FIRE: THE FIRST-YEAR INNOVATION & RESEARCH EXPERIENCE

QUANTUM MACHINE LEARNING

Quantum Finance: Exploring Asset Management with QAOA

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Introduction

In an era where technology is the key factor to developing a competitive advantage in the financial markets, advanced computational tools and algorithms have shaped trading strategies, risk management, portfolio optimization, and financial analysis. As data continues to pour into the markets and influencing decisions, developing cutting edge technology is of utmost importance. This necessity has started driving the industry toward the next frontier: Quantum Asset Management.

Asset Management

Asset management firms oversee trillions of dollars invested across a multitude of funds and portfolios for institutional and individual clients. A critical task is to periodically rebalance these portfolios to maintain their intended risk/return characteristics and investment objectives. Currently, this is tackled using classical optimization methods like quadratic programming and heuristic algorithms. However, these classical approaches can struggle to find optimal solutions, especially as the number of assets and constraints grows, leading to suboptimal portfolio performance.

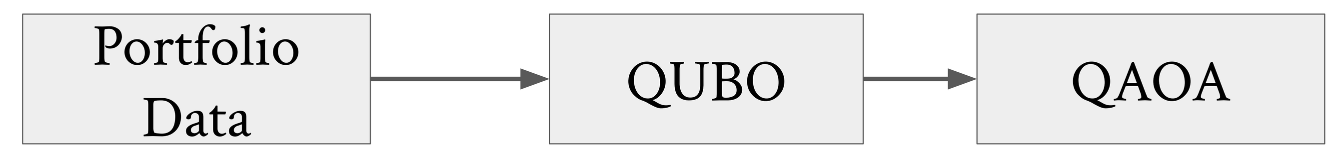
Quantum Benefits

Our research explores how quantum algorithms like QAOA may be able to more efficiently solve these portfolio rebalancing problems by leveraging quantum parallelism to evaluate exponentially more potential solutions simultaneously. By encoding portfolio objectives and constraints into QUBOs, QAOA could identify higher-quality rebalancing strategies, unlocking improved risk-adjusted returns for asset managers and their clients.

Quantum Limitations

While QAOA and QUBO offer promising approaches for tackling difficult optimization problems like the one we've taken on here, there are still some key limitations to consider. In general, these variational quantum algorithms provide *approximate* solutions and do not guarantee finding the true global optimal solution, especially as problem sizes scale up. Their performance heavily depends on choosing good initial parameter values and employing effective optimization routines for the classical outer loop. We used equally weighted portfolios to simplify the process of embedding the problem into a QUBO.

Problem Flowchart

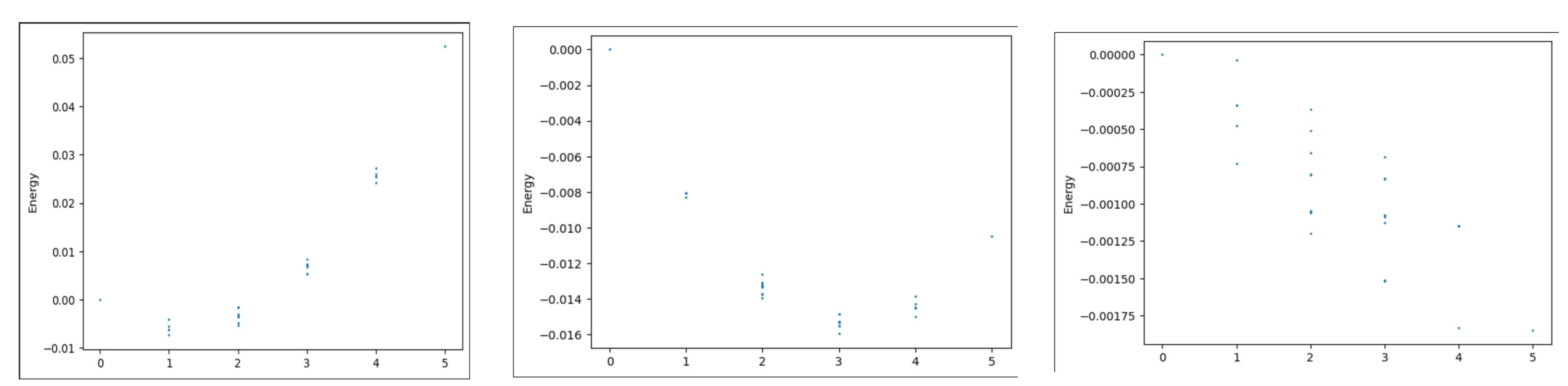


QUBO

The Quantum Unconstrained Binary (QUBO) is a optimization problem that is used in areas such as finance to find optimal solutions. The QUBO will generate a matrix Q which contains all the possible outcomes, of which QAOA will optimize for the best possible solution. The QUBO formulation for the portfolio optimization problem with the constraint of N asset portfolios is below:

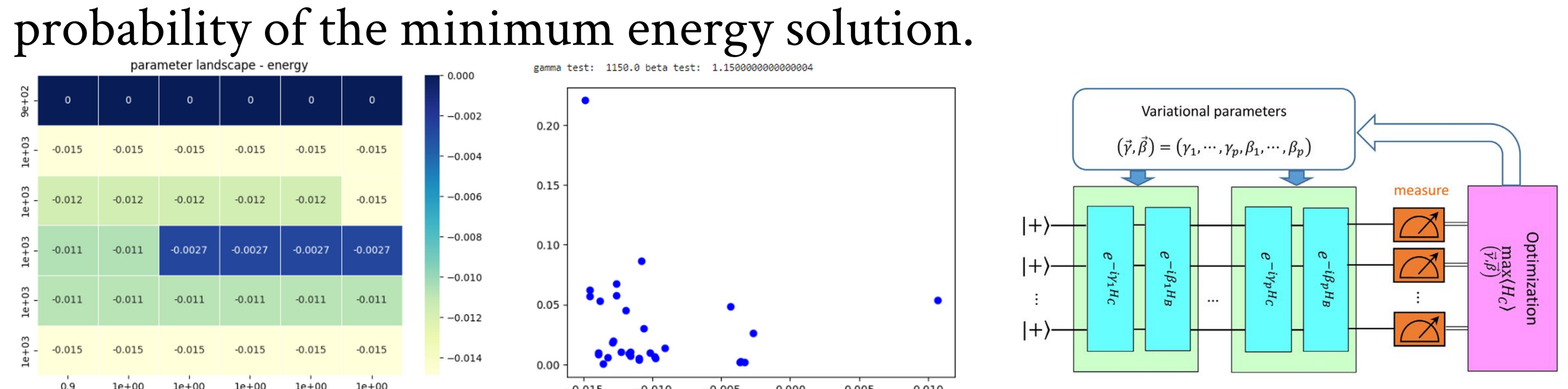
$$\frac{1}{2N^2}\left(\sum_{i=0}^{M-1} x_i\sigma_i^2 + \sum_{i\neq j}^{M-1}\rho_{i,j}x_ix_j\sigma_i\sigma_j\right) - \frac{\lambda}{N}\sum_{i=0}^{M-1}x_ir_i + S\left(\sum_{i=0}^{M-1}x_i - N\right)^2$$

x_i is a binary variable representing whether the asset is included in the solution. The equation's left side quantifies risk by calculating the portfolio's return variance, where higher variance indicates greater risk. We subtract the portfolio's expected return, computed as the weighted average of the investments. The final term penalizes deviations from the target allocation. These components together determine the optimal investment distribution.



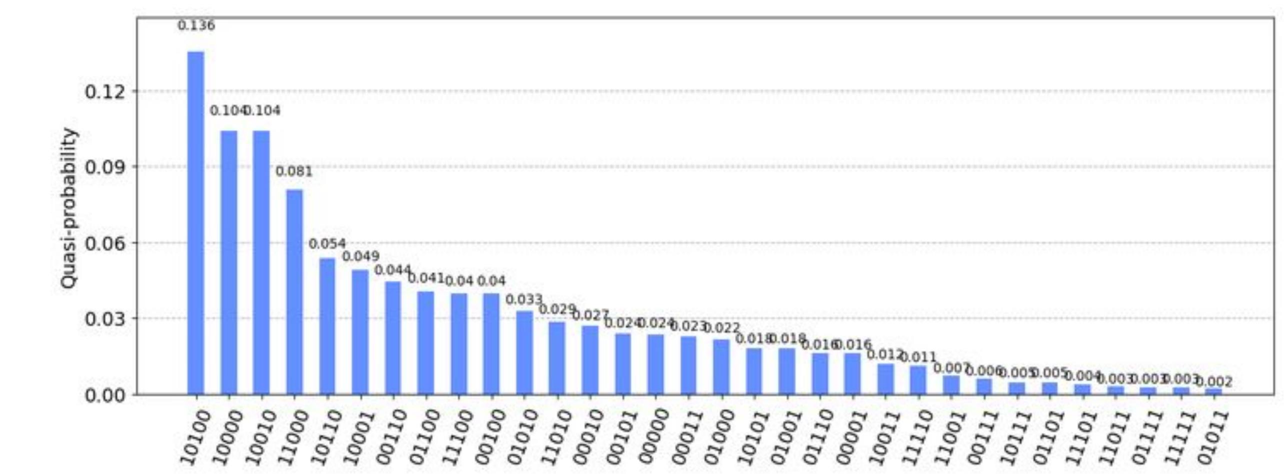
QAOA

The Quantum Approximation Optimization Algorithm (QAOA) is a quantum-classical algorithm that is used to solve any optimization problem such as the traveling salesman problem, molecular chemistry simulation problems, supply chain management, and network optimization. QAOA gets values from rotations representing the terms in the QUBO matrix which help the quantum computer get closer to the solution. The two plots on the left below are the energy of the parameter tuning landscape and scatterplot showing increased probability of the minimum energy solution.

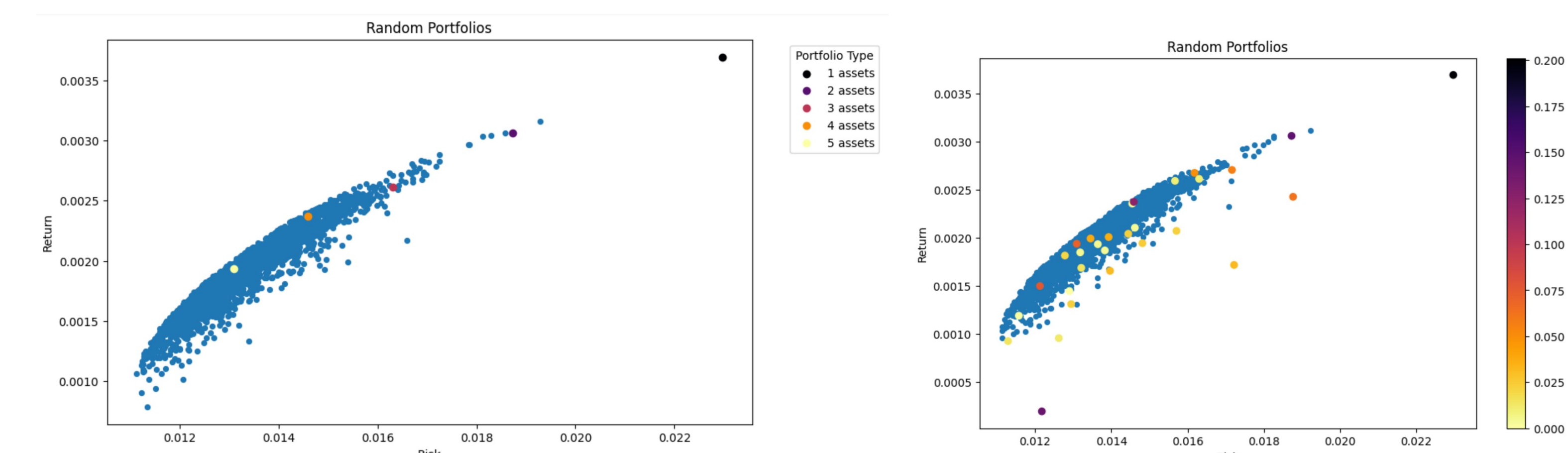


Source: https://arxiv.org/abs/1812.01041

Results



Quantum solutions plot, solutions are listed from highest to lowest probability



Colored dots are the quantum solutions, blue dots are the classical solutions. Left chart shows best quantum solutions for each asset amount, right side shows probability of each solution.

Discussion

Quantum computers surpass classical computers in some optimization problems. Variational algorithms currently do not have a proven advantage, however, researchers are looking at how they can be used to gain an advantage on difficult problems. Continued research and work behind quantum would help algorithms such as QAOA enhance their performance. As quantum computing progresses, QAOA and QUBO would become more enhanced and could eventually work their way into a more diverse range of fields solving more problems other than the current problems that they can currently solve. All in all, as the world progresses with quantum, more discoveries and enhancements can be made relating to current algorithms such as QAOA or discovering newer algorithms.

References

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