^{i,\theta}, \ell_{s1}^{i,\theta}, \ell_{s2}^{i,\theta}\}_{i\in\{b,l\},\theta\in\Theta}$ and prices $\{q^\theta, w_1^\theta, w_2^\theta, m_1^\theta, m_2^\theta\}_{\theta\in\Theta}$, such that agents solve their optimization problems and markets clear. The market clearing conditions are shown formally in the Online Appendix. The solution for the decentralized equilibrium can be obtained via backward induction. Optimal choices at time $t = 2$ are purely intratemporal decisions on consumption and labor supply and demand. In $t = 1$, two sets of variables fully characterize the state of the economy. The first is the holdings of capital by both agents k_1^i. The second one is agents' net worth $n_1^{i,\theta} \equiv e_1^{i,\theta} + x_1^{i,\theta}$.[18] Agents take aggregate states as given so we distinguish individual states $\{n_1^{b,\theta}, n_1^{l,\theta}, k_1^b, k_1^l\}$ from aggregate states $\{N_1^{b,\theta}, N_1^{l,\theta}, K_1^b, K_1^l\}$. We further define $N_1^\theta \equiv \{N_1^{b,\theta}, N_1^{l,\theta}\}$ and $K_1 \equiv \{K_1^b, K_1^l\}$, and note that the equilibrium prices are functions of the aggregate state variables: $q^\theta(N_1^\theta, K_1)$, $m_2^\theta(N_1^\theta, K_1)$, $w_1^\theta(N_1^\theta, K_1)$, and $w_2^\theta(N_2^\theta(N_1^\theta, K_1), K_2(N_1^\theta, K_1)) = w_2^\theta(N_1^\theta, K_1)$. The optimization problem of an individual agent i at time $t = 1$ is

$$V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1) = \max_{\{c_1^{i,\theta}, c_2^{i,\theta}, k_2^{i,\theta}, x_2^{i,\theta}, \ell_{dt}^{i,\theta}, \ell_{st}^{i,\theta}\}} \left\{ u^i(c_1^{i,\theta}, \ell_{s1}^{i,\theta}) + \beta u^i(c_2^{i,\theta}, \ell_{s2}^{i,\theta}) \right\} \tag{2.30}$$

$$\text{s.t.} \;\; c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta} \qquad [\lambda_1^{i,\theta}] \tag{2.31}$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta} \qquad [\lambda_2^{i,\theta}] \tag{2.32}$$

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0 \qquad [\kappa_2^{i,\theta}] \tag{2.33}$$

[18] DK18 include production output as part of net worth. In our model, the quantity $F^i(k_1^i, \ell_{d1}^{i,\theta})$ is not predetermined because labor is chosen during $t = 1$. We therefore do not include it as part of $n_1^{i,\theta}$. In the Online Appendix, we formally verify that this does not alter the original results of DK18.

where $\lambda_1^{i,\theta}$, $\lambda_2^{i,\theta}$, and $\kappa_2^{i,\theta}$ are the Lagrange multipliers. The $t = 0$ optimization problem is

$$\max_{\{c_0^i, k_1^i, x_1^{i,\theta}\}} u^i(c_0^i) + \beta E_0[V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1)] \tag{2.34}$$

subject to (2.25) and (2.28). The Online Appendix presents the agents' first-order conditions.

2.4.1.2 Distributive effects and constraint effects

DK18 show that changes in aggregate states have *distributive effects* and *collateral effects*. We refer to the latter effects with a more general terminology as *constraint effects*.[19] Our Online Appendix formally characterizes the distributive and constraint effects in a symmetric equilibrium in which $n^{i,\theta} = N^{i,\theta}$ and $k_1^i = K_1^i$, by differentiating the indirect utility $V^{i,\theta}$ with respect to $N_1^{j,\theta}$ and $K_1^{j,\theta}$. The first of these derivatives is

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta}\mathcal{D}_{1N^j}^{i,\theta} + \lambda_2^{i,\theta}\mathcal{D}_{2N^j}^{i,\theta} + \kappa_2^{i,\theta}\mathcal{C}_{N^j}^{i,\theta} \tag{2.35}$$

where $\mathcal{C}_{N^j}^{i,\theta}$ is a constraint effect. It collects any derivatives that multiply the shadow price on the financial constraint $\kappa_2^{i,\theta}$, and depends on price changes as follows

$$\mathcal{C}_{N^j}^{b,\theta} \equiv \frac{\partial\Phi_2^{b,\theta}}{\partial q^\theta}\frac{\partial q^\theta}{\partial N_1^{j,\theta}} + \frac{\partial\Phi_2^{b,\theta}}{\partial m_2^\theta}\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}} + \frac{\partial\Phi_2^{b,\theta}}{\partial w_1^\theta}\frac{\partial w_1^\theta}{\partial N_1^{j,\theta}} + \frac{\partial\Phi_2^{b,\theta}}{\partial w_2^\theta}\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}} \tag{2.36}$$

Instead, $\mathcal{D}_{1N^j}^{i,\theta}$ and $\mathcal{D}_{2N^j}^{i,\theta}$ in (2.35) are distributive effects which net out across the agents. Relative to DK18, both constraint and distributive effects feature additional economic forces in our model.

[19]This is because we study credit constraints that do not necessarily contain "collateral" in the sense of physical assets. Alternatively, one could re-label the earnings-based borrowing constraint as a "collateral constraint" in which earnings serve as collateral. We instead refer to collateral more narrowly as the presence of physical k in the borrowing constraint.

In particular, (2.36) makes clear that wages give rise to constraint effects, which we will show leads to pecuniary externalities with earnings-based constraints.

2.4.1.3 Social planner problem and constrained efficient allocation

The social planner chooses allocations in $t = 0$ subject to the same period-0 constraints as the private agents, and subject to optimal behavior of the agents in periods $t = 1, 2$. This corresponds to a constrained Ramsey planner who can levy taxes in $t = 0$. Formally,

$$\max_{\{C_0^i \geq 0, K_1^i, X_1^{i,\theta}\}} \sum_i \alpha^i \{u^i(C_0^i) + \beta E_0[V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)]\} \tag{2.37}$$

$$\text{s.t.} \sum_i [C_0^i + h^i(K_1^i) - e_0^i] \leq 0 \quad (v_0) \tag{2.38}$$

$$\sum_i X_1^{i,\theta} = 0, \quad \forall\theta \quad (v_1^\theta) \tag{2.39}$$

$$\Phi_1^i(X_1^i, K_1^i) \geq 0, \ \forall i \quad (\alpha_i \kappa_1^i) \tag{2.40}$$

Note that α^b and α^l are Pareto weights that the social planner applies to borrowers and lenders, respectively. The variables in brackets denote Lagrange multipliers. The presence of $V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)$ makes clear that the planner takes the private equilibrium of periods $t = 1, 2$ as given and internalizes the impact of changing N^θ and K_1 on prices.

The economy's constrained efficient allocation is described by quantities $(C_0^i, K_1^i, X_1^{i,\theta})$, Pareto weights $\alpha^b/\alpha^l = \lambda_0^l/\lambda_0^b$ and shadow prices v_0, v_1^θ, and κ_1^i satisfying the optimality conditions and constraints of the social planner's problem. This allocation can be implemented with a set of tax rates on financial asset and capital purchases. We relegate the derivations to the

Online Appendix. The tax rate on saving is

$$\tau_x^{i,\theta} = -\Delta MRS_{01}^{ij,\theta}\mathcal{D}_{1N^i}^{i,\theta} - \Delta MRS_{02}^{ij,\theta}\mathcal{D}_{2N^i}^{i,\theta} - \tilde{\kappa}_2^{b,\theta}\mathcal{C}_{N^i}^{b,\theta},\ \forall i,\theta \tag{2.41}$$

$\Delta MRS_{0t}^{ij,\theta} \equiv MRS_{0t}^{i,\theta} - MRS_{0t}^{j,\theta}$ denotes the difference between agents' marginal rate of substitution (MRS) across time, $MRS_{01}^{j,\theta} \equiv \beta\lambda_1^{j,\theta}/\lambda_0^j$, $MRS_{02}^{j,\theta} \equiv \beta\lambda_2^{j,\theta}/\lambda_0^j$. We define $\tilde{\kappa}_2^{b,\theta} \equiv \beta\kappa_2^{b,\theta}/\lambda_0^b$ as the relative shadow price. The $\mathcal{D}$ and $\mathcal{C}$ terms correspond to the distributive and constraint effects discussed above.

2.4.1.4 Nature of externalities and sufficient statistics

The optimal tax (2.41) combined with the constraint effects $\mathcal{C}$ in (2.36) allow us to characterize externalities through a compact list of sufficient statistics. Externalities are determined by the product of the relative shadow price of the financial constraint $\tilde{\kappa}_2^{i,\theta}$, the sensitivity of the financial constraint to the price of capital, asset price and wages $\partial\Phi_2^{i,\theta}/\partial q^\theta$, $\partial\Phi_2^{i,\theta}/\partial m_2^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_1^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_2^\theta$, and the sensitivity of the equilibrium capital price, asset price and wages in periods 1 and 2 to changes in aggregate states $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}$. By analyzing and interpreting price changes, we can study how market outcomes deviate from the constrained efficient allocation and how such distortions are corrected by the planner. A positive $\tau_x^{i,\theta}$ implies that agent i saves too much (borrows too little) in the market outcome, so the planner imposes a tax on savings (subsidy on borrowing).

DK18 show that distributive externalities as well as constraint externalities from changes in aggregate capital cannot generally be signed. In our formal proofs below, we therefore focus on over-/under-borrowing instead of over-/under-investment effects, and on constraint externalities rather than distributive externalities. In the numerical application, we allow for all possible forces, so the planner chooses a tax on capital purchases τ_k^i in addition to $\tau_x^{i,\theta}$, and internalizes both $\mathcal{D}$ and $\mathcal{C}$ effects.

2.4.2 Formal proofs for pecuniary externalities

The following conditions specialize the economic setting enough to determine the sign of the constraint externalities for the financial constraints of interest.

$$\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0, \ \forall i \tag{2.42}$$

$$\frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0, \ \forall i \tag{2.43}$$

We interpret these conditions in Sections 2.2 and 2.3.1. In our numerical application below, we verify the conditions under specific functional forms for preferences and technology. We can now formally derive efficiency properties of different forms of the financial constraint (2.33). Consider first the case of an asset-based collateral constraint. (2.33) becomes

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \phi_k q^\theta k_2^{b,\theta} \geq 0 \tag{2.44}$$

Proposition 1 *A collateral constraint as defined by (2.44), as long as it binds, gives rise to non-negative constraint externalities. This implies that there is an over-borrowing effect that operates through the constraint externalities.*

Proof. From (2.44), $\phi_k > 0$ and $k_2^{b,\theta} \geq 0$ it follows that $\frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \geq 0$. According to condition (2.43), $\frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0$. Therefore $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-negative, that is, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \geq 0$. This implies that there is over-borrowing operating through the constraint externalities: as is visible in equation (2.41), the social planner imposes subsidies on savings $\tau_x^{i,\theta}$ in order to induce less borrowing. ■

Next, consider an earnings-based borrowing constraint. (2.33) is specified as

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \phi_\pi (F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta}) \geq 0 \tag{2.45}$$

Proposition 2 *An earnings-based borrowing constraint as defined by (2.45), as long as it binds, gives rise to non-positive constraint externalities. This implies that there is an over-saving (under-borrowing) effect that operates through the constraint externalities.*

Proof. From (2.45), $\phi_\pi > 0$ and $\ell_{d1}^{b,\theta} \geq 0$ it follows that $\frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \leq 0$. According to (2.42), $\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0$. Therefore, $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \leq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-positive, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \leq 0$. This implies that there is over-saving (under-borrowing) operating through the constraint externalities: as is visible in equation (2.41) the planner imposes taxes on savings (subsidies on borrowing) $\tau_x^{i,\theta}$ in order to induce less saving (more borrowing). ■

Propositions 1 and 2 underscore the insights of our simple model in Sections 2.2 and 2.3.1 more formally. The Online Appendix provides a graphical illustration of our proofs.[20]

[20] In an earlier version of this paper (Drechsel and Kim, 2022b), we also study interest coverage constraints, which restrict the ratio of interest payments to earnings. See also Greenwald (2019). An interest coverage constraint leads to either over-borrowing or under-borrowing, and can be interpreted as a mixture between an asset-based and earnings-based constraint from a welfare point of view.

2.4.3 Numerical application

This section conducts policy experiments in a parameterized version of the model. We quantify the welfare loss that arises from imposing an 'incorrect' macroprudential policy, where the true model is an economy with earnings-based borrowing constraints, but we impose tax rates that are computed as optimal under the assumption that agents face asset-based constraints. In this experiment, both distributive and constraint externalities, as well as both under- and over-borrowing and under- and over-investing, are at play.

2.4.3.1 Model specification

There is no uncertainty and no period-0 financial constraint. We consider the case where labor supply is inelastic and the case where it is optimally chosen. In the case of inelastic labor supply, the period utility function follows the log-utility specification $u^i(c^i_t, \ell^i_{st}) = \log(c^i_t)$. In the case of endogenously determined labor supply, the period utility function follows a standard separable utility specification with wealth effects on labor supply $u^i(c^i_t, \ell^i_{st}) = \log(c^i_t) - \frac{1}{1+\psi}(\ell^i_{st})^{1+\psi}$ for $t \geq 1$. We assume a constant to returns to scale (CRS) and a decreasing returns to scale (DRS) production function for the borrower and the lender, respectively. Formally, $F^b(k^b_t, \ell^b_{dt}) = z_b(k^b_t)^\alpha(\ell^b_{dt})^{1-\alpha}$ and $F^l(k^l_t, \ell^l_{dt}) = z_l((k^l_t)^\alpha(\ell^l_{dt})^{1-\alpha})^\nu$ where we assume $z_b > z_l$ and $\nu < 1$. Following DK18, $h^i(k) = \frac{\eta}{2}k^2$.

2.4.3.2 Parameterization

Table 2.1 summarizes our parameterization. We set β to 0.9752 following Drechsel (2023b) who targets average US corporate loan rates. The Frisch elasticity ψ and returns to scale ν are set to 2 and 0.75 as in Jungherr and Schott (2021). We set the tightness parameter of the asset-based constraint ϕ_k following Bianchi (2016), who uses the average leverage ratio of US non-financial corporations of 46% as a target. We then calibrate ϕ_π to ensure that the debt-to-output ratio is the same across the economies in which we calculate the optimal tax rates and the one in which we impose them. We do this separately for the case with inelastic labor supply and the case with endogenous labor supply. We set the remaining parameters to ensure that the borrower has a superior production technology ($z_b > z_l$), but lacks the endowments to make capital investment relative to the lender.

Table 2.1: Calibration of the model

Parameter	**Description**	**Value**	**Source / Target**
α	Capital share	0.33	Standard for US case
β	Discount factor	0.9752	Drechsel (2023b)
ψ	Labor supply elasticity	2	Jungherr and Schott (2021)
ν	Returns to scale - lender	0.75	Jungherr and Schott (2021)
ϕ_k	Borrowing limit - asset	0.46	Bianchi (2016)
ϕ_π	Borrowing limit - earnings (inelastic labor)	0.534	Match debt-to-output, $\frac{-x_2^b}{y_1^b+y_1^l}$
	Borrowing limit - earnings (endogenous labor)	0.617	Match debt-to-output, $\frac{-x_2^b}{y_1^b+y_1^l}$
η	Investment technology	1	Normalization
(z_b, z_l)	Productivity	(2,1)	
(e_0^b, e_1^b, e_2^b)	Endowments - borrower	(0,0,0)	
(e_0^l, e_1^l, e_2^l)	Endowments - lender	(1,1,0)	

Validity of model restrictions Based on the parameterization of the model, we verify numerically that the model restrictions required to derive our formal theoretical analysis above, indeed hold. That is, the calibration of the model implies $\frac{\partial q}{\partial N_1^i} \geq 0$, $\frac{\partial w_1}{\partial N_1^i} \geq 0, \ \forall i$.

2.4.3.3 Determining the tax schedule in asset-based economy

We first solve the planner problem in an economy with asset-based borrowing constraints. We set (α_b, α_l) to achieve the same ratio of period-0 consumption as in the corresponding decentralized equilibrium. This leads to $(\alpha_b, \alpha_l) = (0.05, 0.95)$ for the case with inelastic labor supply and $(\alpha_b, \alpha_l) = (0.20, 0.80)$ for the case with endogenous labor supply. We then compute the optimal corrective taxes $(\tau_x^b, \tau_x^l, \tau_k^b, \tau_k^l)$ at the constrained efficient allocation.[21] To separate distributive and constraint externalities, we also compute that component of optimal taxes on borrowing/saving that arises from the constraint externalities at the constrained efficient allocation, $\tau_x^{i,c.e.} = -\tilde{\kappa}_2^b \mathcal{C}_{N^i}^b$. $\forall i$.

2.4.3.4 Imposing the 'wrong' tax schedule in earnings-based economy

Next we consider the 'true' economy with earnings-based borrowing constraints. First, we compute the welfare gain from moving from the decentralized equilibrium to the constrained efficient allocation in this economy. This is done with the same welfare weights as in the asset-based economy. We call this the 'right' policy. Second, we compute the welfare change from imposing the corrective taxes that we optimally derived in the economy with asset-based constraints above. We call this the 'wrong' policy. Following Jones and Klenow (2016), we compute how much of permanent consumption should be inflated or deflated when we change from allocation B to allocation A, by finding λ such that

$$
\begin{aligned}
SW^{B,\lambda} &\equiv \alpha_b \sum_{t=0}^{2} \beta^t u((1+\lambda)c_{bt}^B, \ell_{bt}^B) + \alpha_l \sum_{t=0}^{2} \beta^t u((1+\lambda)c_{lt}^B, \ell_{lt}^B) \\
&= \alpha_b \sum_{t=0}^{2} \beta^t u(c_{bt}^A, \ell_{bt}^A) + \alpha_l \sum_{t=0}^{2} \beta^t u(c_{lt}^A, \ell_{lt}^A) \equiv SW^A.
\end{aligned}
$$

Under log-utility assumption, λ is derived as $\lambda = \exp\left((SW^A - SW^B)\frac{1-\beta}{1-\beta^3}\right) \times 100$ (%), where $SW^B \equiv \alpha_b \sum_{t=0}^{2} \beta^t u(c_{bt}^B, \ell_{bt}^B) + \alpha_l \sum_{t=0}^{2} \beta^t u(c_{lt}^B, \ell_{lt}^B)$. Finally, similar to Lanteri and Rampini

[21] Savings taxes τ_x^i are determined by (2.41). The optimal tax on capital investment is derived in an analogous way in the Online Appendix.

(2021) we assume that agents are reimbursed a lump-sum amount that corresponds to the amount they paid or received through distortionary taxes.

2.4.3.5 Optimal corrective taxes in different economies

Table 2.2 shows the tax rates that implement constrained efficient allocation for each economy. The subscripts x and k indicate taxes on saving in the financial asset and saving in capital, respectively. The table shows these two tax rates separately for the lender and the borrower, and additionally reports the component of the corrective taxes on saving due to constraint externalities only, $\tau_x^{b,c.e.}$ and $\tau_x^{l,c.e.}$. The negative sign of these tax rates in the asset-based economy, and the positive sign in the earnings-based economy with endogenous labor supply confirm our findings from above. There is over-borrowing with a collateral constraint, so the social planner levies a negative tax on saving, $\tau_x^{i,c.e.} < 0$. There is over-saving (under-borrowing) with the earnings-based constraint, so the social planner taxes saving (subsidizes borrowing) through $\tau_x^{i,c.e.} > 0$. If labor is inelastic, however, the allocation with the earnings-based constraint is already constrained efficient, so $\tau_x^{i,c.e.} = 0$.

Table 2.2: Optimal corrective taxes in different economies (in $\%$)

Economy	$\boldsymbol{\tau_x^b}$	$\boldsymbol{\tau_x^l}$	$\boldsymbol{\tau_k^b}$	$\boldsymbol{\tau_k^l}$	$\boldsymbol{\tau_x^{b,c.e.}}$	$\boldsymbol{\tau_x^{l,c.e.}}$
Collateral constraints, inelastic labor	-21.1	4.0	-29.1	-29.4	-0.3	-0.1
Earnings-based constraints, inelastic labor	-8.2	-1.3	-26.7	-12.4	0.0	0.0
Collateral constraints, endogenous labor	-1.6	-3.4	-1.0	0.6	-1.9	-3.2
Earnings-based constraints, endogenous labor	0.3	0.4	-2.6	-7.1	0.9	0.3

Table 2.2 also shows that the fully optimal taxes $(\tau_x^b, \tau_x^l, \tau_k^b, \tau_k^l)$ are large compared to the components that address the constraint externalities only. This indicates that distributive externalities and over- and under-investment forces, which cannot be signed in general, are

quantitatively large. This is in line with the findings of Lanteri and Rampini (2021).

2.4.3.6 Results of numerical policy experiment

We calculate how much macroprudential policy designed under imprecise assumptions about financial constraints deteriorates social welfare. Table 2.3, Panel (a) shows the welfare results when both distributive and constraint externalities are operational. With earnings-based borrowing constraints, the constrained efficient allocation leads to a 0.60% higher permanent consumption than the decentralized equilibrium. Importantly, when the wrong policy is rolled out, consumption equivalent welfare decreases by 1.95% and 0.52% relative to the decentralized equilibrium for the economy with inelastic and endogenous labor supply. The table also reports the difference in consumption equivalents between imposing the right and the wrong policy, which amounts to as much as 2.55% in the economy where labor supply is inelastic. To put these magnitudes into context, in Bianchi (2011) the welfare gains from correcting the externality are 0.135% of permanent consumption. In Bianchi and Mendoza (2018) the welfare gain from implementing the optimal policy is 0.3% in permanent consumption. The wrong policy thus worsens social welfare significantly, relative to the market allocation and even more so relative to the optimal policy.

Table 2.3: Consumption equivalent welfare change in different counterfactuals

Panel (a): all types of externalities			
Economy	**Right policy, $\lambda(\%)$**	**Wrong policy, $\lambda(\%)$**	**$\Delta(\%)$**
Earnings-based, inelastic labor	0.60	-1.95	-2.55
Earnings-based, endogenous labor	0.60	-0.52	-1.12

Panel (b): constraint externalities only			
Economy	**Right policy, $\lambda(\%)$**	**Wrong policy, $\lambda(\%)$**	**$\Delta(\%)$**
Earnings-based, inelastic labor	0.00	-0.01	-0.01
Earnings-based, endogenous labor	0.06	-0.47	-0.53

Notes. The table shows the welfare impact of policies carried out in the 'true' economy, which features earnings-based constraints. The right policy is the solution to the social planner's problem in that economy. It moves the allocation in the decentralized equilibrium to the constrained efficient allocation. The wrong policy is calculated under the incorrect assumption that agents face asset-based borrowing constraints. It moves the allocation in the decentralized equilibrium to allocation that arises from the wrong policy.

Panel (b) separately breaks out results for the effects of constraint externalities only. As there is no inefficiency through constraint externalities in the earning-based economy with inelastic labor supply, social welfare is not altered through the right policy. With endogenous labor supply, the right policy increases permanent consumption only marginally, by 0.06%. However, the wrong policy decreases permanent consumption by 0.01% and 0.47% for the economy with inelastic and endogenous labor supply. Compared to the optimal policy, a consumption loss of as much as 0.53% is incurred by the agents. These effects are still meaningful, and larger than some results in the literature. The Online Appendix provides robustness checks for the calibration underlying our numerical experiments.

2.5 Conclusion

This paper examines normative implications of earnings-based credit constraints. Our results have important implications for the design of an effective regulatory system. Macroprudential policy guided solely by an asset-based collateral mechanism might be counterproductive in credit markets where earnings-based borrowing constraints are dominant. The evidence motivating our analysis focuses on nonfinancial companies, so the regulation of corporate credit is where our insights are most applicable. Collateral constraints are a more central force in household mortgage markets, where real estate serves as collateral, or in trade between financial institutions, where financial assets are pledged in repurchase agreements. This paper makes the case for studying carefully which pecuniary externalities are critical in which types of credit markets, and shows that the distinction between asset and input prices in credit constraint is of first-order importance for optimal policy.

Chapter 3: The Welfare Consequences of a Bankruptcy Reform - Evidence from the 2020 Small Business Reorganization Act

3.1 Introduction

Chapter 11 reorganization in the U.S. Bankruptcy Code was designed to rehabilitate operationally efficient but financially insolvent businesses. Legal experts have long argued that Chapter 11's high costs lead small but viable businesses to instead choose Chapter 7 liquidation (American Bankruptcy Institute (2014)). In response to this argument, Congress amended the existing Bankruptcy Code by passing the Small Business Reorganization Act (SBRA) to reduce the costs of Chapter 11 for small business debtors. This law became effective on February 19, 2020. This recent policy reform raises an interesting economic question: what are the impacts of a *size-dependent* bankruptcy reform such as the SBRA on aggregate consumption, output, and productivity? This paper answers this question using a quantitative general equilibrium model in which heterogeneous firms make bankruptcy decisions.

Before exploring the quantitative implications of the SBRA, I introduce novel data to study (direct) bankruptcy costs in Chapter 7 and 11. By combining the U.S. Trustee Chapter 7 Final Reports, Lexis Public Records, and Federal Judicial Center's Integrated Database, I establish two empirical facts. First, direct bankruptcy costs, measured as a ratio of monthly administrative expenses to total assets or total liabilities, are higher in Chapter 11 than Chapter 7, after correcting for potential selection biases due to endogeneity of the choice of bankruptcy chapter. Second, I find no evidence that the gap between the costs under Chapters 7 and 11 is higher when debtors have a smaller *debt*. This result casts doubt on the rationale for the SBRA. However, the gap between Chapters 7 and 11 is higher when debtors have smaller *assets*. Thus, the definition of firm size matters for whether Chapter 11 is relatively more costly than Chapter 7 for small businesses.

By further utilizing Federal Judicial Center's Integrated Database, I investigate how the composition of bankruptcy filing changed after the SBRA. Using 2019 Chapter 11 share as counterfactual to control for any seasonal variations, I find that Chapter 11 share is higher for small businesses after the SBRA effective date and before mid-March while Chapter 11 share does not change significantly for large businesses for the same period. However, after the national emergency had been declared in mid-March, Chapter 11 share for small businesses significantly dropped. As this pandemic period brings a number of factors that affect business bankruptcy margin, it would be hard to identify the pure effect of the SBRA on bankruptcy Chapter composition. Thus, the results before mid-March might say the SBRA worked as intended, but we need more evidences from data that is going to be generated in a normal economy.

Armed with these facts, I use a general equilibrium model with heterogeneous firms that have bankruptcy options to quantitatively explore the impact of the SBRA on aggregate consumption, output, and productivity. I study the effects of the SBRA in the model of Corbae and D'Erasmo (2020), but I made three important modifications. First, I re-formulate the bankruptcy costs to depend on the level of debt and allow small and large businesses (in terms of their debts) to have different cost functions. Second, I assume that bankruptcy costs are transfers from bankrupt firms to households, so there is no direct deadweight loss from bankruptcy costs. This assumption allows me to focus on the impact of changing bankruptcy costs on aggregate consumption only through indirect equilibrium effects, by not treating the lower bankruptcy costs from the SBRA as a direct reduction in deadweight losses in the economy. Lastly, I calibrate the model parameters with data moments that come from samples including small operating and bankrupt firms.

I conduct a counterfactual analysis to quantify the impact of the SBRA. The SBRA has important implications on bankruptcy decisions and aggregate variables. First, the SBRA expands the range of firms that file for Chapter 11 reorganization as bankruptcy costs fall. Interestingly, both firms that choose Chapter 7 liquidation and firms that would not declare bankruptcy absent the SBRA change their decisions to choose Chapter 11 reorganization under the SBRA.

Second, the SBRA *increases* average debt financing costs for firms. The interest rate of defaultable debt is determined by expected payoffs to lenders. Two countervailing forces are at play. The SBRA decreases the interest rates by raising payoffs to lenders. This comes from the fact that some previously liquidated firms now choose to reorganize. However, at the same time, the SBRA increases the interest rate by prompting otherwise solvent firms to choose reorganization, in which the lenders are worse off. In addition, the reform affects lenders' recovery rates in Chapter 11. Under the calibrated parameter values in this paper, we observe a higher debt financing cost for firms on average.

Lastly, the SBRA increases aggregate consumption (welfare) (+0.10%), while it decreases aggregate output (-0.01%) and aggregate productivity (-0.02%). Lower Chapter 11 costs cause both otherwise-solvent firms and those in Chapter 7 liquidation to choose Chapter 11. Reorganizing inefficiently liquidated firms makes the economy better off, while reorganizing otherwise-solvent firms moves the economy away from the efficient allocation. In addition to these channels, changes in the investment decisions of firms in normal operation and Chapter 11 could also change welfare, as investment decisions are generally inefficient in the model due to financial frictions. The decrease in aggregate output is caused by less entry due to the higher debt financing costs. Aggregate productivity decreases as less entry and exit shifts the productivity composition toward low productivity firms.

This chapter proceeds as follows. In Section 3.2, I provide some background on the U.S. Bankruptcy Code and the SBRA, and review the existing literature. In Section 3.3, I describe the novel data and document summary statistics. In Section 3.4, I discuss my methodology to correct for potential selection biases and present empirical results. In addition, I investigate how composition of bankruptcy filings changed after the SBRA. In Section 3.5, I outline a general equilibrium model with bankruptcy choices. Section 3.6 presents the calibration, bankruptcy cost estimates, and the quantitative impacts of the SBRA. Section 3.7 concludes.

3.2 Background

3.2.1 Chapter 7 liquidation vs. Chapter 11 reorganization

Under current U.S. law, businesses have two bankruptcy options: Chapter 7 liquidation and Chapter 11 reorganization. The main difference between Chapter 7 and Chapter 11 concerns whether a bankrupt firm can maintain its operations. Under Chapter 7 liquidation, a trustee is appointed to sell the company's assets to pay off debt. The company stops all of operations and goes out of business. In contrast, a firm that files under Chapter 11 intends to reorganize existing debts and continue in business, with the hope of returning to normal operations in the future.

It is worthwhile to note that Chapter 11 in general is better for creditors. First, debt recovery rates are higher in Chapter 11 than Chapter 7 for both secured and unsecured creditors (Bris et al. (2006)). Second, there is a "best interest of creditors" test in Chapter 11 in which debtors must guarantee to repay at least the amount that creditors would get if the case were converted to Chapter 7. One might wonder why creditors do not voluntarily waive Chapter 11 provisions such as creditors committees, which are costly to debtors, to encourage debtors to choose Chapter 11. The answer is that creditors committees "(1) consult with the debtor in possession on administration of the case (2) investigate the debtor's conduct and operation of the business and (3) participate in formulating a plan," so they are important safeguards to creditors.[1] Therefore, if creditors were to waive such a provision, the high recovery rates observed in Chapter 11 might no longer be guaranteed to creditors.

3.2.2 The SBRA

Chapter 11 was designed for complex debt reorganization. Legal commentators have long argued that Chapter 11's high costs and complexities make it difficult for small businesses to choose Chapter 11 reorganization over Chapter 7 liquidation (American Bankruptcy Institute (2014)). To address this concern, Congress amended the existing Bankruptcy Code by passing the SBRA, which became effective on February 19, 2020.

The SBRA includes several key provisions that are applicable to firms filing for Chapter 11 with debt below a threshold level. Under the SBRA, a creditors committee need no be formed, and a disclosure statement is not required. Second, under the SBRA, only debtors may file a reorganization plan. In a typical Chapter 11 case, the debtor's exclusive right to

[1] Source: https://www.uscourts.gov/services-forms/bankruptcy/bankruptcy-basics/chapter-11-bankruptcy-basics

file a reorganization plan is guaranteed only for an exclusivity period. The SBRA's elimination of potential competing plans from creditors prevents conflicts that prolong the reorganization process and increase costs for debtors. I note that the "best interest of creditors" still applies under the SBRA. Lastly, the SBRA relaxes the requirements to confirm a reorganization plan. In a typical Chapter 11 case, it is usually hard to confirm a reorganization plan over creditors' objections. However, the SBRA makes confirmation easier. A reorganization plan will be confirmed so long as it earmarks all of the small business debtor's projected disposable income to make payments under the plan for a period of 3 to 5 years.

Interestingly, under the CARES Act of March 2020, Congress increased the debt threshold for applying the SBRA from $2,725,625 to $7,500,000 until March 27, 2021, to alleviate the negative economic shock from the pandemic. Although bankruptcy judges sometimes exercise discretion and take employment effects of a business bankruptcy into account, the increase in the debt ceiling for SBRA eligibility is the first instance of an explicit bankruptcy policy change adopted to counteract a recession.[2] It might be worthwhile to think about whether bankruptcy policy could be used more generally as a macro-stabilization policy in other recessions.[3] However, in this paper I consider only the long-run implications of the SBRA and use only pre-SBRA data.

In this draft, I model the SBRA as a lower fixed cost of filing Chapter 11 for small businesses. However, some provisions, such as making it harder for creditors to object to a reorganization plan, could be modeled as an increase in debtor's bargaining power in in the reorganization process. I will consider this channel in a later draft, but the economic impacts of a lower fixed cost and a higher bargaining power for debtors in Chapter 11 should be similar,

[2] For more discussion of judges' discretion, see Liscow (2016).

[3] See Auclert et al. (2019) for the possibility of using time-varying consumer bankruptcy policy as a macro-stabilization tool.

as both channels lead debtors to choose Chapter 11 more often.

3.2.3 Contributions and Literature Review

This paper contributes to the scarce literature studying how firm dynamics are affected by bankruptcy institutions.[4] Corbae and D'Erasmo (2020) study a hypothetical bankruptcy reform suggested by Aghion et al. (1992) and the American Bankruptcy Institute (2014) using a firm dynamics model with both Chapter 7 and Chapter 11 bankruptcy options.[5] They show that the hypothetical reform increases aggregate productivity and welfare. Although Corbae and D'Erasmo (2020) provide a good benchmark, one assumption of their model makes it inconvenient to analyze a size-dependent bankruptcy reform. The cost of filing bankruptcy in their model is a function of a firm's productivity, which is a primitive of the model. This makes it hard to decrease the cost of bankruptcy filing specifically for small firms, as small firms can either be firms with low productivity or firms with high productivity that are small due to adjustment costs. In this sense, their model is not well-suited to experiment with a size-dependent bankruptcy reform as long as the cost is linked to exogenous productivity. In addition, their model parameters are calibrated by large firms data, which is inappropriate to study a bankruptcy reform focusing on small businesses. Peri (2020) studies the impact of pro-creditor bankruptcy reform on firm dynamics when a Chapter 11 filing firm can reorganize not only its debt but also its labor contracts. He shows that the change in expected recovery rates to creditors from a pro-creditor

[4]The literature is broadly connected to the vast literature that studies the impact of financial frictions on firm dynamics and aggregate productivity. See Cooley and Quadrini (2001), Arellano et al. (2012), D'Erasmo and Boedo (2012), and Midrigan and Xu (2014b).

[5]The hypothetical bankruptcy reform analyzed by Corbae and D'Erasmo (2020) proposes a fresh start for a firm. This means that existing debt is discharged, and the new all-equity firm is distributed to former claim holders by absolute priority rules. The hypothetical reform is different from Chapter 11 as (1) Chapter 11 discharges only part of existing debt and (2) creditors who forgive their debt are not assigned equity in Chapter 11.

bankruptcy reform depends on the degree of labor market imperfections. Tamayo (2017) studies firm dynamics and alternative bankruptcy institutions with financial constraints that are micro-founded from the assumptions of limited commitment and asymmetric information. I contribute to the literature by studying the impact of a size-dependent bankruptcy reform on firm dynamics and aggregate productivity using a quantitative general equilibrium model that is calibrated by small and large bankrupt firms data.

This paper is also related to the finance literature documenting direct bankruptcy costs in Chapter 7 and 11. Measuring bankruptcy costs is a first-order issue in the corporate finance literature. According to the trade-off theory of capital structure, firms balance the tax shield benefits and bankruptcy costs of issuing debt to determine their optimal capital structure. For example, if bankruptcy costs are small, firms should have a large amount of debt. Bris et al. (2006) provide a comprehensive study of the costs of Chapter 7 and Chapter 11 in a sample of 300 public and private businesses between 1995 and 2001 in the Federal Bankruptcy Courts of Arizona (AZ) and Southern New York (SNY). Although their data cover both public and private firms, there are two weaknesses in their sample. First, Chapter 11 comprises 80% of their sample, while in the U.S. Courts statistics the fraction of Chapter 11 business bankruptcies was roughly 26.5% for the year 2001.[6] Second, SNY is well-known as a venue in which many large firms file for Chapter 11 reorganization. Besides, some of the Chapter 7 cases in their sample originally filed for Chapter 11 and later converted. Therefore, it is likely that their sample is biased toward large firms. I contribute to the literature by estimating direct bankruptcy costs using a much larger sample with broader coverage of small firms, using novel data combining

[6]Source: https://www.uscourts.gov/statistics/table/f-2/statistical-tables-federal-judiciary/2001/12/31.

the U.S. Trustee Program Chapter 7 Final Report, Lexis Public Records, and the Federal Judicial Center's Integrated Database.

Lastly, this project is potentially related to the literature studying how the COVID crisis affected small and medium-sized enterprises (SMEs). Gourinchas et al. (2020) estimate the impact of the COVID-19 crisis on business failures among SMEs, and they document a 9% increase in SMEs failure rate for firms without government support. Greenwood et al. (2020) project that bankruptcy filing increased by 140% in 2020 compared with the previous year. They argue that restructuring subsidies and payment deferrals could be suitable options for small businesses, as court congestion and excessive liquidation will be severe problems. Hanson et al. (2020) discuss the Federal Reserve's Main Street Lending Program for small and medium-sized businesses and why the government needs to intervene using business credit programs. Drechsel and Kalemli-Özcan (2020) propose direct cash transfers to SMEs experiencing liquidity shortfalls. As eligibility for the SBRA was expanded under the CARES Act, the structural framework in this paper could shed light on the effectiveness of countercyclical policy that helps insolvent SMEs, if the framework is properly modified to include business cycle aspects.

3.3 Novel Data on Business Bankruptcy

3.3.1 Bankruptcy Costs Data

I obtain data on bankruptcy costs from the U.S. Trustee Program (USTP) Chapter 7 Final Report.[7] USTP is the component of the Department of Justice that supervises private trustees across the U.S. In a Chapter 7 bankruptcy case, a private trustee is appointed by the USTP

[7] Antill (2020) uses the USTP to analyze if trustee fees in Chapter 7 are excessively high.

representative. The private trustee's role is to liquidate the debtor's assets and distribute them to creditors.

When a Chapter 7 case with distributable assets is closed, the private trustee has to file a report that accounts for the distribution of the debtor's assets to creditors and administrative expenses. In each Chapter 7 asset case, I observe the total administrative fees paid. These *direct* bankruptcy costs include court, legal, and accounting fees and expenses paid to the trustee and other professionals.

Interestingly, for Chapter 7 cases that converted from other bankruptcy chapters (e.g., Chapter 11), the final report has information about *unpaid* administrative fees and expenses from the prior chapter. This information allows me to estimate a *lower bound* of direct bankruptcy costs during Chapter 11. In doing so, we should keep in mind that a converted case usually spends a shorter time in Chapter 11 than a case that starts and closes in Chapter 11. Based on the sample, which will be introduced below, I confirm that, on average, cases that start and close with Chapter 11 last 716.7 days, while converted cases spend only 349.3 days in Chapter 11 bankruptcy. If we do not consider the shorter duration of Chapter 11 in converted cases, we would *under-estimate* bankruptcy costs in Chapter 11 using this data. From now on, I am going to call firms that file and close with Chapter 11 as pure Chapter 11 firms, firms that file Chapter 11 but convert to Chapter 7 as converted Chapter 11 firms, and firms that file and close with Chapter 7 as pure Chapter 7 firms. I note that some firms convert from Chapter 7 to Chapter 11, but they are very rare.

3.3.2 Detailed Bankruptcy Cases Data

I gather unique data on Chapter 7 and Chapter 11 bankruptcy filings from Lexis Public Records (hereafter Lexis), which compiles bankruptcy filing information from the U.S. court system.[8] The data contains legal information about each filing, including the date the case was filed, the court in which it was filed, the judge assigned to the case, an indicator for whether the filing was involuntary, the Employer Identification Number (EIN) of the filing firm, and status updates on the case. According to Bernstein et al. (2019b), the Lexis bankruptcy data covers essentially 100% of all court cases across all bankruptcy districts from 1995. To the best of my knowledge, this is unique data covering the universe of bankrupt firms excluding sole proprietorships.

This comprehensive coverage is the main strength of the Lexis bankruptcy data. Previous empirical studies on business bankruptcy mostly rely on The UCLA-LoPucki Bankruptcy Research Database (BRD), BankruptcyData from New Generation Research, and Compustat. These data only cover large public firms, while the Lexis bankruptcy data also includes small and medium-sized enterprises. It might be problematic to analyze firm dynamics or business cycle implications of various bankruptcy institutions based on samples of mostly large firms, as large bankrupt firms mostly file for Chapter 11 reorganization.

There are two caveats regarding Lexis. First, Lexis does not include much firm-level information. Therefore, it has to be linked to other data such as Compustat, BvD Orbis, or the Census Longitudinal Business Database (LBD) through various firm identifiers. Second, businesses in the Lexis data are either partnerships or corporations. The Lexis bankruptcy data

[8] Iverson (2018), Bernstein et al. (2019b), and Bernstein et al. (2019a) use Lexis. However, their data set contains only Chapter 11 bankruptcy cases.

does not include sole proprietorships. According to the 2018 County Business Patterns (CBP), individual proprietorship firms comprise 3.7% and 10.8% of the total number of employees and establishments, respectively. Therefore, insufficient coverage of sole proprietorships might not be a first-order issue when analyzing the macroeconomic consequences of bankruptcy institutions.

In this draft, I utilize Lexis to extract information on filing, conversion, and closing dates of bankruptcy cases, which are used to compute monthly bankruptcy costs.

3.3.3 Bankruptcy Filers Data

I obtain data on bankruptcy filers from the Federal Judicial Center Integrated Database (hereafter, FJC). The FJC includes all bankruptcy petitions filed after October 1, 2007 and any petitions filed before October 1, 2007 that were still pending on that date.

As FJC contains the universe of bankruptcy cases filed or continuing after October 1, 2007, it is comparable with Lexis. Some advantages and disadvantages make both data sets useful. First, FJC has information about the debtor's total assets, total liabilities, and secured and unsecured debt claims, while Lexis does not. Second, FJC does not have information to identify debtors such as EIN and debtor name, while Lexis has those debtor identifiers. Lastly, FJC does not have data about conversion dates for converted cases, while Lexis has detailed date information.

Figure 3.1 illustrates the procedure for merging the three data sets. First, I merge USTP into Lexis using four identifiers (case number, state, firm name, and closing date), as there is no unique identifier encompassing both data sets. The case number is a unique identifier within a very detailed court office, e.g., California Eastern - Sacramento. USTP does not have detailed court information. Instead, it has information about the state of the USTP regional office that oversees a Chapter 7 bankruptcy case. The state of the USTP regional office could be different from the state of the bankruptcy court in which a debtor files. For example, some bankruptcy cases from DC are handled by the USTP regional office in Virginia. Therefore, I complement the merging procedure by utilizing fuzzed matching with the firm name and closing date.[9]

Having merged USTP and Lexis data, I combine them with FJC. The USTP-Lexis merged data has more detailed information about bankruptcy courts than FJC. For example, we know from FJC that a bankruptcy case is filed at California Eastern, but only Lexis would report that it is filed at California Eastern - Sacramento. Therefore, I utilize case number, filing court district, and the bankruptcy filing date to merge USTP and Lexis with FJC.

I further clean the data by excluding pre-packs, dismissed, and open cases and some cases without any status information.[10] In addition, I exclude cases involving subsidiaries of the same company. Thus, an observation represents a firm, not a single case. These procedures are consistent with Bris et al. (2006). Lastly, I exclude observations that have internal inconsistency. For example, the variable *easst* categorizes a filing firm's assets with some ranges, and the variable *totassts* presents an actual asset value. I drop all observations in which the actual asset value does not belong to the reported range in *easst*. The same consistency check is also done for liabilities. I treat zero liabilities and bankruptcy costs as missing and winsorize all variables at the 3% and 97% level.[11]

[9]The fuzzed matching procedure for firm names is done using the *matchit* function in Stata.

[10]Some Chapter 11 cases are "pre-packaged" in the sense that a plan of reorganization is confirmed by creditors prior to bankruptcy filing.

[11]Zero liabilities and bankruptcy costs do not make sense as we study "bankruptcy" cases, so I treat them as mis-reported.

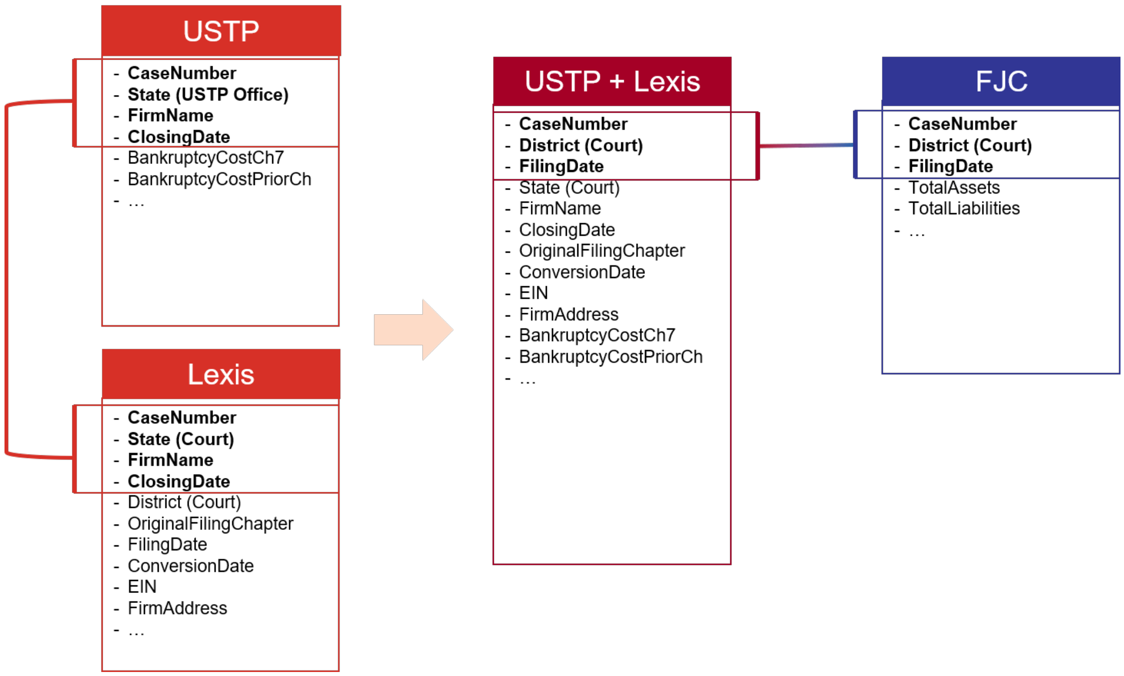


Figure 3.1: Data Merge Diagram

3.3.4 Summary Statistics

Table 3.1 presents summary statistics for pure Chapter 7 firms, converted Chapter 11 firms, and pure Chapter 11 firms.[12] The average number of creditors is higher for firms filing Chapter 11, and is higher for firms staying in Chapter 11 than for firms converting to Chapter 7. Chapter 11 firms have a higher share of secured debt, and pure Chapter 11 firms have a higher share of secured debt than converted Chapter 11 firms. Differences in indebtedness are surprising. Chapter 7 firms, on average, have a much higher leverage ratio than Chapter 11 firms. This fact is in contrast to Bris et al. (2006) and Corbae and D'Erasmo (2020), which found a modestly higher level of indebtedness for Chapter 11 firms than Chapter 7 firms.[13] Total

[12]Note that the number of observations for monthly bankruptcy costs is lower than the number of observations for other variables as (i) we have bankruptcy costs data only for asset cases and (ii) we do not have information about bankruptcy costs for cases closed after 2015.

[13]It is worth noting that the model of Corbae and D'Erasmo (2020) implies a higher leverage ratio for Chapter 7 firms, which is inconsistent with their findings from the Compustat sample. This is possible as they did not target the differences in indebtedness between Chapter 7 and 11 firms when calibrating their model parameters. However,

assets and liabilities are larger for firms in Chapter 11 than Chapter 7, and for pure Chapter 11 firms than converted Chapter 11 firms. On average, monthly bankruptcy costs are four times larger in Chapter 11. Similarly, the monthly bankruptcy costs scaled by total liabilities are higher for Chapter 11 firms. However, monthly bankruptcy costs scaled by total assets are higher for Chapter 7 firms. This is because firms in Chapter 7 have a much higher leverage ratio. We have to be careful in interpreting these facts at face value. Chapter choice and conversion decisions are endogenous, so further in-depth analysis correcting for selection bias is crucial. I formally confront this issue below. As mentioned above, duration in Chapter 7 is shorter than in pure Chapter 11, while converted Chapter 11 cases spend much less time prior to conversion than pure Chapter 11 cases. Thus, we have to account for the shorter duration of the converted Chapter 11 cases when comparing bankruptcy costs.

One might wonder if the coverage of small firms in the data is broad enough, as this paper intends to study a bankruptcy reform targeting small businesses. Calibrating a model with data that only contains large firms such as Compustat would be inappropriate. However, Table 3.1 indicates that many firms in my sample are smaller than the SBRA threshold. To further illustrate this point, Figure 3.2 shows the distribution of (log) total assets and liabilities. Separate distributions are drawn for all observations and for observations with a non-missing ratio of bankruptcy cost / total liabilities, as we only use the observations with non-missing bankruptcy cost data to run the (second-stage) regression in the later section.

We see that a substantial portion of firms are below the SBRA threshold. In the full sample,

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the result from their model is consistent with the summary statistics in this paper.

[14]The number of creditors comes from the variable *ecrdtrs* in FJC. This variable categorizes the number of creditors. For example, "A" in *ecrdtrs* represents that the number of creditors is between 1 and 49. I take the median value for each category. This explains why the values at the 10th and 90th percentiles are identical.

A. Pure Chapter 7	N	Mean	SD	P10	P50	P90
Number of Creditors / 100[14]	39,622	0.819	21.505	0.250	0.250	0.745
Secured Liabilities / Total Liabilities	35,090	0.181	0.299	0.000	0.000	0.731
Total Liabilities / Total Assets	29,360	245.897	699.626	1.261	10.712	536.611
Total Assets (2005 $mil.)	40,039	0.203	0.778	0.000	0.006	0.377
Total Liabilities (2005 $mil.)	35,090	0.921	2.049	0.048	0.289	2.019
Monthly Bankruptcy Costs (2005 $)	6,299	728.510	1,330.041	44.891	263.029	1,759.548
Monthly Bankruptcy Costs / Total Assets (%)	6,270	1.959	4.858	0.036	0.450	3.563
Monthly Bankruptcy Costs / Total Liabilities (%)	6,215	0.157	0.242	0.007	0.064	0.413
Time Spent (days)	40,039	503.071	538.550	70	298	1,216

B. Converted Chapter 11	N	Mean	SD	P10	P50	P90
Number of Creditors / 100	4,038	1.013	3.879	0.250	0.250	1.495
Secured Liabilities / Total Liabilities	3,626	0.538	0.366	0.000	0.597	0.984
Total Liabilities / Total Assets	3,576	20.352	191.637	0.576	1.576	10.997
Total Assets (2005 $mil.)	4,071	1.882	2.461	0.000	0.718	6.708
Total Liabilities (2005 $mil.)	3,626	3.339	4.145	0.246	1.665	9.852
Monthly Bankruptcy Costs (2005 $)	583	3,002.166	2,953.786	154.087	1,697.226	7,790.595
Monthly Bankruptcy Costs / Total Assets (%)	577	0.954	3.008	0.009	0.160	1.866
Monthly Bankruptcy Costs / Total Liabilities (%)	567	0.229	0.328	0.005	0.082	0.792
Time Spent (days)	4,069	349.261	329.316	73	257	719

C. Pure Chapter 11	N	Mean	SD	P10	P50	P90
Number of Creditors / 100	6,784	3.525	43.056	0.250	0.250	1.495
Secured Liabilities / Total Liabilities	6,692	0.621	0.356	0.000	0.744	0.993
Total Liabilities / Total Assets	6,684	21.915	212.295	0.564	1.416	8.289
Total Assets (2005 $mil.)	6,845	2.597	2.811	0.063	1.319	7.885
Total Liabilities (2005 $mil.)	6,692	4.129	4.846	0.269	2.001	15.001
Time Spent (days)	6,845	716.746	483.991	288	586	1,325

Table 3.1: Firm-Level Summary Statistics

90.7% and 82.3% of firms have assets and liabilities below $2,725,625, respectively.[15] In the sample with non-missing data on the bankruptcy cost ratio, 93.0% and 83.3% of firms have assets and liabilities below the SBRA threshold, respectively.

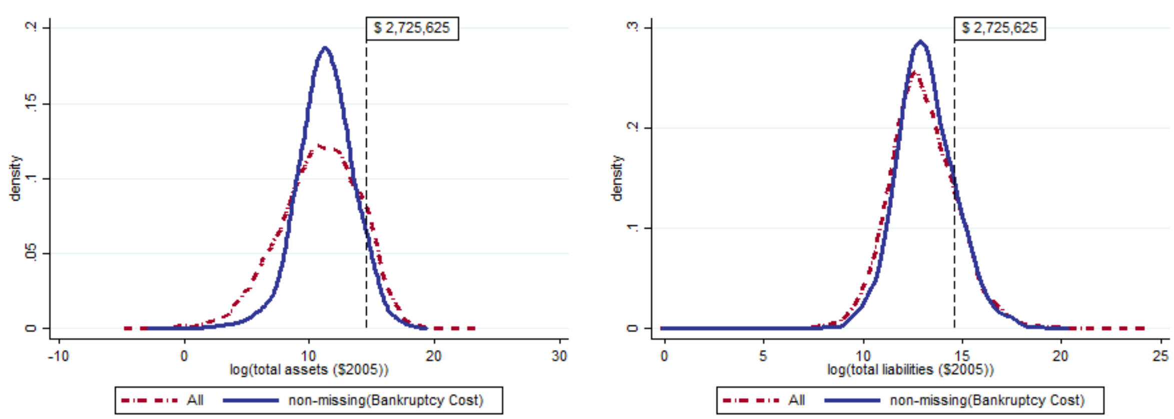


Figure 3.2: Distribution of Assets and Liabilities

[15]The $2,725,625 threshold is in terms of 2019 dollars. When I compute the share of the firms below the SBRA threshold, I deflate the threshold to 2005 dollars.

3.4 Empirical Analysis

Armed with the data described above, this section explores empirically (i) whether Chapter 11 reorganization is more costly than Chapter 7 liquidation, especially for small businesses, and (ii) how composition of bankruptcy chapters changes after the SBRA.

3.4.1 Is Chapter 11 More Costly than Chapter 7 for Small Businesses?

There has been an argument from the legal side that Chapter 11 reorganization is particularly costly for small businesses, but there is no formal empirical evidence to support this argument. A naive approach would be to compare bankruptcy costs for firms in Chapter 7 and Chapter 11, and looking at how the gap in bankruptcy costs between Chapter 7 and 11 varies by firm size. However, the choice of bankruptcy chapter is endogenous, so firms that go through Chapter 7 might be different from firms in Chapter 11. Therefore, bankruptcy costs in Chapter 11 might be higher not because Chapter 11 is inherently more expensive but because larger firms file Chapter 11. In addition, the bankruptcy costs for Chapter 11 in this sample come from firms that initially file Chapter 11 but convert to Chapter 7 later. Just as Chapter 11 firms can be very different from Chapter 7 firms, converted Chapter 11 firms can be different from pure Chapter 11 firms. Thus, it is important to deal with this two-layer selection issue to correctly estimate how the difference in bankruptcy costs for Chapter 7 and 11 varies by firm size.

3.4.1.1 Framework

Ideally, what I want to do is to compare the bankruptcy costs for firms that file and close in Chapter 11 and Chapter 7. Unfortunately, due to data limitations, I only have bankruptcy cost information for firms that start and end in Chapter 7 and firms that start with Chapter 11 and convert to Chapter 7. However, I can compare some observables between pure Chapter 11 firms and converted Chapter 11 firms.

To grasp what I can do given the limitations, I start with the most ideal case. If I had information on bankruptcy costs for pure Chapter 11 firms and pure Chapter 7 firms, I would run the following regression:

$$Y = \beta' X + \eta_1 Q + \eta_2 (Q \times log(size)) + \epsilon,$$

$$Q = \begin{cases} 1, & \text{if a firm files and closes with Chapter 11,} \\ 0, & \text{if a firm files and closes with Chapter 7,} \end{cases}$$

where Y is the monthly bankruptcy cost divided by firm size, as measured by total assets or total liabilities, X is a set of controls, and $size$ is the level of assets or liabilities.[16] As bankruptcy chapter is endogenously chosen by firms, we need to deal with potential selection bias. By using conventional methods for correcting selection bias, such as Heckman 2-stage correction, I can estimate η_1 and η_2 consistently.

As mentioned above, I do not have bankruptcy cost data for pure Chapter 11 firms. Therefore, I cannot estimate η_1 and η_2, and comparing bankruptcy costs for pure Chapter 11 firms and pure Chapter 7 firms is impossible. Instead, I can analyze differences in bankruptcy costs for firms that file Chapter 11, including both pure and the converted Chapter 11 cases, and Chapter 7. I now lay out an econometric strategy to accomplish this task.

[16]The set of controls includes a dummy variable for a bankruptcy case filed by creditors (forced petition), the number of creditors, the share of secured debt, and a dummy for a debt-to-asset ratio greater than 1. I follow Bris et al. (2006) in my choice of control variables for comparison.

My regression specification is as follows,

$$Y = \beta' X + \delta_1 Q_{11} + \delta_2 (Q_{11} \times log(size)) + \epsilon, \tag{3.1}$$

$$Q_{11} = \begin{cases} 1, & \text{if a firm initially files for Chapter 11,} \\ 0, & \text{if a firm initially files for Chapter 7,} \end{cases}$$

where Y, X, and $size$ are the same as the previous notation.

I model the endogenous choice of initial chapter as a function of latent variables:

$$Q_{11} = \begin{cases} 1, & S_1^* = \gamma_1' X_{sub} + u_1 \geq 0, \\ 0, & S_1^* < 0. \end{cases}$$

I apply a similar model to the conversion decision $Q_c : \Omega_c = \{\omega \in \Omega : Q_{11}(\omega) = 1\} \to \{0, 1\}$ where Ω is the original sample space,

$$Q_c = \begin{cases} 1, & S_2^* = \gamma_2' X_{sub} + u_2 \geq 0, \\ 0, & S_2^* < 0, \end{cases}$$

where $Q_c = 1$ represents conversion from Chapter 11 to Chapter 7. Note that X_{sub} is a subset of X including the number of creditors, the share of secured liabilities, the leverage ratio, and the logarithm of total assets or liabilities. I pick the set of variables following Bris et al. (2006). Given Q_{11} and Q_c, I define a variable for sample selection, s, to match the regression model (3.1) to my sample,

$$s = \begin{cases} 1, & (Q_{11} = 1, Q_c = 1) \text{ OR } (Q_{11} = 0), \\ 0, & (Q_{11} = 1, Q_c = 0), \end{cases}$$

where $s = 1$ represents converted Chapter 11 or pure Chapter 7 firms, and $s = 0$ represents pure Chapter 11 firms, which are not observed in my bankruptcy cost sample.

Multiplying s by the regression model (3.1) allows us to identify potential selection bias in my sample,

$$sY = s\beta' X + \delta_1 s Q_{11} + \delta_2 s(Q_{11} \times log(size)) + s\epsilon. \tag{3.2}$$

If $E(s\epsilon | sQ_{11}) = 0$, the errors in Equation (3.2) are orthogonal to sQ_{11}, so there is no bias from the sample selection. To check this condition, I have to determine $E(s\epsilon | Q_{11} = 1, Q_c = 1)$ and $E(s\epsilon | Q_{11} = 0)$, which represent the conditional expectations of errors for converted Chapter 11 and pure Chapter 7 firms, respectively.

(ϵ, u_1, u_2) follows a trivariate normal distribution,

$$\begin{pmatrix} \epsilon \\ u_1 \\ u_2 \end{pmatrix} \sim N(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} , \begin{pmatrix} \sigma_\epsilon^2 & \rho_1 \sigma_\epsilon & \rho_2 \sigma_\epsilon \\ \rho_1 \sigma_\epsilon & 1 & 0 \\ \rho_2 \sigma_\epsilon & 0 & 1 \end{pmatrix}).$$

I assume that the correlation between u_1 and u_2 is zero. This assumption means that the unobserved characteristics that affect the chapter choice and the conversion decision are uncorrelated.

I assume this for tractability, but it could be reasonable as the decisions regarding initial chapter choice and conversion might be made separately. It is worth noting that the variances of u_1 and u_2 cannot be estimated, as the latent variables S_1^* and S_2^* are not directly observed. Therefore, I normalize the variances to one, following the standard approach (Greene (2018)).

Under Assumption 3.4.1.1,

$$E(s\epsilon|Q_{11}=1,Q_c=1)=\rho_1\sigma_\epsilon\frac{\phi(\gamma_1' X_{sub})}{\Phi(\gamma_1' X_{sub})}+\rho_2\sigma_\epsilon\frac{\phi(\gamma_2' X_{sub})}{\Phi(\gamma_2' X_{sub})},$$
$$E(s\epsilon|Q_{11}=0)=\rho_1\sigma_\epsilon\frac{-\phi(\gamma_1' X_{sub})}{1-\Phi(\gamma_1' X_{sub})},$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the PDF and CDF of the standard normal distribution.

Proof. By Assumption 3.4.1.1,

$$E(\epsilon|u_1,u_2)=\rho_1\sigma_\epsilon u_1+\rho_2\sigma_\epsilon u_2.$$

By the law of iterative expectations,

$$\begin{aligned}E(s\epsilon|Q_{11}=1,Q_c=1)&=E(E(\epsilon|u_1,u_2)|Q_{11}=1,Q_c=1)\\&=E(\rho_1\sigma_\epsilon u_1+\rho_2\sigma_\epsilon u_2|Q_{11}=1,Q_c=1)\\&=\rho_1\sigma_\epsilon E(u_1|u_1\geq-\gamma_1' X_{sub})+\rho_2\sigma_\epsilon E(u_2|u_2\geq-\gamma_2' X_{sub})(u_1\perp u_2)\\&=\rho_1\sigma_\epsilon\frac{\phi(\gamma_1' X_{sub})}{\Phi(\gamma_1' X_{sub})}+\rho_2\sigma_\epsilon\frac{\phi(\gamma_2' X_{sub})}{\Phi(\gamma_2' X_{sub})}.\end{aligned}$$

Similarly,

$$E(s\epsilon|Q_{11}=0)=\rho_1\sigma_\epsilon\frac{-\phi(\gamma_1'X_{sub})}{1-\Phi(\gamma_1'X_{sub})}.$$

■

From Lemma 3.4.1.1, we have to control for $\frac{\phi(\gamma_1'X_{sub})}{\Phi(\gamma_1'X_{sub})}$ and $\frac{\phi(\gamma_2'X_{sub})}{\Phi(\gamma_2'X_{sub})}$ for converted Chapter 11 firms and for $\frac{-\phi(\gamma_1'X_{sub})}{1-\Phi(\gamma_1'X_{sub})}$ for pure Chapter 7 firms when estimating equation (3.2) to get unbiased estimators of δ_1 and δ_2.

One can implement the methodology by the following procedure:

(1) Estimate γ_1 and γ_2 using probit analysis for the full sample and for the subsample of Chapter 11 filing firms, respectively.

(2) From the estimates of γ_1 and γ_2, one can compute $\frac{\phi(\gamma_1'X_{sub})}{\Phi(\gamma_1'X_{sub})}$, $\frac{\phi(\gamma_2'X_{sub})}{\Phi(\gamma_2'X_{sub})}$, and $\frac{-\phi(\gamma_1'X_{sub})}{1-\Phi(\gamma_1'X_{sub})}$.

(3) The estimated values of the correction terms are used as additional regressors in equation (3.2) for the converted Chapter 11 and the pure Chapter 7 firms.

3.4.1.2 Results

Table 3.2 reports the first-stage probit regressions for the chapter choice and conversion decisions. First, firms with a higher share of secured debt, lower leverage ratio, and higher assets or liabilities are more likely to choose Chapter 11 over Chapter 7. Second, a firm with fewer creditors, a lower share of secured debt, and lower assets or liabilities is more likely to convert to Chapter 7 conditional on initially choosing Chapter 11. The leverage ratio is not a significant predictor of the case conversion decision.

	Chapter 11		Conversion	
Number of Creditors / 100	-0.00	-0.00	-0.01***	-0.01***
	(0.00)	(0.00)	(0.00)	(0.00)
Secured Liabilities / Total Liabilities	0.65***	1.03***	-0.35***	-0.37***
	(0.03)	(0.02)	(0.04)	(0.04)
Total Liabilities / Total Assets > 1 [Y/N]	-0.21***	-0.92***	-0.03	0.00
	(0.02)	(0.02)	(0.03)	(0.03)
log(Total Assets)	0.33***		-0.02**	
	(0.00)		(0.01)	
log(Total Liabilities)		0.37***		-0.02*
		(0.01)		(0.01)
Constant	-4.69***	-5.27***	0.12	0.11
	(0.06)	(0.08)	(0.11)	(0.14)
R squared	0.3652	0.3140	0.0128	0.0126
Observations	39,326	39,326	10,203	10,203

Table 3.2: Probit Regressions for Chapter Choice and Conversion

Notes. * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$. Standard errors in parentheses.

Table 3.3 reports the main regression results for monthly bankruptcy costs scaled by firm size. It shows OLS and selection-corrected results when the size variable is total assets or total liabilities. Three things are notable. First, Chapter 11 reorganization is more costly than Chapter 7 liquidation after we correct for selection bias. The signs of the OLS results are similar to those from Table 3.1, where monthly bankruptcy costs over total assets were higher for Chapter 7 firms while the monthly bankruptcy costs over total liabilities were higher for Chapter 11 firms. However, the coefficients on Chapter 11 are significantly positive for both measures of size after we correct for selection bias. One might wonder why the coefficient in OLS regressions is underestimated. This is not easy to explain, as we do not have a good theory of how the bankruptcy costs and chapter choice and conversion decisions are influenced by unobserved variables. What we can learn from the regressions at best is as follows. According to Lemma 3.4.1.1, the signs of the coefficients of the Mills ratios represent the correlation between u and ϵ.

As the estimated coefficients of the Mills ratios are negative, we infer that the unobserved factors determining bankruptcy costs and the chapter choice and conversion decisions are negatively correlated. In other words, $\delta_1 + \delta_2$ should be underestimated in the OLS regressions, which is what we observe.

The result that bankruptcy costs are higher for Chapter 11 is worthy of further comment, as it contrasts with Bris et al. (2006). They find that " $\cdots$ Chapter 11 cases consume more fees proportionally, not because Chapter 11 is intrinsically the more expensive procedure, but rather only because self-selecting Chapter 11 firms intrinsically require more expenses." This finding has important implications for parametrizing bankruptcy costs in a structural model. If we believe the results from Bris et al. (2006), there would be no reason to differentiate bankruptcy cost parameters for Chapter 7 and 11. However, the results in this paper argue that we should differentiate them.

Second, the coefficient for the interaction between choosing Chapter 11 and total assets is significantly negative after correcting for selection bias, while the estimated impact of the interaction between Chapter 11 and total liabilities is insignificant. This implies that Chapter 11 is disproportionately costly for businesses with low assets, but there is no evidence that businesses with low debts pay a relatively higher cost for Chapter 11. This suggests that the special treatment for low debt firms in the SBRA may have a weak empirical justification. One might wonder if the insignificance of debt stems from the fact that the measured bankruptcy costs in Chapter 11 in this sample represent only the "unpaid" costs in Chapter 11 prior to conversion. This might be true if the share of costs under Chapter 11 that was unpaid at the time of conversion is particularly large or small for businesses with high debt.

Lastly, the coefficients for the Mills ratios are significant, suggesting that OLS results are significantly affected by selection bias. The selection problem only comes from the choice of bankruptcy chapter when we study bankruptcy costs relative to total assets. However, the selection problem arises both in the chapter choice and conversion decisions in the regression for bankruptcy costs relative to liabilities.

3.4.2 Composition of Bankruptcy Chapters after the SBRA

The objective of the SBRA is to decrease inefficient liquidation among small firms by prompting them to file Chapter 11 reorganization. It is expected to have less Chapter 7 cases and more Chapter 11 cases among small businesses after the SBRA. Thus, it is natural to explore how composition of bankruptcy chapters change after the SBRA.

It is important to note that, right after the effective date of the SBRA (Feb. 19, 2020), massive COVID-19 shocks and associated policy responses hit the economy. These shocks make researchers hard to study the impact of the SBRA on the economy. To account for the pure impact of the SBRA, we need bankruptcy data after the economy returns to its normal status. Thus, I want to emphasize that the results in this section should be regarded as preliminary and suggestive.

3.4.2.1 Framework

Main empirical objective is to document the effect of the SBRA on bankruptcy filings. By following Wang et al. (2021), I estimate weekly changes in chapter 11 shares where bankruptcy filings in 2019 are regarded as the counterfactual. Therefore, my sample period is from January 1, 2019 to December 31, 2020. I conduct the analysis solely based on FJC data as it contains

	Assets		Liabilities	
	OLS	Selection	OLS	Selection
Chapter 11 [Y/N]	-3.02	48.88***	0.31*	1.05***
	(2.56)	(2.55)	(0.16)	(0.22)
log(Total Assets)	-1.48***	-3.06***		
	(0.05)	(0.09)		
Chapter 11 $\times$ log(Total Assets)	0.40**	-1.36***		
	(0.18)	(0.26)		
log(Total Liabilities)			-0.07***	-0.09***
			(0.00)	(0.01)
Chapter 11 $\times$ log(Total Liabilities)			-0.01	0.01
			(0.01)	(0.01)
Forced Petition [Y/N]	3.09***	2.18**	0.06*	0.06*
	(0.81)	(0.86)	(0.03)	(0.03)
Number of Creditors / 100	0.05***	0.02***	0.00***	0.00***
	(0.01)	(0.01)	(0.00)	(0.00)
Secured Liabilities / Total Liabilities	1.56***	-4.58***	-0.03***	-0.08***
	(0.15)	(0.23)	(0.01)	(0.02)
Total Liabilities / Total Assets $>$ 1 [Y/N]	-1.04***	1.70***	-0.20***	-0.15***
	(0.10)	(0.10)	(0.01)	(0.02)
Mills Ratio (Chapter)		-15.91***		-0.13***
		(0.56)		(0.04)
Mills Ratio (Conversion)		-2.54		-0.75***
		(2.10)		(0.15)
Constant	18.88***	29.31***	1.28***	1.44***
	(0.60)	(0.77)	(0.04)	(0.06)
Adjusted R squared	0.3788	0.5202	0.2410	0.2468
Observations	6,713	6,713	6,713	6,713

Table 3.3: Determinants of Monthly Bankruptcy Costs / Firm Size

Notes. * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$. Standard errors in parentheses.

the universe of bankrupt firms including sole proprietorship businesses. The specification is as follows:

$$y_t = \alpha + \sum_{\tau=2020\text{w}1}^{2020w52} \beta_\tau \mathbb{1}\{t=\tau\} + \gamma_{wom} + \gamma_{month} + \epsilon_t, \tag{3.3}$$

where γ_{wom} and γ_{month} are week-of-the-month and month-of-the-year fixed effects, respectively. These fixed effects control for seasonal variations. The dependent variable y_t is the share of firms that file Chapter 11 among all bankrupt firms. I split the sample by the level of liabilities. I define small businesses as firms that have less than 10 million dollars.[17] I am interested in $\hat{\beta}_\tau$, which estimate differences in Chapter 11 share in 2020 related to 2019 after controlling for the seasonal variations.

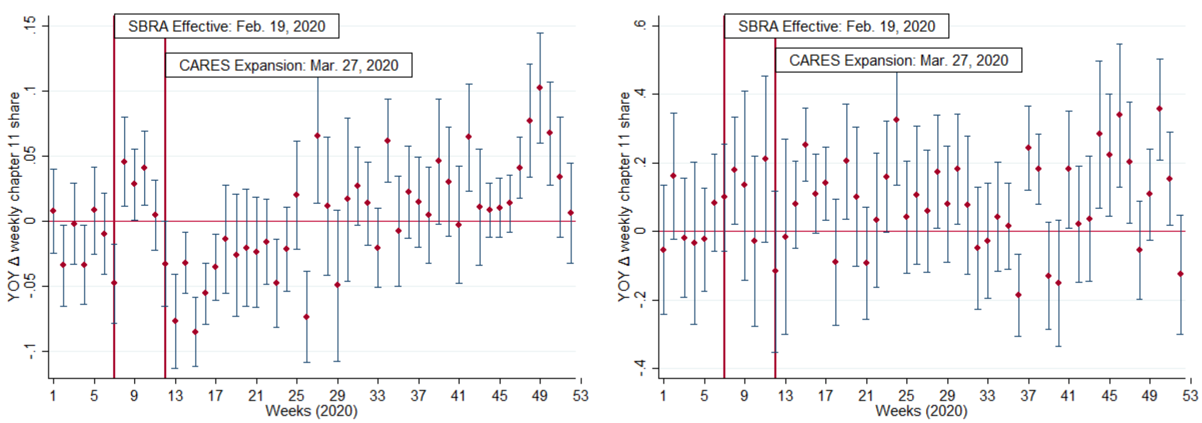
3.4.2.2 Results

I plot these results in Figure 3.3. First, the estimates before the SBRA effective date are insignificant, so there are no systematic pre-trends in both small and large businesses' chapter 11 shares. Thus, 2019 chapter 11 share would be a reasonable counterfactual for those in 2020. Second, for small businesses, Chapter 11 share had increased right after the SBRA effective date, but there was a clear decline between March and May. Although the estimates are noisy, Chapter 11 share among small businesses increased after May. Third, for large businesses, it is hard to observe any significant trends after the SBRA and CARES Act effective dates.

The increases in chapter 11 share for small businesses and the insignificant change in

[17] Ideally, the small businesses should be defined according to the SBRA thresholds, i.e., \$2,725,625 or \$7,500,000 after the CARES Act. However, there are many missing values in a continuous variable that reports liabilities (*totlblts*). Instead, I use a categorical variable that reports liabilities (*orgelblts*), and \$10 million is the closest value to the SBRA thresholds.

chapter 11 share for large businesses for a period after the SBRA effective date and before mid-March might indicate that the SBRA leads more Chapter 11 filing and less Chapter 7 filing only for the small businesses.[18] However, the period is less than 1 month, so it would be hard to conclude that we clearly see the intended effect of the SBRA. After mid-March, the analysis would be much more incomplete as the U.S. economy have been experiencing unprecedented changes that affect business bankruptcy margins. As discussed in Wang et al. (2021), physical constraints by court shutdowns, liquidity constraints, government subsidies, and uncertainty could affect bankruptcy filings during this pandemic period. Thus, it is hard to disentangle the pure effect of the SBRA on bankruptcy filings. Bankruptcy filing data generated in a normal course of the economy would be needed to say something more clearly about the impact of the SBRA.



(a) Small Businesses (b) Large Businesses

Figure 3.3: YoY Change in Chapter 11 Shares (2019 - 2020) - Small and Large Businesses

Notes. The sample consists of the universe of business bankruptcy filing reported by the FJC. The red points represent estimates of the β_τ coefficients in equation (3.3). The blue capped spikes provide the 95-percent confidence interval for each point estimate. The dependent variable in each panel is the share of Chapter 11 filing firms among all bankrupt firms for small (Panel (a)) and large businesses (Panel (b)). Small businesses are those with less than \$10 million in liabilities at filing. All business filings are consolidated to remove subsidiary filings. February 19, 2020 is the date when the SBRA started to be effective. March 27, 2020 is the date when the eligibility for the SBRA was expanded from \$2,725,625 to \$7,500,000 due to the CARES Act.

[18] Note that the national emergency was declared on March 13, 2020.

3.5 Model

I study the effects of the SBRA in the model of Corbae and D'Erasmo (2020), so I will closely follow their notation. This is a discrete time, general equilibrium model with four types of agents: heterogeneous firms, representative households, competitive lenders, and the government. Firms are heterogeneous in terms of productivity. They produce a homogeneous good with capital and labor. They finance their capital investment and dividends using both defaultable debt and equity. There is a standard representative household whose income comes from wages, dividends, and bankruptcy costs paid by firms. Competitive lenders set the price of debt considering default probabilities and payoffs. The government has a passive role to balance tax revenue and transfers.

The model has several differences from Corbae and D'Erasmo (2020). First, I design bankruptcy costs to depend on the endogenous choice of debt instead of exogenous productivity to model the SBRA experiment correctly. Second, I assume bankruptcy costs are transfers from bankrupt firms to households, so there are no direct deadweight losses from bankruptcy costs. This assumption allows me to analyze the impact of the SBRA on household consumption only through the *indirect* general equilibrium effects of the change in bankruptcy costs, excluding the *direct* reduction of deadweight losses in the economy. Lastly, I add taste shocks to value function to improve the convergence properties of the model, following Dvorkin et al. (2021).

3.5.1 Firms

Firms produce a homogeneous good using a standard decreasing returns to scale production technology:

$$y_{it} = z_{it}(k_{it}^{\alpha} n_{it}^{1-\alpha})^{\nu}, \tag{3.4}$$

where $\alpha, \nu \in (0, 1)$. The decreasing returns to scale technology allows us to generate a firm size distribution in equilibrium by having a concave profit function. $z_{it} \in \{z^1, \cdots, z^n\}$ is an idiosyncratic productivity shock, i.i.d. across firms, which follows a first-order Markov process with transition probability $G(z_{jt+1}|z_{jt})$. k_{jt} and n_{jt} are capital and labor input, respectively. Firms pay a fixed cost c_f to produce. Firms pay standard capital adjustment costs $\Psi(i_{it}, k_{it}) = \frac{\psi}{2}(\frac{i_{it}^g}{k_{it}})^2 k_{it}$, where $i_{it}^g = k_{jt+t} - (1-\delta)k_{jt}$ is gross investment.

Production inputs, investment, and dividends are financed using defaultable debt and equity issuance. To ensure that firms issue both debt and equity at an optimum, I assume that (1) there is a tax deduction for interest expense; (2) the price of debt is discounted to reflect the probability of default; and (3) equity issuance is costly. Taxable income is $\Gamma_{it} = \pi_{it} - \delta k_{it} - (\frac{1}{q_{it}} - 1)\frac{b_{it+1}}{1+r}$, where operating income is given by $\pi_{it} = y_{it} - w_t n_{it} - c_f$, w_t is the real wage, b_{it+1} is one-period non-contingent debt issued at t and repaid at $t+1$, and r is the interest rate for a risk-free bond. The corporate tax rate is τ_c, so tax revenue is $T_{it}^c = \tau_c \Gamma_{it} \mathbb{1}_{\{\Gamma_{it} \geq 0\}}$. The price of debt q_{it} is determined from the competitive lender's problem later. External equity is issued at cost $\lambda(e_{it})$ for equity issuance $e_{it} < 0$, which will be described in detail later.

New firms can enter by paying a one-time sunk cost κ. Entrants choose their initial capital level and their mix of debt and equity financing prior to learning their initial productivity. The initial productivity of entrants is drawn from the stationary distribution of the Markov process $G(z_{jt+1}|z_{jt})$, which I denote as $\bar{G}(z)$. I denote the mass of entrants as M. I assume $k_{jt} \in \mathcal{K}$ and $b_{jt} \in \mathcal{B}$, where $\mathcal{K}$ and $\mathcal{B}$ are finite capital and debt grids. For notational convenience, the k-th component of the set $\mathcal{K} \times \mathcal{B}$ is denoted by (k_k, b_k). By abusing notation, I denote the dimensions $|\mathcal{K}|$ and $|\mathcal{B}|$ as $\mathcal{K}$ and $\mathcal{B}$, respectively and I define $\mathcal{N} = \mathcal{K} \times \mathcal{B}$ as the dimension of the endogenous state space.

Incumbent firms have four operation options given idiosyncratic states (z_{it}, k_{it}, b_{it}): (i) normal operation (N), (ii) exit without bankruptcy (X), (iii) Chapter 7 liquidation (7), and (iv) Chapter 11 reorganization (11). In normal operation and Chapter 11 reorganization, firms choose (k_{it+1}, b_{it+1}) to maximize their value function, while firms that exit or liquidate have no such (k_{it+1}, b_{it+1}) choice. From now on, I denote $x_t = x$ and $x_{t+1} = x'$ for a recursive representation.

Incumbent firms receive a vector

$$\epsilon = (\epsilon(N,1), \cdots, \epsilon(N,\mathcal{N}), \epsilon(X), \epsilon(7), \epsilon(11,1), \cdots, \epsilon(11,\mathcal{N}))$$

of action specific, additively separable taste shocks each period. These taste shocks follow a generalized extreme value distribution whose distribution function is given by

$$F(\epsilon) = exp(-[\sum_{j=1}^{\mathcal{N}} exp(-\frac{\epsilon(N,j)-\mu}{\sigma})] - exp(-\frac{\epsilon(X,j)-\mu}{\sigma}) \\ - exp(-\frac{\epsilon(7,j)-\mu}{\sigma}) - [\sum_{j=1}^{\mathcal{N}} exp(-\frac{\epsilon(11,j)-\mu}{\sigma})])$$

where $\sigma > 0$ is a parameter governing the variance of the shocks. I set $\mu = -\sigma\gamma_E$ to make the shocks mean zero, where $\gamma_E = 0.56767...$ is Euler's constant.

Similarly, entrants receive a vector

$$\epsilon_E = (\epsilon(E,1), \cdots, \epsilon(E,\mathcal{N}))$$

that has the same extreme value distribution as that of incumbent firms, where ϵ_E and ϵ are independent. Note that ϵ and ϵ_E is independent across incumbent firms and entrants, respectively.

The *ex-post* value of a firm $V(z,k,b;\epsilon)$ is defined by:

$$V(z,k,b;\epsilon) = \max(V_N(z,k,b;\epsilon), V_X(z,k,b;\epsilon), V_7(z,k,b;\epsilon), V_{11}(z,k,b;\epsilon)) \qquad (3.5)$$

where V_N, V_X, V_7, and V_{11} are the ex-post value functions of choosing normal operation, exit without bankruptcy, Chapter 7 liquidation, and Chapter 11 reorganization, respectively. By ex-post, I mean after the realization of the taste shocks ϵ.

The *ex-ante* value of a firm with state (z,k,b) is defined by

$$W(z,k,b) = \int V(z,k,b;\epsilon)dF(\epsilon). \qquad (3.6)$$

3.5.1.1 Normal Operation (N)

If the firm chooses normal operation, then $V_N(z,k,b;\epsilon)$ is defined as follows:

$$V_N(z,k,b;\epsilon) = \max_{n,(k_j,b_j)\in\mathcal{K}\times\mathcal{B}} \{v_N^j(z,k,b) + \epsilon(N,j)\} \tag{3.7}$$

s.t.

$$v_N^j(z,k,b) = d_j + \frac{1}{1+r} E_{z'|z} W(z',k_j,b_j)$$

$$e_j = \pi - T_j^c - i_j^g - b + q(z,k_j,b_j)b_j - \Psi(k_j,k)$$

$$d_j = \begin{cases} e_j & e_j \geq 0 \\ e_j - \lambda(e_j) & e_j < 0. \end{cases}$$

As corporate tax payments and gross investment depend on the choice of (k_j, b_j), they are subscripted by j. Note that firms discount future expected values by $\frac{1}{1+r}$, as the consumer's stochastic discount factor equals $\frac{1}{1+r}$ in a stationary equilibrium.

3.5.1.2 Exit without Bankruptcy (X)

If the firm exits without bankruptcy, it sells all of its capital at price s_x and repays all of its existing debt.

$$V_X(z,k,b;\epsilon) = v_X(z,k,b) + \epsilon(X) \tag{3.8}$$

$$v_X(z,k,b) = \begin{cases} s_x k - b, & s_x k - b \geq 0 \\ s_x k - b - \lambda(s_x k - b), & s_x k - b < 0 \end{cases}$$

3.5.1.3 Chapter 7 Liquidation (7)

If the firm chooses to liquidate in Chapter 7, it sells all of its capital at price s_7 and repays its debt until it can enjoy the benefit of limited liability. The bankruptcy cost in Chapter 7 is denoted by $c_7(b)$. Note that the bankruptcy costs are paid before debt when $s_7k - b - c_7(b) < 0$.

$$V_7(z, k, b; \epsilon) = v_7(z, k, b) + \epsilon(7) \tag{3.9}$$
$$v_7(z, k, b) = \max(s_7k - b - c_7(b), 0)$$

3.5.1.4 Chapter 11 Reorganization (11)

If the firm chooses to reorganize in Chapter 11, some of its debt will be written off at the rate $(1 - \phi(z, k, b))$. The recovery rate $\phi(z, k, b)$ will be endogenously determined by Nash bargaining between firms and lenders. The firm pays bankruptcy costs $c_{11}(b)$. In addition, there are further debt financing costs λ_{11}^b and equity issuance costs λ_{11} that are different from those in normal operation. Capital is sold at a discount s_x.[19] Lastly, I assume firms in Chapter 11 are not allowed to distribute dividends.[20] These features reflect the higher financing costs and frictions

[19] I assume that capital is sold at price s_x. In principle, the sale price under Chapter 11 does not have to be the same as the sale price in exit. However, I want to impose a penalty for capital sales in Chapter 11 reorganization, as firms need to have a court's approval to sell their capital, which can be costly. At the same time, I do not want to increase the number of parameters.

[20] Instead of assuming $e_j \leq 0$, I express the constraint for dividends as $e_j + \phi(z, k, b)b + c_{11}(b) \leq 0$. $e_j + \phi(z, k, b)b + c_{11}(b) \leq 0$ implies $e_j \leq 0$, so dividend distribution is still limited. There are two reasons why I impose the stronger constraint. First, by including ϕ in the constraint, the Nash bargaining game (3.14) has a closed form solution, so there is a huge computational benefit. Second, by including $c_{11}(b)$, I simplify the impact of a change in $c_{11}(b)$ on the value of Chapter 11 reorganization. If I exclude $c_{11}(b)$, a decrease in $c_{11}(b)$ makes the dividend constraint tighter by mechanically increasing e_j (note that $c_{11}(b)$ has a negative sign in the definition of e_j), so it can decrease the value of Chapter 11. I do not want to have this property in this model.

observed for firms in Chapter 11 in reality.

$$V_{11}(z,k,b;\epsilon) = \max_{n,(k_j,b_j)\in\mathcal{K}\times\mathcal{B}} \{v_{11}^j(z,k,b) + \epsilon(11,j)\} \tag{3.10}$$

s.t.

$$v_{11}^j(z,k,b) = d_j + \frac{1}{1+r} E_{z'|z} W(z',k_j,b_j)$$

$$e_j = \pi - T_j^c - i_j^g \mathbb{1}_{\{i_j^g \geq 0\}} - s_x i_j^g \mathbb{1}_{\{i_j^g < 0\}} - \phi(z,k,b)b + \lambda_b^{11} q(z,k_j,b_j)b_j - \Psi(k_j,k) - c_{11}(b)$$

$$e_j + \phi(z,k,b)b + c_{11}(b) \leq 0$$

$$d_j = e_j - \lambda_{11}(e_j).$$

As mentioned above, $\phi(z,k,b)$ is determined by Nash bargaining between firms in Chapter 11 and lenders. For a given recovery rate ϕ, the ex-post value of a bargaining agreement for the firm in Chapter 11 is:

$$V_R(z,k,b;\phi,\epsilon) = \max_{n,(k_j,b_j)\in\mathcal{K}\times\mathcal{B}} \{d_j + \frac{1}{1+r} E_{z'|z} W(z',k_j,b_j) + \epsilon(11,j)\} \tag{3.11}$$

s.t.

$$e_j = \pi - T_j^c - i_j^g \mathbb{1}_{\{i_j^g \geq 0\}} - s_x i_j^g \mathbb{1}_{\{i_j^g < 0\}} - \phi b + \lambda_b^{11} q(z,k_j,b_j)b_j - \Psi(k_j,k) - c_{11}(b)$$

$$e_j + \phi b + c_{11}(b) \leq 0$$

$$d_j = e_j - \lambda_{11}(e_j).$$

I assume that the threat points in the Nash bargaining procedure are the payoffs in Chapter 7, as bankruptcy law imposes a minimum payoff to lenders in Chapter 11 reorganization as the payoff they would get if the case converted to Chapter 7 ("Best Interest Test"). Therefore, the (ex-ante) surplus for the firm in Chapter 11 is:

$$W_R(z,k,b;\phi) = E_\epsilon V_R(z,k,b;\phi,\epsilon) - \max(s_7 k - b - c_7(b), 0). \tag{3.12}$$

Similarly, the surplus for the lender is:

$$W_L(z,k,b;\phi) = \phi b - \min(b, \max(s_7 k - c_7(b), 0)). \tag{3.13}$$

Given these definitions, the recovery rate $\phi(z,k,b)$ is determined by the Nash bargaining procedure as follows:

$$\phi(z,k,b) =_{\phi\in[0,1]} W_R(z,k,b;\phi)^\theta W_L(z,k,b;\phi)^{1-\theta}, \tag{3.14}$$
$$s.t. \;\; W_R(z,k,b;\phi), W_L(z,k,b;\phi) \geq 0,$$

where θ is the bargaining power of the debtor firm. Note that taste shocks do not affect the recovery rate.

3.5.1.5 Entrants

Entrants choose capital and debt after paying an entry cost κ, but prior to learning their initial productivity. Entrants know taste shocks when choosing k and b.

$$V_E(\epsilon_E) = \max_{(k_j,b_j)\in\mathcal{K}\times\mathcal{B}} \{v_E^j + \epsilon(E,j)\} \tag{3.15}$$

$$v_E^j = d_{j,E} + \frac{1}{1+r} E_{z'} W(z', k_j, b_j)$$

$$d_{j,E} = -k_j + q_E(k_j, b_j)b_j - \kappa - \lambda(-k_j + q_E(k_j, b_j)b_j - \kappa).$$

Note that $\lambda(-k_j + q_E(k_j, b_j)b_j - \kappa)$ is the equity issuance cost of entrants. I assume that entrants do not know taste shocks when they make entry decision, so free entry of firms should imply $E_\epsilon V_E = 0$.

3.5.2 Competitive Lenders

Risk-neutral competitive lenders borrow funds from households using risk-free bonds and lend to firms. The lenders set the price of defaultable debt $q(z, k', b')$ by considering expected payoffs, which depend on the probability of bankruptcy and returns for each case. The lender's expected profit from lending to a firm that wants to borrow b' with productivity z and future capital k' (collateral) is

$$\begin{aligned}\Pi^L(z, k', b') = -q(z, k', b')b' + \frac{1}{1+r} E_{z'|z}[&P_N(z', k', b')b' + P_X(z', k', b')b' \\ &+ P_7(z', k', b') \min(b', \max(s_7 k' - c_7(b'), 0)) \\ &+ P_{11}(z', k', b')\phi(z', k', b')b'],\end{aligned} \tag{3.16}$$

where P_N, P_X, P_7, and P_{11} are the ex-ante choice probabilities of normal operation, exit, Chapter 7 liquidation, and Chapter 11 reorganization, which will be described in detail in Section 3.5.7. As the lenders are competitive, the expected profit $\Pi^L(z, k', b')$ is zero in equilibrium. As a result, the equilibrium debt price schedule is derived as follows:

$$
\begin{aligned}
q(z, k', b') = \frac{1}{1+r} E_{z'|z}[P_N(z', k', b') + P_X(z', k', b') \\
+ P_7(z', k', b') \frac{\min(b', \max(s_7 k' - c_7(b'), 0))}{b'} \\
+ P_{11}(z', k', b') \phi(z', k', b')].
\end{aligned}
\tag{3.17}
$$

3.5.3 Households

A representative household solves the following standard problem:

$$
V_H(B, \{S_i\}) = \max U(C) + \beta E V_H(B', \{S_i'\})
\tag{3.18}
$$

$$
s.t.
$$

$$
C + \int p_i S_i' di + q^B B' = w + \int (p_i + d_i) S_i di + B + \int (c_{11,i} + \lambda_{11}(c_{11,i}) + c_{7,i}) di + T^h,
$$

where p_i is the price of stocks of firm i, S_i is the shares of stock of firm i held by the household, q^B is the price of the risk-free bond, B is the amount of risk-free bonds held by households, $c_{11,i}$ and $c_{7,i}$ are the bankruptcy costs paid by firm i, and T^h is the lump-sum transfer from the government. Unlike Corbae and D'Erasmo (2020), I assume that the bankruptcy costs and the part of equity issuance costs due to Chapter 11 bankruptcy paid by firms are transferred to the households. This assumption allows me to focus on the indirect effects of the SBRA without having direct changes in deadweight losses due to lower bankruptcy costs. Note that the household inelastically supplies one unit of labor to firms.

3.5.4 Government

The government collects corporate income tax from firms and makes lump-sum transfers to the household,

$$T^h = \int T_i^c di. \tag{3.19}$$

3.5.5 Law of Motion for Cross-Sectional Distribution

The law of motion for the cross-sectional distribution of firms is as follows:

$$\Gamma(z', k', b') = \sum_{z,k,b} [P_N(z, k, b)\sigma_N(z, k, b, k', b') + P_{11}(z, k, b)\sigma_{11}(z, k, b, k', b')]G(z'|z)\Gamma(z, k, b) \tag{3.20}$$

$$+ M\sigma_E(k', b')\bar{G}(z'),$$

where $\sigma_N(z, k, b, k', b')$ and $\sigma_{11}(z, k, b, k', b')$ are the ex-ante choice probabilities of choosing (k', b') in normal operation and Chapter 11 reorganization, respectively, for firms in state (z, k, b), which will be described in detail in Section 3.5.7. Similarly, $\sigma_E(k', b')$ is the ex-ante choice probability of (k', b') for entrants.

3.5.6 Definition of Equilibrium

A stationary recursive equilibrium is a list $\{W^*, w^*, r^*, q^{B*}, q^*, \phi^*, \{p_j^*\}, \Gamma^*, M^*, C^*, B'^*, S'^*\}$ that satisfies:

1. Given w^*, r^*, q^*, and ϕ^*, the value function W^* is the solution for the firm's problem in (3.7),(3.8),(3.9), and (3.10).

2. Given W^*, w^*, r^*, and q^*, the recovery rate ϕ^* solves the Nash bargaining problem (3.11), (3.12), (3.13), and (3.14).

3. Given the decision rules implied by W^*, q^* solves the zero-profit condition of the lenders (3.16).

4. Free entry of firms implies $E_\epsilon V_E^* = 0$ in (3.15).

5. Γ^* and M^* in (3.20) are consistent with decision rules implied by W^* and the exogenous stochastic process $G(\cdot|\cdot)$.

6. Given $\{p_j^*\}$, $q^{B,*}$, w^*, and $T^{h,*}$, the household solves (3.18).

7. Labor, bond, and stock markets clear at w^*, q^{B*}, and $\{p_j^*\}$,

$$\text{labor:} \sum_{z,k,b} n^*(z,k,b)[P_N^*(z,k,b) + P_{11}^*(z,k,b)]\Gamma^*(z,k,b) = 1$$

$$\text{bond:} \sum_{z,k,b}[(P^*_N(z,k,b) + P^*_X(z,k,b))b + P^*_7(z,k,b)min(b, max(s_7k - c_7(b), 0))$$

$$+ P^*_{11}(z,k,b)\phi^*(z,k,b)b]\Gamma^*(z,k,b) = B$$

$$\text{stock} : S^{',*} = 1.$$

8. The government budget balance (3.19) is satisfied.

3.5.7 Ex-Ante Value Functions and Choice Probabilities

Assuming that the taste shocks ϵ and ϵ_E follow generalized extreme value distributions gives us closed-form solutions for the ex-ante value function W and choice probabilities P_N, P_X, P_7, P_{11}, σ_N, σ_{11}, and σ_E (see, for instance, Rust (1987)). The reason why I employ these taste shocks is described in Section 3.6.2.

The ex-ante value functions W and $E_\epsilon V_E$ are as follows:

$$W(z,k,b) = \sigma log(\sum_j [exp(\frac{v^j_N(z,k,b)}{\sigma})] + exp(\frac{v_X(z,k,b)}{\sigma}) + exp(\frac{v_7(z,k,b)}{\sigma}) + \sum_j [exp(\frac{v^j_{11}(z,k,b)}{\sigma})]),$$

$$E_\epsilon V_E = \sigma log(\sum_j [exp(\frac{v^j_E}{\sigma})]).$$

The operation choice probabilities P_N, P_X, P_7, and P_{11} are as follows:

$$P_N(z,k,b) = \frac{\sum_j [exp(\frac{v_N^j(z,k,b)}{\sigma})]}{\sum_j [exp(\frac{v_N^j(z,k,b)}{\sigma})] + exp(\frac{v_X(z,k,b)}{\sigma}) + exp(\frac{v_7(z,k,b)}{\sigma}) + \sum_j [exp(\frac{v_{11}^j(z,k,b)}{\sigma})]},$$

$$P_X(z,k,b) = \frac{exp(\frac{v_X(z,k,b)}{\sigma})}{\sum_j [exp(\frac{v_N^j(z,k,b)}{\sigma})] + exp(\frac{v_X(z,k,b)}{\sigma}) + exp(\frac{v_7(z,k,b)}{\sigma}) + \sum_j [exp(\frac{v_{11}^j(z,k,b)}{\sigma})]},$$

$$P_7(z,k,b) = \frac{exp(\frac{v_7(z,k,b)}{\sigma})}{\sum_j [exp(\frac{v_N^j(z,k,b)}{\sigma})] + exp(\frac{v_X(z,k,b)}{\sigma}) + exp(\frac{v_7(z,k,b)}{\sigma}) + \sum_j [exp(\frac{v_{11}^j(z,k,b)}{\sigma})]},$$

$$P_{11}(z,k,b) = \frac{\sum_j [exp(\frac{v_{11}^j(z,k,b)}{\sigma})]}{\sum_j [exp(\frac{v_N^j(z,k,b)}{\sigma})] + exp(\frac{v_X(z,k,b)}{\sigma}) + exp(\frac{v_7(z,k,b)}{\sigma}) + \sum_j [exp(\frac{v_{11}^j(z,k,b)}{\sigma})]}.$$

The capital and debt choice probabilities σ_N, σ_{11}, and σ_E are as follows:

$$\sigma_N(z,k,b,k_j,b_j) = \frac{exp(\frac{v_N^j(z,k,b)}{\sigma})}{\sum_j [exp(\frac{v_N^j(z,k,b)}{\sigma})]},$$

$$\sigma_{11}(z,k,b,k_j,b_j) = \frac{exp(\frac{v_{11}^j(z,k,b)}{\sigma})}{\sum_j [exp(\frac{v_{11}^j(z,k,b)}{\sigma})]},$$

$$\sigma_E(k_j,b_j) = \frac{exp(\frac{v_E^j}{\sigma})}{\sum_j [exp(\frac{v_E^j}{\sigma})]}.$$

3.5.8 Resource Constraint

Combining the household budget constraint with the labor, bond, and stock market clearing conditions leads to the following resource constraint.

$$\text{Consumption} \quad C = Y - CF - I - \Psi - \Lambda - E + X, \tag{3.21}$$

where

$$\text{Output } Y = \sum_{z,k,b} y(z,k,b)[P_N(z,k,b) + P_{11}(z,k,b)]\Gamma(z,k,b),$$

$$\text{Fixed costs } CF = \sum_{z,k,b} c_f[P_N(z,k,b) + P_{11}(z,k,b)]\Gamma(z,k,b),$$

$$\begin{aligned}\text{Investment } I = \sum_{z,k,b}\{&P_N(z,k,b)\sum_{k',b'}[\sigma_N(z,k,b,k',b')i_N^g(z,k,b,k',b')] \\ &+ P_{11}(z,k,b)\sum_{k',b'}[\sigma_{11}(z,k,b,k',b')i_{11}^g(z,k,b,k',b')]\}\Gamma(z,k,b),\end{aligned}$$

$$\begin{aligned}\text{Capital adjustment costs } \Psi = \sum_{z,k,b}\{&P_N(z,k,b)\sum_{k',b'}[\sigma_N(z,k,b,k',b')\psi_N(z,k,b,k',b')] \\ &+ P_{11}(z,k,b)\sum_{k',b'}[\sigma_{11}(z,k,b,k',b')\psi_{11}(z,k,b,k',b')]\}\Gamma(z,k,b),\end{aligned}$$

$$\begin{aligned}\text{External financing costs } \Lambda = \sum_{z,k,b}\{&P_N(z,k,b)\sum_{k',b'}[\sigma_N(z,k,b,k',b')\lambda_N(z,k,b,k',b')] \\ &+ P_{11}(z,k,b)\sum_{k',b'}[\sigma_{11}(z,k,b,k',b')\lambda_{11}(z,k,b,k',b')] \\ &+ P_X(z,k,b)\lambda_x(z,k,b)\}\Gamma(z,k,b) + M\sum_{k',b'}\sigma_E(k',b')\lambda_E(k',b'),\end{aligned}$$

$$\text{Entrants } E = M\sum_{k',b'}\sigma_E(k',b')(k'+\kappa),$$

$$\text{Value of resold capital } X = \sum_{z,k,b}(P_X(z,k,b)s_x k + P_7(z,k,b)s_7 k)\Gamma(z,k,b).$$

3.6 Quantitative Analysis

3.6.1 Mapping the Model to Data

I assume that firm productivity follows an AR(1) process: $log(z') = \rho_z log(z) + \eta$ with $\eta \sim N(0, \sigma_\eta^2)$. I use Tauchen's method to discretize this process into a 7-state Markov chain with support $\{z_1, \cdots, z_7\}$.

I assume that the bankruptcy costs are functions of debt and that the cost structures are different for small and large businesses.

$$c_7(b) = \begin{cases} \max(0, c_{7,0S} + c_{7,1S}b + c_{7,2S}b^2), & b \leq \hat{b} \\ \max(0, c_{7,0L} + c_{7,1L}b + c_{7,2L}b^2), & b > \hat{b} \end{cases} \tag{3.22}$$

$$c_{11}(b) = \begin{cases} \max(0, c_{11,0S} + c_{11,1S}b + c_{11,2S}b^2), & b \leq \hat{b} \\ \max(0, c_{11,0L} + c_{11,1L}b + c_{11,2L}b^2), & b > \hat{b} \end{cases} \tag{3.23}$$

where $\hat{b}$ is the threshold for the SBRA.

In this model, there are 24 parameters. I divide the parameters into three groups. The first group is a set of parameters that are fixed by standard values in the literature. The second group is the set of bankruptcy costs parameters in (3.22) and (3.23), which I estimate directly from data. Parameters in the last group are calibrated within the model by targeting informative data moments.

Table 3.4 shows the externally calibrated parameters. Most of them are standard, but some are worth discussion. First, I set the value of the returns to scale parameter ν as 0.85.

This parameter is set to 0.75 and 0.95 in Bloom et al. (2018) and Bartelsman et al. (2013b), respectively, so I choose the average of them. Second, I take the parameter values for the idiosyncratic productivity process from Khan and Thomas (2013b). Ideally, these parameters should be internally calibrated by targeting informative moments such as dispersion in investment rates, or directly estimated using the method in Cooper and Haltiwanger (2006b). The latter method would be preferred in terms of computation as the first method changes a productivity grid for different ρ_z and σ_η. As I do not have good enough micro data to implement the latter method, I want to keep it as the next to-do list. Lastly, the variance of the taste shocks σ is taken from Dvorkin et al. (2021) who develop a computational method that adds taste shocks following an extreme value distribution to smooth decision rules. This value is consistent in scale ($\sim 10^{-3}$) with Chatterjee et al. (2020), who also include these taste shocks in their model. In a later draft, I will calibrate σ by myself.

Parameter	Explanation	Value	Source
r	Risk-free Rate	0.020	T-bill rate
τ_c	Corporate Tax Rate	0.300	Hennessy and Whited (2005)
δ	Depreciation Rate	0.100	Bartelsman et al. (2013b)
α	Capital Share	0.330	Standard
ν	Returns to Scale	0.850	Bartelsman et al. (2013b), Bloom et al. (2018)
ρ_z	Autocorrelation z	0.659	Khan and Thomas (2013b)
σ_η	Std. Dev. Shock	0.118	Khan and Thomas (2013b)
s_7	Ch. 7 Capital Price	0.380	Bris et al. (2006)
σ	Taste Shock Variance	0.001	Dvorkin et al. (2021)

Table 3.4: Externally Calibrated Parameters

Next, I directly estimate the cost parameters in (3.22) and (3.23) by using the bankruptcy cost data in Section 3.3. I assume the cost functions are the same for small and large businesses in the pre-SBRA economy, as there was no separate bankruptcy procedure for small businesses. To match the scale of the data to the above model, I re-scale the data by matching the maximum value of debt observed in the data to the maximum value of the model. In addition, I multiply by the average observed durations in Chapter 7 and 11 to convert monthly bankruptcy costs to total bankruptcy costs. Using this re-scaled data, I run regressions of total bankruptcy costs for each Chapter on a constant, the level of debt, and the square of debt. I note that the level of debt in the data is what is owed just prior to bankruptcy filing. As in Section 3.4, I deal with the selection problem by controlling for the Mills ratios generated from the first-stage probit regressions.

Table 3.5 presents the estimates of the bankruptcy cost functions. Correcting for selection bias decreases the estimate of the fixed cost for Chapter 7 and increases the estimate for Chapter 11. It is interesting to note that the fixed cost estimates of Chapter 7 and 11 in Corbae and D'Erasmo (2020) are 0.001 and 0.128, respectively. The fixed cost for Chapter 11 is thus 128 times higher than that for Chapter 7 in their paper. However, the selection corrected estimates in this paper are 0.0010 and 0.0157 for Chapter 7 and 11, respectively, so the fixed cost for Chapter 11 is only 16 times higher than that for Chapter 7. The fixed cost estimates for Chapter 7 are in the same ballpark, but the Chapter 11 fixed cost estimate is much higher in Corbae and D'Erasmo (2020). I use the selection corrected estimates as the parameter values for the bankruptcy cost functions in (3.22) and (3.23).

	Ch. 7			Ch. 11		
	OLS	Selection		OLS	Selection	
b	0.0064***	$[c_{7,1S} = c_{7,1L}]$	0.0042***	0.0050***	$[c_{11,1S} = c_{11,1L}]$	0.0052***
	(22.03)		(11.60)	(4.45)		(3.95)
b^2	-0.0008***	$[c_{7,2S} = c_{7,2L}]$	-0.0005***	-0.0007***	$[c_{11,2S} = c_{11,2L}]$	-0.0007**
	(-9.80)		(-5.59)	(-2.63)		(-2.40)
Const.	0.0015***	$[c_{7,0S} = c_{7,0L}]$	0.0010***	0.0056***	$[c_{11,0S} = c_{11,0L}]$	0.0157**
	(17.06)		(9.57)	(8.46)		(2.07)
Mills Ratio (Chapter)			-0.0031***			-0.0006
			(-10.23)			(-0.41)
Mills Ratio (Conversion)						-0.0094
						(-1.54)
Observations	6,215	6,150		567	565	

Table 3.5: Bankruptcy Cost Estimates

Notes. t statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Lastly, I internally calibrate the remaining parameters (Table 3.6). I now discuss the selection of data moments to identify these parameters. The fixed cost c_f can be identified by the firm exit rate. The sales price of capital at exit s_x is related to the bankruptcy rate. The share of Chapter 7 liquidation among all bankruptcies can be informative to identify the equity issuance cost in Chapter 11 reorganization λ_{11}^1, as a higher equity issuance cost in Chapter 11 makes Chapter 7 more attractive. The creditor recovery rate in Chapter 11 is related to the debtor's bargaining power θ. The equity issuance cost in normal operation, λ_1, and the debt cost in Chapter 11, λ_{11}^b, can be identified using the debt-to-assets ratio for operating firms and Chapter 11 firms. As λ_1 goes up, firms want to substitute away from equity towards debt. Similar logic applies to λ_{11}^1. The average investment rate is informative for adjustment costs ψ. The SBRA threshold $\hat{b}$ is set to match the share of businesses that have liabilities less then $2,725,625. Lastly, the entry cost κ is set to be consistent with an equilibrium wage of one.

I next discuss how I construct each data moment from each source. I compute the firm exit rate as the average from 2010 to 2017 from the Census Business Dynamics Statistics (BDS). I use data from 2010 to 2017 to exclude the Great Recession, and because the FJC is available for

Parameter	Explanation	Parameter Value	Target (Source)
c_f	Production Fixed Cost	0.295	Firm Exit Rate (BDS 2010 - 2017)
s_x	Exit Capital Price	0.942	Firm Bankruptcy Rate (BDS, FJC 2010 - 2017)
λ_{11}^{1}	Ch.11 Equity Cost	0.987	Chapter 7 Ratio (FJC 2010 - 2017)
θ	Firm Bargaining Power	0.826	Chapter 11 Recovery Rate (Bris et al. (2006))
λ_1	Normal Equity Cost	0.197	Debt-to-Assets Ratio for Operating Firms (Caglio et al. (2021a))
λ_{11}^{b}	Ch.11 Debt Cost	0.411	Debt-to-Assets Ratio for Chapter 11 Firms (Table 3.1)
ψ	Adjustment Cost	0.108	Average Investment Rate (Cooper and Haltiwanger (2006b))
$\hat{b}$	Small Firm Threshold	1.200	Share of Firms having ¡ $2,725,625 liabilities (ORBIS 2005-2012)
κ	Entry cost	0.276	Equilibrium Wage = 1 (Normalization)

Table 3.6: Internally Calibrated Parameters

the period after 2008 and before 2017. I compute the firm bankruptcy rate as the average ratio of the number of bankrupt firms in FJC in a given year to the number of firms in BDS. I want to note how I calculate the number of firms filing for bankruptcy. In the BDS, the total number of firms includes all legal form of organization, such as sole proprietorship, partnership, and corporation. Since there is no distinction between a founder and a firm for a sole proprietorship, proprietors tend to use Chapter 13, a reorganization process for individual bankruptcies that is similar to but cheaper than Chapter 11. Therefore, to be consistent with the BDS, I include firms that file under Chapter 13 in the group of Chapter 11 firms. I also use this definition of Chapter 11 firms to compute the Chapter 7 share of all bankruptcies. I take the Chapter 11 recovery rate, the median debt-to-assets ratio for operating firms, and the average investment rate from Bris et al. (2006), Caglio et al. (2021a), and Cooper and Haltiwanger (2006b), respectively. I take the median debt-to-assets ratio for Chapter 11 firms from Table 3.1. I use the book and market values of assets to compute the leverage ratio for operating firms and Chapter 11 firms, respectively. Caglio et al. (2021a) compute the leverage ratio from a firm's balance sheet as reported to lenders, while assets and liabilities in Chapter 11 are self-reported, and we see many observations in which liabilities are greater than assets. Therefore, I regard the reported value of assets for Chapter 11 firms as their market value. Lastly, I compute the share of small firms that have liabilities less than

Targeted Moments	Data	Model
Firm Exit Rate (%)	7.61	7.69
Firm Bankruptcy Rate (%)	0.61	0.63
Chapter 7 Ratio (%)	71.21	72.35
Chapter 11 Recovery Rate (%)	69.40	58.66
(Median) Debt-to-Assets Ratio for Operating Firms	0.64	0.38
(Median) Debt-to-Assets Ratio for Chapter 11 Firms	1.42	1.15
Average Investment Rate (%)	12.20	12.23
Share of Small Business (%)	69.00	69.73

Table 3.7: Data vs. Model Moments

$2,725,625 in 2019 dollars using ORBIS from 2005 to 2012.

Table 3.7 shows the targeted data moments and their model counterparts. Overall, the calibrated parameters match the model and data moments well. However, the creditor recovery rate in Chapter 11 and leverage ratios for operating firms and Chapter 11 firms are lower in the model. Further improvement in matching these moments should be accomplished in the next draft.

3.6.2 Taste Shocks

I briefly discuss the role of taste shocks. I employ taste shocks to improve the convergence properties of the model solution methods.[21] The bottleneck for convergence is equation (3.17), the debt price schedule. Equation (3.17) can be written as an iterative form,

$$q^{k+1} = H[W^k, q^k, \phi^k], \tag{3.24}$$

where W^k, q^k, and ϕ^k are the ex-post value function, debt price schedule, and Chapter 11 recovery schedule in the k-th iteration. The function H is the right-hand side of equation (3.17).

[21] For recent papers employing such taste shocks, see Chatterjee and Eyigungor (2012), Caliendo et al. (2019), Chatterjee et al. (2020), and Dvorkin et al. (2021).

For the solution method to have good convergence properties, an infinitesimal change in q^k should not induce a large shift in q^{k+1}, i.e., H should be continuous. However, if there are no taste shocks, the continuity of H is not guaranteed. To see this intuitively, assume that there are no taste shocks, i.e., $\sigma \to 0$. In this case, the vector of operation choice probabilities (P_N, P_X, P_7, P_{11}) is either $(1, 0, 0, 0)$, $(0, 1, 0, 0)$, $(0, 0, 1, 0)$, or $(0, 0, 0, 1)$. This means that an infinitesimal change in q^k can induce a *discrete* change in (P_N, P_X, P_7, P_{11}), so there would be a jump in q^{k+1}.

One might think that this should not be a problem as we take expectations with respect to z in the function H, so an infinitesimal change in q^k would result in an infinitesimal change in q^{k+1}. However, there are two problems with this logic. First, we might have to set a high number of grid points for z, which is computationally impractical. Adding more grid points increases the computational burden considerably. Second, even if we run this model with a high number of grid points, the continuity of H still might not be guaranteed. If the measure of firms that are indifferent between two discrete choices, e.g. $(1, 0, 0, 0)$ and $(0, 1, 0, 0)$, is not zero, an infinitesimal change in q^k can still create a jump in q^{k+1}. For these reasons, adding taste shocks allows us to improve the model's convergence properties by smoothing out the problem.

Although there is a computational benefit of using these taste shocks, we need to be cautious in using them. If the variance of the taste shocks goes to infinity, choice probabilities do not depend on economic fundamentals. For example, when the variance goes to infinity, $\sigma_N(z, k, b, k_j, b_j)$ goes to $\frac{1}{N}$ even when $v_N^j(z, k, b) > v_N^k(z, k, b)$, $\forall k \neq j$. Therefore, it is very important to set the variance σ sufficiently low so that taste shocks do not swamp economic fundamentals. I set σ as 0.001, similar to the variance in other papers that use taste shocks with generalized extreme value distribution. I will conduct a robustness check for different values of σ in a later draft.

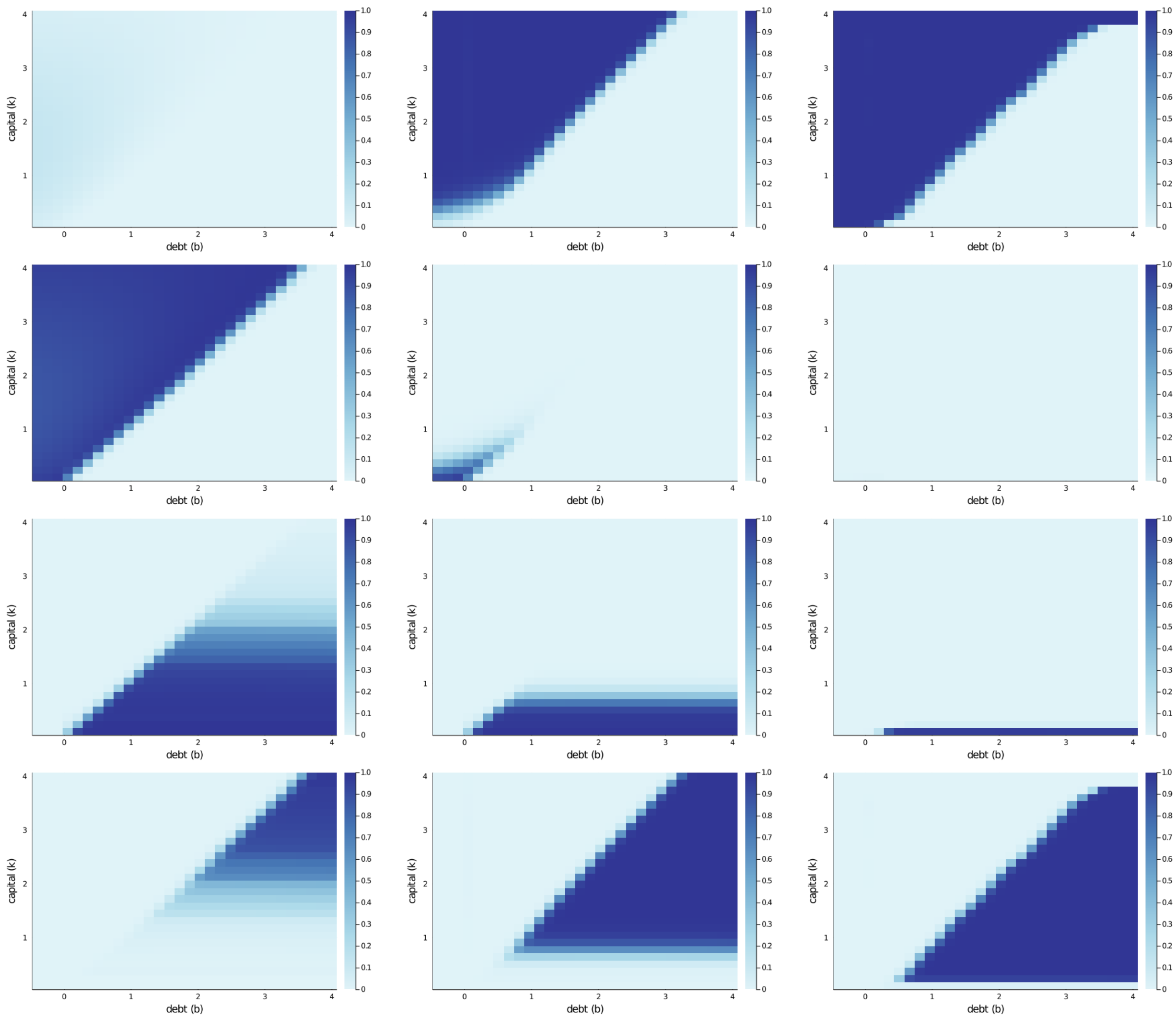
3.6.3 Model Properties

Figure 3.4 presents the bankruptcy decision rules as a function of debt and capital in the pre-SBRA economy. Panels (a), (b), and (c) represent the decision rules of low ($z = z_1$), medium ($z = z_4$), and high productivity firms ($z = z_7$), respectively. The first, second, third, and fourth rows show the probabilities of choosing normal operation ($P_N(z, k, b)$), exit without bankruptcy ($P_X(z, k, b)$), Chapter 7 liquidation ($P_7(z, k, b)$), and Chapter 11 reorganization ($P_{11}(z, k, b)$), respectively.

As shown in Figure 3.4, firms with high productivity tend to maintain their operations, either through normal operation or Chapter 11 reorganization, while firms with low productivity exit or liquidate. Firms with low productivity or low capital prefer to liquidate when they have a large debt. In contrast, firms with high productivity or high capital choose to reorganize when they have a large debt.

I next describe debt price schedules offered to firms conditional on how much they borrow (b'), their next period collateral (k'), and their current productivity (z). Figure 3.5 shows a debt price schedule for firms with median productivity ($z = z_4$). Patterns are similar for other productivity levels. For a given level of next period capital, higher firm debt reduces what lenders recover. Meanwhile, for a given level of borrowing, higher firm capital increases lenders' repayment. Thus, firms with high debt and low capital face higher real interest rates on their borrowing.

Figure 3.6 graphs equilibrium debt price menus offered to firms with low, medium, and high productivity for different levels of capital. For a lower level of capital ($k = 0.64$), firms with higher productivity borrow at lower real interest rates, as higher productivity firms are less



(a) $z = z_1$ (b) $z = z_4$ (c) $z = z_7$

Figure 3.4: Choice Probabilities

Notes. The first, second, third, and fourth rows show the probabilities of choosing normal operation ($P_N(z, k, b)$), exit without bankruptcy ($P_X(z, k, b)$), Chapter 7 liquidation ($P_7(z, k, b)$), and Chapter 11 reorganization ($P_{11}(z, k, b)$), respectively. The first, second, and third columns are the choice probabilities for $z = z_1$, $z = z_4$, and $z = z_7$, respectively.

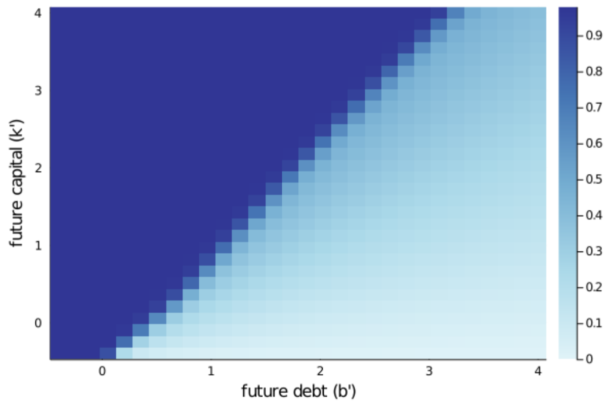


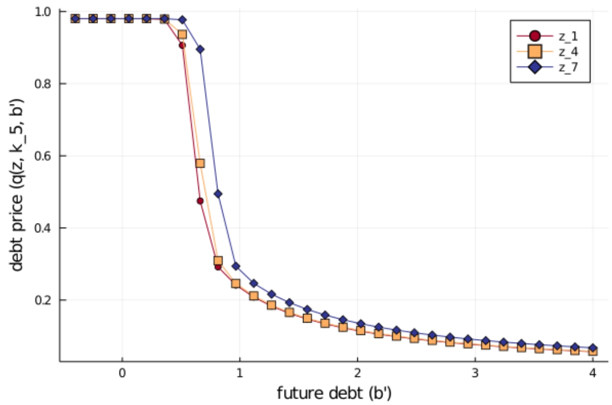
Figure 3.5: Debt Price Schedule ($z = z_4$)

likely to go bankrupt at this level of capital. For medium ($k = 1.98$) and high ($k = 3.33$) capital, firms with the highest productivity borrow at the lowest real interest rates. However, interestingly, at these levels of future capital, medium productivity firms pay higher real interest rates than low productivity firms for a low level of borrowing while medium productivity firms pay lower interest rates for a high level of borrowing. Creditor recovery can be lower for medium productivity than low productivity, as firms with low productivity, medium to high capital, and low debt choose to exit without declaring bankruptcy, while firms with medium productivity, medium to high capital, and low debt are more likely to choose Chapter 11 reorganization, which implies lower recovery to creditors. Lastly, as in Figure 3.5, firms that have lower future capital and that borrow face lower (higher) debt prices (interest rates).

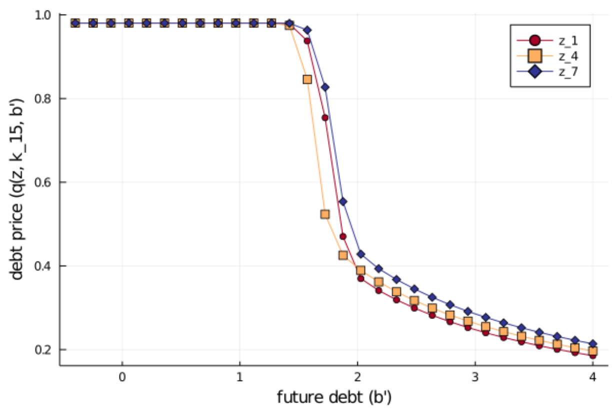
3.6.4 Policy Experiment

3.6.4.1 Policy Shock

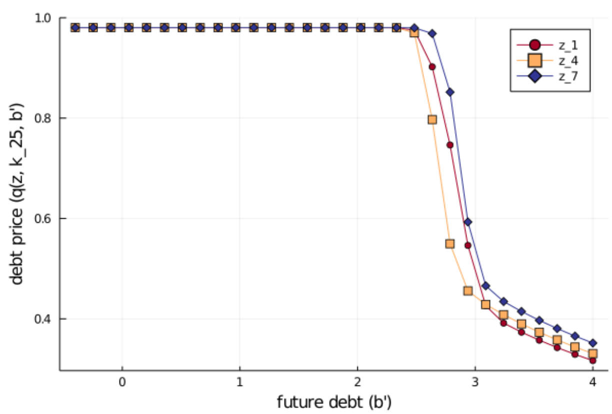
I model the SBRA as a decrease in the fixed cost of Chapter 11 only for small businesses. To be precise, I decrease $c_{11,0S}$ by 75% while $c_{11,0L}$ stays the same. Therefore, the fixed cost of



(a) $k = 0.64$



(b) $k = 1.98$



(c) $k = 3.33$

Figure 3.6: Debt Price Schedule

Chapter 11 for small businesses under the SBRA is $0.0157/4 = 0.0039$. The 75% reduction in reorganization costs for small businesses is meant to capture the fact that (i) a debtor's maximum exclusivity period for filing a reorganization plan in Chapter 11 is 18 months for medium-to-large businesses, while the period is 300 days ($\approx$ 50% of 18 months) for small businesses and (ii) per period bankruptcy costs are lower for small businesses under the SBRA as various requirements for Chapter 11 filers are waived (see Section 3.2.2).

Figure 3.7 presents the estimated bankruptcy cost functions from (3.22) and (3.23). The bankruptcy costs for Chapter 11 are higher than for Chapter 7 for all debt levels in the model. I model the SBRA as a 75% decrease in the fixed cost of filing for Chapter 11 only for small businesses. Chapter 11 reorganization costs under the SBRA are still larger than Chapter 7 bankruptcy costs.

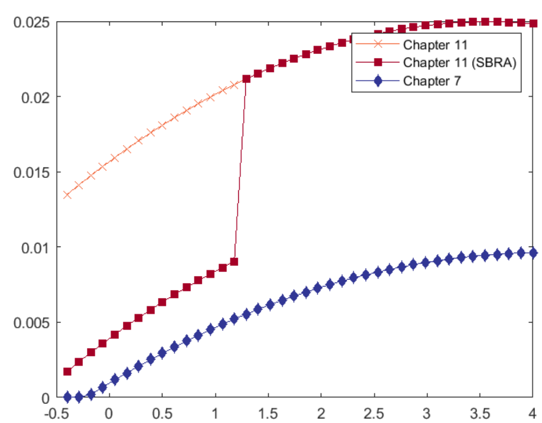


Figure 3.7: Estimated Bankruptcy Cost – Model

3.6.4.2 Results

I first describe how operation decision rules change after the SBRA. Table 3.8 shows the shares of each decision for the entire economy and for different productivity levels. I compute these shares by assuming a uniform distribution over (z, k, b), so these shares represent the area for each operation choice in Figure 3.4. This exercise thus just captures how decision rules would change after the SBRA; it does measure how shares of different choices would change in the new stationary equilibrium. Three results are notable. First, the range of Chapter 11 reorganization expands after the SBRA, as bankruptcy costs in reorganization are lower. Second, the SBRA causes some firms that choose Chapter 7 before the SBRA to choose Chapter 11 instead. At the same time, the SBRA causes some firms that operate normally or exit without declaring bankruptcy prior to the SBRA to choose Chapter 11 reorganization instead. Lastly, the overall share of firms in bankruptcy (Chapter 7 + Chapter 11) increases after the SBRA, which has an implication for debt price schedules as creditor recovery can be lower.

	z_1		z_4		z_7		All	
	pre-SBRA	post-SBRA	pre-SBRA	post-SBRA	pre-SBRA	post-SBRA	pre-SBRA	post-SBRA
N (%)	2.56	2.55 ▼	45.06	45.00 ▼	54.38	54.29 ▼	36.38	36.33 ▼
X (%)	48.68	48.67 ▼	1.77	1.76 ▼	0.00	0.00	13.46	13.46 ▼
7 (%)	32.91	32.88 ▼	13.99	13.89 ▼	2.83	2.81 ▼	15.69	15.63 ▼
11 (%)	15.86	15.89 ▲	39.18	39.35 ▲	42.79	42.89 ▲	34.47	34.59 ▲
7 + 11 (%)	48.77	48.77 ▲	53.17	53.24 ▲	45.62	45.70 ▲	50.16	50.21 ▲

Table 3.8: Policy Experiment: Decision Rules

Notes. ▲: increase, ▼: decrease.

I next describe how the SBRA affects the key aggregate moments that I used to calibrate the parameters (Table 3.9). First, after the SBRA, the firm exit rate decreases as fewer firms choose to exit or file for Chapter 7 liquidation. The firm bankruptcy rate increases, as the number of firms in Chapter 11 increases more than the decrease in Chapter 7 firms. As intended, the SBRA decreases the ratio of Chapter 7 filing among bankrupt firms. The SBRA increases the creditor recovery rate in Chapter 11, as lower Chapter 11 bankruptcy costs increase the debtor's surplus in the bargaining game (3.14), so the Nash bargaining solution moves in favor of creditors. The leverage ratios for operating firms and Chapter 11 firms decrease because investment increases, as will be shown later. One might wonder how the average investment rate decreases if investment increases. As the investment rate is defined by $\frac{i}{k}$, and the increase in investment shifts the distribution of firms towards high capital, the average $\frac{i}{k}$ can decrease.

	pre-SBRA	post-SBRA	% change
Firm Exit Rate (%)	7.69	7.66	-0.41
Firm Bankruptcy Rate (%)	0.63	0.64	2.68
Chapter 7 Share (%)	72.35	69.71	-3.65
Chapter 11 Recovery Rate (%)	58.66	61.46	4.77
(Median) Debt-to-Assets Ratio for Operating Firms	0.38	0.38	-0.25
(Median) Debt-to-Assets Ratio for Chapter 11 Firms	1.15	1.07	-7.40
Average Investment Rate (%)	12.23	12.22	-0.09

Table 3.9: Policy Experiment: Moments

Lastly, I analyze the impact of the SBRA on aggregate variables. Table 3.10 presents aggregate variables before the SBRA and how much the variables change after the SBRA. I want to mainly focus on consumption, output, and productivity. The SBRA increases consumption (+0.10%), so there is a welfare gain from the SBRA. However, the SBRA has small negative impacts on aggregate output (-0.01%) and productivity (-0.02%). The diverging impact on consumption, output and productivity are not immediately intuitive, so I explain them further below.

First, I discuss the welfare implication of the SBRA. This policy experiment is basically a comparative statics exercise. Baqaee and Farhi (2020) show that the impact of changes in parameters on welfare can be decomposed into two parts: (i) the direct change in available resources through the change in production possibilities and (ii) the improvement in allocational efficiency through resource reallocation. As I assume that bankruptcy costs are transfers from firms to the household, there is no direct change in available resources from the change in bankruptcy costs. One important lesson from Baqaee and Farhi (2020) is that to have a first-

	pre-SBRA	post-SBRA	% change
Aggregate Consumption, C	0.5922	0.5928	0.0957
Aggregate Output, Y	1.7559	1.7557	-0.0130
Fixed Cost, CF	0.5315	0.5310	-0.1005
Investment, I	0.3232	0.3234	0.0680
Adjustment Costs, Ψ	0.0319	0.0318	-0.1802
External Finance Costs, Λ	0.1326	0.1325	-0.0627
Entry Costs, E	0.3557	0.3549	-0.2303
Exit Value, X	0.2113	0.2108	-0.2273
TFP (= $Y/K^{\alpha\nu}$)	1.1628	1.1626	-0.0204
Output-weighted Mean Prod., $\hat{z}$	1.1288	1.1286	-0.0112
Unweighted Mean Prod., $\bar{z}$	1.0447	1.0447	-0.0068
Olley-Pakes Covariance	0.0840	0.0840	-0.0656
Mass (entrants)	0.1502	0.1494	-0.5482
Mass (total)	1.9531	1.9504	-0.1350
Wage, w	1.0000	0.9999	-0.0130
Capital, K	4.3464	4.3476	0.0265
Bankruptcy Costs, BC	0.0001	0.0001	-12.8484
Average Spread (%)	20.1701	20.1785	0.0419

Table 3.10: Policy Experiment: Aggregate Variables

Notes. Resource constraint, $C = Y - CF - I - \Psi - \Lambda - E + X$.

order welfare impact from resource reallocation the initial equilibrium must be inefficient. Also, it is important to identify where inefficiencies come from and how a change in parameter affects the inefficiencies in the initial equilibrium.

In the pre-SBRA economy, inefficiencies come from the fact that firms can enjoy the benefits of limited liability in Chapter 7 and charge off some of their debts in Chapter 11. To see the inefficiencies intuitively, I here define some notations and concepts. For simplicity, I assume the variance of the taste shocks $\sigma = 0$. Let $R_N(z, k, b)$ and $R_{11}(z, k, b)$ be the discounted sum of consumption (net resources) that firms with productivity z, capital k, and debt b in normal operation and Chapter 11 can deliver in the pre-SBRA economy. If we ignore $c_7(b)$ and $c_{11}(b)$ for the moment to clarify the argument, R_N and R_{11} have the following relationships with V_N and V_{11} in (3.7) and (3.10).

$$V_N(z, k, b) = R_N(z, k, b) - b \tag{3.25}$$

$$V_{11}(z, k, b) = R_{11}(z, k, b) - \phi b \tag{3.26}$$

Next I define an inefficient liquidation or reorganization in the pre-SBRA economy. An inefficient liquidation occurs when a firm chooses to liquidate to enjoy the benefits of limited liability even though its resources can contribute more to the economy under other operation choices.

[inefficient liquidation] A firm with (z, k, b) liquidates inefficiently if $R_N(z, k, b) > s_7k$, $R_{11}(z, k, b) > s_7k$, or $s_xk > s_7k$; and $max(s_7k - b, 0) \geq V_N(z, k, b)$, $max(s_7k - b, 0) \geq V_{11}(z, k, b)$, and $max(s_7k - b, 0) \geq s_xk - b$. According to this definition of inefficient liquidation, Chapter 7 liquidation is always inefficient in my calibrated model as the calibrated s_x is greater than s_7. If a firm chooses to liquidate when $R_{11}(z,k,b) > s_7 k$, the SBRA increases welfare by decreasing inefficient liquidation.

Similarly, an inefficient reorganization occurs when a firm chooses to reorganize to write off existing debts even though its resources can contribute more to the economy under other operation choices. The SBRA can increase inefficient reorganization by inducing otherwise non-distressed firms to reorganize.

[inefficient reorganization] A firm with (z,k,b) reorganizes inefficiently if $R_N(z,k,b) > R_{11}(z,k,b)$ or $s_x k > R_{11}(z,k,b)$; and $V_{11}(z,k,b) \geq V_N(z,k,b)$ and $V_{11}(z,k,b) \geq s_x k - b$. Note that $V_{11} \geq max(s_7 k - b, 0)$ cannot happen when $s_7 k > R_{11}(z,k,b)$, as firms always write off more debt in Chapter 7 than Chapter 11 due to the"Best Interest Test" in Chapter 11. Therefore, if $s_7 k > R_{11}(z,k,b)$, then firms always choose to liquidate.

One important lesson is that, given $R_N(z,k,b)$ and $R_{11}(z,k,b)$, the SBRA decreases inefficient liquidation while it increases inefficient reorganization. The net welfare impact from these countervailing channels is ambiguous, so a quantitative analysis is necessary.

In addition to the margins of inefficient liquidation and inefficient reorganization, the SBRA affects welfare through other channels. Since there are financial frictions (corporate income tax, equity issuance costs, limited liability, and Chapter 11) in the pre-SBRA economy, $R_N(z,k,b)$ and $R_{11}(z,k,b)$ themselves are not efficiently determined. For example, as the debt price schedule $q(z,k',b')$ depends on future capital k' and there are equity issuance costs, investment decisions under normal operation and Chapter 11 deviate from optimal investment in the frictionless world (e.g. a general equilibrium version of Hopenhayn (1992) with capital). This means that, as long as the debt price schedule $q(z,k',b')$ changes due to the lower bankruptcy costs in Chapter 11, $R_N(z,k,b)$ and $R_{11}(z,k,b)$ can be closer to or farther from their efficient values under the SBRA.

This effect depends on whether a firm with a (z, k, b) over- or under-invests prior to the SBRA, and how a change in $q(z, k', b')$ affects its investment decision. In short, there are various channels by which the SBRA affects welfare, so a serious quantitative analysis is essential. Ideally, I should decompose the increase in welfare from the SBRA into these countervailing forces (inefficient liquidation, inefficient reorganization, changes in $R_N(z, k, b)$, and changes in $R_{11}(z, k, b)$), but I will leave it as a future task.

I note that entry also endogenously changes due to the change in the debt price schedule. Since the average spread increases, the value of entry decreases, so there are fewer entrants under the SBRA. However, I should emphasize that entry margin is itself efficient, in the sense that, given $R_N(z, k, b)$, $R_{11}(z, k, b)$, and $q_E(k', b')$, an infinitesimal change in the mass of entrants does not change welfare. This is because the free entry condition $E_\epsilon V_E(\epsilon) = 0$ holds and there is no debt overhang problem for entrants.

I next describe how each of the four welfare-relevant margins and the entry margin affect the resource constraint (Table 3.11). First, less inefficient liquidation leads to higher aggregate output and fixed costs as the number of operating firms increases. The change in aggregate investment is ambiguous, as some firms in Chapter 11 have positive investment while others have negative investment. Aggregate adjustment costs and external financing costs increase, as firms do not adjust capital or issue equity in Chapter 7 liquidation. The value of capital recovered in exit decreases as there is less Chapter 7 liquidation.

The analysis of inefficient reorganization is more complicated than for inefficient liquidation. Increased inefficient reorganization can occur under the SBRA both for firms in normal operation and firms exiting without bankruptcy. We need to distinguish these two cases. When the SBRA causes firms that otherwise exit without bankruptcy to instead choose Chapter 11 reorganization.

The impact on the resource constraint is the same as the effect of lower inefficient liquidation. However, when the SBRA brings firms in normal operation into Chapter 11 reorganization, the impact on resources is theoretically ambiguous, as some firms in Chapter 11 invest more than firms in normal operation and some firms do not. Therefore, the consequences for Y, CF, I, Ψ, and Λ cannot be analytically determined, and depend on parameter values.

When there is less entry, there will be fewer operating firms. As a result, output, fixed costs, adjustment costs, and external finance costs decrease. However, the effect on investment is ambiguous as it depends on what (k', b') the entrants optimally choose. The value of capital for exiting firms would decrease, as reduced entry must imply less exit in steady state.

Lastly, the impact of changes in R_N and R_{11} on the aggregate resource constraint is ambiguous as I do not have clear analytic predictions on how R_N and R_{11} should move. As mentioned above, because $q(z, k', b')$ depends on investment and there exist equity issuance costs, it is not clear whether investment is excessive or not prior to the SBRA, from the viewpoint of an efficient equilibrium in which there are no financial frictions. For example, for a firm with high capital under normal operation, equity issuance costs should not be a concern. As higher investment decreases loan rates ($q(z, k', b')$ increases with k'), such a firm tends to over-invest prior to the SBRA. In contrast, for a low capital firm, paying equity issuance costs can be a concern, so it might under-invest pre-SBRA. As explained above, changes in R_N and R_{11} under the SBRA may be heterogeneous, so the impact of lower Chapter 11 bankruptcy costs on the aggregate resource constraint through R_N and R_{11} is theoretically ambiguous.

	C	Y	CF	I	Ψ	Λ	E	X
Less Inefficient Liquidation	▲	▲	▲	?	▲	▲	-	▼
More Inefficient Reorganization	▼	(?,▲)	(?,▲)	(?,?)	(?,▲)	(?,▲)	-	(?,▼)
Less Entry	-	▼	▼	?	▼	▼	▼	▼
Change in R_N	?	?	?	?	?	?	-	?
Change in R_{11}	?	?	?	?	?	?	-	?

Table 3.11: Policy Experiment: Resource Constraint

Notes. ▲: increase, ▼: decrease, -: no effect, ?: ambiguous. The first element in the 2-tuples in the "more inefficient reorganization" is for when inefficient reorganization occurs from normal operation while the second element is for when inefficient reorganization occurs from an exit without bankruptcy.

Aggregate productivity measures (TFP and output-weighted productivity) decline under the SBRA as there is less entry and exit. On average, entrants have the median productivity z_4, as $\bar{G}(\cdot)$ has the highest probability at the median. However, exiting firms have low productivity on average, and rescuing these firms through Chapter 11 reorganization shifts the productivity distribution toward low productivity firms.

3.7 Conclusion

This paper studies the long-run aggregate implications of a recent corporate bankruptcy reform in the U.S., known as the SBRA. Congress amended the existing bankruptcy code by introducing a streamlined debt reorganization process specifically tailored for small businesses. I develop a firm dynamics model with bankruptcy choices of insolvent firms and estimate this model using novel bankruptcy data I have uncovered. Traditionally, information about bankrupt firms has been primarily sourced from Compustat, which predominantly covers large public firms. Given that the SBRA is specifically designed to target small firms, utilizing data from large firms may not be as relevant. This paper's primary contribution lies in gathering information about the universe of bankrupt firms in the U.S. and employing this unique dataset to estimate the quantitative model. The findings of this paper indicate that the SBRA leads to an improvement in the long-run welfare of the U.S.

Appendix A: Appendix for Chapter 1

A.1 Additional details for the simple theoretical model

A.1.1 Proof for Proposition 1

Proof. Equation (1.5) implies that $logMP_i \propto -log\xi_i$, thus $\sigma^2_{logMP_i} = \sigma^2_{log\xi_i}$ hold. When $\log \xi_i$ is close to $\mathbb{E}[\xi_i]$, $log\xi_i \approx log\mathbb{E}[\xi_i] + \frac{1}{\mathbb{E}[\xi_i]}(\xi_i - \mathbb{E}[\xi_i])$. Thus,

$$\begin{aligned}\sigma^2_{logMP_i} = \sigma^2_{log\xi_i} = Var(log\xi_i) \approx Var(log\mathbb{E}[\xi_i] + \frac{1}{\mathbb{E}[\xi_i]}(\xi_i - \mathbb{E}[\xi_i])) \\ = \frac{1}{\mathbb{E}[\xi_i]^2}Var(\xi_i),\end{aligned}$$

holds.

When $\mathbb{E}[m_i] = 0$,

$$\frac{1}{\mathbb{E}[\xi_i]^2}Var(\xi_i) = \frac{1}{(1-\mathbb{E}[\tau_i])^2}(\sigma^2_{\tau_i} + \tau_c^2\sigma^2_{m_i} + 2\tau_c Cov(\tau_i, m_i)).$$

This implies that,

$$\frac{\partial \sigma^2_{logMP_i}}{\partial \tau_c} = \frac{1}{(1-\mathbb{E}[\tau_i])^2}(2\tau_c\sigma^2_{m_i} + 2\rho\sigma_{\tau_i}\sigma_{m_i}),$$

where $\rho = \rho(\tau_i, m_i)$ is the correlation between emission intensity and distortions.

Thus,

$$\frac{d\sigma^2_{logMP_i}}{d\tau_c} = \begin{cases} \geq 0, \text{if } \rho \geq 0 \text{ or } (\rho < 0, \tau_c \geq \frac{\rho\sigma_{\tau_i}}{\sigma_{m_i}}) \\ < 0, \text{if } (\rho < 0, \tau_c < \frac{\rho\sigma_{\tau_i}}{\sigma_{m_i}}). \end{cases}$$

■

A.1.2 Economic intuition and numerical illustration for $E(m) > 0$

When the mean of emission intensity is positive, an increase in carbon tax influences the dispersion in the logarithm of marginal products in two ways. First, a carbon tax treats firms differently due to variations in emission intensity. It penalizes dirtier firms relatively more than cleaner ones, leading to a greater increase in the marginal products of dirtier firms compared to cleaner ones, as explained in Section 1.3. I refer to this channel as the ’differential treatment channel’. Second, an increase in carbon tax implies a general rise in the level of distortions, which is termed the ’level channel.’ To illustrate this more clearly, I decompose the emission intensity (m_i) as the sum of the mean ($\bar{m}$) and a mean-zero component (ϵ_i):

$$m_i = \bar{m} + \epsilon_i, \text{ where } \bar{m} > 0, \text{ and } \mu(\epsilon_i) = 0.$$

In this case, the marginal products (MP_i) are proportional to:

$$MP_i \propto \frac{1}{1 - \tau_i - (\tau_c\bar{m} + \tau_c\epsilon_i)}.$$

In Proposition 1, $\bar{m}$ is assumed to be zero, meaning that an increase in carbon tax only affects the marginal products through the differential treatment channel.

To focus on the effect of carbon tax on the dispersion of marginal products through the level channel, I set $\epsilon_i = 0$. The relative ratio of marginal products for a firm i with and without a carbon tax is:

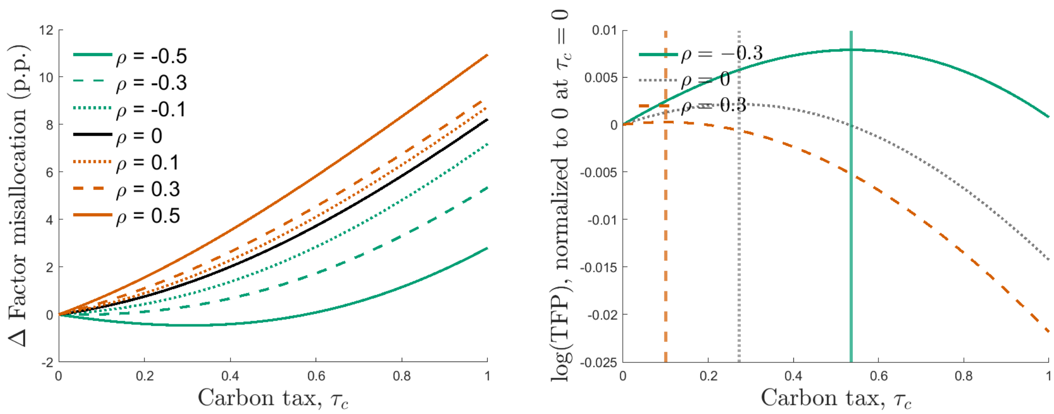
$$\frac{MP_{i,\tau>0}}{MP_{i,\tau=0}} \propto \frac{1-\tau_i}{1-\tau_i-\tau_c\bar{m}} = \frac{1}{1-\tau_c\bar{m}/(1-\tau_i)}.$$

This implies that a carbon tax leading to an overall increase in distortions by $\tau_c\bar{m}$ results in previously more distorted firms (firms with higher τ_i) having relatively higher marginal products. Consequently, the dispersion of $log(MP_i)$ increases with the overall rise in distortions due to an increase in carbon tax. Thus, through the level channel, a carbon tax exacerbates existing misallocation.

The dominance of either the differential treatment channel or the level channel critically hinges on the correlation between emission intensity and distortions. When the correlation is positive or modestly negative, a carbon tax can amplify existing misallocation. However, if the correlation is sufficiently negative, the differential treatment channel dominates, potentially mitigating misallocation. This is shown in Panel (a) of Figure A.1.

Importantly, as the level effect remains the same across different levels of correlation, the differential treatment channel is the primary factor determining the level of optimal carbon taxes. Hence, the assertion that the correlation between emission intensity and distortions is a key metric in determining the optimal carbon taxes holds true even for cases where $E(m) > 0$. Panel (b) of Figure A.1 illustrates this point.

Figure A.1: Factor misallocation and the optimal carbon taxes when $E(m) > 0$



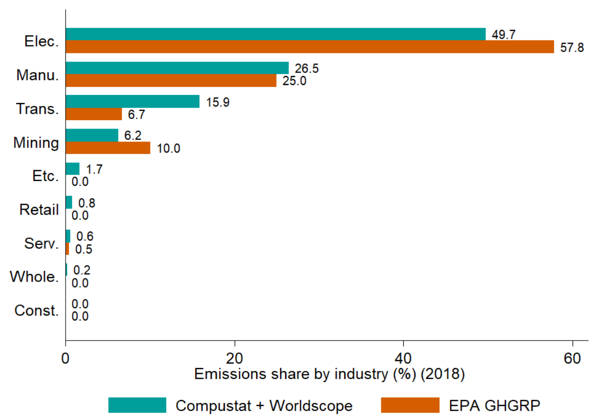
(a) Misallocation (b) Optimal carbon taxes

Notes. I simulate 10,000 firms with different levels of productivity, distortions, and emission intensity, but with varying degrees of correlation between emission intensity and distortions, denoted as ρ. For each level of the ρ and carbon taxes, I calculate the degree of misallocation as $(1 - \text{allocative efficiency})$, and $log(TFP)$ using Equation (1.6). I plot the degree of misallocation and $log(TFP)$ relative to the value when the carbon tax is zero for each value of ρ. In Panel (b), the vertical lines represent carbon taxes that maximize $log(TFP)$. Parameter values: $\alpha = 0.8$, $\gamma = 0.005$, $\mu_{logz} = 0$, $\sigma_{logz} = 0.2$, $\mu_\tau = 0$, $\sigma_\tau = 0.2$, $\mu_m = 0.2$, $\sigma_m = 0.2$, and $\bar{F} = 10$.

A.2 Additional details on the empirical analysis

A.2.1 Compustat-Worldscope vs. EPA GHGRP

Figure A.2: Share of carbon emissions by industry in Year 2018: Compustat-Worldscope vs. EPA GHGRP



Notes. The EPA Greenhouse Gas Reporting Program (GHGRP) mandates the submission of greenhouse gas (GHG) data and pertinent details from significant GHG emission sources, fuel and industrial gas providers, and CO_2 injection sites across the United States. This reporting obligation applies to around 8,000 facilities annually. Generally, facilities are mandated to submit yearly reports if GHG emissions from covered sources surpass 25,000 metric tons CO_2e per year.

A.2.2 Additional regression results

Table A.1: Emission intensity (by COGS) and the measures of distortions

	(1)	(2)	(3)
	log(MRPK)	log(MRPL)	log(TFPR)
log(emissions/cogs)	-0.161***	-0.037	-0.070***
	(0.022)	(0.022)	(0.019)
Adj. R^2	0.827	0.734	0.650
Ind x Year FE			
N	2,847	2,819	2,819

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The table provides the results of empirical analysis, where I conduct regressions of a distortions measure (TFPR) on emission intensities measured by the cost of goods sold (COGS). This analysis incorporates controls for 4-digit SIC industry-year dummies. Standard errors, which are presented in parentheses, are clustered at both the firm and year levels.

Table A.2: Emission intensities (by COGS) and the measures of distortions by industries

log(TFPR)	Mining	Manufacturing	Transportation	Electricity
log(emissions/cogs)	-0.150*	-0.086***	-0.382***	-0.020
	(0.076)	(0.028)	(0.054)	(0.028)
Adj. R^2	0.476	0.537	0.778	0.074
Ind x Year FE				
N	221	1,490	254	335

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The table provides the results of empirical analysis, where I conduct regressions of a distortions measure (TFPR) on emission intensities measured by the cost of goods sold (COGS). This analysis incorporates controls for 4-digit SIC industry-year dummies and conducts separate analyses for firms within the mining, manufacturing, transportation, and electricity-generating sectors. Standard errors, which are presented in parentheses, are clustered at both the firm and year levels.

Table A.3: Emission intensity (by COGS) and productivity

log(emissions/cogs)	(1)	(2)	(3)	(4)	(5)	(6)
log(TFPQ)	-0.581***	-0.518***	-0.585***	-0.746***	-0.509***	-0.629***
	(0.170)	(0.123)	(0.150)	(0.186)	(0.129)	(0.151)
Size				0.174***	-0.035	0.151
				(0.052)	(0.080)	(0.115)
Age				-0.001	0.000	0.000
				(0.003)	(.)	(.)
Adj. R^2	0.787	0.973	0.973	0.792	0.973	0.973
Year FE						
Ind x Year FE						
Firm FE						
N	2,819	3,789	2,671	2,819	3,789	2,671

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The table provides the results of empirical analysis, where I conduct regressions of emission intensities measured by the cost of goods sold (COGS) on firms' productivity. This analysis incorporates controls for 4-digit SIC industry-year dummies. I also include controls for firm size, measured by log(total assets), and firm age. Standard errors, which are presented in parentheses, are clustered at both the firm and year levels.

Table A.4: Emission intensities (by COGS) and productivity by industries

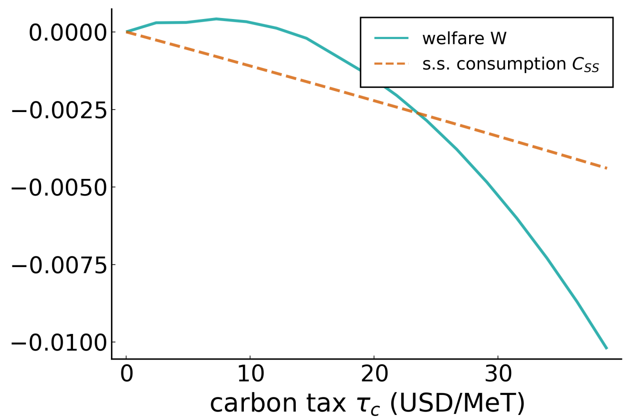
log(emissions/sales)	Mining	Manufacturing	Transportation	Electricity
log(productivity)	-1.053***	-0.944***	-1.446***	-0.340
	(0.338)	(0.278)	(0.158)	(0.378)
Size	0.043	0.194***	0.139*	-0.045
	(0.099)	(0.058)	(0.065)	(0.195)
Age	0.000	0.001	0.005*	0.000
	(0.003)	(0.003)	(0.003)	(.)
Adj. R^2	0.397	0.578	0.961	0.933
Ind x Year FE				
N	221	1490	254	330

Notes. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The table provides the results of empirical analysis, where I conduct regressions of emission intensities measured by the cost of goods sold (COGS) on firms' productivity. This analysis incorporates controls for 4-digit SIC industry-year dummies and conducts separate analyses for firms within the mining, manufacturing, transportation, and electricity-generating sectors. I also include controls for firm size, measured by log(total assets), and firm age. Standard errors, which are presented in parentheses, are clustered at both the firm and year levels.

A.3 Additional details on the quantitative analysis

A.3.1 Welfare over transition path vs. steady state consumption

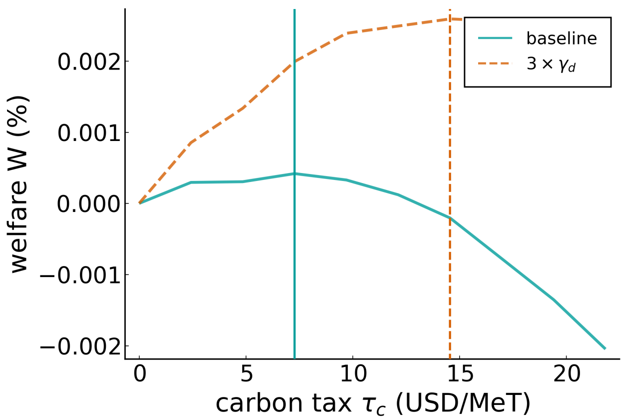
Figure A.3: Welfare over carbon taxes vs. steady state consumption



Notes. The x-axis represents the different levels of carbon taxes. The solid line represents the difference in welfare, which is the lifetime utility of the representative household over a transition path, compared to the case with zero carbon tax. This difference is expressed in units of consumption equivalent welfare. The dashed line represents steady state consumption relative to the case with zero carbon tax.

A.3.2 A higher carbon damage

Figure A.4: Welfare over carbon taxes: baseline vs. $3 \times \gamma_d$



Notes. The x-axis represents the different levels of carbon taxes. The y-axis measures the difference in welfare, which is the lifetime utility of the representative household over a transition path, compared to the case with zero carbon tax. This difference is expressed in units of consumption equivalent welfare. The solid lines correspond to the baseline case, while the dashed lines pertain to the case with a higher carbon damage parameter.

Appendix B: Appendix for Chapter 2

B.1 Derivation for the sufficient condition for case (ii)

In case (ii) where the lender is the only supplier for labor, $h(w, m, x_1^b) = 0$ in Equation (2.8). By solving Equation (2.8) for m and plugging in Equation (2.9),

$$\alpha\phi_\pi \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} K = \frac{1}{m}\left(e_1^l + w\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} K + x_1^l - \left(\frac{w^{(1+\frac{\psi}{\alpha})}}{(1-\alpha)^{\frac{\psi}{\alpha}} K^{\psi}}\right)^{\frac{1}{\gamma}}\right).$$

By differentiating this equation with respect to x_1^l,

$$\left[\frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}-\frac{\psi}{\alpha}}\beta\phi_\pi}{K^{\psi-1}}\left((1+\frac{\psi}{\alpha}) - (\frac{1-\alpha}{\alpha})\right) w^{\frac{\psi}{\alpha}-\frac{1-\alpha}{\alpha}} + (1-\alpha)^{\frac{1}{\alpha}} K(\frac{1}{\alpha}-1) w^{-\frac{1}{\alpha}} \right.$$
$$\left. + \frac{1}{\gamma}(1+\frac{\psi}{\alpha})\left(\frac{w^{(1+\frac{\psi}{\alpha})}}{(1-\alpha)^{\frac{\psi}{\alpha}} K^{\psi}}\right)^{\frac{1}{\gamma}} \frac{1}{w}\right] \frac{\partial w}{\partial X_1^l} = 1.$$

As long as $(1+\frac{\psi}{\alpha}) - (\frac{1-\alpha}{\alpha}) \geq 0$ holds, $\frac{\partial w}{\partial X_1^l} \geq 0$.

This is a sufficient condition, not a necessary condition. To understand what necessity and sufficiency mean in this context, it is helpful to invoke Figure 2.1. Condition (2.12) holds if the function $w = L(m)$ is steeper with respect to m than the function $w = B^{-1}(m, X_1^l)$. The relative steepness of the two functions depends on many model primitives, including γ. However, $w = L(m)$ alone does not depend γ. Under the condition $1 + \frac{\psi}{\alpha} > \frac{1-\alpha}{\alpha}$ this function is so "flat", that the function $w = B^{-1}(m, X_1^l)$ is steeper for any value of γ. In this case, the appropriate relative size of α and ψ alone suffices to fulfill condition (2.12). But this is not necessary. Even when $w = L(m)$ is less "flat" than it is under the sufficient condition, then there are γ values that can make $w = B^{-1}(m, X_1^l)$ steep enough to fulfil condition (2.12).

B.2 Additional details for the small open economy model

B.2.1 SOE model with tradable production and earnings-based constraints

There are two time periods $t = 1, 2$. There is a representative household who consumes tradable goods c_t^T and nontradable goods c_t^N according to a standard CES aggregator. The representative agent starts period 1 with an initial net worth X (see Section 2.2.3 for a discussion of aggregate net worth). The supply of nontradable goods is exogenously determined by an endowment y_t^N while tradable goods y_t^T are produced using capital and labor in period 1 and using only capital in period 2. The agent supplies labor (ℓ^s) in period 1. Capital K is is fixed. We assume risk-neutrality in period 2. International borrowing ($-x_2$) is denominated by tradable goods units with a fixed bond price m. The representative agent is subject to earnings-based borrowing constraints that are discussed in the main text. The price of nontradable goods in period t and wage are denoted by p_t and w, respectively.

The optimization problem of the representative household is

$$\max_{c_1^T, c_1^N, c_2^T, c_2^N, \ell^d, \ell^s, x_2} (u(c_1) - v(\ell^s)) + \beta c_2$$

$$s.t.$$

$$c_1^T + p_1 c_1^N + m x_2 = (y_1^T - w\ell^d) + p_1 y_1^N + w\ell^s + X$$

$$c_2^T + p_2 c_2^N = y_2^T + p_2 y_2^N + x_2$$

$$-x_2 \leq \phi_\pi((y_1^T - w\ell^d) + p_1 y_1^N)$$

where

$$c_t = \left[\theta (c_t^T)^\rho + (1-\theta)(c_t^N)^\rho\right]^{\frac{1}{\rho}}, \quad t \in \{1,2\}, \quad \rho \in (-\infty, 1]$$

$$y_1^T = z_1 K^\alpha (\ell^d)^{1-\alpha}$$

$$y_2^T = z_2 K.$$

The market clearing conditions are:

$$c_1^T + m x_2 = y_1^T + X, \quad c_1^N = y_1^N$$

$$c_2^T = y_2^T + x_2, \quad c_2^N = y_2^N$$

$$\ell^d = \ell^s.$$

When the borrowing constraint binds, (p_1, w) are determined by the following two equations:

$$p_1 = \frac{1-\theta}{\theta}\left(\frac{(1+\alpha m \phi_\pi) y_1^T + X}{y_1^N} + m\phi_\pi p_1\right)^{1-\rho} \tag{B.1}$$

$$\left(\frac{(1-\alpha) z_1}{w}\right)^{\frac{1}{\alpha}} K = \ell^{s^*}, \tag{B.2}$$

where ℓ^{s^*} is the optimal labor supply.

We now show why $sign(\partial p_1/\partial X) = sign(\partial w/\partial X)$ holds when labor supply is endogenously determined. For a general preference $u(c_1) = \frac{1}{1-\gamma}c_1^{1-\gamma}$, $v(\ell^s) = \frac{1}{1+\psi}(\ell^s)^{1+\psi}$, the optimal labor supply ℓ^{s^*} is

$$\ell^{s^*} = \left(w\theta(\theta + (1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}})^{\frac{1-\rho}{\rho}} c_1^{-\gamma}\right)^{\frac{1}{\psi}}, \tag{B.3}$$

where $c_1 = y_1^N[\theta(\frac{\theta p_1}{1-\theta})^{\frac{\rho}{1-\rho}} + (1-\theta)]^{\frac{1}{\rho}}$.

By differentiating Equation (B.2) with respect to X after plugging in equation (B.3), the following relationship holds:

$$\frac{1}{w}\left[\psi + \alpha\right]\frac{\partial w}{\partial X} = \frac{1}{p_1}\left[\alpha\epsilon + \frac{\alpha\gamma}{1-\rho}(1-\epsilon)\right]\frac{\partial p_1}{\partial X}, \tag{B.4}$$

where $\epsilon = \frac{(1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}}}{\theta+(1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}}} < 1$. As $\psi + \alpha > 0$ and $\alpha\epsilon + \frac{\alpha\gamma}{1-\rho}(1-\epsilon) > 0$, $sign(\partial p_1/\partial X) = sign(\partial w/\partial X)$ holds. Note that this result holds even with GHH preferences (when $\gamma = 0$).

B.2.2 SOE model with nontradable production and earnings-based constraints

We also consider the case where nontradable goods are produced and tradable goods are an endowment. (p_1, w) are still the key prices in this case, and we can characterize them with similar equilibrium conditions:

$$p_1 = \frac{1-\theta}{\theta}\left(\frac{(1+m\phi_\pi)y_1^T + X}{y_1^N} + \alpha m\phi_\pi p_1\right)^{1-\rho} \tag{B.5}$$

$$\left(\frac{(1-\alpha)z_1 p_1}{w}\right)^{\frac{1}{\alpha}} K = \ell^{s^*} \tag{B.6}$$

For a general specification of preferences $u(c_1) = \frac{1}{1-\gamma} c_1^{1-\gamma}$, $v(\ell^s) = \frac{1}{1+\psi} (\ell^s)^{1+\psi}$, we derive a relationship between $\partial p_1/\partial X$ and $\partial w/\partial X$

$$\frac{1}{w} \left[\psi + \gamma(1-\alpha) + \alpha\right] \frac{\partial w}{\partial X_1} = \frac{1}{p_1} \left[\psi + \alpha\epsilon + \gamma(1-\alpha) + \frac{\alpha\gamma}{1-\rho}(1-\epsilon)\right] \frac{\partial p_1}{\partial X_1}, \tag{B.7}$$

where $\epsilon = \frac{(1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}}}{\theta + (1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}}} < 1$. As $\psi + \gamma(1-\alpha) + \alpha > 0$ and $\psi + \alpha\epsilon + \gamma(1-\alpha) + \frac{\alpha\gamma}{1-\rho}(1-\epsilon) > 0$, $sign(\partial p_1/\partial X) = sign(\partial w/\partial X)$ also holds under this alternative SOE model with nontradable production. Note that $\partial w/\partial X$ is not zero even with exogenously determined labor supply as labor demand changes with p_1 which changes with X. Thus, it can be shown that $sign(\partial p_1/\partial X) = sign(\partial w/\partial X)$ even in a setting with inelastic labor supply.

B.3 Details about the general model

B.3.1 Market clearing conditions

The model's market clearing conditions are the following:

$$\sum_i [c_0^i + h^i(k_1^i)] \leq \sum_i e_0^i \tag{B.8}$$

$$\sum_i c_t^{i,\theta} \leq \sum_i [e_t^i + F^i(k_t^{i,\theta}, \ell_{dt}^{i,\theta})],\ t = 1, 2,\ \forall\theta \tag{B.9}$$

$$\sum_i k_2^{i,\theta} \leq \sum_i k_1^i,\ \forall\theta \tag{B.10}$$

$$\sum_i \ell_{dt}^{i,\theta} = \sum_i \ell_{st}^{i,\theta},\ t = 1, 2, \forall\theta \tag{B.11}$$

$$\sum_i x_t^{i,\theta} = 0,\ t = 1, 2,\ \forall\theta \tag{B.12}$$

B.3.2 First-order conditions

The first-order conditions for the period-1 maximization problem with respect to $x_2^{i,\theta}$ and $k_2^{i,\theta}$ are

$$m_2^\theta \lambda_1^{i,\theta} = \beta\lambda_2^{i,\theta} + \kappa_2^{i,\theta}\Phi_{2x^\theta}^{i,\theta}, \tag{B.13}$$

$$q^\theta \lambda_1^{i,\theta} = \beta\lambda_2^{i,\theta} F_{2k}^{i,\theta}(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) + \kappa_2^{i,\theta}\Phi_{2k}^{i,\theta},\ \forall i, \theta \tag{B.14}$$

Equations (B.13) and (B.14) are the Euler equations for the financial asset and physical investment. Remember that $\Phi_2^{b,\theta}$ is given by (2.29) and $\Phi_2^{l,\theta} = 0$.

Using the envelope conditions $\frac{\partial V^{i,\theta}(\cdot,\cdot)}{\partial n_1^{i,\theta}} = \lambda_1^{i,\theta}$ and $\frac{\partial V^{i,\theta}(\cdot,\cdot)}{\partial k_1^i} = \lambda_1^{i,\theta}(q^\theta + F_{1k}^{i,\theta}(k_1^i, l_{d1}^{i,\theta}))$, the first-order conditions with respect to the asset holding and capital are derived as

$$m_1^\theta \lambda_0^i = \beta \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i, \tag{B.15}$$

$$h^{i\prime}(k_1^i)\lambda_0^i = E_0[\beta \lambda_1^{i,\theta}(F_{1k}^{i,\theta}(k_1^i, \ell_{d1}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i, \ \forall i, \theta \tag{B.16}$$

where λ_0^i is Lagrange multiplier for (2.25) and κ_1^i is Lagrange multiplier for (2.28).

B.3.3 Derivation of distributive and constraint effects

Lemma 1 characterizes relevant properties of the date 1 equilibrium.

The effects of changes in the aggregate state variables $N_1^{j,\theta}$ and K_1^j on agent i's indirect utility at date 1 are given by

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta}\mathcal{D}_{1N^j}^{i,\theta} + \lambda_2^{i,\theta}\mathcal{D}_{2N^j}^{i,\theta} + \kappa_2^{i,\theta}\mathcal{C}_{N^j}^{i,\theta} \tag{B.17}$$

$$V_{K_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dK_1^j} = \lambda_1^{i,\theta}\mathcal{D}_{1K^j}^{i,\theta} + \lambda_2^{i,\theta}\mathcal{D}_{2K^j}^{i,\theta} + \kappa_2^{i,\theta}\mathcal{C}_{K^j}^{i,\theta} \tag{B.18}$$

where $\mathcal{D}_{1N^j}^{i,\theta}$, $\mathcal{D}_{1K^j}^{i,\theta}$, $\mathcal{D}_{2N^j}^{i,\theta}$ and $\mathcal{D}_{2K^j}^{i,\theta}$ are called the distributive effects

$$\mathcal{D}_{1N^j}^{i,\theta} \equiv -\frac{\partial q^\theta}{\partial N_1^{j,\theta}}\Delta K_2^{i,\theta} - \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}} X_2^{i,\theta} - \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}\ell_{d1}^{i,\theta} + \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}\ell_{s1}^{i,\theta} \tag{B.19}$$

$$\mathcal{D}_{1K_1^j}^{i,\theta} \equiv -\frac{\partial q^\theta}{\partial K_1^j}\Delta K_2^{i,\theta} - \frac{\partial m_2^\theta}{\partial K_1^j} X_2^{i,\theta} - \frac{\partial w_1^\theta}{\partial K_1^j}\ell_{d1}^{i,\theta} + \frac{\partial w_1^\theta}{\partial K_1^j}\ell_{s1}^{i,\theta} \tag{B.20}$$

$$\mathcal{D}_{2N^j}^{i,\theta} \equiv -\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}\ell_{d2}^{i,\theta} + \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}\ell_{s2}^{i,\theta} \tag{B.21}$$

$$\mathcal{D}_{2K^j}^{i,\theta} \equiv -\frac{\partial w_2^\theta}{\partial K_1^j}\ell_{d2}^{i,\theta} + \frac{\partial w_2^\theta}{\partial K_1^j}\ell_{s2}^{i,\theta} \tag{B.22}$$

and $\mathcal{C}^{i,\theta}_{N^j}$ and $\mathcal{C}^{i,\theta}_{K^j}$ are called the constraint effects

$$\mathcal{C}^{b,\theta}_{N^j} \equiv \frac{\partial \Phi^{b,\theta}_2}{\partial q^\theta}\frac{\partial q^\theta}{\partial N^{j,\theta}_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial m^\theta_2}\frac{\partial m^\theta_2}{\partial N^{j,\theta}_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_1}\frac{\partial w^\theta_1}{\partial N^{j,\theta}_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_2}\frac{\partial w^\theta_2}{\partial N^{j,\theta}_1} \tag{B.23}$$

$$\mathcal{C}^{b,\theta}_{K^j} \equiv \frac{\partial \Phi^{b,\theta}_2}{\partial q^\theta}\frac{\partial q^\theta}{\partial K^j_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial m^\theta_2}\frac{\partial m^\theta_2}{\partial K^j_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_1}\frac{\partial w^\theta_1}{\partial K^j_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_2}\frac{\partial w^\theta_2}{\partial K^j_1} \tag{B.24}$$

$$\mathcal{C}^{l,\theta}_{N^j} = \mathcal{C}^{l,\theta}_{K^j} = 0 \tag{B.25}$$

for $i \in \{b,l\}$, $j \in \{b,l\}$ and $\theta \in \Theta$.

Proof. The effects of changes in the aggregate state variables (N^θ_1, K_1) on agents' indirect utility are derived by taking partial derivatives of $V^{i,\theta}$ as defined by equations (2.30) to (2.33). We make use of the envelope theorem, according to which the derivatives of $\left\{u^i(c^{i,\theta}_1, \ell^{i,\theta}_{s1}) + \beta u^i(c^{i,\theta}_2, \ell^{i,\theta}_{s2})\right\}$ with respect to the state variables are 0. We further impose a symmetric equilibrium in which $n^{i,\theta} = N^{i,\theta}$ and $k^i_1 = K^i_1$. ■

Remarks on Lemma 1 $\mathcal{D}^{i,\theta}_{1N^j}$, $\mathcal{D}^{i,\theta}_{1K^j}$, $\mathcal{D}^{i,\theta}_{2N^j}$ and $\mathcal{D}^{i,\theta}_{2K^j}$ are called *distributive effects* because

$$\sum_i \mathcal{D}^{i,\theta}_{1N^j} = \sum_i \mathcal{D}^{i,\theta}_{2N^j} = \sum_i \mathcal{D}^{i,\theta}_{1K^j} = \sum_i \mathcal{D}^{i,\theta}_{2K^j} = 0 \tag{B.26}$$

from the market clearing conditions, that is, they are "zero sum" effects across agents, state by state. Such a relation does not hold for the *constraint effects* $\mathcal{C}^{i,\theta}_{N^j}$ and $\mathcal{C}^{i,\theta}_{K^j}$. These collect any derivatives that multiply the shadow price on the financial constraint $\kappa^{i,\theta}_2$. Comparing Lemma 1 to its analogue in DK18, both our inclusion of labor markets and our more general financial constraint change this characterization. In particular, wage changes generate both distributive effects and constraint effects. This observation will be important for the earnings-based constraint.

Third, we also allow equation (2.29) to include the asset price m_2^θ so the constraint effects include partial derivatives with respect to this variable.

B.3.4 Constrained efficient allocation and implementation

The economy's constrained efficient allocation is described by quantities $(C_0^i, K_1^i, X_1^{i,\theta})$, Pareto weights $\alpha^b/\alpha^l = \lambda_0^l/\lambda_0^b$ and shadow prices v_0,v_1^θ, and κ_1^i satisfying the optimality conditions and constraints of the social planner's problem. This allocation can be implemented with a set of tax rate on financial asset and capital purchases.

Derivation of constrained efficient allocation These derivations correspond to Proposition 1 (a) and the associated proof in DK18. The Lagrangian of the social planner's problem can be written as

$$\mathcal{L} = \sum_i \alpha^i \{u^i(C_0^i) + \beta E_0[V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)] + \kappa_1^i \Phi_1^i(X_1^i, K_1^i)\}$$
$$+ v_0 \sum_i [e_0^i - (C_0^i + h^i(K_1^i))] - \int_{\theta\in\Theta} v_1^\theta \sum_i X_1^{i,\theta} d\theta.$$

The first-order conditions of the social planner are

$$\frac{d\mathcal{L}}{dC_0^i} = \alpha^i u'^i(C_0^i) - v_0 = 0, \ \forall i \tag{B.27}$$
$$\frac{d\mathcal{L}}{dX_1^{i,\theta}} = -v_1^\theta + \alpha^i \beta V_n^{i,\theta} + \alpha^i \kappa_1^i \Phi_{1x}^i + \beta \sum_j \alpha_j V_{N^i}^{j,\theta}, \ \forall i, \theta \tag{B.28}$$
$$\frac{d\mathcal{L}}{dK_1^i} = -v_0 h'^i(K_1^i) + \alpha^i \beta E_0[V_k^{i,\theta}] + \alpha^i \kappa_1^i \Phi_{1k}^i + \beta \sum_j \alpha^j E_0[V_{K^i}^{j,\theta}] = 0, \ \forall i \tag{B.29}$$

Note that there are no expectation terms in the second first-order condition since $X_1^{i,\theta}$ is chosen for each θ.

The first first-order condition in the decentralized equilibrium implies $v_0 = \alpha^i \lambda_0^i$, so $\alpha^b/\alpha^l = \lambda_0^l/\lambda_0^b$. We divide the second FOC by α^i, and use $\alpha^i = v_0/\lambda_0^i$ as well as the envelope condition in the decentralized equilibrium $V_n^{i,\theta} = \lambda_1^{i,\theta}$. This gives us

$$\frac{v_1^\theta}{v_0}\lambda_0^i = \beta_i \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i + \beta \sum_j \frac{\alpha^j}{\alpha^i} V_{N^i}^{j,\theta}, \ \forall i, \theta \tag{B.30}$$

We then use the third first-order condition and the envelope condition to get

$$h^{i\prime}(K_1^i)\lambda_0^i = \beta E_0[\lambda_1^{i,\theta}(F_{1k}^{i,\theta}(K_1^i, l_{1d}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i + \beta \sum_j \frac{\alpha^j}{\alpha^i} E_0[V_{K^i}^{j,\theta}], \ \forall i, \tag{B.31}$$

Equations (B.30) and (B.31), together with the constraints of the social planner's problem describe the constrained efficient allocation. Note that variables in $t \geq 1$ are optimal choices by the agents. Lemma 1 gives more detailed expressions being $V_{N^i}^{j,\theta}$ and $V_{K^i}^{j,\theta}$.

Implementation of constrained efficiency These derivations correspond to Proposition 1 (b) and the associated proof in DK18. The constrained efficient allocation can be implemented by setting taxes on Arrow-Debreu security purchases and capital investment that satisfy

$$\tau_x^{i,\theta} = -\sum_j MRS_{01}^{j,\theta}\mathcal{D}_{1N^i}^{j,\theta} - \sum_j MRS_{02}^{j,\theta}\mathcal{D}_{2N^i}^{j,\theta} - \sum_j \tilde{\kappa}_2^{j,\theta}\mathcal{C}_{N^i}^{j,\theta}, \ \forall i, \theta \tag{B.32}$$

$$\tau_k^i = -\sum_j E_0[MRS_{01}^{j,\theta}\mathcal{D}_{1K^i}^{j,\theta}] - \sum_j E_0[MRS_{02}^{j,\theta}\mathcal{D}_{2K^i}^{j,\theta}] - \sum_j E_0[\tilde{\kappa}_2^{j,\theta}\mathcal{C}_{K^i}^{j,\theta}], \ \forall i \tag{B.33}$$

where $MRS^{j,\theta}_{01} \equiv \beta\lambda^{j,\theta}_1/\lambda^j_0$, $MRS^{j,\theta}_{02} \equiv \beta\lambda^{j,\theta}_2/\lambda^j_0$ and $\tilde{\kappa}^{j,\theta}_2 \equiv \beta\kappa^{j,\theta}_2/\lambda^j_0$. This can be shown as follows. Re-write the period-0 first-order conditions (B.15) and (B.16) by including tax wedges for security purchases ($\tau^{i,\theta}_x$) and capital investment (τ^i_k). This gives

$$(m^\theta_1 + \tau^{i,\theta}_x)\lambda^i_0 = \beta\lambda^{i,\theta}_1 + \kappa^i_1\Phi^i_{1x^\theta} \tag{B.34}$$

$$(h'^i(k^i_1) + \tau^i_k)\lambda^i_0 = \beta E_0[\lambda^{i,\theta}_1(F^{i,\theta}_{1k}(k^i_1, l^{i,\theta}_{d1}) + q^\theta)] + \kappa^i_1\Phi^i_{1k}\ \forall i \tag{B.35}$$

Substituting the above tax rates into these optimality conditions replicates the planner's optimality conditions (B.30) and (B.31). Note that $m^\theta_1 = \frac{v^\theta_1}{v_0}$ in the replicated allocations, i.e., Arrow-Debreu price in the decentralized equilibrium should equal the value of state contingent commodity in the social planner's problem measured by the shadow prices. Importantly, note also that the expressions for the tax rates contain additional terms relative to DK18 due to the presence of labor markets and the more general financial constraint formulation.

Combining equations (B.32) and (B.33) with equation (B.25) and (B.26) gives a set of tax rates

$$\tau^{i,\theta}_x = -\Delta MRS^{ij,\theta}_{01}\mathcal{D}^{i,\theta}_{1N^i} - \Delta MRS^{ij,\theta}_{02}\mathcal{D}^{i,\theta}_{2N^i} - \tilde{\kappa}^{b,\theta}_2\mathcal{C}^{b,\theta}_{N^i},\ \forall i,\theta \tag{B.36}$$

$$\tau^i_k = -E_0[\Delta MRS^{ij,\theta}_{01}\mathcal{D}^{i,\theta}_{1K^i}] - E_0[\Delta MRS^{ij,\theta}_{02}\mathcal{D}^{i,\theta}_{2K^i}] - E_0[\tilde{\kappa}^{b,\theta}_2\mathcal{C}^{b,\theta}_{K^i}],\ \forall i \tag{B.37}$$

$\Delta MRS^{ij,\theta}_{0t} \equiv MRS^{i,\theta}_{0t} - MRS^{j,\theta}_{0t}$ for $t = 1, 2$ denotes the difference between agents in the marginal rate of substitution (MRS) across time, $MRS^{j,\theta}_{01} \equiv \beta\lambda^{j,\theta}_1/\lambda^j_0$, $MRS^{j,\theta}_{02} \equiv \beta\lambda^{j,\theta}_2/\lambda^j_0$. We define $\tilde{\kappa}^{b,\theta}_2 \equiv \beta\kappa^{b,\theta}_2/\lambda^b_0$ as the relative shadow price. A positive $\tau^{i,\theta}_x$ implies that agent i saves too much (borrows too little) in the market outcome. The planner thus wants to impose a tax on savings (remember that $x_1^i > 0$ implies saving, $x_1^i < 0$ borrowing). A positive τ_k^i means that agent i invests too much in capital relative to the constrained efficient allocation, so the planner imposes a tax on investment. In our formal welfare analysis, we focus on over-/under-borrowing since over-/under-investment effects cannot be signed in the DK18 framework. In the numerical application of the model, we do allow for both forces.

Nature of externalities and sufficient statistics The optimal tax wedges, in combination with the distributive effects $\mathcal{D}$ and the constraint effects $\mathcal{C}$ derived in Lemma 1, allow us to characterize the externalities in this economy. In essence, by analyzing and interpreting the different terms in (B.36) and (B.37), we can understand how outcomes in the market economy deviate from the constrained efficient allocation and how such distortions could be corrected. Building on the earlier terminology we distinguish *distributive externalities* and *constraint externalities*.

The sign and magnitude of *distributive externalities* are determined by the product of:

(i) The difference in MRS of agents in periods 1 and 2, $\Delta MRS_{01}^{ij,\theta}$ and $\Delta MRS_{02}^{ij,\theta}$

(ii) The net trading positions on capital $\Delta K_2^{i,\theta}$, financial assets $X_2^{i,\theta}$, labor supply in periods 1 and 2 $\ell_{s1}^{i,\theta}, \ell_{s2}^{i,\theta}$, and labor demand in periods 1 and 2 $\ell_{d1}^{i,\theta}, \ell_{d2}^{i,\theta}$

(iii) The sensitivity of equilibrium prices to changes in aggregate state variables $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial q^\theta}{\partial K_1^j}$, $\frac{\partial m_2^\theta}{\partial K_1^j}$, $\frac{\partial w_1^\theta}{\partial K_1^j}$

The sign and magnitude of *constraint externalities* are determined by the product of:

(i) The relative shadow price of the financial constraint $\tilde{\kappa}_2^{i,\theta}$

(ii) The sensitivity of the financial constraint to the price of capital, asset price and wages for period 1 and 2 $\partial\Phi_2^{i,\theta}/\partial q^\theta$, $\partial\Phi_2^{i,\theta}/\partial m_2^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_1^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_2^\theta$

(iii) The sensitivity of the equilibrium capital price, asset price and wages in periods 1 and 2 to changes in aggregate states $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial q^\theta}{\partial K_1^j}$, $\frac{\partial m_2^\theta}{\partial K_1^j}$, $\frac{\partial w_1^\theta}{\partial K_1^j}$, $\frac{\partial w_2^\theta}{\partial K_1^j}$

Remarks on the externalities The lists above reveal how distortions in the model can be parsed into a compact list of sufficient statistics. Distributive externalities, those driven by effects which are "zero sum," depend on the difference in marginal rates of substitution in combination with the positions that agents take in quantities of capital, labor and financial assets in equilibrium. If these externalities were fully corrected, these quantities would be such that marginal rates of substitution equalize across agents. Logically, constraint externalities depend on the shadow price on the financial constraint, in combination with how the constraint moves with prices changes. Finally, both types of externalities depend on how prices react to changes in the aggregate states, making clear any externalities ultimately operate through price changes.

B.3.5 Insensitivity to re-definition of net worth

In our model, we do not include production output as part of the definition of net worth. This is because output is not predetermined at the beginning of the period due to labor markets clearing during the period. It therefore cannot be a state variable of the model. To ensure that this definitional change does not affect the results, we show in this Appendix that a re-definition of net worth along the same lines gives identical results in the original Dávila and Korinek (2018) (DK18) framework. This is also useful to interpret our Lemma 1 in relation to its analogue in DK18: in our model, we obtain extra terms that contain additional economically meaningful effects.

We proceed by re-defining net worth in DK18 by excluding production output and prove that the *distributive effects* and *collateral effects* in DK18's version of Lemma 1 are identical. We denote net worth as defined by DK18 as $N_{DK}^{i,\theta} \equiv e_1^{i,\theta} + X_1^{i,\theta} + F_1^{i,\theta}(K_1^i)$. The resulting equilibrium capital and debt price are denoted by $q_{DK}^{\theta}(N_{DK}^{\theta}, K_1)$ and $m_{2,DK}^{\theta}(N_{DK}^{\theta}, K_1)$. We define net worth without production output as $N_{WP}^{i,\theta} \equiv e_1^{i,\theta} + X_1^{i,\theta}$ and the resulting equilibrium capital and debt price are denoted by $q_{WP}^{\theta}(N_{WP}^{\theta}, K_1)$ and $m_{2,WP}^{\theta}(N_{WP}^{\theta}, K_1)$. A simple re-definition of the model's state variables cannot change the prices in equilibrium, so that we can set

$$q_{WP}^{\theta}(N_{WP}^{\theta}, K_1) = q_{DK}^{\theta}(N_{DK}^{\theta}, K_1) \tag{B.38}$$

$$m_{2,WP}^{\theta}(N_{WP}^{\theta}, K_1) = m_{2,DK}^{\theta}(N_{DK}^{\theta}, K_1) \tag{B.39}$$

Noting that $N_{DK}^{i,\theta} = N_{WP}^{i,\theta} + F_1^{i,\theta}(K_1^i)$, we differentiate both sides of (B.38) and (B.39) with respect to $N_{(\cdot)}^{i,\theta}$ and K_1^i, in order to determine how the derivatives of prices with respect to net worth and capital are related across models. This gives us

$$\frac{\partial q_{WP}^{\theta}}{\partial N_{WP}^{i,\theta}} = \frac{\partial q_{DK}^{\theta}}{\partial N_{DK}^{i,\theta}} \tag{B.40}$$

$$\frac{\partial m_{2,WP}^{\theta}}{\partial N_{WP}^{i,\theta}} = \frac{\partial m_{2,DK}^{\theta}}{\partial N_{DK}^{i,\theta}} \tag{B.41}$$

$$\frac{\partial q_{WP}^{\theta}}{\partial K_1^i} = \frac{\partial q_{DK}^{\theta}}{\partial N_{DK}^{i,\theta}} \frac{\partial N_{DK}^{i,\theta}}{\partial K_1^i} + \frac{\partial q_{DK}^{\theta}}{\partial K_1^i} = \frac{\partial q_{DK}^{\theta}}{\partial N_{DK}^{i,\theta}} F'(K_1^i) + \frac{\partial q_{DK}^{\theta}}{\partial K_1^i} \tag{B.42}$$

$$\frac{\partial m_{2,WP}^{\theta}}{\partial K_1^i} = \frac{\partial m_{2,DK}^{\theta}}{\partial N_{DK}^{i,\theta}} \frac{\partial N_{DK}^{i,\theta}}{\partial K_1^i} + \frac{\partial m_{2,DK}^{\theta}}{\partial K_1^i} = \frac{\partial m_{2,DK}^{\theta}}{\partial N_{DK}^{i,\theta}} F'(K_1^i) + \frac{\partial m_{2,DK}^{\theta}}{\partial K_1^i} \tag{B.43}$$

where we used the chain rule for the differentiation with respect to capital. (B.42) and (B.43) make clear that the derivatives of prices with respect to capital after the re-definition of net worth "contain" the partial derivatives of $F(\cdot)$ that appear in DK18's Lemma 1. The *distributive effects* in DK18 are the following:

$$\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = -\left[\frac{\partial q^{\theta}_{DK}}{\partial N^{j,\theta}_{DK}}\Delta K^{i,\theta}_{2} + \frac{\partial m^{\theta}_{2,DK}}{\partial N^{j,\theta}_{DK}} X^{i,\theta}_{2}\right] \tag{B.44}$$

$$\mathcal{D}^{DK,i,\theta}_{K^{j}_{1}} = F'(K^{i}_{1})\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} - \left[\frac{\partial q^{\theta}_{DK}}{\partial K^{j}_{1}}\Delta K^{i,\theta}_{2} + \frac{\partial m^{\theta}_{2,DK}}{\partial K^{j}_{1}} X^{i,\theta}_{2}\right] \tag{B.45}$$

The *distributive effects* with the re-definition of net-worth can be derived as

$$\mathcal{D}^{WP,i,\theta}_{N^{j,\theta}_{WP}} = -\left[\frac{\partial q^{\theta}_{WP}}{\partial N^{j,\theta}_{WP}}\Delta K^{i,\theta}_{2} + \frac{\partial m^{\theta}_{2,WP}}{\partial N^{j,\theta}_{WP}} X^{i,\theta}_{2}\right] \tag{B.46}$$

$$\mathcal{D}^{WP,i,\theta}_{K^{j}_{1}} = -\left[\frac{\partial q^{\theta}_{WP}}{\partial K^{j}_{1}}\Delta K^{i,\theta}_{2} + \frac{\partial m^{\theta}_{2,WP}}{\partial K^{j}_{1}} X^{i,\theta}_{2}\right] \tag{B.47}$$

Using (B.40) - (B.43), we obtain

$$\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = \mathcal{D}^{WP,i,\theta}_{N^{j,\theta}_{WP}} \tag{B.48}$$

$$\mathcal{D}^{DK,i,\theta}_{K^{j}_{1}} = \mathcal{D}^{WP,i,\theta}_{K^{j}_{1}} \tag{B.49}$$

Similarly, it can be shown that

$$\mathcal{C}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = \mathcal{C}^{WP,i,\theta}_{N^{j,\theta}_{WP}} \tag{B.50}$$

$$\mathcal{C}^{DK,i,\theta}_{K^{j}_{1}} = \mathcal{C}^{WP,i,\theta}_{K^{j}_{1}} \tag{B.51}$$

This shows that a re-definition of net worth in the original DK18 model gives identical results.

Furthermore, these derivations show that Lemma 1 in our model would be identical to Lemma 1 to its counterpart in DK18 if we did not include labor markets and did not have a more general definition of the financial constraint.

B.4 More details on model results

B.4.1 Intuition for Proposition 1

Proposition 1 confirms one of the main insights of DK18 and the existing literature more generally. The borrower's decisions exert an externality through the market price of capital. As borrowers increase their debt position in period $t = 0$, they reduce aggregate net worth in the borrowing sector in period $t = 1$. Since the price of capital positively depends on sector-wide net worth by condition (2.43), it falls in $t = 1$.[1] Through the collateral constraint, the lower price of capital limits the ability to borrow between $t = 1$ and $t = 2$. As borrowers in $t = 0$ do not internalize this negative effect on future borrowing capability, the amount of debt taken on in $t = 0$ is suboptimally high, that is, there is over-borrowing. The social planner internalizes this relation, and thus discourages borrowing in $t = 0$ through subsidies on saving (for any given level of distributive externalities).

Graphical representation Figure B.1 provides the intuition behind Proposition 1 graphically. This graphical analysis will be especially helpful as a benchmark for the results with the earnings-based constraint below. It shows the period-0 credit market, period-1 capital market, and period-1

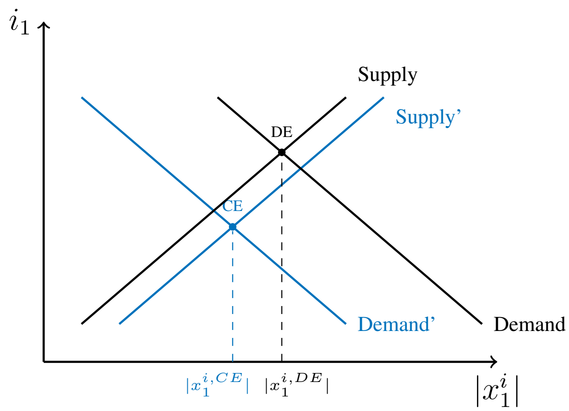
[1]While borrowing more reduces future aggregate net worth in the borrowing sector, it also increases future net worth in the lending sector. By condition (2.43), the latter effect actually puts upward pressure on the price of capital. However, the net effect of changes in borrower and lender net worth leads to a fall in the price of capital. We highlight this in the graphical illustration we provide further below.

credit market. In each panel, points CE and DE represent the constrained efficient allocation and the decentralized equilibrium, respectively. The figure conveys how externalities emerge from borrowing decisions in $t = 0$, which through changes in the price of capital affect credit constraints in $t = 1$.

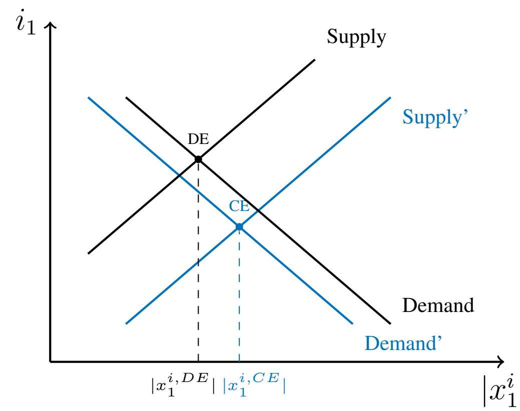
To explain Figure B.1, we focus first on the decentralized equilibrium, point DE across Panels (a)-(d). The difference between Panels (a) and (b) only becomes relevant for implementing constrained efficiency, so for now consider Panel (a) to understand the period-0 credit market. The horizontal axis depicts the financial asset position of each agent in absolute value, that is, borrowing or credit demand $-x_1^{b,\theta}$, and saving or credit supply $x_1^{l,\theta}$. The vertical axis captures the interest rate between periods 0 and 1, $i_1^\theta = 1/m_1^\theta - 1$. Due to market clearing, saving and borrowing positions net out to 0, so $x_1^{b,\theta,DE} + x_1^{l,\theta,DE} = 0 \Rightarrow |x_1^{b,\theta,DE}| = |x_1^{l,\theta,DE}|$. Decisions on the credit market in $t = 0$ impact future net worth and thereby affect investment decisions in period $t = 1$. This is visible in Panel (c), which plots the capital supply curve (given by the vertical line indicating K_1) and the capital demand curve (given by the downward sloping relation between K_2^θ and q_1^θ). Capital supply is in general governed by an upward sloping relationship between K_1 and $q_1^\theta, \forall\theta$. However, since the analysis in the figure traces out the effects of period-0 borrowing externalities, and how these result from changes in period-1 net worth, capital supply is effectively predetermined at the beginning of period $t = 1$.[2] The location of the demand curve does depend on the realization of aggregate net worth. Finally, the capital market equilibrium is linked to the period-1 credit market through the collateral constraint. Panel (d) shows credit supply and credit demand in period 1, by plotting $-x_2^{b,\theta}$ and $x_2^{l,\theta}$ in absolute value against the

[2] This would be different in a graphical analysis of pecuniary externalities that result from over- and under-investment between $t = 0$ and $t = 1$.

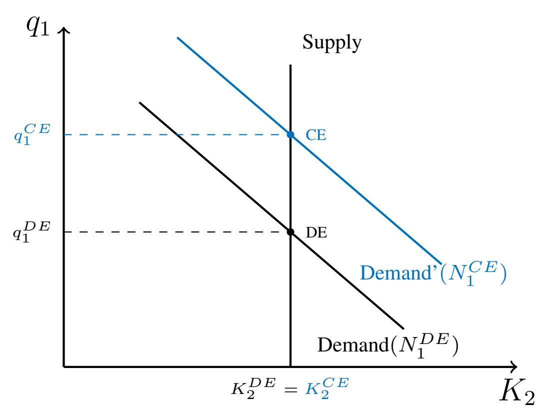
Figure B.1: Market vs. planner allocations: collateral constraint



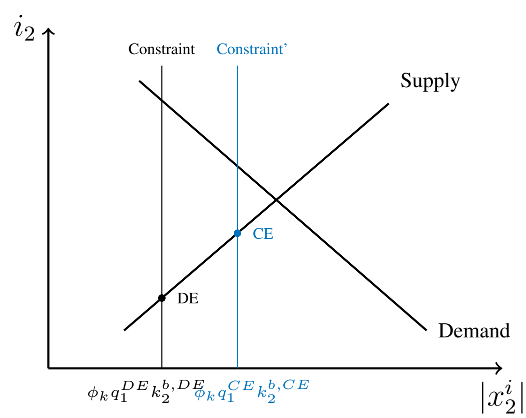
(a) Period-0 credit market (case 1: $|\tau_x^b| > |\tau_x^l|$)



(b) Period-0 credit market (case 2: $|\tau_x^b| < |\tau_x^l|$)



(c) Period-1 capital market (both cases)



(d) Period-1 credit market (both cases)

Notes. Decentralized equilibrium (DE) and constrained efficient equilibrium (CE) in the period-0 credit market, period-1 capital market and period-1 credit market of the model. State θ is omitted from the notation in the labeling. The figure distinguishes case 1 ($\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$) and case 2 ($\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$) as described in the text. In both cases, the social planner internalizes that period-0 borrowing decisions reduce equilibrium prices in the market for physical capital in period 1, which tightens the collateral constraint. The constrained efficient allocation features higher capital prices and more credit in period 1, as more saving (less borrowing) is incentivized through taxes/subsidies in period 0.

interest rate i_2^θ. The collateral constraint (2.44) puts a cap $\phi_k q_1^{\theta,DE} k_2^{\theta,DE}$ on the amount of credit, represented by a vertical line. Importantly, its location is determined by the market clearing price of capital $q_1^{\theta,DE}$. The decentralized equilibrium in the period-1 credit market is given by the intersection of the constraint and the credit supply curve.

By Proposition 1, the decentralized equilibrium is not efficient: the social planner distorts borrowing decisions in period 0 to drive up capital prices and thereby relax borrowing constraints in period 1. Under condition (2.43), sector-wide net worth of both borrowers and lenders positively impacts the price of capital. For the graphical analysis of the constrained efficient allocation, point CE across Panels (a)-(d), two finer cases can be distinguished: in case 1 the impact of the borrower sector net worth on wages is stronger than that of net worth in the lender sector ($\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta}$) and in case 2, the opposite is true ($\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta}$). In both cases, the social planner alters borrower and lender equilibrium net worth such that capital prices increase in $t = 1$. However, depending on the relative impact of net worth in the different sectors on the price of capital, the planner will tax borrowing (subsidize saving) more heavily for either the borrower or the lender to achieve the desired increase in the price of capital: in case 1, $|\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$, while in case 2, $|\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$. In other words, the planner reverts the over-borrowing of that agent more heavily whose decisions have a stronger impact on capital prices, making capital prices in period 1 rise in either case.[3] This is visible in Panels (a) and (b) which show the constrained efficient equilibrium for cases 1 and 2. In both cases, the planner incentivizes lenders to save more and borrowers to borrow less, to counteract the *over-borrowing*

[3]This can be seen as follows. According to Proposition 1, the constraint externality from the collateral constraint is non-negative, meaning that through equation (B.36) the planner desires a negative $\tau_x^{i,\theta}$ for $i \in \{b, l\}$. By equation (B.36), the size of the tax rate the planner chooses to implement the constrained efficient equilibrium is proportional to the size of the derivative of capital prices to sector wide net worth, that is, $\tilde{\kappa}_2^{b,\theta} \mathcal{C}_{N^i}^{b,\theta} \propto \partial q_1^\theta / \partial N_1^{i,\theta}$. As a result, when constraint externalities are corrected by the planner, the relative magnitude of $\partial q_1^\theta / \partial N_1^{b,\theta}$ and $\partial q_1^\theta / \partial N_1^{l,\theta}$ determines the relative magnitude of $\tau_x^{b,\theta}$ and $\tau_x^{l,\theta}$.

motive of both agents.[4] As a result, the credit supply curve is located to the right, and the credit demand curve to the left relative to their counterparts in the decentralized case. However, in Panel (a) (case 1), $|\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$, so the decrease in demand from the borrower is larger than the increase in supply from the lender, and the equilibrium quantity of credit is below that of the decentralized equilibrium. With a smaller amount of equilibrium borrowing, borrower net worth in period 1 will be higher while lender net worth will be lower relative to the decentralized equilibrium. Since $\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta}$, capital prices are higher. In Panel (b) (case 2), $|\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$ so there is a greater amount of equilibrium borrowing, and borrower net worth in period 1 will be lower while lender net worth will be higher. Since $\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta}$, capital prices are higher, as in case 1. This makes clear that while the collateral constraint induces over-borrowing motives (borrowers want to borrow too much, savers want to save too little), a corrective policy may actually increase or decrease equilibrium credit.

In both cases 1 and 2, the corrective wedges introduced by the planner lead capital demand to shift upward, while changes the net worth induced by the planner do not move the capital supply curve, all else equal. These effects, shown in Panel (c), are the graphical counterpart to our discussion of condition (2.43) above.[5] As a result, capital prices in the constrained efficient equilibrium in period $t = 1$ are higher relative to the decentralized equilibrium. As in the decentralized case, the period-1 credit market, shown in Panel (d), is connected to the capital market through the price of capital. An increase in the price of capital loosens the collateral

[4]This explanation highlights that in principle, in the case of the lender one could alternatively call the over-borrowing force an 'under-saving' effect.

[5]Recall that in the formal welfare analysis we focus on pecuniary externalities that operate through changes in net worth, and do not characterize over- or under-investment effects. In the graphical depiction, we therefore abstract from any difference in investment in $t = 0$ that may occur between the decentralized equilibrium and the constrained efficient allocation that the planner implements. In the numerical application of the model in Section 2.4.3, we also allow for over- und under-investment.

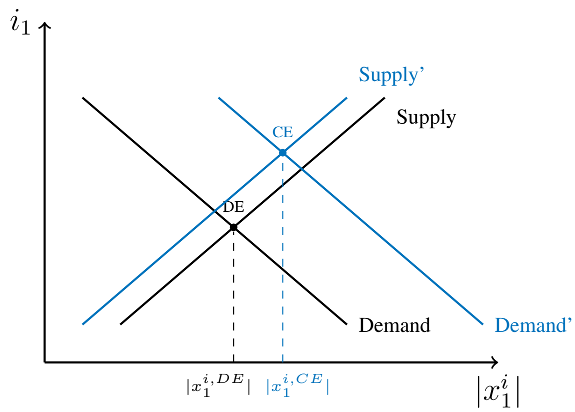
constraint, moving the intersection of the vertical line with the credit supply curve in Panel (d) to the right relative to the decentralized equilibrium. The planner internalizes the effect of period-0 borrowing decisions on future prices, and in turn on future borrowing space. The over-borrowing force in $t = 0$ is corrected through a tax wedge so that borrowers can obtain more credit between period 1 and 2 in the constrained efficient economy.

B.4.2 Intuition for Proposition 2

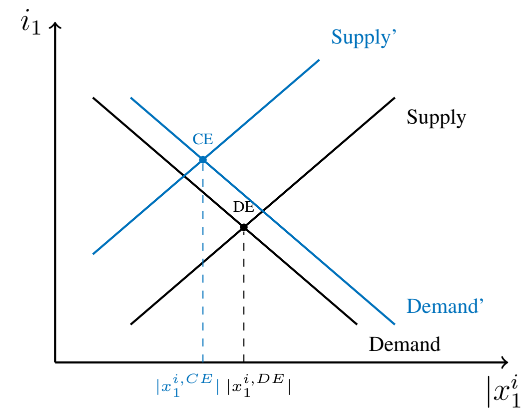
Proposition 2 delivers one of our main theoretical insights. An earnings-based borrowing constraint implies that the borrower takes a debt position that is too small relative to the social optimum. The mechanics of the model are similar to our explanation of Proposition 1, but operate through the real wage rate rather than the price of capital. A larger debt position in $t = 0$ reduces net worth in the borrowing sector in $t = 1$, which in turn reduces wages due to condition (2.42) (recall the discussion around labor demand and labor supply). Borrowers in $t = 0$ do not internalize that lower wages increase earnings and provide slack in the borrowing limit in $t = 1$. Therefore, in the market economy, agents under-borrow. The social planner internalizes the positive effect of borrowing in $t = 0$ on debt capacity in $t = 1$ through wages, and subsidizes (lowers the tax on) borrowing in period $t = 0$ (for a given level of distributive externalities).

Graphical representation Figure B.2 presents a graphical analysis for the case of the earnings-based borrowing constraint. As in Figure B.1, points CE and DE represent the constrained efficient allocation and the decentralized equilibrium. The figure conveys how externalities emerge from borrowing decisions in $t = 0$, which through wage determination in the labor market affect credit constraints in $t = 1$. Relative to the case of the collateral constraint, Panel

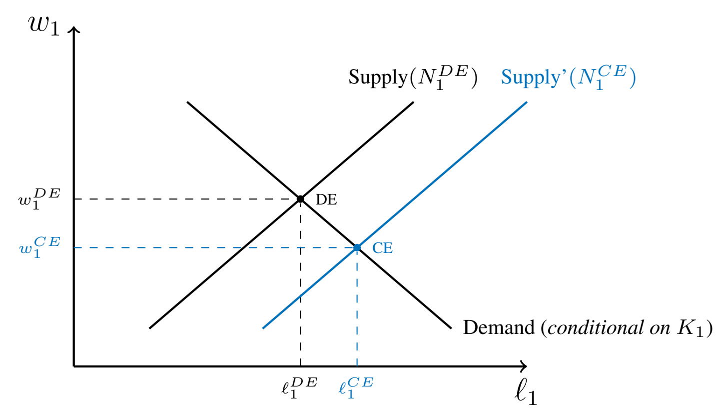
Figure B.2: Market vs. planner allocations: earnings-based borrowing constraint



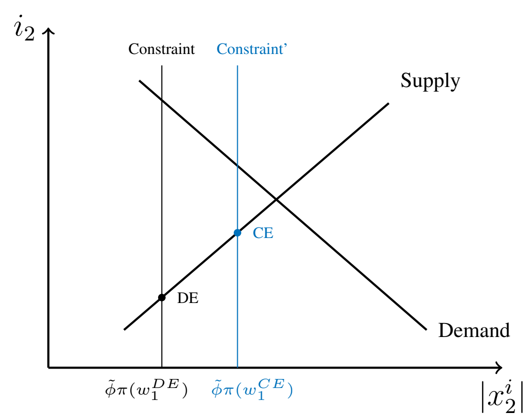
(a) Period-0 credit market (case 1: $|\tau_x^b| > |\tau_x^l|$)



(b) Period-0 credit market (case 2: $|\tau_x^b| < |\tau_x^l|$)



(c) Period-1 labor market (both cases)



(d) Period-1 credit market (both cases)

Notes. Decentralized equilibrium (DE) and constrained efficient equilibrium (CE) in the period-0 credit market, period-1 labor market and period-1 credit market of the model. State θ is omitted from the notation in the labeling. The figure distinguishes case 1 ($\partial w_1^\theta / \partial N_1^{b,\theta} > \partial w_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$) and case 2 ($\partial w_1^\theta / \partial N_1^{b,\theta} < \partial w_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$) as described in the text. In both cases, the social planner internalizes that period-0 borrowing decisions reduce equilibrium wages in period 1, which relaxes the earnings-based borrowing constraint. The constrained efficient allocation features lower wages and more credit in period 1, as less saving (more borrowing) is incentivized through taxes/subsidies in period 0.

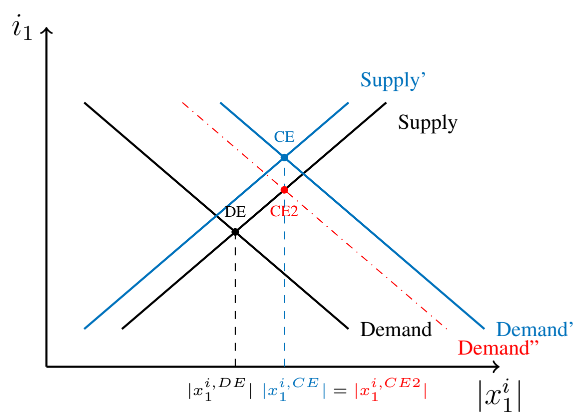
(c) now depicts the labor market in $t = 1$ rather than the market for physical capital. The earnings-based constraint (2.45) is represented by a vertical line in Panel (d), putting a cap $\phi_\pi \pi(w_1^\theta) = \phi_\pi(F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta})$ on the amount of credit. Its location is affected by the market clearing wage. Similar to the collateral constraint and Figure B.1, there is a refinement of condition (2.42) on the response of wages to changes in net worth. In both cases, according to Proposition 2, the decentralized equilibrium features under-borrowing and the social planner subsidizes borrowing (taxes saving) in $t = 0$. In period $t = 0$ agents do not internalize that by reducing net worth in period 1 wages are reduced and this relaxes future borrowing constraints. To lower wages and thus create space for the constrained optimal amount of period-1 credit, the planner induces more debt in period 0 through corrective tax wedges.

The graphical representation of the economy with earnings-based borrowing constraint highlights the new insights that come with signing pecuniary externalities in our model with labor markets. The condition that wages increase with sector wide net worth in $t = 1$ requires understanding the response of labor demand as well as labor supply. Given that the capital available for production (K_1) is predetermined at the beginning of the period, labor demand is already pinned down, while labor supply responds to changes in sector-wide net worth (see Panel (c) of Figure B.2). This is different in the market of capital relevant for the collateral constraint case, where the supply of capital is fixed, but the demand for new capital (K_2) increases with net worth (compare Panel (c) of Figure B.1). In the presence of earnings-based constraints the planner can therefore induce more borrowing in the initial period, and thereby reduce borrower net worth in $t = 1$ to increase labor supply. This leads wages to fall.

Take-aways from graphical analysis of both constraints In conclusion to the graphical analysis, the differences between Figures B.1 and B.2 reveal the sharp contrast between the normative consequences of the earnings-based and the collateral constraint. In the earnings-based constraint an *input price* (through the wage bill) enters with the opposite sign to how an *asset price* (the value of capital) enters the collateral constraint. Since wages and the price of capital respond with the same sign to changes in borrower net worth, all else equal, the implications in terms of whether agents borrow to much or too little in period $t = 0$ from a normative standpoint are the opposite for the two constraint types.

Alternative implementations of constrained efficiency The set of tax rates τ_x^i, $i \in \{b, l\}$ that implements the constrained efficient equilibrium is not unique. There is an infinite number of combination of τ_x^b and τ_x^l that will alter $N_1^{b,\theta}$ and $N_1^{l,\theta}$ such that the same changes in period-1 prices and credit access are achieved. For the case of the earnings-based borrowing constraint we illustrate this in Figure B.3, which is constructed as Panel (a) of Figure B.2 but also plots an alternative implementation of the constrained efficient equilibrium (denoted $CE2$). This equilibrium represents the polar case in which only the borrower's financial asset position is taxed (borrowing is subsidized), while the lender is not taxed, $\tau_x^l = 0$. As the graph conveys, there is a choice for τ_x^b that achieves the identical equilibrium credit amount as point CE. As a result, the labor and credit market outcomes in period 1 would be the same as in Figure B.2. A similar argument can be made for case 2 in Figure B.2 and for both cases of the collateral constraint analyzed in Figure B.1.

Figure B.3: Non-uniqueness of implementation



Period 0 Credit Market ($|\tau_x^b| > |\tau_x^l|$)

Notes. This figure repeats Panel (a) of Figure B.2 but also plots an alternative implementation of the constrained efficient equilibrium (denoted $CE2$). Constrained efficiency can be achieved with different sets of tax rates $\tau_x^{i,\theta}$, $i \in \{b, l\}$, which give rise to the same change in aggregate net worth (and resulting wage reduction) in the constrained efficient relative to the decentralized equilibrium. In this case, only the borrowers' savings decisions are taxes (borrowing is subsidized), while $\tau_x^{l,\theta} = 0$. State θ is omitted from the notation in the labeling of the graph.

B.5 Robustness of numerical model experiments

To explore robustness of our model parameterization, we construct variations of Tables 2.2 and 2.3 from the main text in which we change the values of key parameters and then report the resulting optimal tax rates and welfare losses. We focus on the capital share α and the labor supply elasticity ψ. These parameters are of particular interest, since the sufficient condition we can derive for our main assumption to hold (see Section 2.2, case (ii) of the main text) depends on these two parameters. For each parameter, we solve the model for a 20% larger and a 20% smaller value relative to the baseline calibration, which sets $\alpha = 0.33$ and $\psi = 2$. In the case of α we can do this for the model version with inelastic labor supply as well as the one with endogenous labor supply. The variation of ψ only applies in the model version where labor supply is chosen by the agents.

Table B.1 reports the resulting optimal tax rates. The table is constructed in the same way as Table 2.2 in the main text, but each panel corresponds to a different parameter variation. The important take-away from this table is that our main assumption holds also for variations in the parameter values. In particular, the signs of $\tau_x^{b,c.e.}$ $\tau_x^{l,c.e.}$ are the same as in the analysis in the main text, indicating that our assumptions on the derivatives of the price of capital and wage with respect to changes in net worth are also satisfied for a higher and lower capital share and labor supply elasticity.

Table B.1: Optimal corrective taxes in different economies (in $\%$)

Economy ($\alpha = 0.33 \times 1.2$)	τ_x^b	τ_x^l	τ_k^b	τ_k^l	$\tau_x^{b,c.e.}$	$\tau_x^{l,c.e.}$
Collateral constraints, inelastic labor	-21.6	4.0	-33.3	-30.6	-0.7	-0.3
Earnings-based constraints, inelastic labor	-9.7	-2.2	-31.9	-10.5	0.0	0.0
Collateral constraints, endogenous labor	-2.1	-3.5	-1.5	0.8	-2.3	-3.4
Earnings-based constraints, endogenous labor	0.2	0.3	-3.2	-5.9	0.9	0.3
Economy ($\alpha = 0.33 \times 0.8$)						
Collateral constraints, inelastic labor	-17.6	3.5	-22.1	-23.6	-0.0	-0.0
Earnings-based constraints, inelastic labor	-5.7	-0.6	-20.1	-12.5	0.0	0.0
Collateral constraints, endogenous labor	-1.3	-3.2	-0.4	0.7	-1.5	-3.1
Earnings-based constraints, endogenous labor	0.4	0.4	-1.5	-8.0	0.9	0.3
Economy ($\psi = 2 \times 1.2$)						
Collateral constraints, endogenous labor	-1.9	-3.4	-1.4	0.6	-2.1	-3.3
Earnings-based constraints, endogenous labor	0.1	0.3	-2.9	-7.1	0.8	0.3
Economy ($\psi = 2 \times 0.8$)						
Collateral constraints, endogenous labor	-1.4	-3.3	-0.6	0.6	-1.7	-3.2
Earnings-based constraints, endogenous labor	0.5	0.5	-2.1	-7.1	1.0	0.3

Table B.2 presents the results of our experiment of rolling out the wrong policy. It reveals that we find significant welfare losses across the parameter variations we introduce. A higher capital share makes the welfare even larger than in the main text, reaching up to over 3% in consumption equivalents for the model with inelastic labor supply. When the capital share is decreased, the welfare losses are smaller but still substantial with more than 1% welfare loss. For the labor supply elasticity, it is visible that a lower parameter value increases the strength of the negative welfare consequences. With a higher labor supply elasticity, the effect is still strong, again around 1% in consumption equivalents, so not very different for the effect in the main text when labor supply is endogenous. Finally, As in the experiment in main text, the welfare losses coming from the constraint externality by itself are smaller. This highlights again that distributive externalities are important in the general model.

Table B.2: Consumption equivalent welfare change in different counterfactuals

Panel (a): all types of externalities			
Economy ($\alpha = 0.33 \times 1.2$)	**Right policy, $\boldsymbol{\lambda}$(%)**	**Wrong policy, $\boldsymbol{\lambda}$(%)**	**$\boldsymbol{\Delta}$(%)**
Earnings-based constraints, inelastic labor	0.89	-2.28	3.16
Earnings-based constraints, endogenous labor	0.60	-0.54	1.14
Economy ($\alpha = 0.33 \times 0.8$)			
Earnings-based constraints, inelastic labor	0.39	-0.97	1.36
Earnings-based constraints, endogenous labor	0.61	-0.51	1.12
Economy ($\psi = 2 \times 1.2$)			
Earnings-based constraints, endogenous labor	0.49	-0.50	0.99
Economy ($\psi = 2 \times 0.8$)			
Earnings-based constraints, endogenous labor	0.77	-0.55	1.32

Panel (b): constraint externalities only			
Economy ($\alpha = 0.33 \times 1.2$)	**Right policy, $\boldsymbol{\lambda}$(%)**	**Wrong policy, $\boldsymbol{\lambda}$(%)**	**$\boldsymbol{\Delta}$(%)**
Earnings-based constraints, inelastic labor	0.00	-0.04	0.04
Earnings-based constraints, endogenous labor	0.06	-0.50	0.56
Economy ($\alpha = 0.33 \times 0.8$)			
Earnings-based constraints, inelastic labor	0.00	-0.00	0.00
Earnings-based constraints, endogenous labor	0.05	-0.45	0.51
Economy ($\psi = 2 \times 1.2$)			
Earnings-based constraints, endogenous labor	0.04	-0.45	0.50
Economy ($\psi = 2 \times 0.8$)			
Earnings-based constraints, endogenous labor	0.08	-0.50	0.58

Notes. The table shows the welfare impact of policies carried out in the 'true' economy, which features earnings-based constraints. The right policy is the solution to the social planner's problem in that economy. It moves the allocation in the decentralized equilibrium to the constrained efficient allocation. The wrong policy is calculated under the incorrect assumption that agents face asset-based borrowing constraints. It moves the allocation in the decentralized equilibrium to allocation that arises from the wrong policy.

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