

ABSTRACT

Title of Dissertation: DETECTION AND CHARACTERIZATION
OF LAVA TUBES FROM SURFACE
SIGNATURES: TWO NEW APPROACHES

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Lava tubes are sinuous volcanic caves formed when a lava flow develops a solidified crust before draining out. On other planets and moons, lava tubes could be ideal sites for future exploration or habitation, providing practical and scientific benefits. Lava tubes are difficult to study without *in-situ* exploration, which is impossible for extraterrestrial, inaccessible, or filled tubes. We present two new techniques to constrain the internal morphologies of lava tubes, relying only on remotely observable surface features. Through modeling, we predict the surface features that lava tubes produce and quantify how the dimensions of these surface features depend on the dimensions of the tubes. Looking for these features in real data reveals several candidate tubes and allows us to constrain their dimensions. Large tubes on the Moon should cause tensile cracks in predictable patterns that reflect tube dimensions. Such cracks near discontinuities in Rimae Mairan, Sharp, and Marius indicate lava tubes with widths less than 2 km. Cracking inside lunar tubes would impair the usability of the floor for exploration, leaving less than 400 m of contiguous pristine floor in these three cases. Inflation ridges above tubes can

also reveal tube morphologies. By modeling how Valentine Cave inflated, we constrain what conditions and age it must have had to produce the observed ridge. It and two similar examples must have inflated within the first 400 hours after roof formation. We build from this idea and discover a new, widely applicable relationship between the dimensions of an inflation ridge and the height, roof thickness, and width of its tube. From ridge topography alone, this relationship constrains a range of possible tube shapes. Searching for inflation ridges in a LiDAR dataset of the Lava Beds National Monument reveals eighteen candidate tubes. Most of these tubes share a similar range of possible shapes; one ridge shows a consistently different shape range, possibly indicating different formation conditions. Combined with constraints from other sources on two of these tubes, the shape ranges narrow to more specific results. These methods constitute two novel tools to find and study lava tubes on Earth and beyond.

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by

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Table of Abbreviations

GEODES: Geophysical Exploration of the Dynamics and Evolution of the Solar System
LiDAR: Light Detection and Ranging
NCALM: National Center for Airborne Laser Mapping
SSERVI: Solar System Exploration Research Virtual Institute
LRO: Lunar Reconnaissance Orbiter
LROC: Lunar Reconnaissance Orbiter Camera
LROC NAC: Lunar Reconnaissance Orbiter Camera Narrow Angle Camera
 c : Tube height
 W : Tube or ridge width
 T_{ve} : Tube viscoelastic roof thickness, also referred to as tube effective roof thickness
 T_f : Tube full roof thickness
 T_a : Apparent roof thickness: Roof thickness of an idealized tube model that produces the same inflation ridge height as a realistic model with dimensions c , W , T
 T_e : Effective roof thickness, calculated from an integral of the dimensions of a realistic (semi-elliptical) tube model
 H : Ridge height
MHH: Marius Hills Hole
LBNM: Lava Beds National Monument
NAIP: National Agriculture Imagery Program
 D_b : Distance from the center of a lunar lava tube to the surface bulges it causes
 D_c : Distance from the center of a lunar lava tube to the surface cracking it causes
 R_{cW} : Ratio of c to W : Aspect ratio of a tube
 R_{TW} : Ratio of T to W : Scaled roof thickness of a tube
 R_{HW} : Ratio of H to W : Ratio of inflation ridge height to tube width
GPR: Ground penetrating radar
LRS: Lunar Radar Sounder
GRAIL: Gravity Recovery and Interior Laboratory
Mini-RF: Miniature Radio-Frequency instrument
CTX: Mars Reconnaissance Orbiter Context Camera
GSI: Geological Strength Index
 R_r : Tube roof width-to-height ratio
 R_f : Tube floor width-to-height ratio
 R_{HB} : Hoek-Brown ratio, where values closer to 1 are closer to compressive failure
IMU: Inertial Measurement Unit
 L : A lengthscale we define to capture how a tube's inflation conditions control its inflation ridge height

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Chapter 1 Introduction

Lava tubes are long, often sinuous underground void spaces created by some lava flows. A lava tube forms when a lava flow forms a solidified crust over its entire surface, the crust becomes stiff enough to support its own weight, and the internal lava subsequently drains out leaving a cave (Figure 1-1; Peterson & Swanson, 1974; Sakimoto & Zuber, 1998; Huff & Owen, 2013; Gregg, 2017).

Tubes are found in many locations on Earth, including at least seven continental US states, the Canary, Galapagos, and Hawaiian islands, Iceland, Italy, and Australia (Waters et al., 1990; Calvari and Pinkerton, 1998; L  veill   and Datta, 2010; Gregg, 2017; Theinat et al., 2019). The lengths of terrestrial tubes can reach up to kilometers, and their widths and heights up to a few tens of meters (Gregg, 2017; Pipan and Culver, 2019; Sauro et al., 2020). Tube systems can be longer than individual intact tubes.

Tubes most often occur in p  hoehoe flows (Peterson and Swanson, 1974; Greeley, 1987; Peterson et al., 1994; Kauahikaua et al., 1996; Kauahikaua et al., 1998; Byrnes and Crown, 2001; Kauahikaua et al., 2003; Tomasi et al., 2022; Calvari et al., 2024), but can also be found in flows of ‘a‘  (Ponte, 1949; Guest et al., 1980, 1984; Cavallaro et al., 1985; Corsaro and Cristofolini, 1989; Kauahikaua et al., 2003; Guest et al., 2012; Calvari et al., 2024).



Figure 1-1: Interior of Sunshine Cave from Medicine Lake volcano, within Lava Beds National Monument, with Dr. N. Schmerr for scale (~1.8 m). Several blocks of tube roof have fallen to the floor. Image taken by Edward Williams, 2024.

1.1 Lava Tubes

1.1.1 Lava Tube Formation

The defining characteristic of a lava tube is that a conduit of flowing lava has a crust of solidified rock on all sides. However, there are several formation mechanisms that can accomplish this.

If a flow is already channelized, forming a crust of lava over its top surface encloses it completely and can create a tube. The top surface of a channelized flow can crust over from the

sides inwards or from the center outwards. Floating rafts of crust can collide and merge to form a roof; this is especially likely to happen at places where the channel changes direction sharply or becomes narrower. Once a roof has begun to form at one point, it can spread simultaneously up- and down-stream. A channelized flow can also roof over by forming levees on either side, created by accumulated lava spatter and lava that overflows from the channel, that eventually extend far enough over the channel to meet in the middle. These processes can happen in combination in the same channelized flow (Greeley, 1971, 1972; Peterson et al., 1994; Cashman et al., 2006; Valerio et al., 2008; Sauro et al., 2020).

Alternatively, lava tubes can form in broad pāhoehoe flows. Over time, a narrow, preferred flow pathway can form in a wide active flow. If the movement of lava becomes sufficiently concentrated to only this pathway, the rest of the flow stagnates and cools. This eventually results in a still-flowing conduit in an otherwise solidified flow (Hon et al., 1994; Peterson et al., 1994; Anderson et al., 2005; Diniega et al., 2015; Gregg, 2017).

Lava tubes play a critical role in the development of many lava flow fields. With the thermal insulation provided by tubes, lava can flow much greater distances without cooling down than it could on the surface (Greeley, 1987; Keszthelyi, 1995; Gregg & Fornari, 1998; Keszthelyi & Self, 1998; Zimbelman, 1998; Stephenson et al., 1998; Gregg, 2017; Michael et al., 2024). Flows can thus spread significantly farther from the vent than they could without tubes. Tubes are integral in the formation of long flow fields and the assessment of potential hazards since lava can escape from a tube significantly farther from the vent than expected from surface flows alone (Calvari & Pinkerton, 1999a; Chen et al., 2016; Calvari et al., 2024). During the 1991-1993 eruption of Mt. Etna the town of Zafferana Etnea was threatened by a lava flow, and countermeasures were taken to protect the town (Barberi et al., 1993; Stevens et al., 1997; Bisson

et al., 2009). The encroaching flow had an internal network of lava tubes, and likely could not have reached the same distances if it did not. In the same eruption, tube-fed flows were found to reach 3 km farther from the vent than the longest channel-fed flow (Calvari and Pinkerton, 1998). The longest channelized lava flow known on Earth is 27 km long; in comparison, Kazumura Cave is a 60 km long lava tube (Lipman and Banks, 1987; Allred and Allred, 1997; Gregg, 2017). Lava tubes allowed a flow from the Undara volcano in Queensland to reach a length of 160 km (Stephenson et al., 1998; Whitehead et al., 2010; Gregg, 2017). Understanding lava tubes is also essential to understanding some flow fields on Earth, even those that are no longer active (Calvari & Pinkerton, 1999b). In the case of an old, long-solidified flow, where direct observations of the developing flow field are impossible, tubes can provide insight into how the lava flowed, its source, and its preferred pathways. For instance, interpreting the sudden appearance of a lava fan on changes in slope as breakouts from a ruptured lava tube enhances our understanding of the development of these flow fields. This knowledge, in turn, helps predict how similar flow fields may behave in the future.

Drained tubes are also important for ecological reasons. Lava tube caves often harbor cave ecosystems that would not develop on the surface, in some cases possibly including unique species (Taylor & Krejca, 2004; Wilkens et al., 2009).

While they are still filled with flowing lava, tubes can be modified by thermal and mechanical erosion. Erosion can cut downwards and sideways into the surrounding material, deepening or widening the tube (Fagents & Greeley, 2001; Greeley et al., 1998; Kauahikaua et al., 1998). Partial collapse of the walls and roofs can also occur, when active or once cooled. In some terrestrial cases there are clearly large pieces missing from the roof or walls of an intact tube, but no debris present on the floor. This implies the tube was active when this partial failure

occurred, and the debris chunks were carried downstream by the flow. These blocks may have been rafted by the initial flow or by subsequent flows reusing the tube.

Inflation is a process that can occur in some tubes while active (Walker, 1991; Self et al., 1996; Hoblitt et al., 2012; Hamilton et al., 2020). When the internal pressure of lava increases, the roof of a lava tube can be forced upwards into a ridge (Figure 1-2). The roof can further solidify while inflated, with the consequence that when the lava pressure decreases or the tube drains, the ridge remains in place (Kempe et al., 2010; Orr et al., 2015; Tonello, 2017; Sauro et al., 2020; Tomasi et al., 2021). This creates a lasting topographic ridge on the surface above the tube (Hróarsson & Jónsson, 1991; Kempe, 2019; Sauro et al., 2019; Sauro et al., 2020). Not every tube has an inflation ridge, as the process and conditions required do not always occur; however, inflated tubes are believed to exist on every major shield volcano on Earth (Sauro et al., 2020). The process of inflation is a relatively common one on Earth, and can occur in a tube or more generally in a lava flow. Even where inflation does not create inflation ridges, specifically above lava tubes, it can form other lava flow inflation features such as tumuli, lava rises, inflated flows, and inflation pits (Walker, 1991; Peterson et al., 1994; Hon et al., 1994; Walker, 2009; Bernardi et al., 2015; Sauro et al., 2020). In some cases, inflation in a thin flow by subsequent pulses of lava form it into a much thicker flow, in which lava tubes eventually develop (Gregg, 2017).

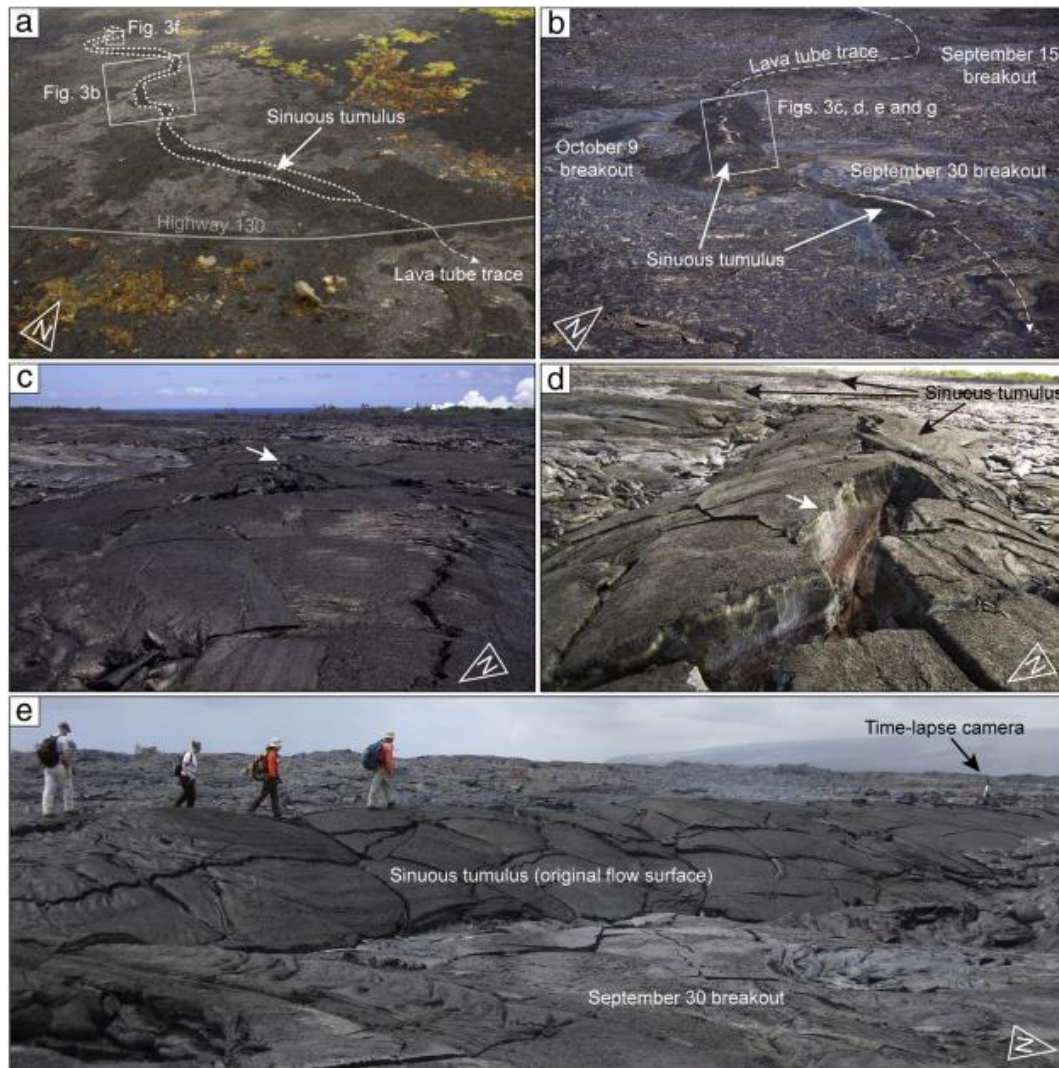


Figure 1-2: Example of an inflation ridge above an inflated lava tube, in a Kīlauea volcano 2010 lava flow. Panels a-e show different views of the same feature. Figure taken from Orr et al., 2015.

1.1.2 Skylights and Collapses

Failure in a tube can cause partial or full collapse of the roof and walls. In its largest form, this constitutes the entire roof collapsing inwards. A collapsed lava tube is clearly visible from the surface as a trench, typically filled with debris. Roof collapse in most cases is caused by the roof collapsing under its own weight (Greeley and Hyde, 1972; Greeley, 1987; Dragoni et al.,

2005; Valerio et al., 2008; Sauro et al., 2020). Less often, failure can be triggered by internal lava pressure of an active tube rupturing the roof (Sauro et al., 2020). Collapses into a lava tube are known as pukas and jameos in Hawai'i and the Canary Islands, respectively (Jackson, 2005; Sauro et al., 2020). Parts of a tube can collapse while other sections remain stable. This causes a collapse chain, one of the most common surface signs of a lava tube. Chains of collapses following a sinuous pattern are a strong indicator that a lava tube lies beneath the surface between these depressions (Greeley, 1971; Calvari and Pinkerton, 1999; Valerio et al., 2008; Sauro et al., 2020; Sawłowicz, 2020) (Figure 1-3). Collapse chains can be created in other ways, such as by tectonic processes. However, tectonic chains are linear and can have greater lengths; sinuous chains of collapses confined to a single flow must be formed by a lava tube (Wyrick and Buczkowski, 2022).

When the roof of a lava tube only partially collapses, it can create a collapse pit into an otherwise intact tube. These holes allow access from the surface to many lava tubes on Earth. The floor of the tube below such collapses can be covered by debris from the collapse (Larson, 1991; McGown et al., 2002). If the collapse occurred when the tube was still active, the debris may have been carried away downstream on the flow of lava.

Skylights are another form of hole in a lava tube roof, with a different formation mechanism to collapses. Skylights occur when a lava tube roof never fully forms, instead of a fully formed roof partially collapsing. Despite their different origins these two types of hole can be difficult to tell apart in remote sensing data, and the word “skylight” is sometimes used to refer to any hole into an intact tube regardless of formation mechanism (Witter and Harris, 2007; Haruyama, 2009; Sauro et al., 2020).



Figure 1-3: Sinuous chain of collapses into a lava tube in the Lava Beds National Monument. Image center: -121.534°, 41.732°. NAIP imagery data (EROS, 2017) visualized through QGIS.

1.1.3 Locating Lava Tubes on Earth

Locating unexplored lava tubes often relies on finding collapse chains, collapse pits, and skylights. However, other methods are available. Geophysical techniques can detect lava tubes from the surface. This has been an important research topic of the GEODES and TubeX teams (Young et al., 2020; Schmerr et al., 2020; Esmaili, 2021; Schmerr et al., 2023). Ground-penetrating radar (GPR) can probe the subsurface to a depth of a few meters, depending on the radar frequency. The interior wall of a lava tube is a sharp boundary where rock transitions suddenly to void, and thus creates a radar reflection. Reflections in GPR data from the lava tube roof, and potentially the floor, can reveal where the tube is and its depth beneath the surface (Esmaili, 2021; Esmaili et al., 2020, 2022). A large void space underground would also create a magnetic anomaly due to the missing volume of rock. Magnetometry surveys reveal magnetic signatures of lava tubes such as Skull, Incline, and Ship Caves (Bell et al., 2022). The sharp

boundary and void space beneath it would also have a distinctive signature in seismic data, allowing seismic surveys to detect tubes (Harmon et al., 2024; Wike, 2024). Collecting data over a sufficiently large tube such as Skull Cave with a gravimeter shows the drop in gravity caused by a large missing mass below the surface. It is worth noting that Skull Cave's width and height reach up to 20 meters each, making it larger and more easily detectable than many lava tubes. Combinations of these methods can be particularly effective at characterizing subsurface voids (Coonan, 2023).

1.2 Lava Tubes on Other Planetary Bodies

1.2.1 Evidence for Lunar Tubes

The maria of the Moon were formed by vast flows of basaltic lava (Head, 1976; Neal and Taylor, 1992; Hiesinger et al., 2011; Head et al., 2023), making them a likely geologic setting to find lava tubes. Sinuous rilles, a type of winding depression, are found prevalently on the surface of the Moon and predominantly in the maria (Hurwitz et al., 2013). These channels are believed to have formed from lava flows through similar formation mechanisms to channelized flows and lava tubes on Earth (Greeley, 1971; Gornitz, 1973; Hulme, 1982; Spudis et al., 1988; Hurwitz et al., 2013; Roberts and Gregg, 2019). Some sinuous rilles may in fact be lava tubes that have collapsed (Greeley et al., 1969; Greeley, 1971; Hurwitz et al., 2013; Roberts and Gregg, 2019). These similar formation mechanisms mean that a tube can transition into a rille, and a rille could continue under the surface into a tube. Rilles such as Rima Marius have discontinuities, where the rille disappears before resuming a short distance away; these might be intact lava tube segments. Hurwitz et al. (2013) document a number of sinuous rilles that either have discontinuities or are associated with collapse chains, implying a partially collapsed lava tube.

Several openings on the lunar surface are interpreted as skylights or collapse pits into lava tubes (Oberbeck et al., 1969; Boston et al., 2004; Haruyama et al., 2009; Robinson et al., 2012; Wagner and Robinson, 2022). Angled imagery into a subset of steep-sided pits in the lunar mare revealed that they extend into void spaces. The full dimensions of these void spaces cannot be measured from the imagery, as most of the floors are out of view, but the underground spaces are visibly wider than the holes that lead into them (Haruyama et al., 2009; Wagner and Robinson, 2014, 2022; Robinson et al., 2012). One particularly well-studied example is the Marius Hills Hole (MHH, alternatively called the Marius Hills Pit) (Figure 1-4). The MHH is on a sinuous rille and leads into a void space of unknown dimensions. Ground-penetrating radar data from the Lunar Radar Sounder (LRS) shows possible evidence of large lava tubes in several locations on the lunar surface, including one near the MHH (Kaku et al., 2017). Angled radar data from the Mini-RF onboard the Lunar Reconnaissance Orbiter (LRO) confirms that the Mare Tranquillitatis Pit, a similar feature to the MHH, leads into a void space (Carrer et al., 2024). Gravity data from the GRAIL mission provides another way to probe the subsurface of the Moon. This data shows signatures of large voids under the lunar surface, including one near the MHH (Chappaz et al., 2017; Zhu et al., 2024). None of these methods can reveal the full dimensions of the possible tubes, but all suggest their presence.

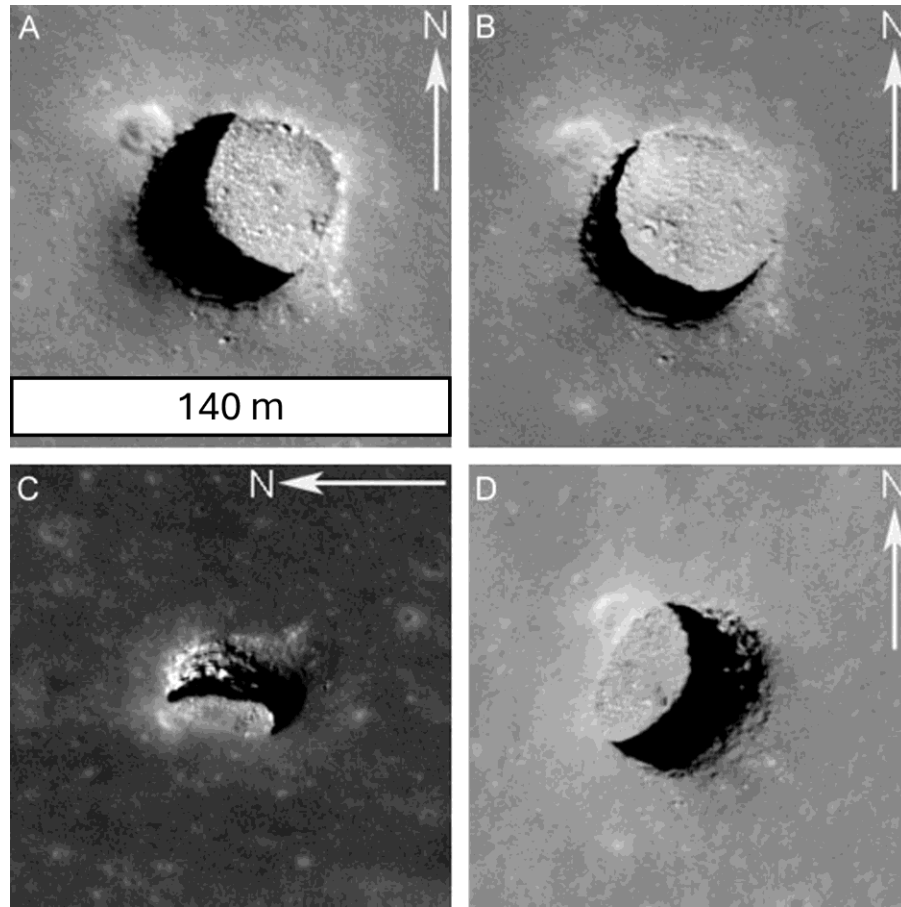


Figure 1-4: Angled LROC imagery of the Marius Hills Hole on the lunar surface reveals that it leads into a void space of unknown dimensions. Figure taken from Robinson et al., 2012. Image center: 303.2301° , 14.0908° . LROC image numbers: (a.) M122584310L, (b.) M155607349R, (c.) M137929856R, (d.) M155614137R (Robinson et al., 2012).

Lava tubes on the other planetary bodies such as the Moon can be much larger than they are on Earth. Due to the lower gravity of the Moon, lunar tubes can be potentially stable with widths measuring several kilometers (Blair et al., 2017; Theinat et al., 2018, 2020). Irregular shapes of the tube walls may reduce the maximum size of tube that can be stable; however, even with this effect lunar tubes could be an order of magnitude larger than terrestrial ones (Chwała et al., 2024). The very large eruptive events on the Moon that formed the mare are also conducive

to forming large tubes. Sinuous rilles on the Moon can be several kilometers wide and hundreds of kilometers long (Schubert et al., 1970; Head, 1976; Hurwitz et al., 2013), suggesting that tubes might reach similar widths. The voids detected in GRAIL data are several kilometers wide (Chappaz et al., 2017; Zhu et al., 2024).

1.2.2 Evidence for Martian Tubes

Mars has sinuous depressions on the flanks of shield volcanoes much as the lunar surface does on its mare. Some of these channels, such as on Arsia Mons, are interrupted by discontinuities that are likely to be uncollapsed tube segments (Carr et al., 1977; Boston et al., 2004; Cushing et al., 2012) (Figure 1-5). Ridges with a sinuous shape are also found on the Martian surface. Southeast Tharsis, Eastern Tharsis, and Alba Mons all have sinuous rilles that transition into sinuous ridges. In some cases the same feature transitions back and forth several times between a surface ridge and a rille. These appear to be inflated, partially collapsed Martian lava tubes (Bleacher et al., 2017; Zhao et al., 2017; Crown et al., 2022).

Sinuous collapse chains and apparent skylights on Mars further indicate that the planet has large lava tubes (Sauro et al., 2020; Sharma et al., 2025). (Figure 1-6). Figure 1-6 I and L (Sauro et al., 2020) show a small hole on the same sinuous path as several wider depressions. This is consistent with a large lava tube, fully collapsed in some places, with one small collapse pit in an otherwise intact section. Examples of sinuous chains on Mars are found on Arsia, Alba, and Olympus Mons (Léveillé and Datta, 2010; Sauro et al., 2020; Mari et al., 2021; Crown et al., 2022).

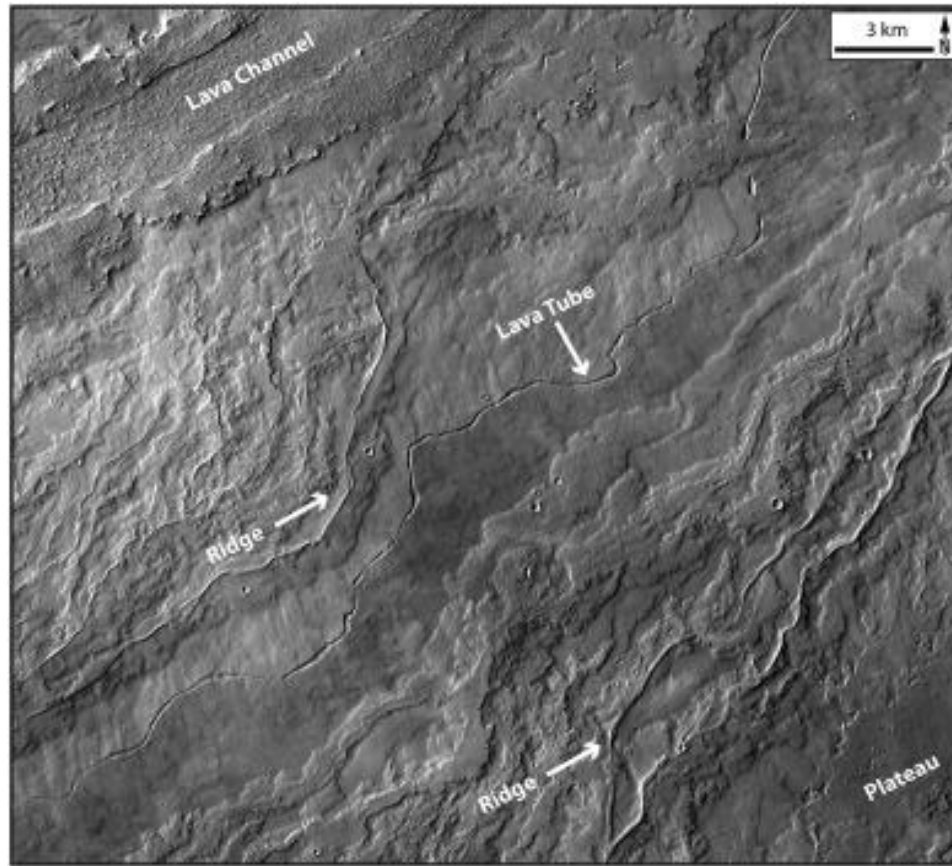


Figure 1-5: Lava flow features on the Tharsis Plains. Included are a lava channel, sinuous ridges, and a lava tube. Figure taken from Bleacher et al., 2017. Approximate image center: 249.92°, -2.83°. Mars Reconnaissance Orbiter Context Camera (CTX) image G02_018929_1772_XN_02S110W (Bleacher et al., 2017).

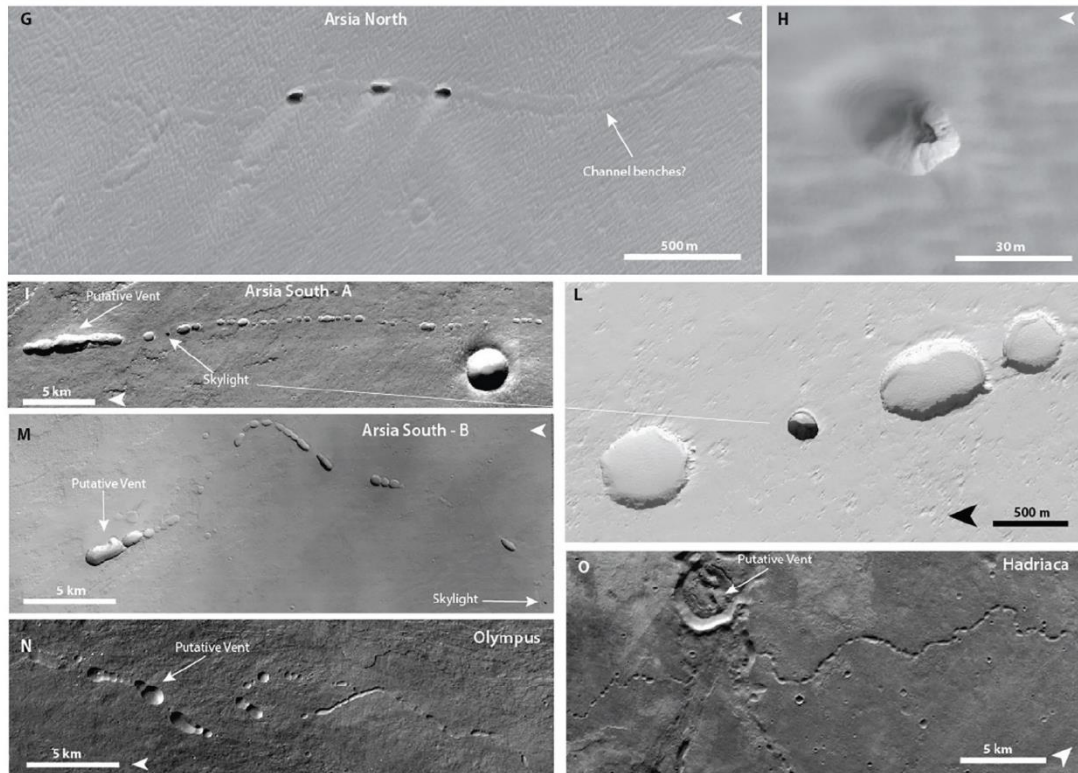


Figure 1-6: Chains of collapses and holes in the Martian surface that may lead into lava tubes.

Figure taken from Sauro et al., 2020.

1.2.3 Benefits of Planetary Lava Tubes

Tubes on these planetary bodies could have great scientific and practical benefits to future missions (Hörz, 1985; Coombs & Hawke, 1992; Lévêillé & Datta, 2010; Blamont, 2014; Titus et al., 2021; Wynne et al., 2022). The surfaces of the Moon and Mars are constantly subjected to surface hazards, including micrometeoroids, radiation, temperature extremes, and/or dust storms. The solid roof of a lava tube would protect anything inside from such hazards (Hörz, 1985; De Angelis et al., 2002). The benefits this presents are twofold. Firstly, any astronauts, equipment, or habitats placed inside a tube would be protected from surface dangers without the heavy constructed shielding that would otherwise be required. A roof 6 m thick would

completely block incoming hazardous radiation, and even a roof only 1-2 m thick would block more than enough to be considered safe (De Angelis et al., 2002). The thermal environment inside a lunar lava tube would be stable, despite the harsh temperature fluctuations on the surface (Hörz, 1985; Horvath et al., 2022; Schörghofer, 2022). Secondly, the environment inside a tube would be preserved very well without weathering by wind, impact gardening, and other alternative processes that affect the surface. Lava tubes therefore present an opportunity to look backwards in time to the unaltered volcanic history of the body at the time the tube formed. This could include volatiles that would not survive on the surface, such as water ice in lunar tubes (Daga et al., 1992; Boston et al., 2004; Haruyama et al., 2012; Blamont, 2014; Titus et al., 2021; Schörghofer, 2022; Wilcoski et al., 2023). Martian caves can contain water ice as hoarfrost (Williams et al., 2020; Schörghofer, 2021). Lava tubes on Earth are often host to life and have distinct biosignatures of it (Boston et al., 2001; Ni, 2019; Kopacz et al., 2022; Weng et al., 2022). If life ever existed on Mars, it is likely that it also inhabited tubes (Boston et al., 2001; Léveillé and Datta, 2010; Weng et al., 2022; Palma et al., 2024; Medley et al., 2024). Biosignatures of this potential past life would be well preserved in the protected environment inside a tube, making lava tubes an excellent study site in the search for life beyond Earth (Léveillé & Datta, 2010; Sauro et al., 2020).

1.2.4 Considerations for Exploration

Robotic missions have been discussed to explore inside a lava tube on another planetary body such as the Moon or Mars (Whittaker, 2012; Fink et al., 2015; Kalita et al., 2018; Olson and Alexis, 2023; Nesnas et al., 2023; Di et al., 2024; Morrell et al., 2024). Traversing inside a lava tube presents logistical challenges, such as unknown terrain and inability to communicate with Earth while shielded by the roof. Even lava tubes on Earth can be difficult to traverse due to

debris. It is useful for any future missions aiming to explore lava tubes to have as much information about the tube interiors as possible beforehand (Meglich et al., 2003; Miyamoto et al., 2005; Esmacili et al., 2020; Torrese et al., 2021; Titus et al., 2021; Wynne et al., 2022; Nesnas et al., 2023). Geophysical methods like seismic surveys and ground-based GPR require sending a dedicated rover to the area. However, detailed satellite imagery and topographic information is available for much of the lunar and Martian surfaces (Christensen et al., 2009; Robinson et al., 2010, 2012). Being able to constrain the dimensions of lava tubes and how damaged their interiors are using only these remote observations would be a valuable asset to any attempts to explore or utilize them.

1.3 Project Aims

Lava tubes are topics of scientific interest on the Earth and beyond; however, it is challenging with current methods to accurately determine their internal morphologies. In this research, we aim to find new ways of characterizing the shapes, sizes, depths, histories, and internal conditions of lava tubes using only their observable surface features.

Numerical modeling of lava tubes gives useful insights into how their surface features reflect tube shape and thus how they can be used to gain new information about the tubes. In the case of large lunar tubes, we find that the tubes create regions of failure on the lunar surface at predictable distances that depend on tube dimensions. Cracking also occurs on the internal surfaces of lunar tubes, affecting their usability for possible future missions (Chapter 2).

In the case of terrestrial tubes these surface cracks are not expected to occur, as these smaller tubes produce less intense stresses. Instead, we study inflation ridges. Using an inflation ridge above Valentine Cave, we find ways to constrain the tube's formation history informed by the ridge's dimensions (Chapter 3). Further investigating this idea we discover a broadly

applicable relationship between the shape, size, and depth of any inflated lava tube and the inflation ridge it would cause (Chapter 4). Applying this method to the Lava Beds National Monument yields new information about some candidate tubes found there (Chapter 5).

Chapter 2 Crack Development Inside and Around

Lunar Lava Tubes

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2.1 Abstract

Evidence shows that lava tubes on the Moon, which constitute intriguing targets for exploration and long-term habitation, could be much larger than terrestrial tubes. However, existing observation strategies cannot confidently constrain the exact dimensions of lunar tubes in anticipation of *in situ* exploration. We produced and analyzed numerical models of the elastic gravity-induced deformation field around lunar lava tubes to determine where failure may occur on their internal surfaces and how the lunar surface above them may deform. This analysis suggests ways to determine tube dimensions from lunar surface morphological observations. The models predict extensive failure on the tube floors and roofs, leaving relatively small sections of pristine floor near the tube walls even if the tube itself may not collapse. The most extensive type of internal failure, tensile floor failure, depends on the tube shape. Additionally, the lunar surface around lava tubes develops cracks and subtle topographic bulges running parallel to the tube axis at distances proportional to tube width. If observed, these could be used to determine the tube's width remotely. We examined several lunar sinuous rilles using orbital data to search for the surface features we expect near tubes. Linear cracks consistent with the presence of lava tubes

are present on the possible extensions of three rilles: Rimae Mairan, Sharp, and Marius. Other locations did not show similar features, possibly because of the resolution of available data, tube depths, or surface degradation. However, surface or low-altitude observations would be more sensitive to the expected structures.

2.2 *Plain Language Summary*

Lava tubes are underground caves present inside some lava flows on the Earth. Lava tubes probably exist on the Moon and may be related to ancient lava channels called sinuous rilles. Lunar lava tubes could be scientifically and practically useful locations for exploring the geology of the Moon or establishing lunar bases. However, current observation strategies provide only limited information on tube dimensions and their internal conditions. We complement previous analyses of lunar lava tube stability by modeling and analyzing how the ground around lava tubes deforms and determining where cracks may develop both on the lunar surface and on tube floors and roofs. The models predict extensive failure in the interiors of large lunar lava tubes, leaving only a small part of their floors free of cracks or fallen debris. At the lunar surface, linear cracks and faint topographic bulges may develop parallel to the tubes. The positions of these features help locate undiscovered tubes and constrain tube dimensions based solely on geomorphological observations. Examples of such linear crack regions can be seen near three sinuous rilles, possibly indicating that lava tubes with widths of less than two kilometers are present connected to these rilles.

2.3 *Introduction*

Lava tubes are long underground void spaces created by lava flows. Lava tubes are observed in many volcanic fields on Earth and are suspected on the Moon as well (Waters et al.,

1990; Calvari & Pinkerton, 1998; Fornari et al., 2004; Haruyama et al., 2009; L  veill   & Datta, 2010; Robinson et al., 2012; Gregg, 2017; Chappaz et al., 2017; Theinat et al., 2019; Wagner & Robinson, 2022). Because of their large sizes, potential to provide protection from surface hazards, and even the possibility of hosting ice deposits, lunar lava tubes are an intriguing target for exploration and long-term human activity (Blamont, 2014; H  rz, 1985; L  veill   & Datta, 2010; Blamont, 2014; Titus et al., 2021; Titus et al., 2021; Wynne et al., 2022). However, the dimensions and state of the interior of lunar lava tubes, which are important information to consider during mission planning, are difficult to evaluate in the existing data. Here we propose, based on elastic numerical models, that cracks likely develop on the floor and roof of large lunar lava tubes, presenting a series of challenges for future exploration. Further, we show that cracks may form on the surface of the Moon near these tubes and help constrain tube dimensions from remote sensing data. We apply this strategy to known rilles and propose that at least three of them are associated with lava tubes several hundred meters wide.

A lava tube may form when a lava flow develops a continuous crust of solid rock on its sides and roof, and the fluid interior lava subsequently flows out and drains the space (Peterson & Swanson, 1974; Sakimoto & Zuber, 1998; Huff & Owen, 2013; Gregg, 2017). If the crust is structurally sound enough to remain in place after the tube is drained, this process leaves a long, empty tube along the path of the flow. Thermal and mechanical erosion can also deepen and widen lava tubes (Greeley et al., 1998; Kauahikaua et al., 1998; Fagents & Greeley, 2001), and failure can cause regions of cracking or full collapses of the tube roof (Theinat et al., 2019). Lava tube formation is common on Earth, and examples have been studied and explored in many locations (Waters et al., 1990; Calvari & Pinkerton, 1998; Fornari et al., 2004; L  veill   & Datta,

2010; Gregg, 2017; Theinat et al., 2019). Lava tubes on Earth typically have widths up to a few tens of meters and can stretch for tens of kilometers (Pipan & Culver, 2019).

Multiple lines of evidence indicate that lava tubes exist on other bodies such as the Moon. These include holes in some locations on the lunar surface that are interpreted as skylights into lava tubes (Oberbeck et al., 1969; Carr et al., 1977; Boston et al., 2004; Cushing et al., 2007; Haruyama et al., 2009; Robinson et al., 2012). Skylights are openings in the roof of a tube where part of a lava tube roof has either collapsed or never fully formed, allowing access from the tube interior to the surface (Larson, 1992; Okubo & Martel, 1998; Halliday et al., 2012; Wagner & Robinson, 2022). Here we use the term “skylight” to mean any opening that leads into a lava tube, regardless of formation mechanism. Several lunar holes have observable overhangs between the walls and floor, confirming that they connect to underground void spaces of unknown sizes (Wagner and Robinson, 2014, 2022). These include the Marius Hills Hole (MHH, alternatively called Marius Hill Pit), a well-studied example of a suspected lunar lava tube skylight (Haruyama et al., 2009; Robinson et al., 2012). Holes in the lunar surface sometimes form what appear to be skylight chains, possibly tracing the path of a tube as skylights can do on Earth (Cruikshank & Wood, 1972; L  veill   & Datta, 2010; Whitten & Martin, 2019; Theinat et al., 2020; Sauro et al., 2020; Torrese et al., 2021; Wyrick & Buczkowski, 2022). Wider collapse features such as the aligned depressions at the end of Rima Mairan may also indicate concealed, partially collapsed lava tubes (Figure 2.1). Other lines of evidence support the presence of lunar lava tubes, sometimes in the same locations where tubes are already suspected from skylights. Gravity data from the Gravity Recovery and Interior Laboratory (GRAIL) contains signatures of several long, wide subsurface voids, one in the region of the MHH. It is difficult to determine exact tube dimensions from this data, and gravity methods cannot detect tubes narrower than

approximately 1 km (Chappaz et al., 2017). Ground Penetrating Radar (GPR) data from the Lunar Radar Sounder (LRS) onboard Kaguya and angled radar data from the Mini-RF onboard the Lunar Reconnaissance Orbiter (LRO) also suggests the existence of lunar lava tubes, including one near the MHH, but cannot reveal their exact dimensions (Kaku et al., 2017; Carrer et al., 2024).

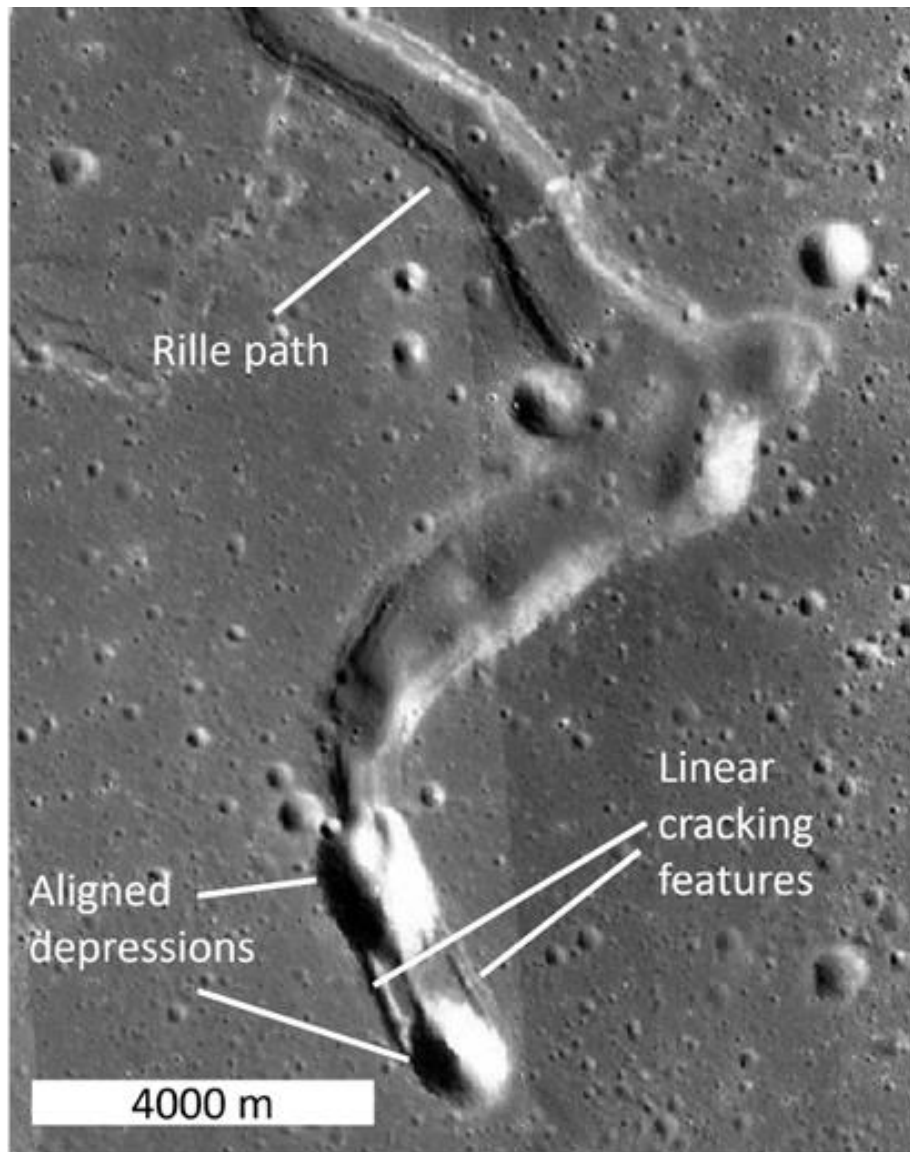


Figure 2-1: Southern end of Rima Mairan, a lunar sinuous rille in the Oceanus Procellarum region. The rille ends in two aligned depressions linked by narrow cracks. These depressions may be large collapse pits over a lava tube extension of the rille. Image source: LROC NAC large inclination West illumination imagery, obtained through LROC Quickmap. Center of image: 36.95°N, 46.45°W.

Sinuuous rilles are winding channels found on the lunar maria and may be closely related to lava tubes, as they likely also formed from lava flows through related formation mechanisms (Greeley, 1971; Gornitz, 1973; Hurwitz et al., 2013). These features can have widths of up to several kilometers and stretch for hundreds of kilometers (Schubert et al., 1970; Head, 1976; Hurwitz et al., 2013). Like lava tubes, sinuous rilles may have formed by combining levee formation and downward erosion into older lava flows (Hulme, 1982; Hurwitz et al., 2013; Roberts & Gregg, 2019). Rilles could also be formed by the collapse of large lava tubes (Hurwitz et al., 2013). Because of these linked formation mechanisms, both sinuous rilles and tubes may be present in the same continuous flow (Greeley, 1971). Therefore, one channel on the lunar surface may comprise rilles and tubes leading into each other, depending on whether the rille is roofed over in some places or if part of a tube roof collapsed. For example, one of the most widely accepted examples of a lunar lava tube skylight, the MHH, sits along the path of a sinuous rille (Haruyama et al., 2009; Robinson et al., 2012). Moreover, some sinuous rilles, such as Rima Mairan, in Oceanus Procellarum, appear discontinuous; the channel may be roofed over or continue underground as a lava tube between sinuous rille segments (Figure 2-1). Discontinuities such as this are therefore promising locations to look for lunar lava tubes, as are the termini of rilles where the channel could continue into an intact tube.

Lava tubes on Mars and the Moon would be potentially very valuable locations for exploration and habitation because of the protection provided by the tube roofs, which has both practical and scientific benefits (Blamont, 2014; Hörz, 1985; Léveillé & Datta, 2010; Titus et al., 2021; Wynne et al., 2022). The lunar surface is constantly bombarded by micrometeorite impacts and radiation. It also undergoes extreme temperature changes of approximately 280 °C between day and night (Hörz, 1985). Inside a lava tube, however, the rock of the roof is expected to block

potentially harmful impacts and radiation (De Angelis et al., 2002), and lava tube interiors are predicted to remain at a constant temperature (Hörz, 1985; Schörghofer, 2022; Horvath et al., 2022). A lava tube would protect any astronauts, equipment, and structures inside from these hazards, aiding crew safety and removing the necessity of manmade shielding. Tubes formed on the Moon or Mars could be much larger than on the Earth while remaining stable, due to the significantly lower gravities on these bodies (Blair et al., 2017; Theinat et al., 2020). These tubes could therefore be wide enough for structures such as large habitats to be built inside them. The protected environment inside a tube would also preserve geologic features and deposits (Haruyama et al., 2012; Boston et al., 2014), making it a valuable scientific target for study. A tube could preserve basalts and flow features unmodified by surface processes and volatiles such as water ice that would not survive on the surface, providing a unique opportunity to study the geologic history of the Moon (Haruyama et al., 2012, Boston et al., 2014, Schörghofer, 2022, Wilcoski et al., 2023). The possible presence of preserved water could also be valuable for *in-situ* resource utilization for a long-term lunar base (Daga et al., 1992; Hörz, 1985; Blamont, 2014; Titus et al., 2021).

Although lunar lava tubes are potentially ideal locations for future exploration and habitation, we will show here that failure zones of crack development and debris fields should be expected inside the tubes, presenting new challenges to exploration even if these features do not threaten the structural integrity of the tubes. Although some of the difficulties presented by these obstacles could be mitigated, logistical limitations make careful mission planning necessary. Thus, it is essential to know the conditions of a tube's interior and its geometry before entering it. It would also be of great use to tube mission planning to be able to predict the dimensions of a target tube as accurately as possible before *in-situ* exploration (Melgich et al., 2003; Miyamoto et

al., 2005; Esmaeili et al., 2020; Torrese et al., 2021; Titus et al., 2021; Wynne et al., 2022; Nesnas et al., 2023). Our study suggests a way to constrain the dimensions of a lava tube from the pattern of cracks expected to develop at the lunar surface near large tubes.

Here, we model the deformation and stress fields expected on the walls and in the surroundings of stable lunar lava tubes. Our objectives for this are twofold. Firstly, we predict how much cracking occurs on the internal surfaces of the tubes, and discuss what difficulties this would pose for their exploration. Secondly, we predict what crack patterns may appear on the lunar surface around lunar tubes, and how these features may be used to constrain tube dimensions. To avoid confusion, the surfaces of the roof, walls, and floor inside lava tubes will be referred to as “internal surfaces,” and the surface of the Moon above lava tubes will be referred to as “the lunar surface.” We then search for candidate lava tubes near sinuous rilles based on the surface crack pattern predicted in our models. Using LROC (Lunar Reconnaissance Orbiter Camera) images (Robinson, 2011), we detected tube candidates connected to Rima Mairan, Rima Sharp, and Rima Marius. Other datasets and locations studied did not provide clear evidence of tube-related features, although subsequent surface processes could mask such evidence.

We note that other researchers have previously modeled the stability of lunar lava tubes (Blair et al., 2017; Modiriasari et al., 2018; Theinat et al., 2020). The modeling processes of these researchers inspired some aspects of our modeling, such as the half-elliptical tube shape used for most of our models and the reasonable possible dimension ranges of stable tubes. Our goal is distinct and complementary from those of these studies. Instead of the overall tube stability, we focus on where cracks in surfaces may form, regardless of the tube’s structural stability. In fact, since our goals are to understand how tubes could be used for exploration or

how concealed tubes could be found, our work is most relevant for tubes that have survived to the present day without collapsing. We refer to the results of prior authors who have studied tube stability to inform our choices of tube dimensions (Blair et al., 2017; Modiriasari et al., 2018; Theinat et al., 2020), ensuring that the tubes we consider could reach the large dimensions relevant for lunar exploration without collapsing. We build on this basis to fulfill our dual objectives: understanding the conditions of the floors of large lava tubes and developing strategies to detect and characterize the size of lunar lava tubes before *in-situ* exploration.

2.4 Modeling Process and Methods

We modeled the deformation and stress fields around them using COMSOL Multiphysics®, a finite-element modeling software, and its LiveLink™ for MATLAB feature. Livelink™ for MATLAB allows COMSOL Multiphysics® to be controlled by MATLAB code instead of manually, making it possible to automate the creation, running, and analysis of the models (Figure 2-2). The model simulates half of a tube in cross-section, assuming symmetry along the tube axis (Figure 2-3). We choose for simplicity to ignore along-strike variations in tube shape. Thus, the model is constructed under plane-strain assumptions. This assumption is justified because known lava tubes are much longer than they are wide, and though their geometries can change slowly along their path, these variations take place over distances longer than the tube's width.

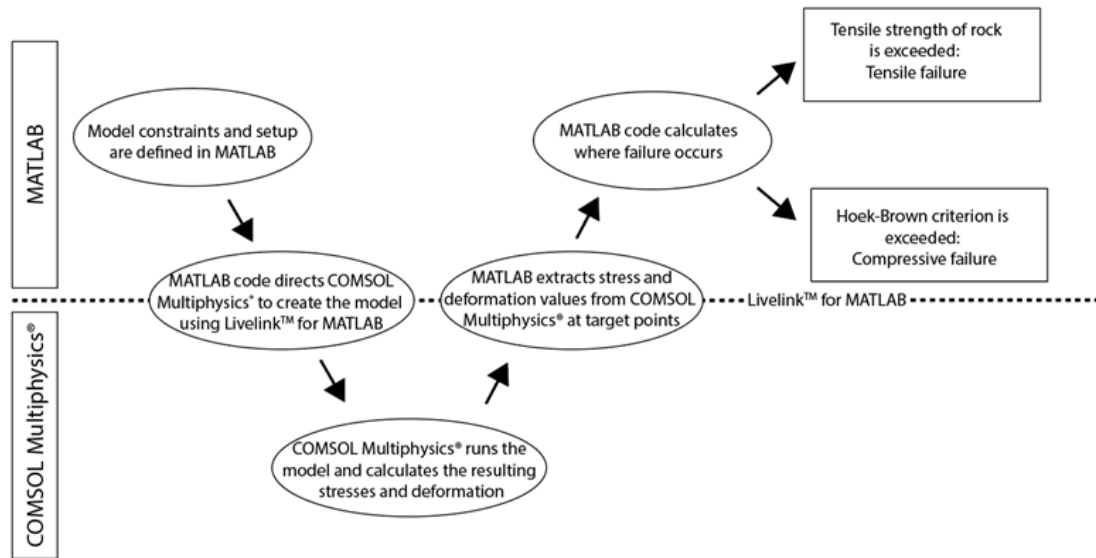


Figure 2-2: Flow chart showing the sequence of events between MATLAB and COMSOL Multiphysics® that leads from defining model parameters to finding failure extents.

Each half-tube model consists of a void space embedded in a 20 km by 20 km block of material. The model is solved over a COMSOL-defined “very fine” grid for numerical accuracy (Supporting Information Figures 7.1.S1 to 7.1.S4), but the results are interpolated over a coarser grid to facilitate visualization and analysis (Figure 2-2). The model setup and mesh generation scheme are described in Supporting Information Text 7.1.2 (See Chapter 7: Appendix). The principal stress values are interpolated at these grid points and saved using LiveLink™ for failure analysis using MATLAB (Figure 2-2).

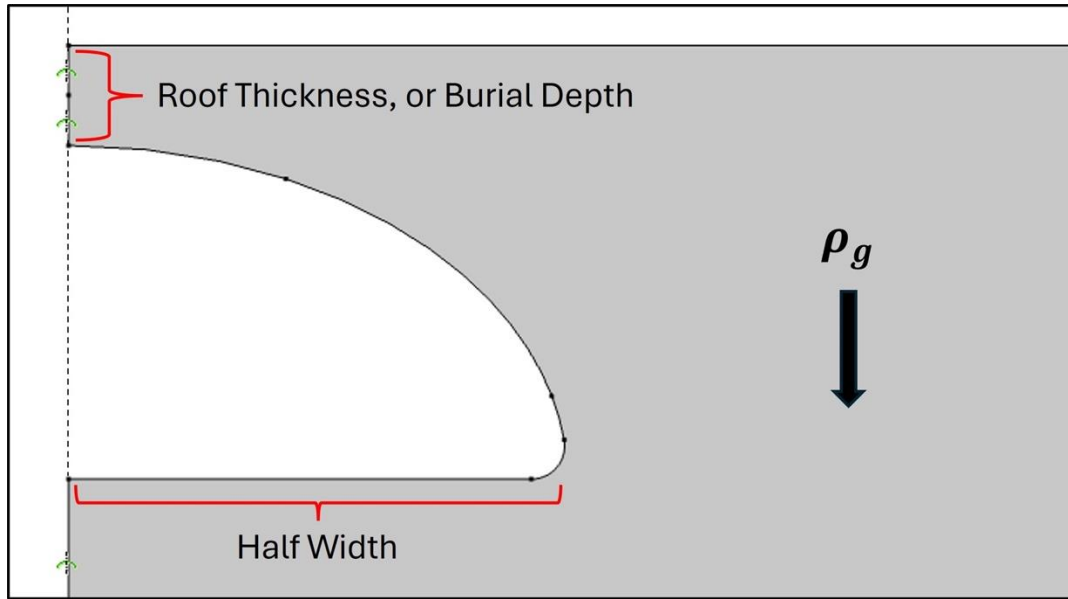


Figure 2-3: Example diagram of tube model geometry, showing the roof thickness, the tube half-width, and the direction of gravity. The model is symmetric (denoted by green half-circular symbols) across the dotted line on the left vertical axis. The computation domains extend further than shown here to avoid edge effects (See Supporting Information Figure 7.1.S2). An associated computational mesh is shown in Supporting Information Figure 7.1.S1.

The tube shape used in our reference model setup consists of a half-ellipse representing the tube roof, a flat floor, and a rounded corner joining the two (Figure 2-3). The width-to-height ratio of these half-elliptical tubes was 3:1, corresponding to a full ellipse with a 3:2 aspect ratio. This shape is similar to that used by Blair et al. (2017) and Theinat et al. (2020) and observed in terrestrial lava tubes by Theinat et al. (2019). The rounded corners add realism to the model geometry and prevent the development of stress singularities at sharp corners. Additional models, presented in Section 2.5.3, were constructed with a circular or shallower roof shape (2:1 or 6:1 width-to-height ratio, respectively) or an elliptical or circular floor (3:1 or 2:1 width-to-height ratio, respectively). The tubes' widths and roof thicknesses were varied systematically

(Table 2.1) to determine their effects on the stress, deformation, and failure patterns produced. These dimensions were chosen based on the widths of lunar features such as sinuous rilles (Hurwitz et al., 2013), tube candidates (Chappaz et al., 2017), prior modeling results (Blair et al., 2017; Theinat et al., 2020) concerning stable tube dimensions, and further experimentation with different ranges of dimensions.

The block in which the tube is embedded represents a lunar rock mass, not intact basalt. Its rheology is elastic, with the material properties reported in Table 2-1. A gravitational acceleration of $g = 1.662 \frac{\text{m}}{\text{s}^2}$, equivalent to lunar gravity, is applied to the block. The bottom boundary of the model is fixed in place, and the right edge boundary is allowed to move vertically, while the left edge is an axis of symmetry. The remaining surfaces are stress-free. We also modeled a set of smaller tubes under a gravitational acceleration of $g = 9.8 \frac{\text{m}}{\text{s}^2}$ to evaluate whether the patterns of failure documented here in lunar lava tubes would be expected in terrestrial lava tubes.

The possibility of failure is evaluated based on the von Mises criterion in tension and the Hoek-Brown criterion under compression (Marinos & Hoek, 2000). Failure strength is calculated based on the Geological Strength Index (GSI) representing a lunar rock mass (Blair et al., 2017; Theinat et al., 2018; Theinat et al., 2019) and other parameters representing intact lunar basalts (Table 2-1).

The GSI value and these intact rock parameter values were used to calculate two unitless parameters for the rock mass, s and m_b , using Equations 1 and 2:

$$s = \exp\left(\frac{\text{GSI} - 100}{9}\right) \quad (2.1)$$

$$m_b = m_i \exp\left(\frac{\text{GSI} - 100}{28}\right) \quad (2.2)$$

In our nominal models, $GSI = 70$ and $m_i = 25$ (Table 2-1), giving $s = 0.0357$ and $m_b = 8.5630$. These values were calculated using Equations 1 and 2 and the nominal values given in Table 2-1. Both tensile and compressive failure criteria are linked to these parameters and the intact compressive strength σ_{ci} (Table 2-1).

Tensile failure follows the Von Mises criterion:

$$\sigma_1 = \sigma_t \equiv \frac{\sigma_{ci}}{2} \left(m_b - \sqrt{m_b^2 + 4s} \right) \quad (2.3)$$

where σ_1 is the first principal stress (most tensile stress), and σ_t is the tensile strength of the rock mass. In the nominal case, $\sigma_t = \frac{\sigma_{ci}}{240} = 1.11$ MPa.

Compressive failure obeys the Hoek-Brown failure criterion, which relates the first and third principal stresses, σ_1 and σ_3 , at failure:

$$\sigma_1 = \sigma_3 + \sqrt{(m_b \sigma_{ci} \sigma_3) + s \sigma_{ci}^2} \quad (2.4)$$

Table 2-1: Elastic parameters and tube shape parameters

Symbol	Property	Nominal Value (variations)	References
<i>W</i>	Tube width	6.5 (0.5–6.5) km	Hurwitz et al., 2013, Chappaz et al., 2017, Blair et al., 2017, Theinat et al., 2020

T	Roof thickness	650 (50–950) m	Blair et al., 2017, Theinat et al., 2020,
R_r	Roof width-to-height ratio	3:1 (2:1–6:1)	Blair et al., 2017, Theinat et al., 2019, and Theinat et al., 2020
R_f	Floor width-to-height ratio	Flat floor (2:1–6:1)	Blair et al., 2017, Theinat et al., 2019, and Theinat et al., 2020
G	Shear modulus	12.2 GPa	Marinos & Hoek (2000)
ν	Poisson ratio	0.3	Theinat et al., (2020) Schultz (1995)
ρ	Density	3100 kg/m ³	Blair et al. (2017) Kiefer et al., (2012)
GSI	Geological Strength Index	70 (60–80)	Blair et al. (2017) Theinat et al. (2018) Theinat et al. (2019)

σ_{ci}	Compressive strength	266 MPa	Schultz (1995)
m_i	Intact strength parameter	25 (20, 22)	Marinos & Hoek (2000) Blair et al. (2017)
s	Strength parameter	Equation 2.1	Marinos & Hoek (2000)
m_b	Bulk strength parameter	Equation 2.2	Marinos & Hoek (2000)
σ_t	Tensile strength	Equation 2.3	Marinos & Hoek (2000)

In Figure 2-4, the tensile and compressive failure regions at the tube surfaces are marked by red and blue circles, respectively. Regions experiencing tensile stress are highlighted in red. The blue-hued black-to-white color scale indicates proximity to compressive failure, quantified by the Hoek-Brown ratio R_{HB} :

$$R_{HB} \equiv \frac{(\sigma_1 - \sigma_3)}{\sqrt{(m_b \sigma_{ci} \sigma_3) + s \sigma_{ci}^2}} \quad (2.5)$$

Compressive failure takes place where $R_{HB} = 1$ (white in Figure 2-4). For example, the rock is close to failure (pale blue color) along the top surface of the model directly above the center of the tube, where the downward flexure of the roof compresses the upper surface. The tube walls have high Hoek-Brown ratio values because they are compressed as the tube roof deforms downwards from the center, causing compressive failure at the tube corner.

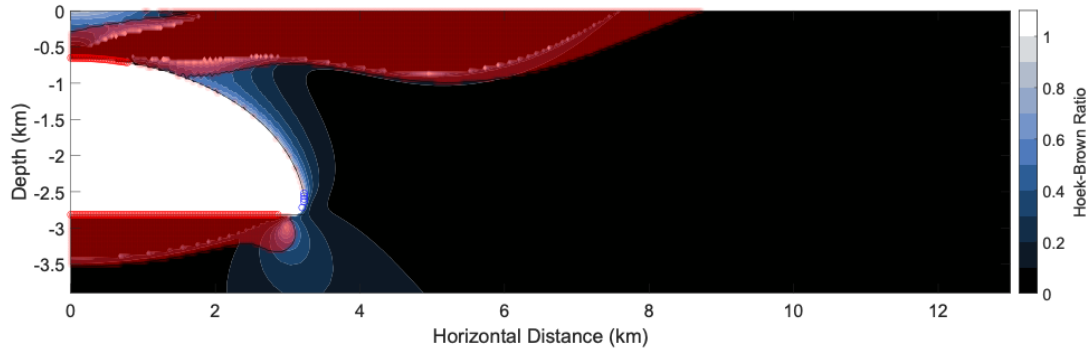


Figure 2-4: Model output for our nominal lunar tube model (full width: 6.5 km; roof thickness: 650 m). The blue color scale follows the Hoek-Brown ratio (Equation 5). Translucent red areas indicate where the rock is in tension. Circles on the tube edges show tensile (red) and compressive (blue) failure

2.5 Results

Under the influence of gravity, the block surrounding the lava tube shortens uniaxially in the vertical direction. The tube itself acts as a mass deficit and rises slightly upward compared to its surroundings. However, the roof of the tube is supported only at its edges, so it sinks slightly under its own weight into the tube. Thus, the presence of a tube results in the complex deformation field and associated stresses depicted in Figure 2-4.

2.5.1 Failure inside the tube

Three regions inside the tube consistently experience failure: 1) the tube floor is under tension as it bows upward, and stress can exceed the tensile strength; 2) the underside of the tube roof can also undergo tensile failure as it bows downward; 3) the tube walls are compressed due to uniaxial shortening, enhanced by the corner between the tube roof and floor to the point that they can experience compressive failure.

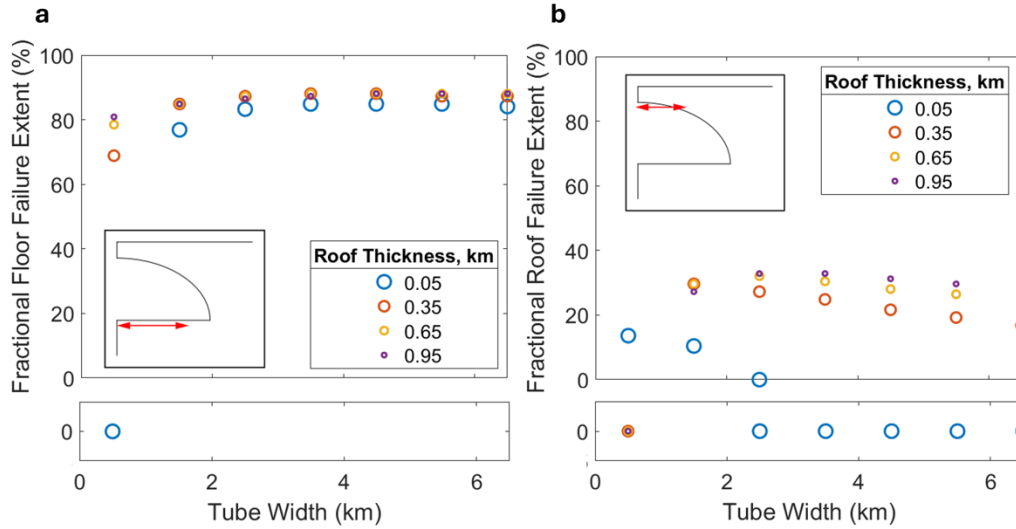


Figure 2-5: Fractional extent of the tube floor (a) and roof (b) that undergoes tensile failure reported against tube width. Symbol color and size indicate the roof thickness. The inset shows a simplified visualization of where this failure takes place. The bottom panels show cases where the failure extent is zero.

The tensile failure on the roof of the tube is likely the most important type for overall tube stability and determining whether a skylight may form, but all three failure types must be considered when describing the characteristics of the insides of a stable tube. Floor failure is often most important for this purpose, as it typically covers the largest horizontal extent. Below, we report the fractional extents of each failure domain, defined as the horizontal extent of the zone of failure as a fraction of the tube width. Figure 2-5 shows these fractional failure extents for the two most prevalent failure types: tensile floor failure (Figure 2-5a) and tensile roof failure (Figure 2-5b).

The most common failure type in the modeled tubes is tensile failure on the tube floor. The zone of failure starts at the center of the tube floor and spreads outwards towards the sides of the tube, leaving relatively small unaffected regions near the tube walls. Tensile failure is caused

by the tube floor rising upward due to the lack of density in the void space compared to the surrounding rock. As the floor flexes upwards, especially towards the tube center, the rock is stretched and put under tension. The tensile stress on the rock can lead to cracking in a type of failure referred to here as extrados cracks. An extrados is defined as the outside surface of an arch (Goode, 2009). Extrados surfaces can often be in tension and have tensile cracking (e.g., Borghi et al., 2015). When the tube floor flexes upwards into an arch shape, the surface is stretched into an extrados and put under tensile stress. We refer to the resulting tensile failure on this type of geometry as extrados cracking; this includes the tensile floor failure we predict. When present, this extrados failure is expected on at least 65% and usually over 80% of the tube floor in our models. Model symmetry dictates that these cracks occur parallel to the tube axis. As this floor failure is due to the mass deficit in the tube void space, it is most widespread in wider and deeper tubes (Figure 2-5a), but the dependence on roof thickness is not strong. Only the narrowest and shallowest tube studied has no floor failure.

Tensile failure of the tube roof is the second most prevalent type of failure observed in the models. Like floor tensile failure, roof tensile failure starts at the center of the tube and spreads outwards towards the sides. This failure is caused by the tube roof flexing downward into the void space because it is not supported from below like the surrounding rock is. As the roof bends downwards, the bottom internal surface is stretched and may undergo tensile failure if the tensile stresses are large enough. This again forms extrados cracks (i.e., Borghi et al., 2015) but in a different setting; instead of the floor flexing upwards to create an extrados under tension, the roof is flexing downwards to create one. Tensile roof failure is generally less extensive than tensile floor failure, with a maximum fractional extent of under 35%. The resulting cracks also run parallel to the tube axis. This kind of failure does not affect most of the narrowest or

shallowest tubes (Figure 2-5b). Roof failure is most extensive when the roof is thicker (Figure 2-5b), which shows that, at least for tubes the size of lunar tubes, failure is related to the unsupported weight of the roof, not its elastic flexibility or rigidity. If roof failure results in the formation of a skylight, that skylight would be about one-third the size of the tube at maximum.

Compressive wall failure occurs in only a few modeled tubes, those with widths of at least 4.5 km and roof thicknesses of at least 350 m. This failure type occurs at the tube corner and spreads up the wall. The fractional extent of this failure increases with tube width and depth, indicating that it is linked to the torques on the corner from both the void mass deficit and the roof weight. The extent of compressive wall failure is always relatively small and depends on the arbitrary rounding of the tube corner, so we do not include it in Figure 2-5.

The three failure types described above are combined to determine the extent of the pristine floor for each tube. The floor is considered pristine if it has not undergone tensile failure, which may otherwise produce large crevasses, and is not directly underneath a region of the roof or the wall that has undergone any type of failure, which could drop significant debris onto the floor below. The pristine part of the tube would be most suitable for exploration or habitation.

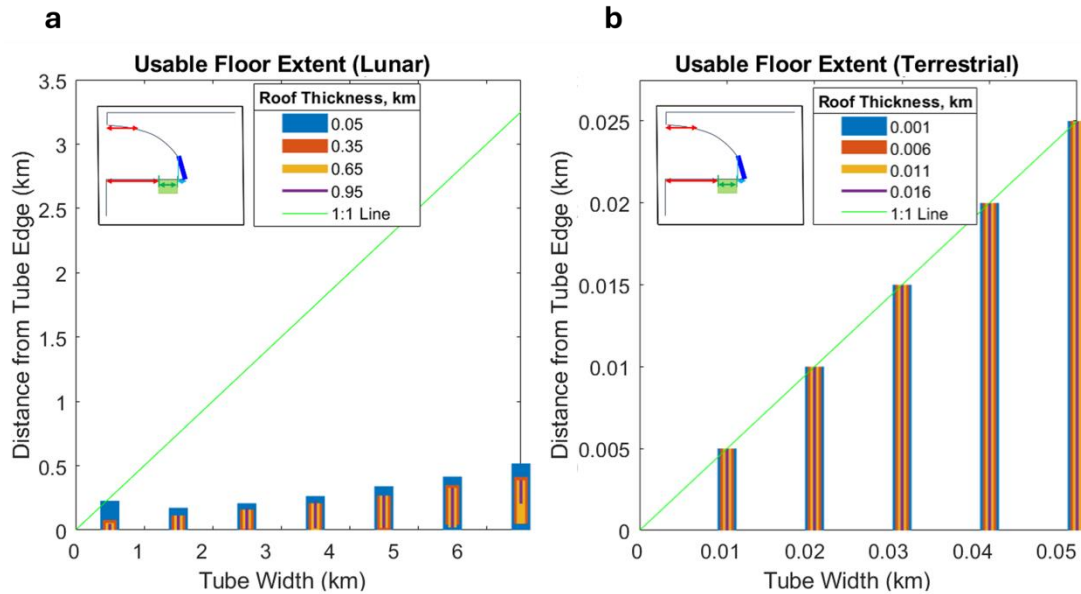


Figure 2-6: Pristine floor extents vs. tube widths for lunar (a) and terrestrial (b) tubes. Colors and line widths indicate different roof thicknesses. The inset shows a simplified visualization of where the failure types take place and where pristine floor (green) is found. The green line shows the full tube width

In most tubes, tensile failure on the floor leaves only a small unaffected region near the tube edge; in some cases, compressive failure spreading on the wall also eliminates a portion of the floor close to the tube edge. The combination of these factors leaves little pristine floor in the modeled lunar tubes, reaching a maximum contiguous extent of 500 m (Figure 2-6a). The uncertainties in these pristine floor extents are approximately 1.6% of the tube width in each case (Supporting Information Text 7.1.3).

Although we predict widespread failure in large lava tubes of the kind documented on the Moon, no failure is predicted in any of the much smaller terrestrial tubes. As failure is caused by the weight of the roof and the void inside the tube, larger voids and wider unsupported roofs would make failure more likely. The Earth's gravity, which is stronger than the Moon's by a

factor of ~ 6 , should increase the likelihood of failure in terrestrial tubes, but this effect is more than counterbalanced by the much smaller tube sizes on Earth. Terrestrial tubes have widths and heights smaller than modeled lunar tubes by a factor of ~ 100 , making their volumes smaller by a factor of $\sim 10,000$. Thus, the zones of failure presented above are highly unlikely to be observed on Earth. Accordingly, the model results show no floor, wall, or roof failure in terrestrial tubes and predict that their entire floors are pristine (Figure 2-6b). Nonetheless, cracks and debris are commonly observed inside real terrestrial tubes, contrary to model predictions (e.g., Waters et al., 1990). This contradiction indicates that our models do not consider all possible sources of debris. For example, thin lava coatings deposited after tube formation may detach and fall, forming small debris (Waters et al., 1990; Calvari and Pinkerton, 1999). Weathering, subcritical crack growth, and seismic shaking may all play a role in terrestrial tubes. These phenomena should be much less important on the Moon.

2.5.2 Failure at the lunar surface

Tube-induced deformation leaves traces on the surface of the Moon above the tubes at locations that scale linearly with tube width. The tube mass deficit lifts the lunar surface near the tube whereas the roof itself bends downwards at its center, where it is least supported by the surrounding rock. Thus, the transition between uplift and downward bend forms a topographic maximum, or “bulge,” some distance from the edge of the tube (Figure 2-7). The resulting topography puts the lunar surface under tension to the point that tensile failure, again resulting in extrados cracks, sometimes becomes possible.

Tubes are, therefore, associated with a depression above their axis, as well as topographic bulges and possibly cracks running parallel to the tube on either side of its axis. The cracks and the bulges develop at specific distances from the tube center, D_c and D_b , respectively, that

increase linearly with both tube width and roof thickness (Figure 2-8). The following relations fit the model results.

$$D_b = (0.71 \pm 0.006)W + (1.76 \pm 0.037)T - (0.066 \pm 0.032) \text{ km} \quad (2.6)$$

$$D_c = (0.42 \pm 0.010)W + (1.00 \pm 0.061)T - (0.066 \pm 0.052) \text{ km} \quad (2.7)$$

where W and T are the tube width and roof thickness in kilometers.

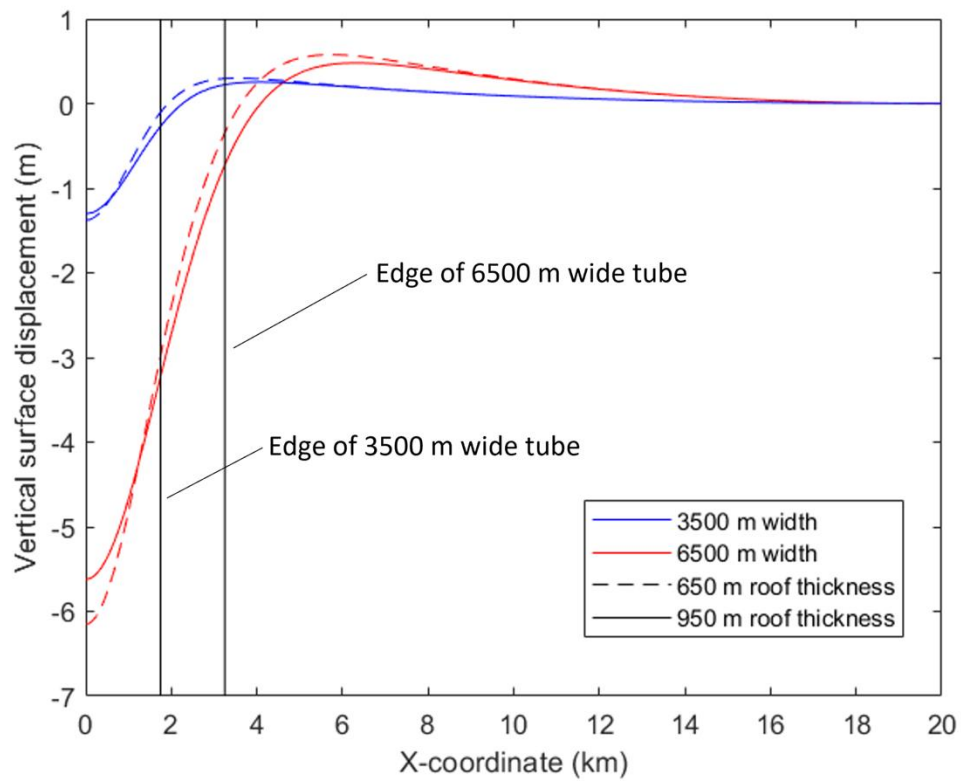


Figure 2-7: Vertical lunar surface displacement vs. horizontal distance from tube center for several example modeled tubes. The lunar surface dips above the tube and forms a slight bulge beyond the tube edges. Tube roof thicknesses and widths are given in the legend. The black vertical lines show where the tube edges would be for the two different tube widths used.

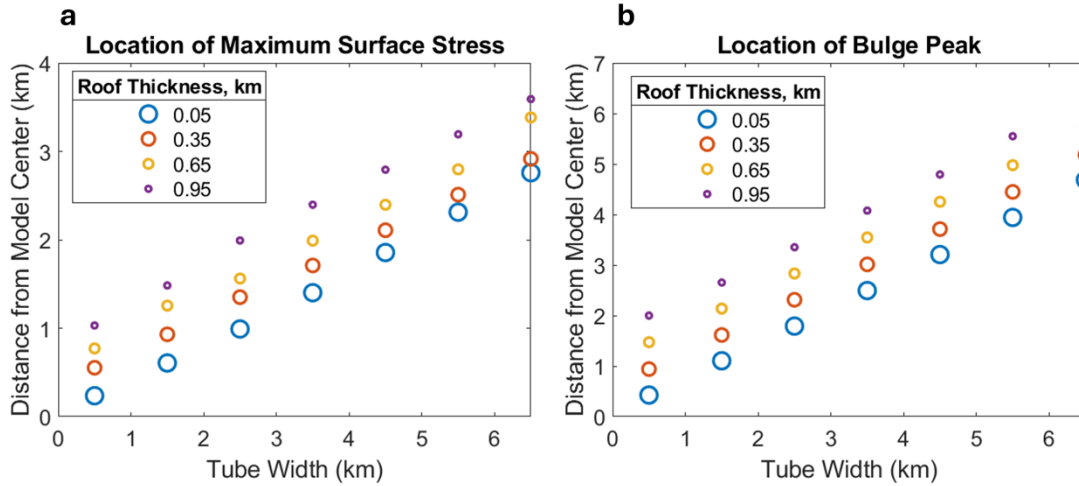


Figure 2-8: Horizontal distances from the tube center to lunar surface features caused by the tube: location of maximum lunar surface tensile stress (a) and the peak of the elastic bulge on the lunar surface (b), plotted vs. tube widths. Symbol color and size indicate the roof thickness.

Uncertainties in these equations correspond to the 95% confidence bounds given by the MATLAB “fit” function. Slight deviations from these linear trends may be seen in our results and should be expected due to the nonlinear effect of roof thickness on flexural rigidity and the way roof thickness changes with distance from the tube’s axis. However, these deviations are very small and cannot be confidently constrained in our numerical models.

Here, we associate cracks with the location of the tensile stress maximum, as this is where the first crack would form. If areas of cracking or a surface bulge running parallel to a tube are observed on the lunar surface, Equations 2.6 and 2.7 could be used to determine the width of the tube. The roof thickness of the tube may be determined by angled imagery if a skylight is present. In the absence of such constraints, the roof thickness may be approximated as the thickness of the lava flows. Robinson et al. (2012) determined that the walls of the MHH are

~80 m thick, similar to the lower end of lava flow thickness determined by Weider et al. (2010).

If both D_b and D_c can be determined, it may be possible to constrain both W and T .

Determining D_b and D_c is best achieved by measuring the distance between similar structures on either side of the tube. However, if only one side is visible, it is possible to use the relative position of the cracks and the topographic bulge. Our models show that the ratio $\frac{D_b}{D_c}$ remains relatively constant at 1.76 ± 0.062 for lunar tubes and 1.80 ± 0.14 for terrestrial ones (Figure 2-9).

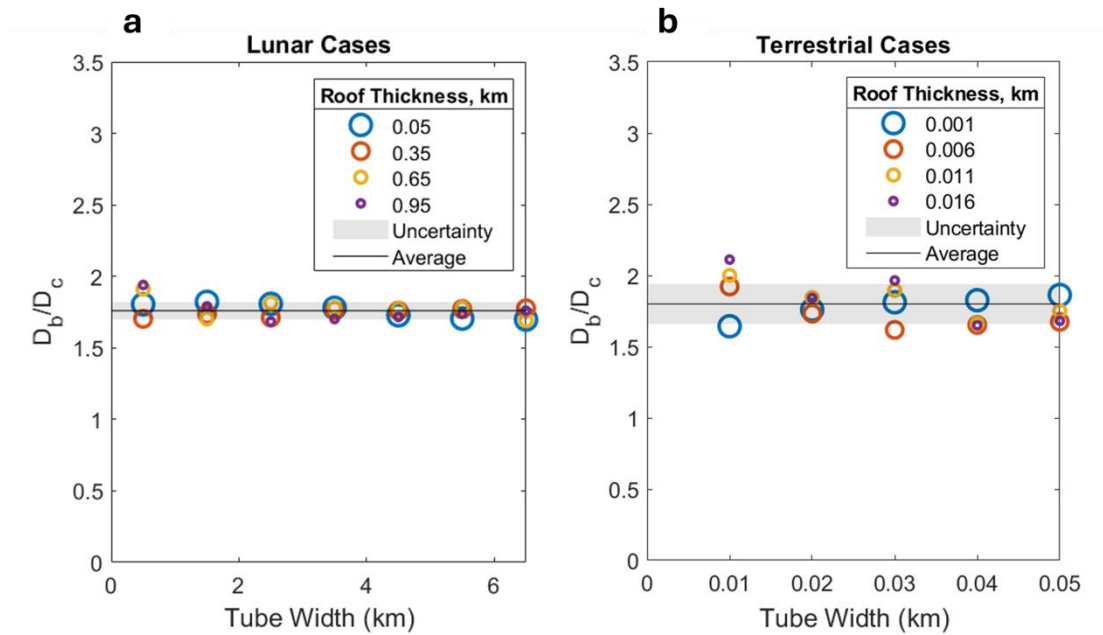


Figure 2-9: Ratio $\frac{D_b}{D_c}$ for modeled lunar (a) and terrestrial (b) tubes vs. tube widths. Symbol color and size indicate the roof thicknesses.

The major difficulty with any of these approaches is recognizing the lunar surface bulge. Its amplitude of a few meters at most would be very difficult to detect, considering the other unrelated lunar surface features present and the roughness of the volcanic constructs.

2.5.3 Effect of tube shape

Some lava tubes on Earth are observed to have curving floors, and lunar tubes may as well. We modeled lunar tubes with curving floors to find if these different geometries significantly affected floor failure. Note, however, that deeply curving floors are not the norm for tubes on Earth, and we do not have reason to assume that they are much more common on the Moon; as such, the flat-floored models are expected to be the best match for most lunar tubes.

The shape of the tube floor affects the stress field and the development of cracks. This effect is minimal for lunar surface cracks and deformation but is important on the tube floor. Figure 2-10 shows that the flat floor of the reference models leads to the highest tensile stresses. Increasing floor curvature decreases the tensile stress to the point that no failure is expected when the floor is nearly circular even though, as tube width is constant in these models, the void mass deficit, which drives floor deformation, is larger for circular tubes. This indicates that the reduction in floor tension is linked to the reduced flexibility of the circular floor, which distributes stress along its entire surface. The shape of the tube roof also affects the stress on the tube's floor, but its effect is secondary compared to that of floor shape. A flatter roof is again more flexible and leads to higher amplitude tension on the tube floor.

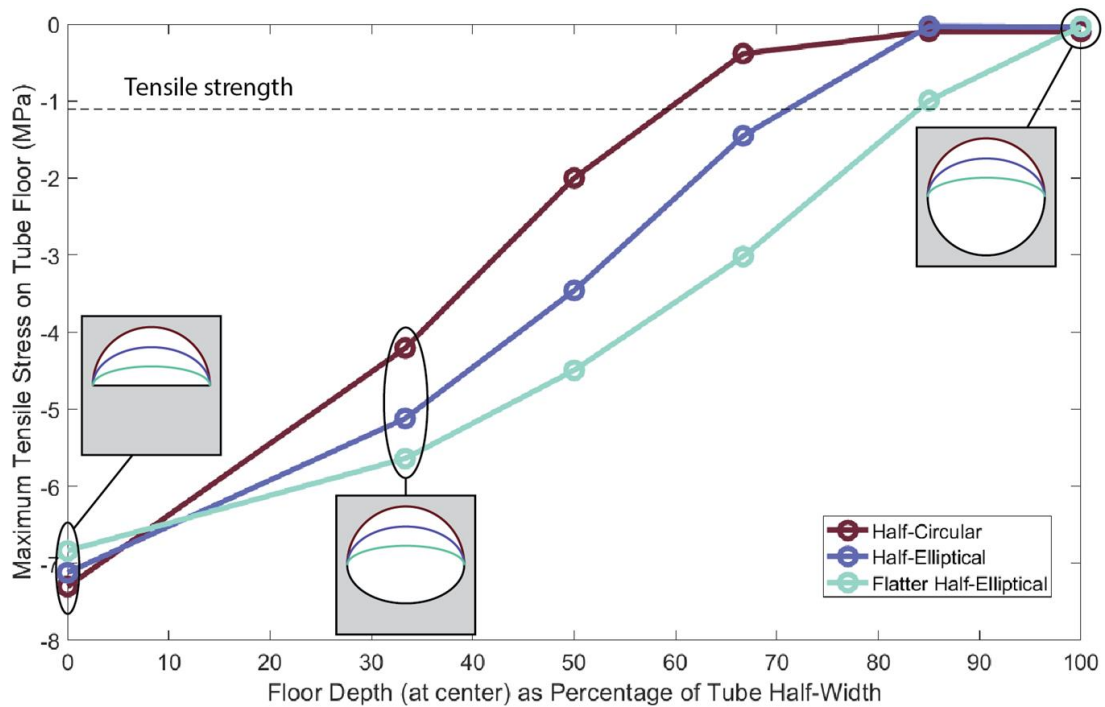


Figure 2-10: Tube shape effects on maximum floor tensile stress. The x-axis shows floor curvature depth. Colors designate different aspect ratios for the top half of the tube (full-width to top-half-height): Half-Circular (6:3), Half-Elliptical (6:2; used for the original models), and Flatter Half-Elliptical (6:1). Diagrams show color-coded visualizations of some tube shapes used. In all cases, the tube width was 5.5 km, and the roof thickness was 0.95 km.

2.6 Examination of lunar sinuous rilles

The model results presented above imply that deflection of the lunar surface around a large lava tube can generate sets of cracks that run parallel to a tube segment on both sides of its axis. The position of these cracks can help constrain the dimensions of the tube. We searched for evidence of such cracks in LROC imagery. To keep this search manageable, we focus on known rilles (Hurwitz et al., 2013), as they may have a genetic link with lava tubes (Greeley, 1971). Cracks may be visible in images of the surface as faint lineaments without obvious topographic

offsets. A chain of pits or depressions may also indicate a crack that formed below a layer of loose sediments or regolith (Tanaka & Golombek, 1989; Whitten & Martin, 2019; Wyrick et al., 2004). A complete list of observations is included in Supporting Information Table 7.1.S1, with illustrations in Supporting Information Figures 7.1.S5 to 7.1.S12 (Chapter 7: Appendix).

Lunar sinuous rilles generally show no clear evidence of cracks running parallel to rilles or candidate tube sections. Occasionally, lineaments are visible close to a rille but are not consistently parallel or symmetric to it. Even if they correspond to shallow cracks, these cracks are probably unrelated to the rille. Sets of parallel lineaments are sometimes present inside the troughs of rilles, close to the upper edges. These are unlikely to represent tube deformation, as they are on the rille walls instead of the surrounding flat lunar surface. They may indicate material collapsing and sliding down the rille slopes or provide evidence for lava flow stratification on the rille walls (e.g., Robinson et al., 2012). Nevertheless, three locations provide possible evidence for the kinds of lunar surface cracks predicted in our models at Rima Mairan (Figure 2-1), Rima Sharp (Figure 2-11), and Rima Marius (Figure 2-12).

Our findings on each of these rilles are summarized in Table 2-2. We use the observed cracks with Equation 2.7 to deduce the expected widths of the tubes. Without skylights, we cannot measure roof thicknesses, so we assume an approximate roof thickness of 80 m, as at the MHH; thicker roofs would imply smaller tube widths. We then compare these with our modeled tubes of similar widths to predict how much pristine floor each tube would have.

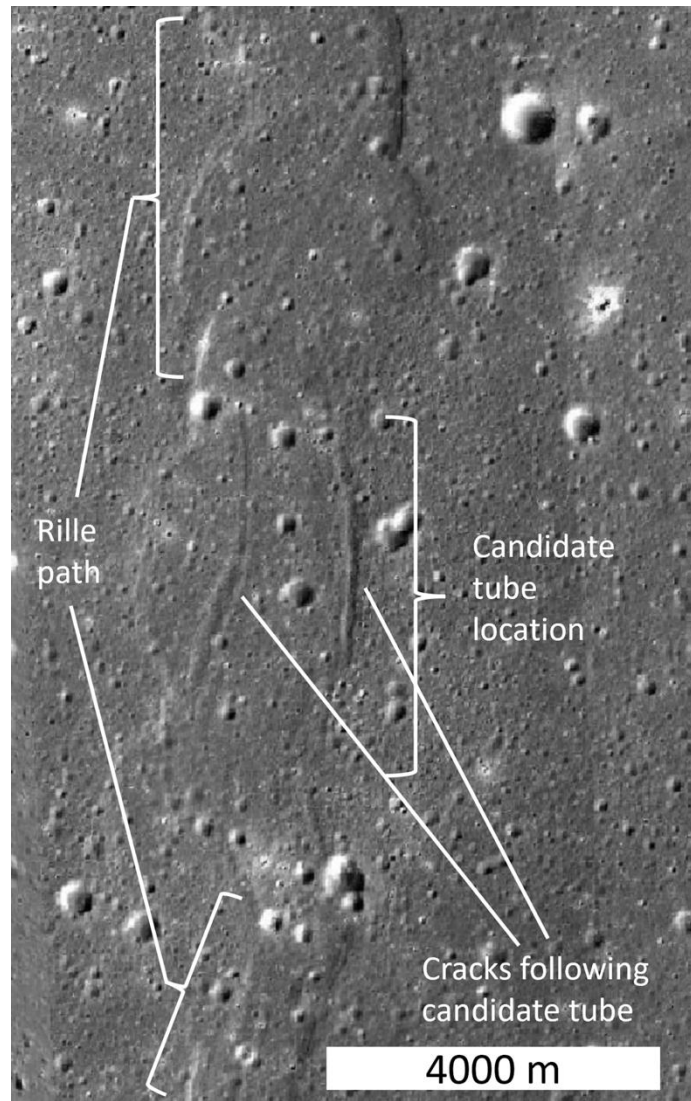


Figure 2-11: Candidate tube segment on the path of Rima Sharp. Parallel linear depressions are present in the rille discontinuity, continuing the path of the rille; these features may be evidence of cracks, suggesting the presence of a lava tube. Image source: LROC NAC imagery, obtained through LROC Quickmap. Center of image: 41.75°N, 50°W.

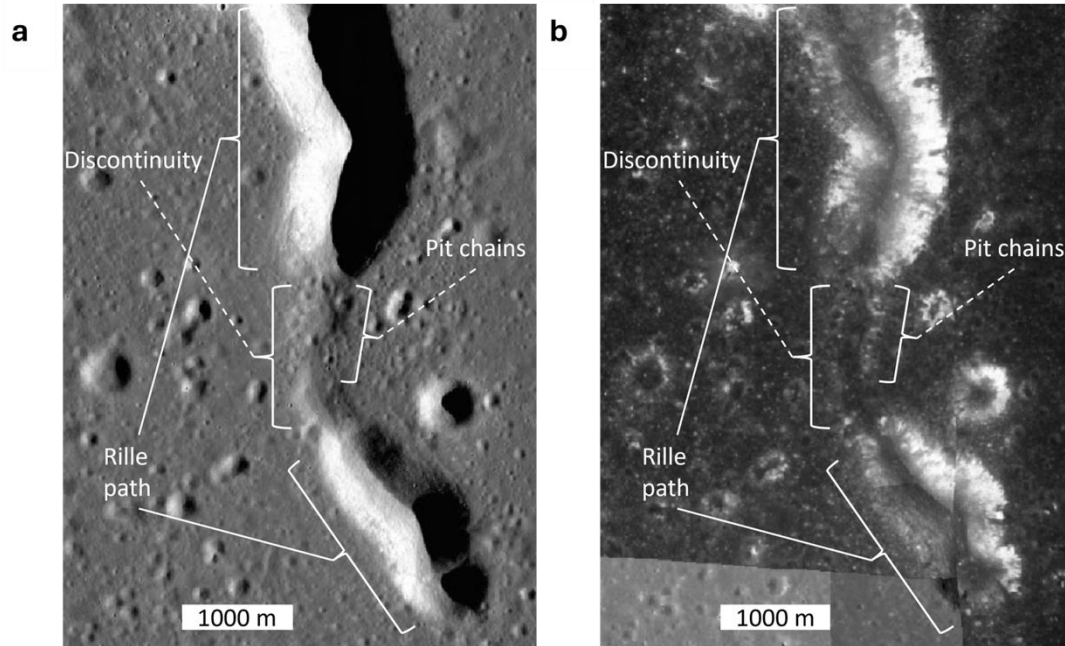


Figure 2-12: Discontinuity in Rima Marius with pit chains running parallel to it on either side.

Two images of the same region with different lighting conditions are shown to make the small depression features as clear as possible: a) LROC Narrow Angle Camera large solar incidence angle (55-80°) East illumination imagery, and b) LROC Narrow Angle Camera small solar incidence angle (0-40°) imagery. Image source: LROC NAC imagery obtained through LROC Quickmap. Center of image: 14.6°N, 48.6° W

Table 2-2: Crack observations above three possible lunar lava tubes, and the characteristics of the tubes they imply. These calculations assume an estimated roof thickness of 80 meters, chosen based on the example of the Marius Hills Hole. Each candidate tube continues the path of a sinuous rille.

Associated Rille Name	Observation	Separation between Parallel Cracks (km)	Distance from Tube Center to Cracks (km) (D_c)	Calculated Tube Width (km)	Expected Pristine Floor Extent (km)
Rima Mairan	Aligned wide depressions and lineaments resembling cracks	1.3	0.65	1.5 ± 0.17	0.10
Rima Sharp (Widest Crack Separation)	Lineaments resembling cracks	1.7	0.85	2.0 ± 0.18	0.13
Rima Sharp (Narrowest Crack Separation)	Lineaments resembling cracks	1.3	0.65	1.5 ± 0.17	0.10
Rima Marius	Linear pit chains	0.28	0.14	0.32 ± 0.14	0.32

Another concealed lava tube that may have produced lunar surface cracks is found along Rima Sharp (Figure 2-11). The rille appears discontinuous at a location where its path becomes very faint or absent on the surface. Cracks in this region may indicate that an underground lava tube links the two rille segments. As in the case of Rima Mairan, narrow linear depressions that may be evidence for major cracks occur on either side of where a candidate tube would be. The crack separation distance varies, giving an upper and lower bound for tube width (Table 2-2). The percentage of the floor that is pristine does not change significantly between these widths (Figure 2-5a), so in this case the wider end of the tube would have a proportionally wider usable floor area. This usable floor would reach the edges of the tube, as even the wider tube is not large enough to have compressive wall failure. Rima Sharp also has several points along its path where less clear examples of linear cracking following the rille are present; these may indicate further candidate tube sections (Supporting Information Table 7.1.S1).

The third possible example of the predicted surface features is found along the path of Rima Marius. This rille has a clear discontinuity near its southern end, where the rille disappears over a distance of approximately 1.5 km. This discontinuity may be caused by the rille transitioning into a tube at this location. A ~350-meter-wide shallow depression follows the trace of the proposed tube (visible in Figure 2-12a). No clear continuous surface cracks are present at this location. However, parallel pit chains are visible on either side of the proposed tube (Figure 2-12), especially at small sun incidence angles (Figure 2-12b). Wyrick et al. (2004) discussed multiple possible formation mechanisms for similar pit chains on planetary bodies and concluded that subsurface extensional faults or fissures likely cause these features. An underground cavity created by a fault or fissure can allow material above to drain into it, creating chains of shallow depressions similar to those observed beside the Rima Marius discontinuity (Tanaka &

Golombek, 1989; Wyrick et al., 2004). Thus, the predicted cracking associated with a tube may be expressed as aligned pits and depressions. The separation between the pit chains implies a smaller tube here than we modeled (Table 2-2), with no expected roof cracking and likely no floor cracking due to its narrow width (Figure 2-5). This tube would be relatively undamaged overall.

The candidate tube at Rima Marius is significantly narrower than the width of the associated rille. The difference in widths could be explained if the rille was once a long continuous lava tube that since collapsed, leaving only a small section intact. Collapse into a deep, narrow lava tube could result in a surface depression wider and shallower than the tube itself due to the slope of the collapse walls. Alternatively, depending on the mechanism of collapse and how much material can move into the void, Rimae Mairan and Sharp could be evidence of large unstable lava tubes that collapsed and left depressions approximately the width of the tube.

The rilles may alternatively have formed only where the flow became too wide for a tube to form and be stable. It may be expected that the tube is related to the width of the rille floor, although it may have been covered by subsequent lava flows. In that sense, it is interesting to note that Rima Marius, where the tube is very small, displays a V-shape without a flat floor, whereas Rima Sharp has a flat floor and can reach a width of up to 1.5 km, similar to the width of the inferred lava tube. Rima Mairan also has a wide flat floor near its Southern end, of a comparable width to the lava tube that we predict there (Figure 2-1).

Of the three rilles that have candidate tube sections, Rima Marius is the only one that is accompanied by lines of depressions instead of cracks. The candidate tube in Rima Marius is also the smallest by a large margin. The size of a tube may dictate the size and depth of the

cracks it causes on the surface, with smaller tubes creating weaker stress fields and shallower and narrower cracks. Therefore, the tubes in Rima Mairan and Sharp would create large cracks that could not be covered by regolith, whereas the smaller Rima Marius tube would cause small cracks that could be covered over. These covered cracks could cause chains of depressions as the covering layer of regolith partially drains into them. The majority of lunar lava tubes may similarly be small, not producing large visible surface cracks and instead creating chains of regolith depressions that are difficult to observe.

Images of the region around the MHH showed no clear evidence of tube-related cracking. Other datasets we studied, including surface roughness, slope, and elevation, also showed no signs of the predicted cracking. The MHH is approximately 25.5 m in radius, and while angled imagery of the hole reveals a void space, it cannot determine the full width of this space (Robinson et al., 2012). The tube may be too narrow to cause cracking on the surface, or the cracks may have been covered or eroded.

The absence of evident tube-related cracks could imply that most tubes are too small to produce cracks or that the cracks have been hidden by subsequent surface processes. The surface of the Moon continues to evolve to this day through a combination of impact gardening and seismic shaking (Senthil Kumar et al., 2016; Costello et al., 2018; Bickel et al., 2021). These may cover thin cracks or craters. We note that the cracks would form immediately after the tube develops and may be buried by subsequent lava flows. Regolith could cover the cracks, but the example of Rima Marius shows that regolith draining into subjacent cracks could also highlight faint structures. In-situ exploration is necessary to ascertain the absence or presence of tube-related cracks.

2.7 Implications for tube exploration and utilization

Plans for future lunar exploration and habitation should consider the patterns of failure exhibited by the models, even if the tubes are overall stable. In large tubes, a large portion of the tube floor is expected to be covered by cracks or debris from internal failure, leaving only a small area of pristine floor, at most 500 m wide and often smaller, near the tube edge (Figure 2-6a). Of course, these features could be covered by subsequent lava flows. If we ignore this possibility, realizing that the floors of lunar lava tubes may be significantly more complex than those of terrestrial tubes could affect the general planning of what kind of structures could be built in a tube.

Some schematics of possible bases inside lunar tubes include large inflatable structures covering most of the tube floor (Daga et al., 1992). As much of the floor may be covered in large cracks and debris fields, which could have sharp edges, care should be taken to develop inflatable structures of strong and flexible materials. Habitation platforms need to function on potentially rough terrain.

The location of the most pristine floor near the tube edges could also inform exploration plans. Future tube exploration missions would likely enter through skylights, which form most easily over the centers of the tubes. Therefore, these missions would arrive on the tube floor in the center of a heavily damaged region. High-resolution skylight images show debris on the underlying cavity's floor (Robinson et al., 2014). Thus, lunar exploration missions need to develop plans for traversing crevasses and debris fields. A potential solution could be to utilize a vehicle capable of crossing uneven and cracked surfaces, possibly by using treads instead of wheels that could get stuck in wide cracks. Another solution could be to cover or repair the

cracked floor of the traverse path to the point that it can be traversed safely. Clearing the debris fields may also be desirable. Either solution requires complex logistics and careful planning.

The knowledge that pristine floor regions are near tube edges and not directly below skylights could also have implications for planned lunar bases. The regions of pristine floor probably offer the best conditions to build a lunar base. Their locations at the edges of the tubes also provide superior sheltering from radiation. However, these locations do not allow direct overhead access to the surface. Long-term missions to the lunar surface may rely on solar power, as many long-term experiments and sensors operating on the lunar surface for years do. Collecting solar power would be difficult from a base near the edge of a lava tube, as the entire base would be in the shadow of the roof. A possible solution to this obstacle would be to set up solar panel arrays on the surface above the tube, with cables transferring power to the base inside the tube. For shorter times spent in tubes, such as for missions to explore and study tube interiors without setting up long-term habitats, battery power may be sufficient, and the lack of solar power may not be an issue; the Apollo missions had operational times of up to several days on the lunar surface and survived on their batteries without requiring solar power. Low-light conditions of the pristine regions of large tubes would also require artificial lighting to be supplied; this would apply to both long-term habitations and exploration missions.

When planning how to lower large pieces of equipment, one must also consider the lack of direct overhead access to a skylight from pristine floor regions. If a base were being constructed directly below a skylight, large segments could be lowered into the tube for use in assembling the base; since the most readily usable floor for construction is near tube edges, such large segments would have to be transported across damaged or interrupted floor regions. This obstacle could be avoided by developing means to render small regions of floor pristine, traverse

disrupted floor, as described above, or by assembling a base from smaller pieces that could be more easily transported over difficult terrain.

Depending on the engineering solutions to these problems available and the desired size of a lunar base, the advantages and disadvantages of small versus large tubes should be considered. Tubes with widths at or below the lower extreme of the width range studied have benefits and drawbacks. The smallest tubes have less floor and wall failure than larger tubes or none at all, potentially making them preferable for exploration, construction, and habitation. For example, the entire ~350 m wide floor of the Rima Marius candidate tube may be useable, in contrast to two ~100 m wide bands of pristine floor at both edges of the larger Rima Mairan and Rima Sharp tube candidates. However, even if such a tube has experienced no failure, its relatively small size may limit its adequacy for long-term habitability. Any settlement would necessarily be long and narrow, extending as deep as possible into the tube with access to only one of its ends. The larger tubes modeled provide long and narrow habitats akin to an alcove (De Hon, 2022). Access would be possible anywhere along its length but only after crossing a crevasse and debris fields. If solutions have been found to the issues of traversing or clearing crevasse and debris fields and establishing a base without direct overhead skylight access and a large base is desired, a wide tube may be preferable. If the damaged floor and the lack of overhead access present significant problems for the mission design and only a relatively small base is needed, a narrower tube could be used, with smaller amounts of pristine floor but easier access to it.

The effects of tube shape on the floor's cracking described in Section 3 indicate that the maximum tensile stress on a tube's floor depends on multiple aspects of the tube's geometry. This stress generally decreases with increasing depth of floor curvature, decreasing width-to-

height ratio, and decreasing width and depth. Choosing a tube with a concave floor could help ensure failure is not present, but working on a sloping floor presents its own difficulties; these factors should be considered when selecting a target tube. Choosing tubes with higher roofs compared to their widths could also help avoid encountering floor failure and may be a valuable consideration to prioritize; this aspect of tube shape might be determined from the surface if tube height could be determined from images of a skylight, and tube width could be determined from a surface feature relationship.

2.8 *Conclusions*

Elastic deformation models show that lunar lava tubes are expected to develop extensive zones of failure on their internal surfaces. This failure falls into three categories: tensile floor failure, tensile roof failure, and compressive wall failure. Such extensive failure leaves the floor free of cracks or debris fields only over relatively small regions near the tube walls, with a maximum extent of 500 m. This geometry could be an important consideration for planning future missions to explore and utilize lunar tubes. Tensile floor failure, the most extensive type, would be reduced in tubes with smaller width-to-height ratios (more circular) and more concave floors. Elastic deformation around tubes also causes distinctive patterns of surface features, specifically a line of tensile cracking and a topographic bulge running parallel to the tube at predictable distances from its axis that depend on tube width. These features, if observed, could be used to deduce tube width from the surface or locate a concealed tube. Examining potential lunar lava tube locations using available datasets and imagery types showed apparent linear cracking, of the type predicted by our models, on the southern end of Rima Mairan, around a discontinuity in Rima Sharp, and possibly around a discontinuity in Rima Marius. These features

may indicate lava tubes present at these locations. Using these features, we calculate the expected widths of these candidate tubes.

2.9 Acknowledgments

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2.10 Open research

Numerical models were developed using COMSOL Multiphysics® (<https://www.comsol.com/>). Input files and MATLAB® postprocessing routines are available at https://osf.io/v9eja/?view_only=b9e488daae4b42b2a5441684d0b181c8. The images of the lunar surface display LRO Camera (LROC) Narrow Angle Camera data (Robinson, 2011) accessed using LROC Quickmap (<https://quickmap.lroc.asu.edu/>).

Chapter 3 Development of Inflation Ridges over Lava Tubes

3.1 Abstract

Lava tubes carry magma over long distances in some flows. The pressure of lava can sometimes inflate active tubes, forming meter-high topographic ridges at the surface above them. We develop scaling relations to link inflation ridges to the internal tube pressure and the mechanical conditions of the flow during inflation, namely the roof thickness, which depends on the age of the tube's roof and its elastic modulus. These relationships imply that inflation ridges found in various lava flows grew within the first 400 hours of roof formation. In two cases, Young's modulus at the time of inflation was no more than a few GPa. This new method and scaling relations open the possibility of using inflation ridges as probes to the structure of the history of both active and inactive lava flows.

3.2 Introduction and Background

Lava tubes are volcanic features formed when a crust of solidified lava over a flow becomes strong enough to support its weight, and the underlying lava drains away. These features are present in many locations on Earth, including Hawaii (Greeley, 1987), Kenya (Forti et al., 2003), Iceland (Pederson et al., 2017), Korea (Ahn & Hwang, 2009), Italy (Calvari & Pinkerton, 1999) and the Canary Islands (Tomasi et al., 2022). Lava tubes also present significant interest in solar system exploration (Titus et al., 2021; Wynne et al., 2022).

Sometimes, the pressure inside a lava tube increases while it is active, forcing the roof to deform upwards. This process, known as inflation, can create an inflation ridge on the surface

above the tube that remains even after the tube is no longer active and has drained (Guest et al., 1984). Inflation ridges have been documented in locations such as the Canary Islands, Iceland, Argentina, Italy, and the Lava Beds National Monument (LBNM) California (Guest et al., 1984; James et al., 2012; Orr et al., 2015; Whelley et al., 2017; Bernardi et al., 2019; Voigt et al., 2021; Tomasi et al., 2022). Here, we contrast inflation ridges, which result from the pressurization of a lava tube or flow interior, to the pressure ridges that come from horizontal compression and buckling of the flow crust (Walker, 1991). Inflation ridges on the ocean floor have also been taken as evidence for lava tubes (Fornari, 1986; Deschamps et al., 2014).

Here, we study inflation ridges that formed in various lava flows to better understand the conditions under which they formed, linking them in particular ridge height to the flow's cooling history and the pressure inside the inflated flow. To do this, we use numerical modeling and scaling analyses of deformation around a pressurized lava tube. We constrain the combinations of internal pressure, effective roof thickness, and elastic modulus that can produce the observed surface ridge. Our principal case study is of Valentine Cave, a lava tube in the LBNM (Figure 3-1). The LBNM, on the flank of the Medicine Lake shield volcano, holds the highest concentration of lava tubes in the continental US (Larson & Larson, 1990; Esmaeili et al., 2020; Bell et al., 2022). LiDAR (Light Detection and Ranging) scans reveal a topographic ridge above the cave (Whelley et al., 2017). The dimensions of this ridge and the internal dimensions of the cave have both been measured in detail, making this an ideal study environment. We discuss other examples of linear inflation ridges to link their height with the possible cooling history of the underlying lava flow, which may contain a lava tube.

Initial analyses showed that the ridge must have formed generally during the early stages of lava tube development when the rock was still warm and relatively soft. The relationships we

present here can be applied widely and make it possible to constrain rough at what stage of lava flow emplacement an inflation ridge may have formed based on a simple measure of ridge height in the field.

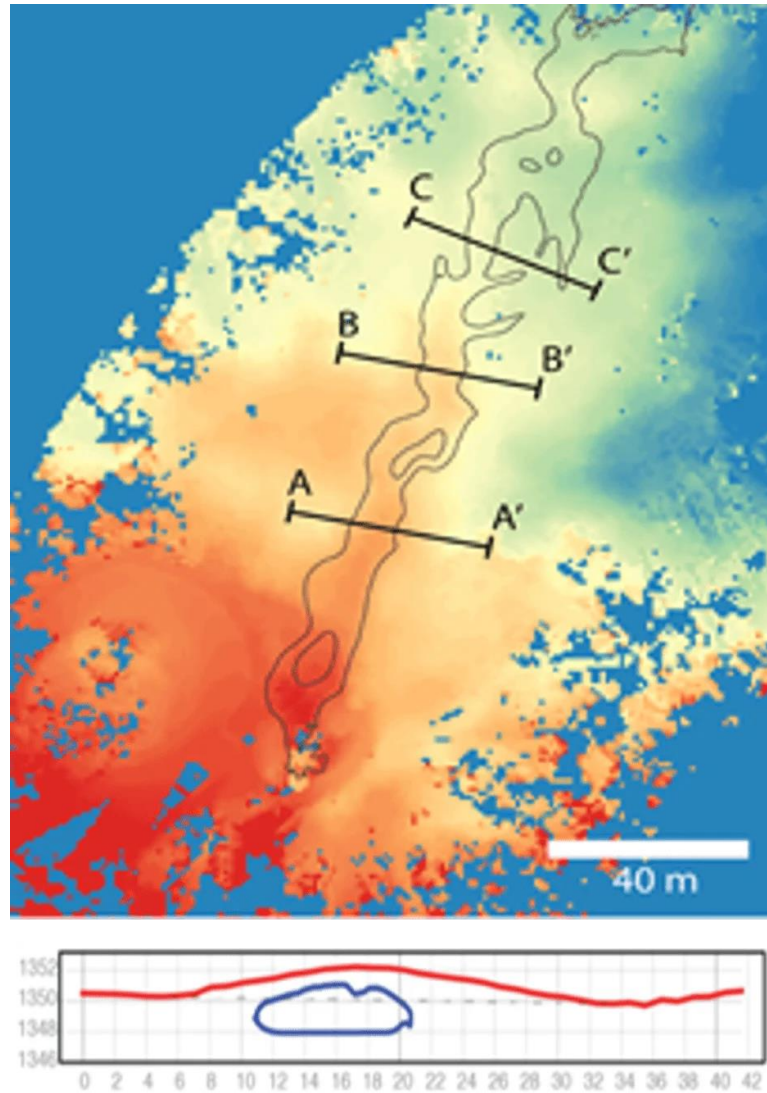


Figure 3-1: Surface elevation above Valentine cave (black outline in map view) and cross-section A-A'. At this location, the ridge height is 2 m, tube width is 11.8 m, roof thickness is 1 m, and tube height is 3 m. Portable LiDAR scanners were used to collect the surface topography data and the internal scans of the tube (Whelley et al., 2017).

3.3 Model Assumptions and Parameter Constraints

We created and analyzed two-dimensional elastic finite element models of lava tube inflation using COMSOL Multiphysics® under plane strain assumptions. Each model comprises a large block with a tube shape cut out of it, ensuring in each case that the width of the tube is always small compared to the width of the block to avoid edge effects. The models were allowed to deform under the influence of an internal pressure applied to the walls of the tube. The tube is idealized as a half-elliptical shape, which is an adequate representation of Valentine Cave (Figure 1) and has been used in previous studies (Blair et al., 2017; Modiriasari et al., 2019; Theinat et al., 2020; Williams & Montesi, 2025). Our model tube's dimensions match Valentine Cave: 11.8 m width, 3 m central height, and 1 m central roof thickness. We systematically vary magma overpressure, P , effective roof thickness, T_{ve} , and the Young's modulus of the material around the tube, E and measure the resulting ridge height (Figure 3-2).

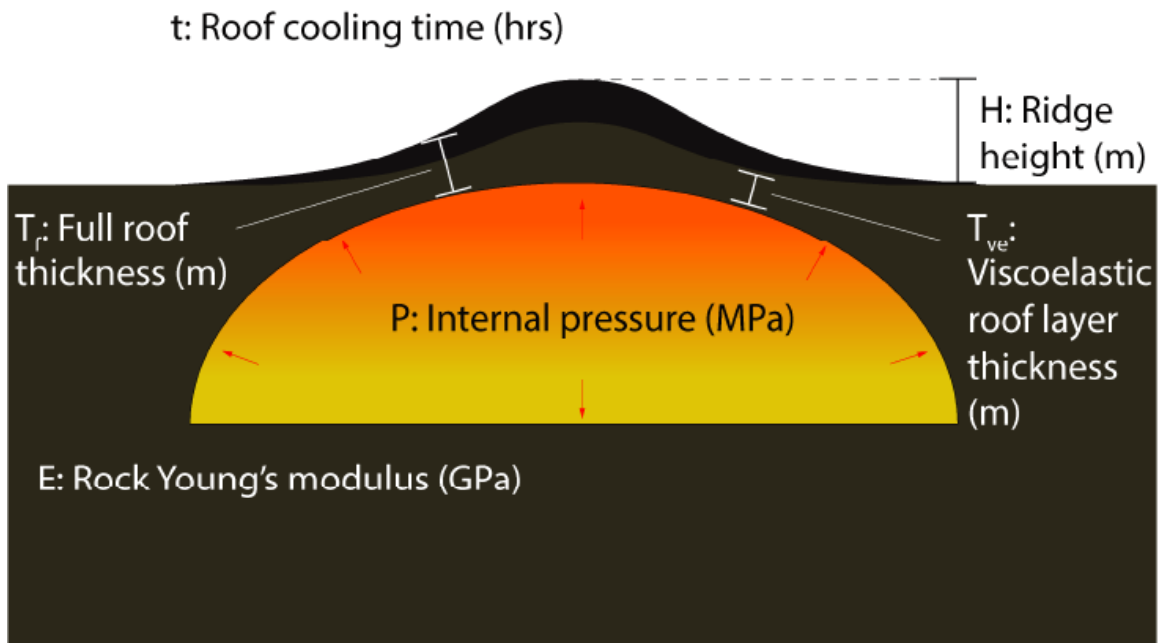


Figure 3-2: Schematic diagram of an idealized lava tube (yellow to orange) with internal pressure P leading to an inflation ridge of thickness H . The parameters involved in the model setup include the Young's modulus of the surrounding material, E and the viscoelastic roof thickness, T_{ve} which is a fraction of the full roof thickness, T_f that depends on the roof cooling time t .

Although the magma overpressure in a tube when it was active cannot be exactly measured today, it may be estimated from the pressure head ψ that arises from the difference of elevation between the source vent and the tube entrance:

$$P = \rho g \psi, \quad (1)$$

where ρ is the density of the lava, g is the acceleration of gravity, and ψ is the elevation offset between the tube entrance and the vent up-slope that the lava flowed from.

The flow in which Valentine Cave formed probably erupted from the Tickner Chimney vents (Donnelly-Nolan, 2010), giving a pressure head of $\psi \approx 270$ m from the difference in elevation. The density of basaltic andesite is $\rho = 2700 \frac{\text{kg}}{\text{m}^3}$, and $g = 9.8 \frac{\text{m}}{\text{s}^2}$, resulting in $P \approx 7.1$ MPa. We used the same technique to estimate pressure for all other examples of inflated tubes that we studied, and find values on the same order of magnitude. These pressure estimates far exceed the weight of the ridge, which is at most $\rho g H \approx 0.05$ MPa. Therefore, tube inflation is balanced by the mechanical resistance of the tube's roof, not its weight. Internal lava pressure is thus the dominant control of inflation ridge height, and the weight of the roof has a negligible effect.

Preliminary models show that ridge formation is infeasible with values of T_{ve} and E similar to modern-day values. The lava is too stiff to develop the observed 2-meter-high ridge. Therefore, inflation must have taken place during the relatively early stages of tube formation,

while the roof had not yet fully solidified and T_{ve} and E could both be significantly lower than observed today. To choose reasonable values for these parameters, we must consider the behavior of the tube as the roof is solidifying.

As the crust solidifies, its brittle outer layer cracks and loses its ability to resist bending. The viscoelastic lava layer beneath the brittle outer crust provides most of the resistance to bending, and its thickness can be regarded as the effective roof thickness (Rossi & Gudmundsson, 1996). As the lava cools and solidifies, the total roof thickness and the effective roof thickness increase. We build models with variable effective roof thicknesses by resizing the tube while preserving its shape and depth. This strategy represents the progressive cooling and inward solidification of the tube's walls and roof while keeping the depth of the tube constant. All the models are designed so that the tube width is 11.8 m when the roof thickness is 1 m, to represent the current conditions at Valentine Cave. We assume that the tube shape remains constant as material accumulates from the flowing lava onto the roof and walls. As the roof thickens from a thin sheet to its current value, solidified lava also accumulates on the walls of the tube, causing the tube to become smaller until it reaches its current shape. The brittle part of the roof, which conceptually does not resist ridge development, is not included in the model. Its main effect is as a load on the surface, which is small compared with the imposed overpressure.

Young's modulus is poorly constrained for the early stages of lava flow emplacement, when the rock of the roof was hot and relatively soft (Rossi & Gudmundsson, 1996; Smith et al., 2009; Lavalley & Kendrick, 2021; Heap et al., 2020). Rossi & Gudmundsson (1996) modeled tumuli inflation with $E = 5$ GPa for a fresh basaltic flow (Rossi & Gudmundsson, 1996), but even lower values may be appropriate when the lava was younger and hotter. At the temperature

of the viscoelastic crust, 800 and 1070 °C (Hon et al., 1994), Young's modulus is between ~20 and ~0.5 GPa (Lavallee & Kendrick, 2021). We adopt this range of possible values for our study.

3.4 Inflation Ridge Relationships

Overpressure lifts the tube's roof to form a ridge with height H (Figure 3-3). We recorded H for every combination of internal pressure, P , Young's modulus, E , and effective roof thickness, T_{ve} . See Table 7.2.1 (Appendix) for these model results.

Ridge height, inflation pressure, and Young's modulus were linearly proportional to each other. This relationship is expected, as the model only considers elastic deformation: strain, associated with the development of the ridge, is proportional to the stress (Figure 3-3b), which scales with the internal pressure. Ridge height for a given pressure and Young's modulus, though, depends on the tube's shape and depth, and so on the roof thickness. We use the linear scaling described above to reduce the number of models run here. In particular, we can calculate the Young's modulus necessary to match the observed ridge height for a given roof thickness and internal pressure.

To isolate the nonlinear effect of effective roof thickness, T_{ve} , on ridge development, we define the inflation length scale:

$$L \equiv \frac{H E}{P} \quad (2)$$

Numerical models show that L depends non-linearly on T_{ve} (Figure 3-4) according to

$$L = 170 T_{ve}^{-1.4} \quad (3)$$

We found this relationship empirically by fitting our model results. Chapter 7 (Appendix)

Section 2 describes in more detail specifically how we determined the parameters in this equation.

Equation 3 matches the expectation that a thinner roof, being more flexible than a thicker one, produces a taller ridge at a given Young's moduli and internal pressure, i.e., is associated with a higher L . Similarly, a thinner roof can produce a ridge of a given height for smaller internal pressure than a more rigid, thick roof. Equation 3 can be used to predict the height of an inflation ridge given E , P , and T_{ve} . Follow-up work will discuss how this relation also depends on the shape of the tube.

The value of T_{ve} is related to the age of the tube roof. Hon et al. (1994) monitored and modeled the thermal evolution of lava flows on Kilauea Volcano. They report that crust thickness, T grows as

$$T = 0.0779\sqrt{t} \quad (4)$$

Where T is in meters and t is the time, in hours. They consider that the load-bearing viscoelastic part of the flow crust is between temperatures of 800° and 1070°C. Combining Equation (4) with their equation for the depth to the 800°C isotherm change over time, we obtain

$$T_{ve} = 0.0306\sqrt{t} - 0.0233 = 0.393 T - 0.0233 \quad (5)$$

This relationship makes it possible to link the inflation length scale L to the time since the lava flow's roof started to cool.

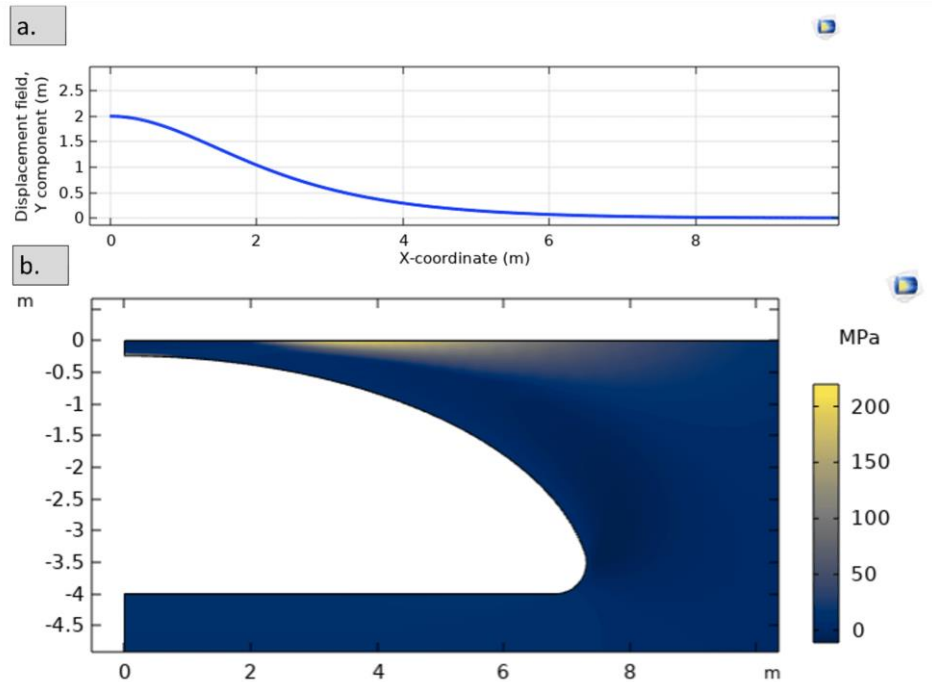


Figure 3-3: a.) Vertical surface displacement profile above a modeled tube, showing the inflation ridge produced by tube pressurization. b.) Compressive stress in the rock surrounding a pressurized tube. Young's modulus: 5 GPa; internal pressure: 7.1 MPa; effective roof thickness: 0.239 m, resulting in a ridge height of 2 m. The model is symmetric about $x = 0$. Colormap used from Crameri (2018).

3.5 Specific Inflation Ridge Results

Through modeling, we have found a new relationship between an inflated tube's surface expression and its inflation history (Equation 3). See Table 7.2.1 (Chapter 7: Appendix) for the raw model results and how they were used to construct this relationship. We now apply this relationship to three real-world examples of inflation. Our results are summarized below and captured in Figure 3-4.

Today, the roof of Valentine Cave is one meter thick (Whelley et al., 2017). Equation 4 implies that the flow cooled over 165 hours. As this is the maximum estimate of the total roof thickness when the flow was active, Equations 3 and 5 imply the ridge formed with $T_{ve} < 37$ cm and $L > 680$. The ridge height requires a minimum Young's modulus of 2.4 GPa (Figure 3-4). The lowest value of Young's modulus considered by Lavalley & Kendrick (2021), 0.5 GPa, would produce a 10-meter-high ridge unless the viscoelastic roof was an impossible 1.3 meters thick. Their highest Young's modulus estimate of 20 GPa forms a 2-meter-high ridge if the viscoelastic roof is 8.2 cm thick, that is, after ~12 hours of cooling. Rossi & Gudmundsson (1996) used $E = 5$ GPa, which implies a viscoelastic roof thickness of 22 cm, obtained after ~64 hours of cooling. Overall, the ridge dimensions are most compatible with inflation occurring two to three days after the initial formation of the crust.

Bernardi et al. (2019) described a 3-meter-high inflation ridge on the El Puesto lava flow. The current roof thickness is 1.5 m, and we estimated a pressure of 10.6 MPa. These yield a minimum Young's modulus of 1.3 GPa, effective roof thicknesses ranging from 0.08 to 0.57 m, and roof ages between 12 and 371 hours (Figure 4). A preferred estimate of $E = 5$ GPa gives $T_{ve} = 22$ cm and cooling for 63 hours. These results are essentially the same as for Valentine Cave, because $\frac{H}{P}$ is almost the same in both examples. This is probably coincidental, as the next example has a different $\frac{H}{P}$.

Voigt et al. (2021) documented a 0.9-meter high inflation ridge in the Holuhraun lava flow, Iceland. However, the flow feature did not drain and thus could not be explored, so the roof thickness remains unknown. The maximum cooling time, approximately 4,300 hours, does not provide a useful constraint on roof thickness during inflation. We estimated the pressure inside this flow, while active, to be 1.06 MPa. If E is between 0.5 and 20 GPa, T_{ve} is between 3.7 and

52 cm, and the total roof thickness ranges from 0.15 to 1.43 m, with cooling times ranging from 4 hours to 13 days. With a preferred Young's modulus of 5 GPa, the roof thickness is 32 cm, indicating that the ridge formed after ~16 hours of cooling, with a viscoelastic roof thickness of 9 cm.

Orr et al. (2015) described a sinuous tumulus on a Kīlauea lava flow. The documented ridge height was 5.5 m, and the full roof thickness was 2.9 m. However, the pressure head cannot be evaluated from the report. We infer an inflation pressure of 19 MPa if $E = 0.5$ GPa, and more likely 189 MPa if $E = 5$ GPa. It is unlikely that inflation occurred with a significantly smaller roof thickness, as the tumulus is ruptured along its length, exposing a cross-section of the roof during inflation, but, supposing that, as in other cases, $E = 5$ GPa and $T_{ve} = 30$ cm, the pressure would be 30 MPa. The large pressure values deduced here far exceed our estimates from pressure head in other examples, but they could still be realistic. Fracturing of the sinuous tumulus and lava breakout may be the consequence of high lava pressure. Alternatively, the mechanical cohesion of the roof may have been lost once it fractured, allowing its side panels to lift without deforming, which could exaggerate inflation ridge height. Nevertheless, a high pressure is likely necessary to fracture the roof, especially given its thickness.

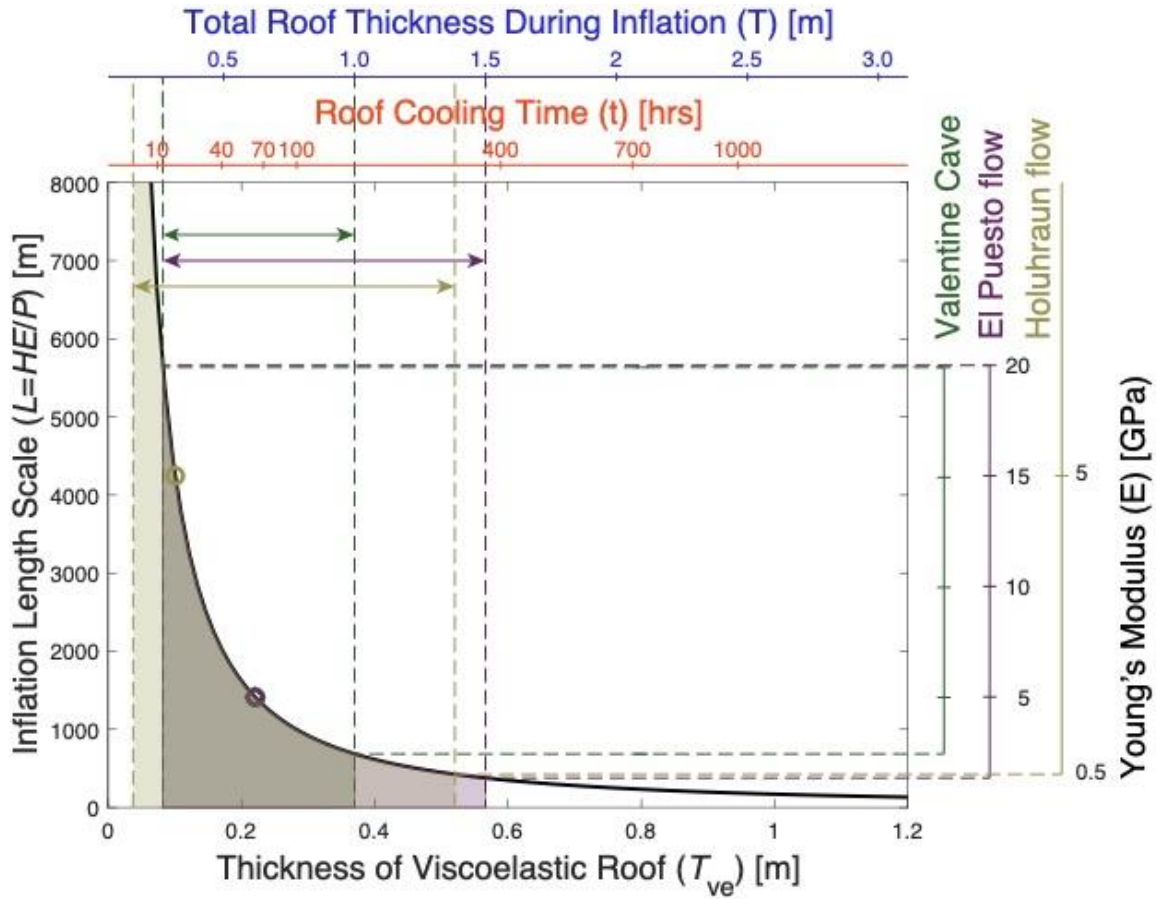


Figure 3-4: Relationship between length scale $L \equiv \frac{HE}{P}$ and effective roof thickness T_{ve} . The shaded regions and dashed lines show the possible ranges of conditions for Valentine Cave (green), El Puesto lava flow (purple), and Holuhraun (beige). The right-hand vertical axes show the Young's modulus values corresponding to the L values, given the pressures and ridge heights we assume in each case. The top horizontal axis shows cooling times associated with effective roof thicknesses. The preferred solution with $E = 5$ GPa is marked with a colored symbol on the black master curve. The black curve and shaded regions show the space of possible results we have found for each ridge; these are new constraints on these tubes' conditions during inflation.

3.6 *Discussion*

Using today's tube shapes, roof thicknesses, and the Young's modulus of rock at ambient temperature, the observed ridges cannot form under realistic inflation pressures. The Kilauea example shows that the lava crust likely fractures when the pressure is high enough to create a large inflation ridge. However, realistic inflation ridges can develop during the first few days of the flow history, when the roof had not yet cooled completely and was both thinner and more deformable than at present. Better constraining rheological parameters for this period would be useful to fully understand the formation of lava tubes.

If the flow crust was simply elastic when the ridge developed, as in our numerical model, the ridge height would decrease as the flow cools and its roof thickens. The existence of inflation ridges today, as in the case of Valentine Cave, shows that the ridge is associated with permanent deformation and is kept in place by subsequent solidification of the flow. Thus, the observed ridges represent a late stage of flow development when the viscoelastic roof becomes rigid. Earlier pressure fluctuations are not recorded by the inflation ridge.

In the cases of Valentine Cave, El Puesto flow, and Holuhraun, the inflation recorded by the observed ridge must have occurred within the first ~400 hours after the flow started to cool. Because the ridge is currently observed, we favor the upper end of this range. This suggests that the pressure was maintained for several days and that the effective roof thickness at the time of inflation was several tens of centimeters. Even if a lava tube remained active for a longer duration while transporting lava to the active parts of a flow field, it would create only smaller inflation ridges due to its increased stiffness. The only exception would be if very high flow pressure develops, in which case the tube could generate a sinuous tumulus like that seen at Kilauea, which would eventually break and be accompanied by lava breakouts. However, in the

case of high pressure, the more prominent inflation features would appear at the younger, more distal ends of the flow, where pressure is smaller. Therefore, the formation of a sinuous tumulus may require blockage of the tube network.

The simple model presented here can be applied to other locations where inflation ridges are suspected. They can provide insight into the history and mechanical properties of the flow. Direct observations of inflation ridge formation during an eruption may help constrain lava crust rheology. Here, we can only say that forming a meter-high ridge with a decimeter-high viscoelastic crust implies a Young's modulus of a few GPa at most, but targeted observations would provide stronger constraints, which should be compared with experimental studies of lava crust rheology before full solidification.

Inflation ridges are visible to the naked eye, but aerial or ground-based LiDAR scans are often necessary to discover and characterize them, as in the case of the ridge above Valentine Cave (Figure 3-1). This inflation ridge is quite broad, making it challenging to detect despite its height. Vegetation and subsequent flows further complicate the identification of inflation ridges in older flows. LiDAR scans covering extensive areas where lava tubes are suspected may reveal many more inflation ridges than are typically recognized. A dataset collected over an entire flow field would aid in uncovering the history of that flow and tracking its internal network of active lava conduits (See Chapter 5). Monitoring the growth and potential decrease of ridges in an active flow would contribute to mapping its pressure distribution, mechanical properties, and predicting its evolution and the likelihood of new breakouts.

Characterizing inflation ridges in an inactive flow field may reveal the timelines of flow emplacement. For example, if ridges reveal a thick roof at one location and progressively thinner

roofs on either side- upstream and downstream- this could indicate that the central region formed first and was cooler during inflation.

Peterson et al. (1994) describe a roof formation mechanism that could produce such a distinctive signature: at a constriction or bend in the channel, crustal rafts can become jammed and accumulate to create a short span of roof. The roof can subsequently grow upstream through the accretion of additional rafts and downstream through rooted crust growth (Peterson et al. 1994). Inflation ridges may indicate where the first crust jam formed and constrain the speed of roof formation upstream and downstream. The ridge above Valentine Cave becomes shallower along the tube path before disappearing, while the roof thickness of the tube increases along its course. This may suggest that roof formation began downstream and progressed upward, meaning that points upstream had a younger, thinner roof and could deform more during inflation.

If different parts of a tube system require significantly different pressures, we can deduce that they were inflated by separate events instead of simultaneously. Combined with estimated dates of the lava flows containing the tubes, these methods may yield insights into a volcano's evolution over time. Constraining inflation pressures in older and younger flows may provide an indication of how the effusion rate changed over time, which can have implications for understanding the source magmatic system (Dietterich et al., 2021). Even if we cannot find well-constrained specific values, relative trends between different parts of a flow system can still offer valuable insights.

3.7 Conclusions

Inflation can realistically cause the ridges observed above several lava tubes or in active lava flows. However, they can achieve the observed heights only soon after the tube roof begins

to form, when the tube's roof is relatively thin and soft. This conclusion is supported by quantitative analyses using the relationships between Young's modulus, effective roof thickness, internal lava pressure, and inflation ridge height presented in Equations 2 to 5. Assuming reasonable values for the rock's strength, the tube's inflation occurred within 400 hours after roof formation began, and the viscoelastic roof layer thickness was a few decimeters. Even if the tube remained active for an extended period, an increase in lava pressure later, when the roof was thicker and stronger, would not be unlikely to form an inflation ridge. The Young's modulus of the viscoelastic roof was probably just a few GPa. The scaling relations presented here can be applied to other inflation ridges to deduce. If applied to multiple inflation ridges on the same lava flow system, this technique could yield insights into the overall flow behavior and how it changed over time.

Chapter 4 Determining Lava Tube Morphologies from Observable Surface Ridges

4.1 Abstract

Lava tubes are long void spaces found inside some lava flows. These tubes are present in many locations on Earth, and evidence suggests they also exist on the Moon and Mars. On Earth, it is possible to constrain the tubes' internal shapes and dimensions, which usually requires exploring inside them; for planetary tubes, where only surface observations are available, this is not currently feasible. A method to determine the depths and dimensions of lava tubes through surface observations would be invaluable for planning expeditions to explore both terrestrial and planetary lava tubes. This advancement would also make it possible to characterize unexplored or unmapped lava tube systems on Earth. Furthermore, this can be applied to undrained tubes, which cannot be explored, to estimate their shapes and sizes while they were active, thus enhancing our understanding of the flow fields in which they were formed. Here, we present a way to achieve this using inflation ridges, which are topographic surface highs that form over lava tubes while they are being inflated. We have established a relationship between the dimensions of the inflation ridge and those of the tube that created it. By utilizing this equation, surface features alone can provide new insights into tube morphologies. Testing this on a terrestrial example yielded results consistent with observations of the tube's actual shape obtained from within the tube.

4.2 Introduction

Lava tubes are long, sinuous caves formed within lava flows. A lava tube develops when a lava flow forms a crust of solidified rock on its surface, sealing the tube and insulating the lava within, and the internal lava drains out to produce a cave (See Chapter 1.1; Figure 4.1; Peterson & Swanson, 1974; Sakimoto & Zuber, 1998; Huff & Owen, 2013; Gregg, 2017). Inflation from the internal lava pressure in active lava tubes can push the roof up into a topographic ridge, which may remain as a permanent surface feature of the tube even after it has drained (See Chapter 1.1.1 pages 5-6; Walker, 1991; Hróarsson & Jónsson, 1991; Self et al., 1996; Hoblitt et al., 2012; Kempe, 2019; Sauro et al., 2019; Hamilton et al., 2020; Sauro et al., 2020).

Lava tubes are found in numerous locations on Earth, and multiple lines of evidence suggest that they also exist on other planetary bodies such as the Moon and Mars (See Chapter 1.2; Waters et al., 1990; Calvari & Pinkerton, 1998; Bleacher et al., 2006; Cushing et al., 2007; Haruyama et al., 2009; Lèveillé & Datta, 2010; Robinson et al., 2012; Hurwitz et al., 2013; Wagner & Robinson, 2014; Haruyama et al., 2016; Sood et al., 2016; Chappaz et al., 2017; Kaku et al., 2017; Gregg, 2017; Theinat et al., 2019; Sauro et al., 2020; Crown et al., 2022; Carrer et al., 2024; Zhu et al., 2024; Williams & Montesi, 2024; Sharma et al., 2025). Inside a planetary lava tube exists a valuable protected environment shielded from surface hazards, where astronauts and equipment would be safe and ancient geological features would be preserved (See Chapter 1.2; De Angelis et al., 2002; Haruyama et al., 2012; Boston et al., 2014; Schörghofer, 2022; Horvath et al., 2022).



Figure 4-1: View down Valentine Cave, a lava tube in the Lava Beds National Monument, CA. The image covers almost the full width of the cave's roof. Image credit: Edward Williams.

Understanding terrestrial tubes is integral to understanding the history of flow fields (See Chapter 1.1.1 pages 3-4). Tubes allow lava to flow much farther without cooling than it could on the surface (Greeley, 1987; Gregg & Fornari, 1998; Keszthelyi & Self, 1998; Zimbelman, 1998; Michael et al., 2024), and are thus essential to understanding the formation of long flow fields and predicting their hazards (Calvari & Pinkerton, 1999a; Chen et al., 2016; Calvari et al., 2024).

Studying tubes can also help understand the history of old, solidified flows where direct observations of the developing flow field are impossible (Calvari & Pinkerton, 1999b).

Fully assessing the role of lava tubes in various volcanological or exploration contexts requires knowledge of their shape and dimensions. Some lava tubes on Earth have been mapped (e.g., Waters et al., 1990) or have had LiDAR data collected within them (Cashman et al., 2013; Whelley, 2017; Esmaeili et al., 2020; Lemaire et al., 2024; Yang et al., 2024), but these methods require sending exploration teams inside the tubes. Finding a way to use a tube's surface features alone to deduce its internal geometry could help quickly learn about these caves without needing to explore them; this would be beneficial when studying inaccessible tubes or multiple tubes at once and could help discover new tubes. Determining the dimensions and depths of tubes on other bodies is particularly challenging, as we currently lack the means to explore them.

We present a technique that uses the dimensions of a lava tube's inflation ridge, observable through remote sensing, to infer the interior dimensions of the tube. We have discovered a new relationship connecting the height of an inflation ridge to the width, height, and burial depth of the tube that created it. This relationship could enhance our understanding of unexplored tubes wherever an inflation ridge is found, on Earth or beyond.

4.3 *Methodology*

Inflation ridges can develop as a result of a pressure increase inside an intact lava tube. The height of the ridge often indicates that it formed while the flow was still active (Chapter 3). During this time, the roof is viscoelastic but responds elastically to the short-duration loading associated with an increase in lava interior pressure. We previously demonstrated that the height of inflation ridges contains information about the thickness of the lava tube's roof at the time of inflation. However, our earlier models considered only one geometric shape, inspired by

Valentine Cave in the Lava Beds National Monument (Chapter 3). Here, we develop a method to link lava ridge height to the geometry of the underlying tube.

The height of an inflation ridge can be predicted analytically for idealized models of the tube's roof behavior, presented below. However, the roof in these models has a uniform thickness over the tube, and the tube's wall rocks do not deform. Thus, the analytical model is highly simplified and does not consider the shape of the underlying tube. Our objective, therefore, is to find an equation that relates a tube's internal dimensions to the roof thickness in the simplified model and, consequently, to the height of the inflation ridge that may develop above the tube.

After presenting an analytical elastic solution from which we can derive the apparent roof thickness based on the ridge dimension, we use finite element modeling to simulate a suite of more realistic tube shapes. We determine the apparent roof height from these models and derive an equation that links this quantity to the parameters controlling tube shape in the numerical models. This relationship transforms a complex, realistic tube shape into a uniform-thickness roof that yields the same ridge height and is therefore regarded as “effective” to the roof of the complex tube. By employing this method, we can estimate the inflation ridge height of a realistic tube without requiring numerical modeling: we first input the dimensions of the realistic tube shape into our equation to find the dimensions of the effective idealized tube, and then we utilize the relationship described below to derive an effective roof thickness and find the expected ridge height. By reversing this process, we can deduce constraints on the dimensions of a real tube using only the dimensions of an inflation ridge.

4.3.1 Idealized Models

Each idealized model consists of a perfectly flat elastic plate of perfectly uniform thickness resting over a void. The plate is pinned at both edges but otherwise allowed to deform. Pressure applied to the underside of the plate simulates internal lava pushing upward. This model best represents a perfectly rectangular lava tube. However, the tube's height has no effect on the results because the edges of the roof are pinned. Only the roof is involved in the analytical solution (Figure 4.2).

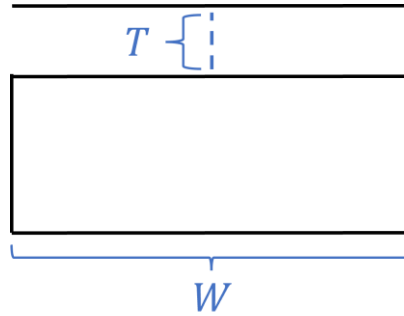


Figure 4-2: Diagram of an idealized tube model, which can be solved analytically. T is the tube roof thickness, W is the tube width.

The roof is represented by a thin elastic plate that flexes and bends in response to loads applied above and below it. The only load considered here is the pressure P exerted on the underside of the plate. The roof is pinned at each end, so that its vertical displacement w and its slope $\frac{dw}{dx}$ are 0 at $x = \pm \frac{W}{2}$, where W is the full width of the tube.

This problem can be solved analytically (Turcotte and Schubert, 2014, section 3.12). The maximum uplift, corresponding to the ridge height H , takes place at the middle of the roof, at $x = 0$. It verifies

$$\frac{PW^4(1 - \nu^2)}{32EH} = T^3 \quad (4.1)$$

where T , E , and ν are the thickness, Young's modulus, and Poisson ratio of the roof.

The idealized model is much simpler than an actual lava tube. Therefore, it is not an accurate representation of a real tube; however, it is analytically solvable and serves as a valuable reference solution. Its usefulness lies in being an intermediary step between a complex tube shape and an observed ridge.

Using this idealized reference model, we define the apparent roof thickness T_a as the roof thickness that would produce a ridge of height H over a width W according to the elastic plate model. Using Equation 4.1, T_a can be determined directly from the height and width of a ridge observed at the surface. Since only the roof of an idealized tube affects its behavior, and width can be approximated using the width of the inflation ridge, T_a is a key variable dictating the behavior of an idealized tube. For this reason, T_a will sometimes be used synonymously with “the dimensions of an idealized tube.” Next, we develop a way to relate the shape and dimensions of a realistic tube to the dimensions of the idealized tube (T_a) that produces the same surface ridge height H .

4.3.2 Complex Models

To model more complex and realistic tube shapes, we use COMSOL Multiphysics® finite element modeling software. We employ half-elliptical lava tube shapes for these models (Figure 4.3). This is still an idealization of a real tube's shape, but it is a reasonable and realistic one that prior researchers have used to simulate lava tube behavior (Blair et al., 2017; Modiriasari et al., 2018; Theinat et al., 2019; Theinat et al., 2020). Additionally, most observed inflated tubes exhibit horizontally elliptical cross-sections (Cooper & Kauahikaua, 1992; Sauro

et al., 2020). Based on our prior research, we do not expect the difference between an elliptical and half-elliptical tube to affect the results significantly.

Our models assume plane strain since lava tubes are very long compared to their widths. It is reasonable to analyze their behavior in cross-section while neglecting along-tube deformation. Each tube is placed in an elastic block that shares the same Young's modulus and Poisson ratio as the roof. The model domain is considerably larger than the tube to avoid edge effects. We apply pressure to all internal surfaces of the tube and compute the resulting heights of the inflation ridges.

Our complex tube models encompass a range of shapes and dimensions. We use three variables to fully characterize the shape of a half-elliptical tube: the full tube width W , the roof thickness at the center of the tube T , and the tube height at the center of the tube c (Figure 4.4). Any half-ellipse can be defined using these parameters. This enables us to systematically vary the tube shape and assess the impacts of different shape characteristics on the ridge height. We modeled an array of tube shapes with c ranging from 1 to 6 m, T from 0.1 to 2 m, and W from 5 to 20 m. We recorded the resulting ridge height H in each instance and the apparent roof thickness T_a associated with this ridge height and width, following Equation 4.1 (Figure 4.6).

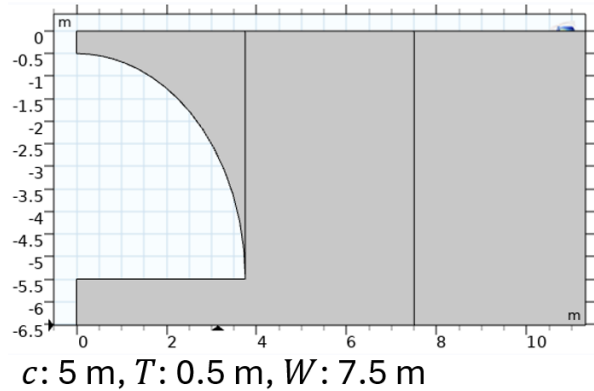
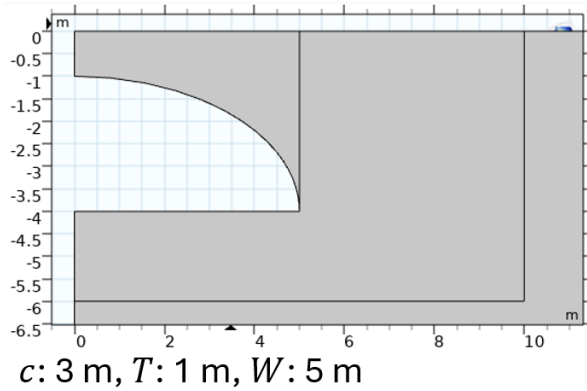


Figure 4-3: Two example tube models with different tube height c , roof thickness T , and tube width W .

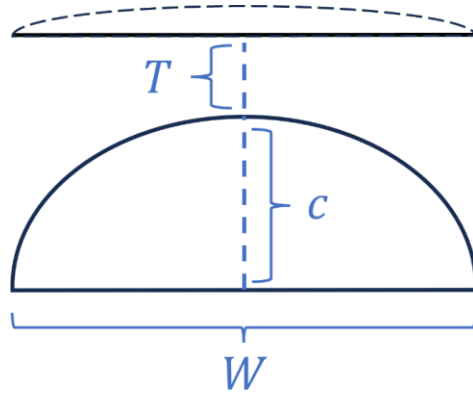


Figure 4-4: Schematic diagram of a complex tube shape, with the definition of the tube shape parameters width W , central roof thickness T , and central tube height c .

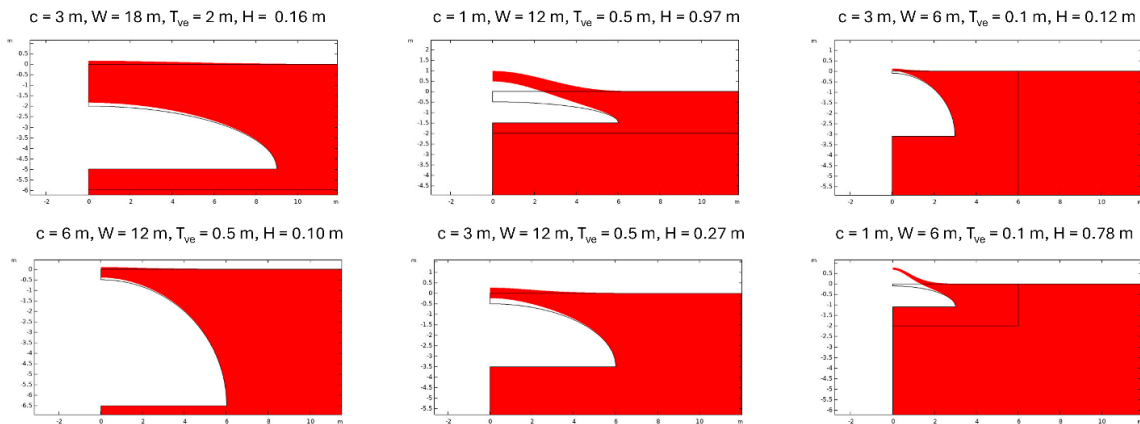
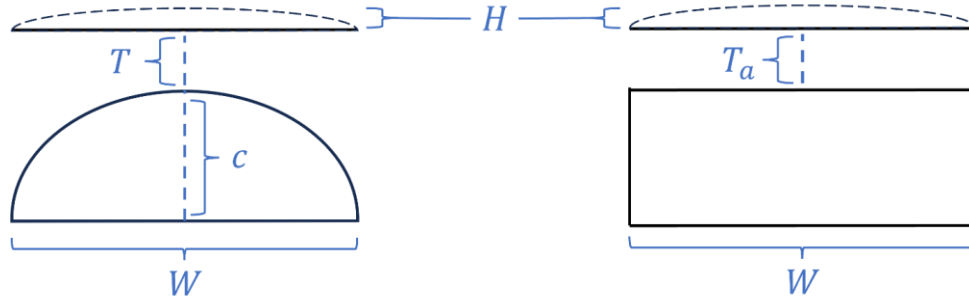


Figure 4-5: Example model outputs for a subset of the tube shapes used. The tube dimensions and resulting ridge height for each case are listed above the panel.



$$T_a = \text{function}(T, c, W)$$

Figure 4-6: Conceptual relation between the tube shape parameters used for the idealized and realistic models. The apparent roof thickness T_a is defined as the roof thickness of an idealized model that produces the same inflation ridge height as a realistic model with dimensions W , c , and T . A function relating T_a to the complex tube dimensions is needed to make a complex tube's inflation ridge height analytically solvable.

4.3.3 Results of Numerical Models

All the model results are shown in Figure 4.7, where line thickness corresponds to tube height and line colors represent tube widths. Figure 4.8 presents a subset of these results, changing only one parameter to illustrate its effect on ridge height better.

The ridge heights produced by numerical models with complex tube shapes follow expected general trends: wider tubes create taller deformation ridges than narrower ones (Figure 4.8b), as a plate with fixed edges can bend more easily in the center if it is wide than if it is not; tubes with smaller roof thicknesses produce taller ridges (Figure 4.8a), as thinner roofs are easier to bend. Additionally, a smaller tube height facilitates the generation of a taller ridge (Figure 4.8), assuming the other variables remain unchanged. This happens because a smaller tube has an overall thinner roof. Roof thickness is measured at the tube center. Therefore, keeping the roof

thickness at the center unchanged while increasing tube height means that most of the tube is situated deeper underground, resulting in an overall thicker roof that is more difficult to deform.

The most flexible roofs are found with small T or c but larger W . They can produce unrealistically high ridges of several tens of meters (Figure 4.7). Such ridges would likely rupture or produce lava breakouts at the surface (Walker, 2009).

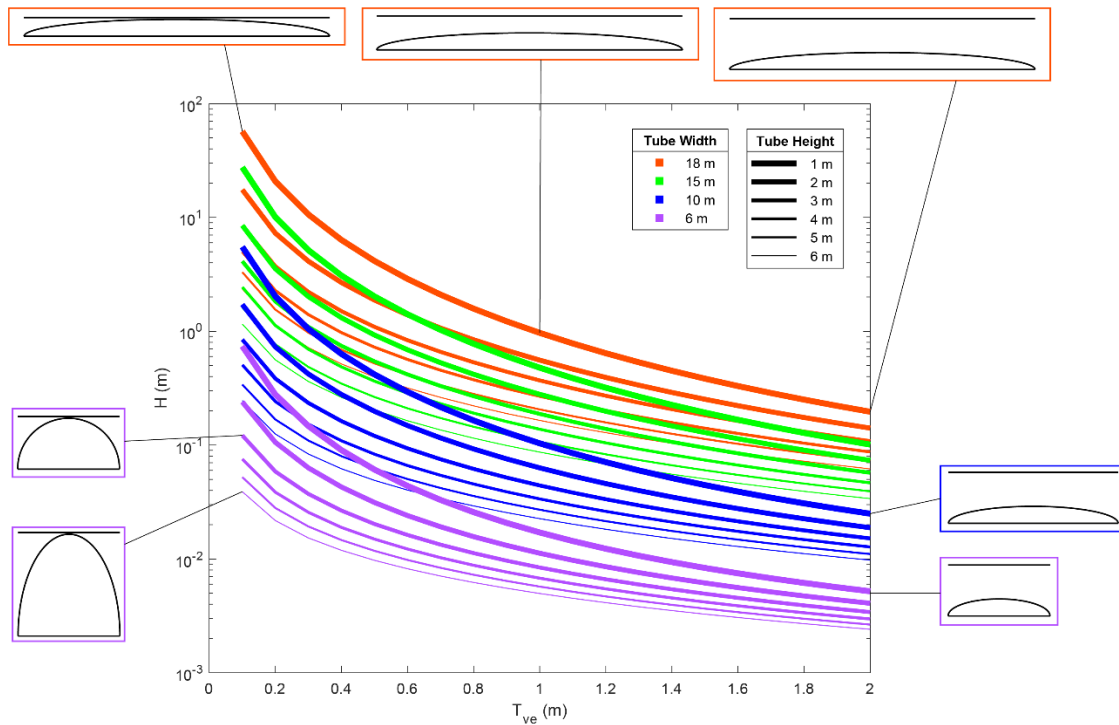


Figure 4-7: Inflation ridge height H from numerical tube models against tube roof thickness T , with other tube dimensions described in the legends. Schematics show scaled diagrams of a subset of tube shapes.

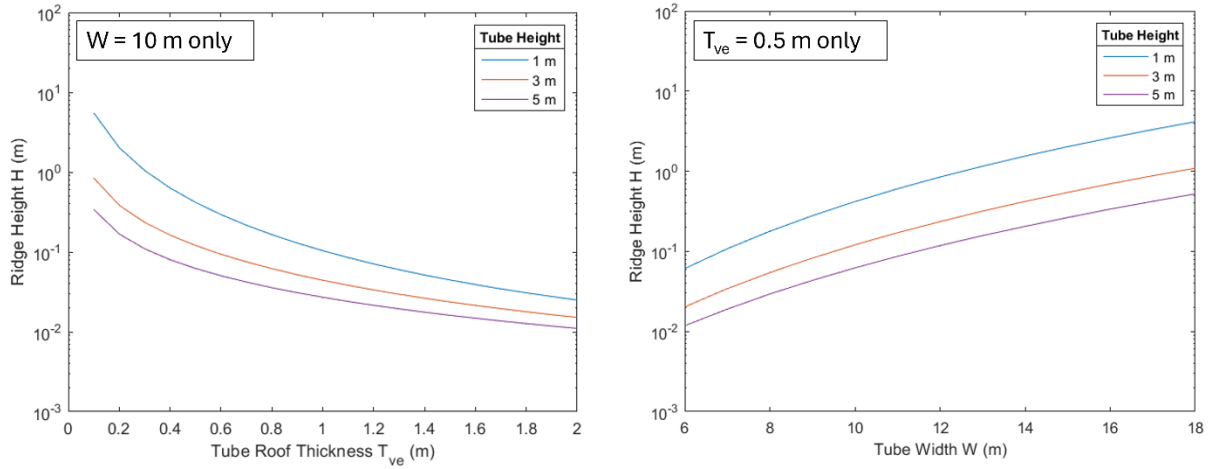


Figure 4-8: Inflation ridge height from a selection of numerical tube models against tube dimensions. a) Ridge height vs. roof thickness for a given width and three values of tube height. b) Ridge height vs. tube width for a given roof thickness and three values of tube height.

When describing tube dimensions and the resulting inflation ridge heights, it is useful to introduce ratios that combine multiple aspects of tube shape into a dimensionless value. Tube width serves as a convenient scaling length, as it is directly linked to the width of the ridge in the analytical model. Therefore, we report in Figure 4.9 ridge height as a function of the tube aspect ratio, R_{cW} , defined as the tube height c divided by width W , or the scaled roof thickness R_{TW} , which is the ratio of roof thickness T to width W , R_{TW} .

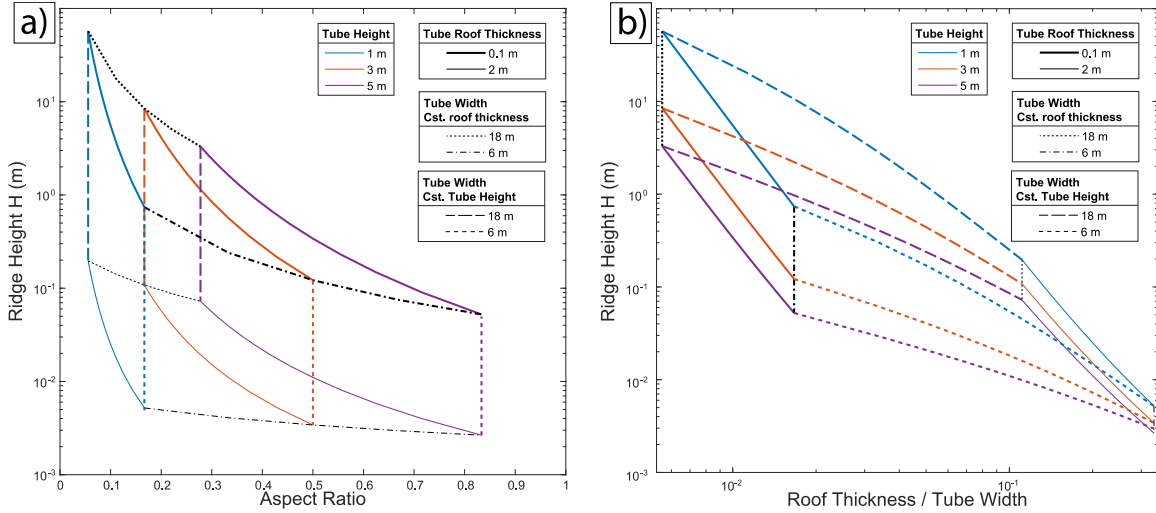


Figure 4-9: Ridge height H produced by various models plotted against a) aspect ratio R_{cW} or b.) scaled roof thickness R_{TW} . The colors of the solid lines correspond to specific values of the tube height c and line thickness to tube roof thickness T . Other line styles connect the results of models with the same tube width W and either the same roof thickness (dots and dash-dots) or tube height (long and short dashes, color-coded according to c).

Ridge height decreases with increasing tube aspect ratio R_{cW} if only one tube shape parameter (c , W , or T) is changed at a time (Figure 4.9a), as expected from the effects that tube width and height have on ridge height individually. However, there isn't a unique solution for the ridge height at a given tube aspect ratio: H can increase even as tube height, tube width, and tube aspect ratio all increase. For instance, compare the high R_{cW} end of the solid blue curve on Figure 4.9a, where $W = 6$ m and $c = 1$ m ($R_{cW} = 0.17$), with the low R_{cW} end of the solid purple curve on Figure 4.9, where $W = 18$ m and $c = 5$ m ($R_{cW} \approx 0.28$). Roof thickness has a strong effect on ridge height, decreasing it by up to two orders of magnitude at a given c and W (Figure 4.9a). This effect is expected because roof thickness controls flexibility of the tube roof.

The parameter R_{TW} can be understood as the relative depth of the tube, showing that deep tubes have a much smaller ridge than shallow ones.

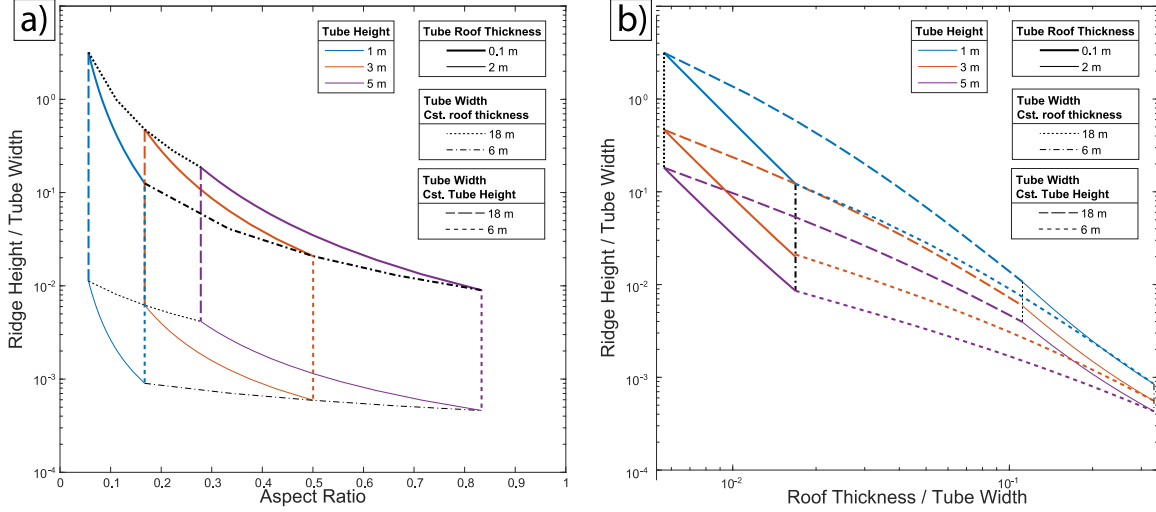


Figure 4-10: Ratio of inflation ridge height to tube width R_{HW} produced by various models plotted against a) aspect ratio R_{cW} or scaled roof thickness R_{TW} . The colors of the solid lines correspond to specific values of the tube height c and line thickness to tube roof thickness T . Other line styles connect the results of models with the same tube width W and either the same roof thickness (dots and dash-dots) or tube height (long and short dashes, color-coded according to c).

Ridge height scales with the internal pressure and Young's modulus following the concept of an inflation length scale $L = \frac{HE}{P}$ introduced in Chapter 3. We do not vary these parameters here, as their effects have been described in Chapter 3. Because of our current focus on tube geometry, we present in Figure 4.10 how the scaled ridge height R_{HW} depends on the non-dimensional ratios discussed above, the aspect ratio R_{cW} , and the scaled roof thickness R_{TW} . The trends are similar to those seen in Figure 4.9 but the range of R_{HW} is slightly smaller.

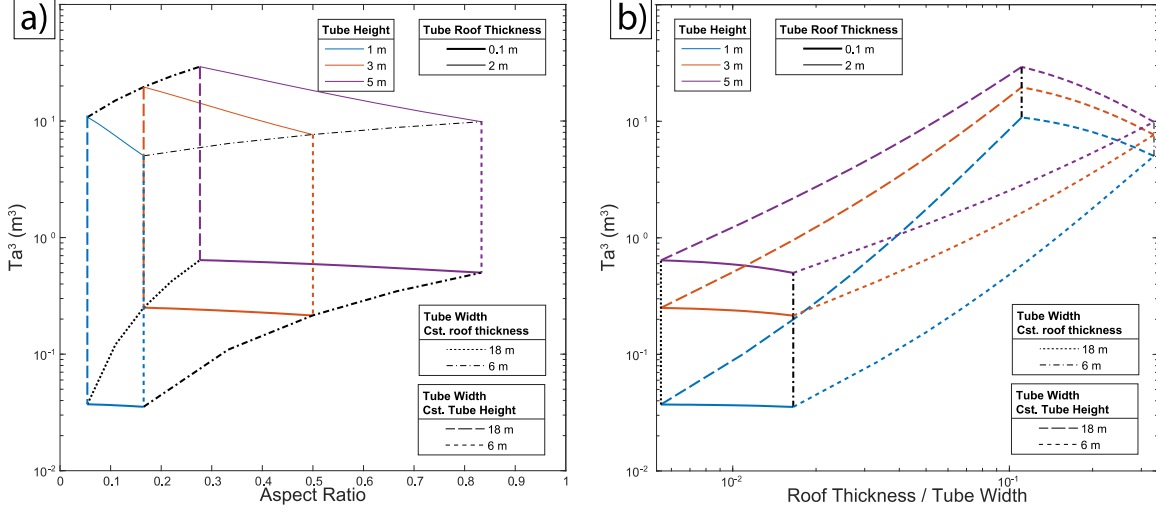


Figure 4-11: Apparent roof thickness cubed T_a^3 produced by various models plotted against a) aspect ratio R_{cW} or scaled roof thickness R_{TW} . The colors of the solid lines correspond to specific values of the tube height c and line thickness to tube roof thickness T . Other line styles connect the results of models with the same tube width W and either the same roof thickness (dots and dash-dots) or tube height (long and short dashes, color-coded according to c).

The analytical model of Section 4.3.1 links ridge height to an apparent thickness T_a . We treat the ridge produced in the models as if it was a ridge observed in nature and calculate T_a for each numerical model using Equation 4.1. Figure 4.11 shows how T_a^3 , which appears directly in the equation, varies with the ratios R_{cW} and R_{TW} against T_a^3 . As the principal length scale in these models is the tube width W , we also show in Figure 4.12 how $(T_a/W)^3$ depends on the ratios R_{cW} and R_{TW} . Equation 4.1 can be rearranged to give directly $(T_a/W)^3$ as a function of the scaled ridge height $R_{HW} = \frac{H}{W}$ and the mechanical properties of the model

$$\left(\frac{T}{W}\right)^3 = \frac{1 - \nu^2}{32} \frac{P}{E} R_{HW}^{-1} \quad (4.2)$$

This equation makes it clear that the trends in Figure 4.12 are simply the inverse of those in Figure 4.10.

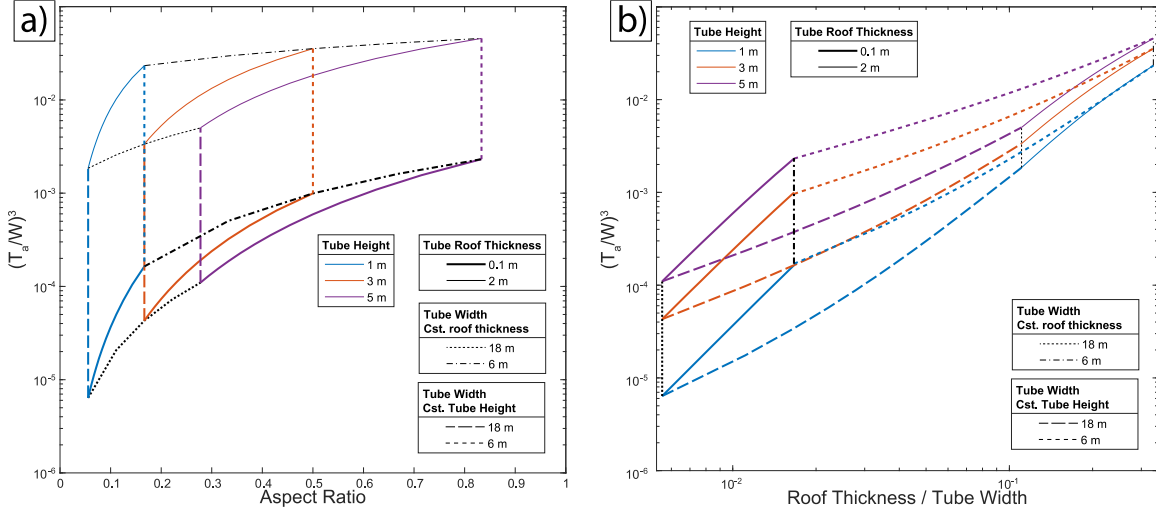


Figure 4-12: Apparent roof thickness scaled by the tube width $(T_a/W)^3$ produced by various models plotted against a) aspect ratio R_{cW} or b.) scaled roof thickness R_{TW} . The colors of the solid lines correspond to specific values of the tube height c and line thickness to tube roof thickness T . Other line styles connect the results of models with the same tube width W and either the same roof thickness (dots and dash-dots) or tube height (long and short dashes, color-coded according to c).

4.3.4 Connecting Ideal and Complex Models

The analytical model of Section 4.3.1 features a constant roof thickness T_a that we want to relate to the laterally variable roof thickness $t(x)$ for a tube of arbitrary shape. To do this, we first define an effective roof thickness T_e derived from the tube's shape and describe an algorithm relating T_e and T_a .

As seen in Equation 4.1, flexure depends on the cube of the elastic plate thickness.

Therefore, the effective roof thickness is defined as the average of the cube of t .

$$T_e^3 \equiv \frac{2}{W} \int_0^{W/2} t^3 dx \quad (4.3)$$

In our numerical models, the tube is half-elliptical. Therefore, t is minimum at $x = 0$, the tube center, where it equals the central roof thickness T , and maximum at $x = \pm \frac{W}{2}$, the edges of the tube, where its value is $T + c$. The equation of an ellipse provides:

$$t = T + c \left(1 - \sqrt{1 - \left(\frac{2x}{W} \right)^2} \right) \quad (4.4)$$

T_e can be calculated by combining Equations 4.3 and 4.4. Due to the complexity of the associated analytical solution and the simplifying assumption that the tube shape is elliptical, we calculate T_e numerically for each numerical model. Then, we compare T_e^3 with T_a^3 obtained from the model solution (Figure 4.13).

The effective and apparent roof thicknesses are generally correlated to one another but not identical (Figure 4.13). However, the difference between T_e^3 and T_a^3 depends systematically on the tube dimensions c and W . T_e^3 is systematically larger than T_a^3 . In other words, the ridges produced numerically are larger than expected given T_e . This vertical offset on Figure 4.13 depends predicably on c , the tube height. Then, the slope on these curves is larger than one, so that the difference between T_e^3 and T_a^3 increases with T_a^3 . This slope is also higher when W is smaller (Figures 4.13).

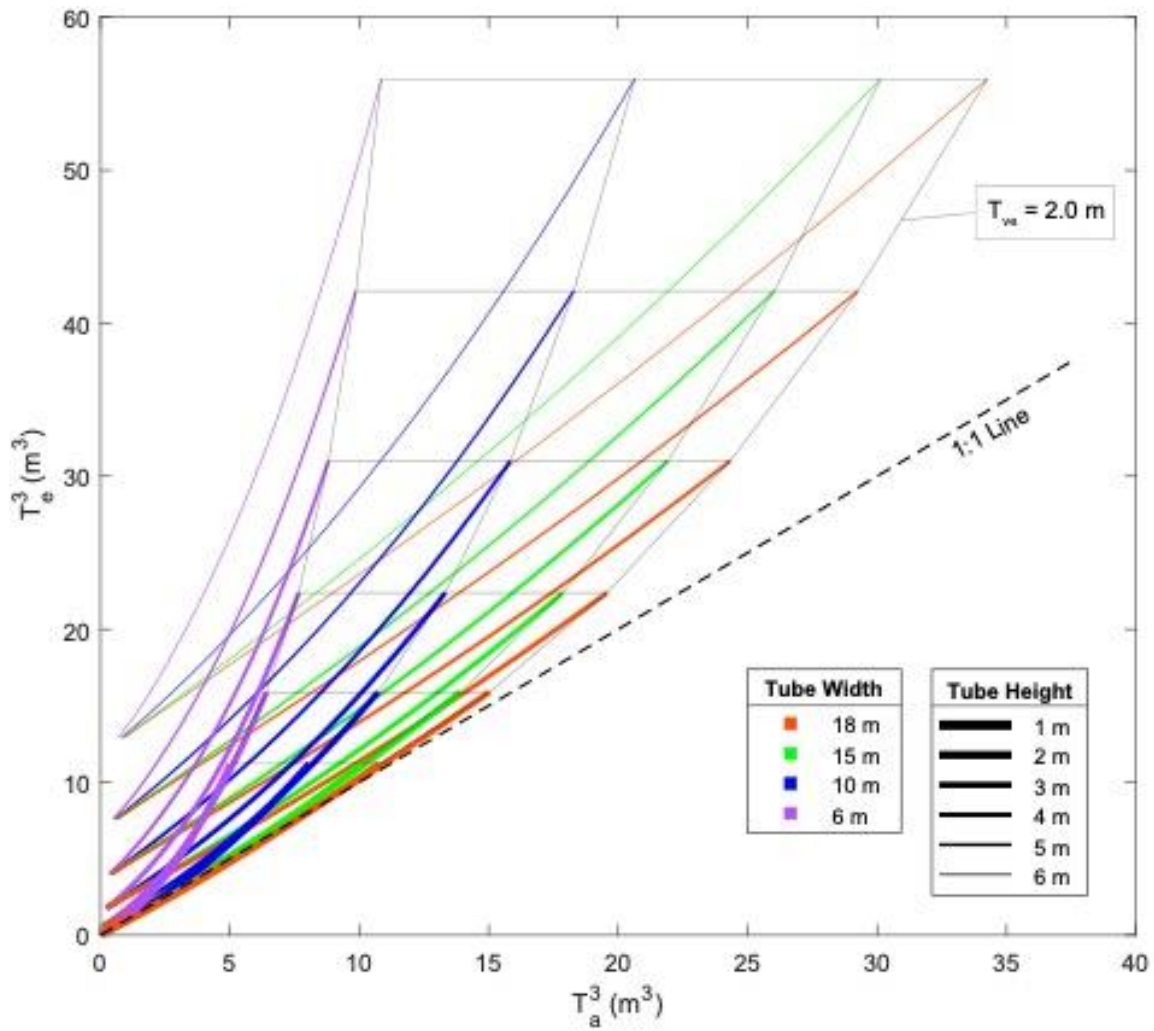


Figure 4-13: Effective roof thickness cubed, T_e^3 , vs. apparent roof thickness cubed, T_a^3 .

Effective roof thickness is calculated from the dimensions of a semi-elliptical tube; apparent roof thickness is calculated from the surface ridge heights in the model results. The gray lines link the models constructed with the maximum tube roof thickness (2.0 m).

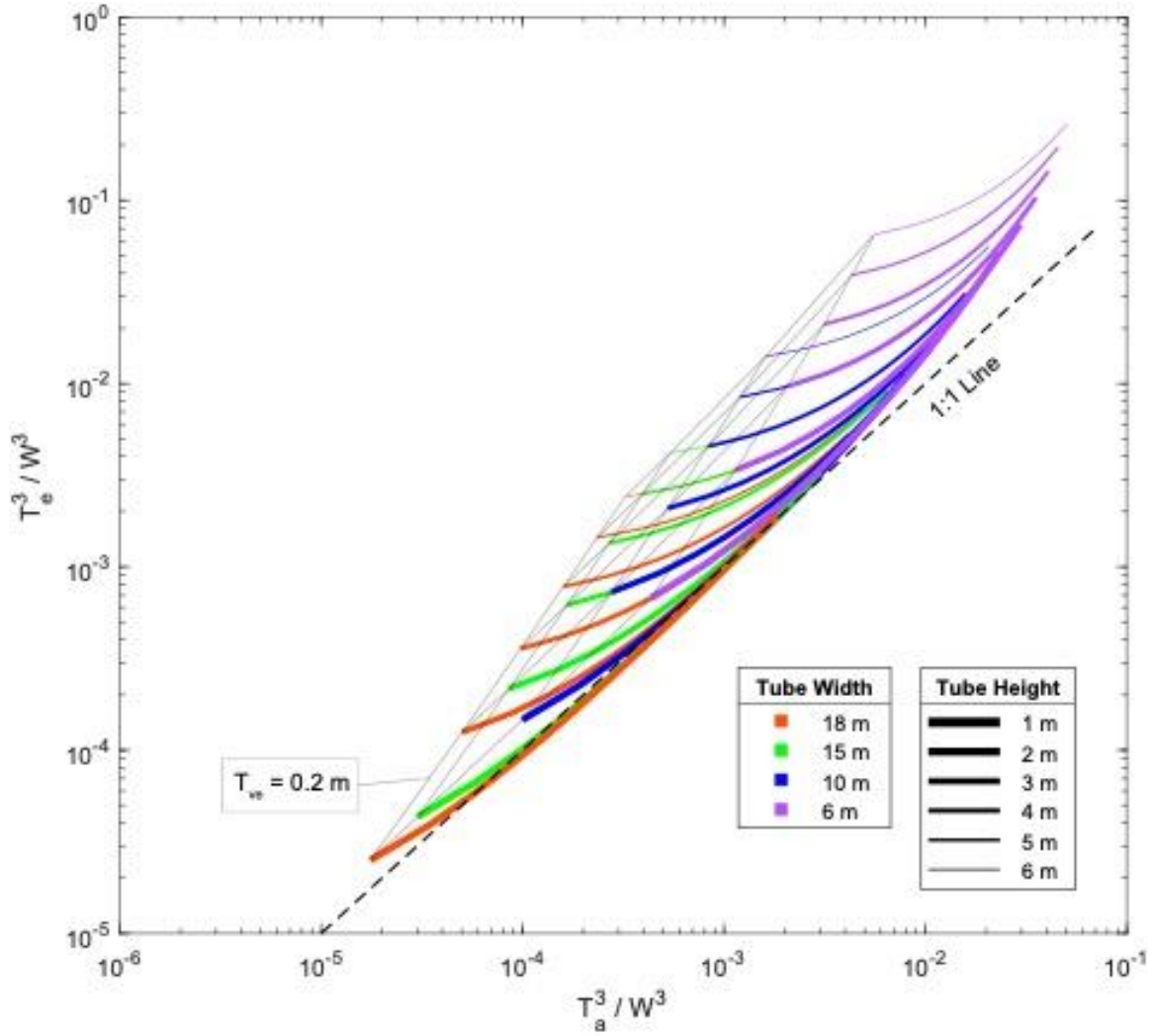


Figure 4-14: Scaled effective roof thickness cubed, $\frac{T_e^3}{W^3}$, vs. scaled apparent roof thickness cubed, $\frac{T_a^3}{W^3}$. The gray lines link the models constructed with a tube roof thickness of 0.2 m.

We use these observations to propose an empirical equation converting T_e^3 , which can be calculated from the tube's shape, and T_a^3 , which controls ridge height. Scaling all quantities with ridge width W helps collapse the curves of Figure 4.13 when presenting $\frac{T_e^3}{W^3}$ vs. $\frac{T_a^3}{W^3}$ (Figure 4.14), but not perfectly.

We first parameterize the effect of tube height on the low- T_a end of the curves of Figure 4.13, where they appear to all have the same slope. We find that the offset is approximately $0.055 c$. After removing this offset, curves with the same tube width appear similar to one another. They are well described by a parabola, so that

$$T_e^3 - 0.055 c^3 = A (T_a^3)^2 + T_a^3 \quad (4.5)$$

We fit Equation 4.5 using the curves with each value of W and find $A = \frac{46}{W^3}$. Putting it all together, we obtain the empirical relation

$$T_e^3 - 0.055 c^3 = T_a^3 \left(1 + \frac{46}{W^3} T_a^3 \right) \quad (4.6)$$

$$\tilde{T}_e^3 - 0.055 \tilde{c}^3 = \tilde{T}_a^3 \left(1 + 46 \tilde{T}_a^3 \right) \quad (4.7)$$

Where the tilde indicates scaling by W . Equation 4.7 then simplifies to:

$$\left(5 - \frac{3}{2} \pi \right) T c^2 + \left(3 - \frac{3}{4} \pi \right) T^2 c + T^3 = T_a^3 \left(1 + 46 \frac{T_a^3}{W^3} \right) \quad (4.8)$$

For comparison purposes, we refer to values of T_a^3 predicted from tube dimensions using Equation 4.6 as $T_a^{3'}$, as opposed to values calculated from the numerical models. Comparing $T_a^{3'}$ against T_a^3 or $\tilde{T}_a^{3'}$ against $\tilde{T}_a^3$ shows good agreement, and plots along a one-to-one equivalence line (Figures 4.15 and 4.16), indicating that this equation works successfully. The coefficient of determination (R^2) for these fits is 0.9949 for $T_a^{3'}$ vs. T_a^3 and 0.9914 for $\tilde{T}_a^{3'}$ vs. $\tilde{T}_a^3$. We can now relate c , W , T_e^3 , and T_a^3 analytically. T_e^3 depends only on the dimensions of a complex tube, T , c , and W , by Equations 4.2 and 4.3. T_a^3 depends on W and the inflation ridge height H by Equation 4.1. Therefore, Equation 4.6 lets us analytically relate the dimensions of a complex tube to the ridge height it would produce on the surface or *vice versa*.

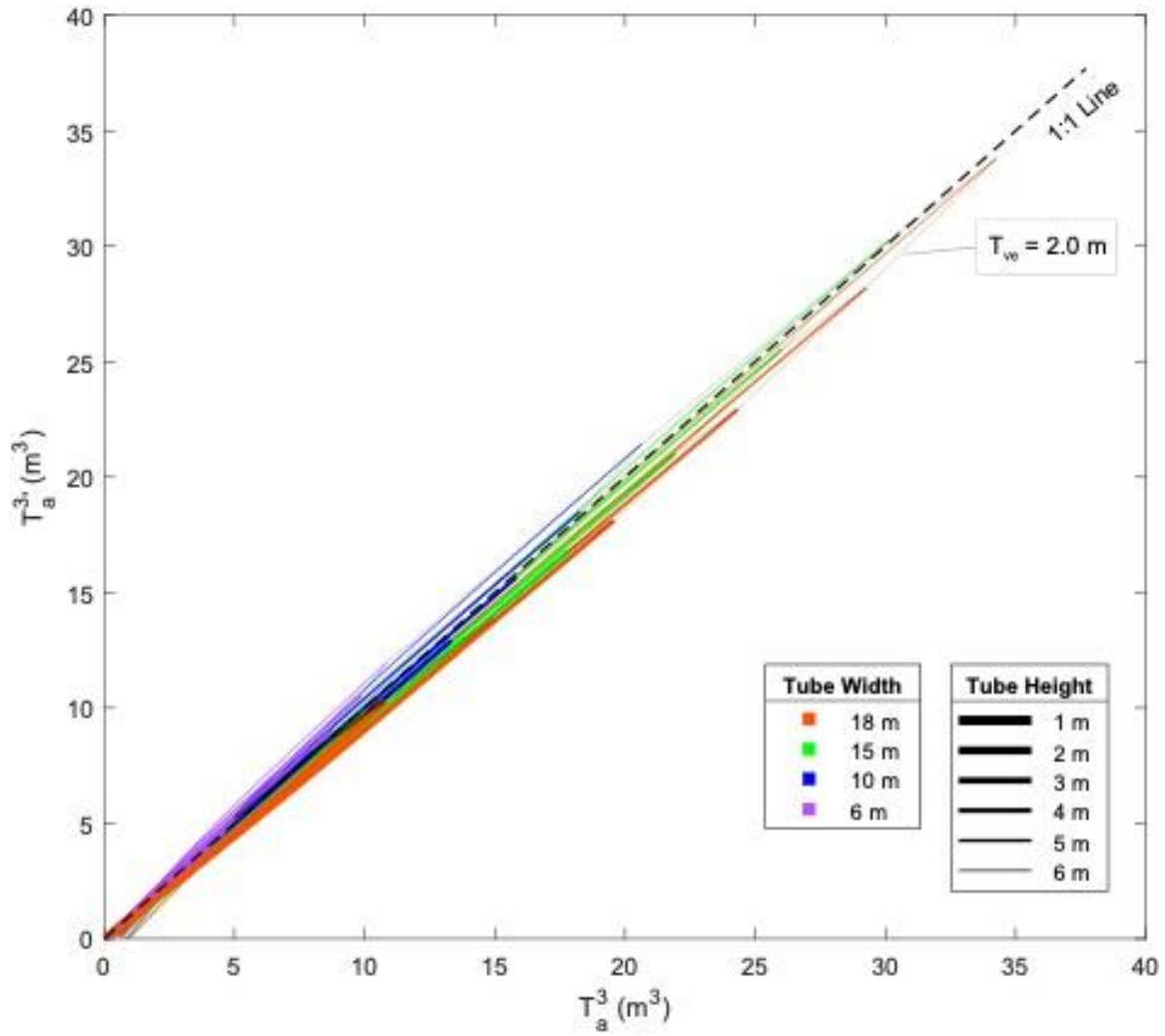


Figure 4-15: Apparent roof thickness $T_a^{3'}$ predicted by the shape of the tube and the conversion of Equation 4.6 vs. T_a^3 obtained from the numerical model results. The differences between the datasets are almost entirely removed in comparison to Figure. 4.13, and they fit close to the one-to-one matching line ($R^2 = 0.9949$). The gray lines connect the models constructed with the maximum tube roof thickness (2.0 m).

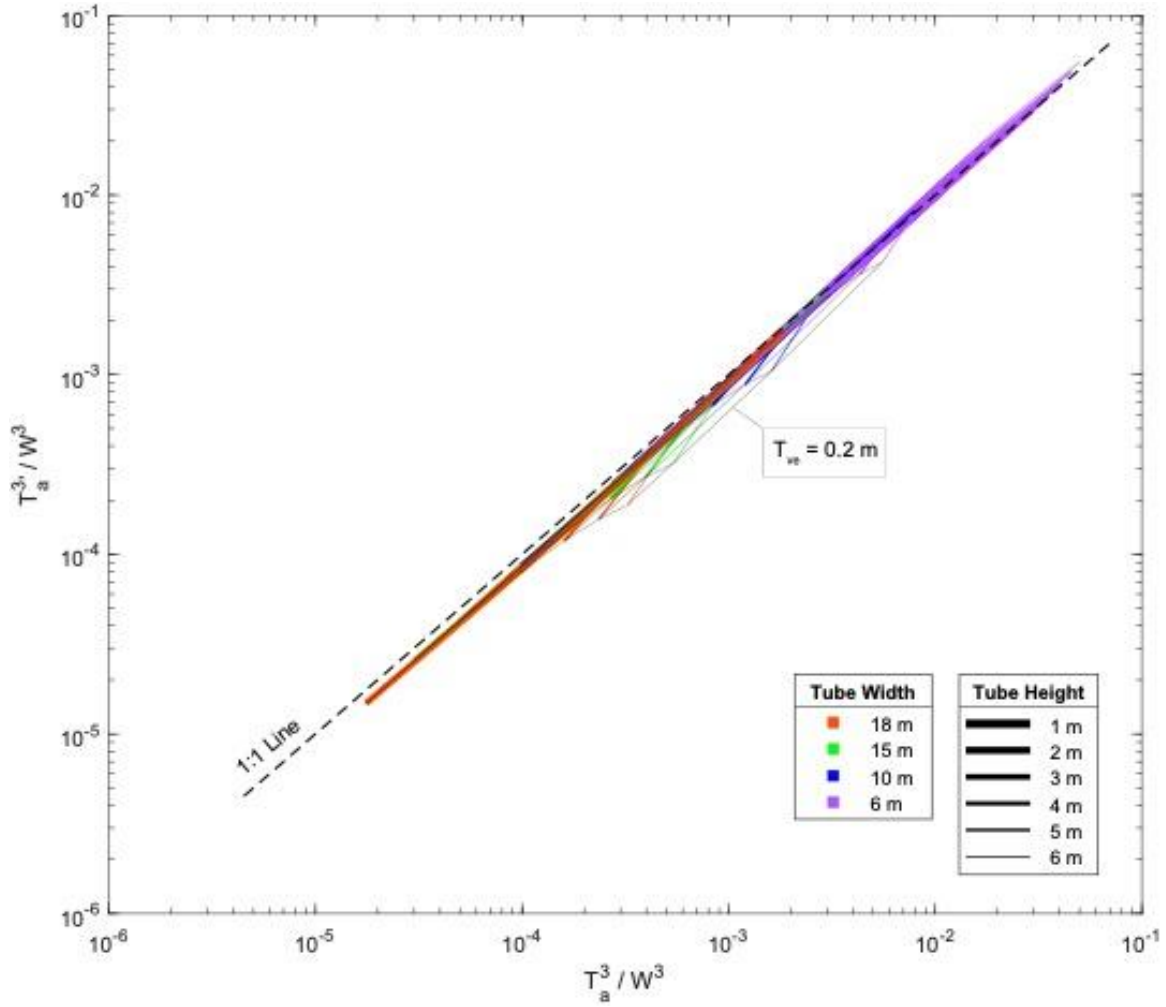


Figure 4-16: Scaled apparent roof thickness $\widetilde{T}_a^{3'}$ predicted by the shape of the tube and the conversion of Equation 4.6 vs. $\widetilde{T}_a^3$ obtained from the numerical model results. The differences between the datasets are almost entirely removed in comparison with Figure. 4.14, and they fit close to the one-to-one matching line ($R^2 = 0.9914$). The gray lines connect the models constructed with a tube roof thickness of 0.2 m, which marks the maximum observed deviation from the one-to-one line.

4.4 *Testing on a Terrestrial Example*

We test Equation 4.6 on Valentine Cave, a terrestrial lava tube located in the Lava Beds National Monument, California. Valentine Cave serves as an ideal example for this purpose because LiDAR scans are available both inside and above the tube (Whelley et al., 2017). Consequently, the cave's internal dimensions and surface expression, the two aspects our equation is designed to relate, are well known. A surface ridge above the tube has been noted by Whelley et al. (2017) and is discussed in Chapter 3. At the point along the tube where we conduct our analysis, Valentine Cave is 11.8 m wide and 3 m tall, with a roof thickness of 1 m. The surface ridge is 2 m high with a width of approximately 27 m; based on the relationship between tube width and ridge width along this tube, we estimate a tube width of about 15 m here. We estimate that the pressure inside the tube while it was active was 7.1 MPa, based on the pressure head and the elevation difference between the tube mouth and the vent from which the lava flowed. We estimate a Young's modulus of solidifying rock at the time of inflation to be 10 GPa, derived from a reasonable range (Lavalley & Kendrick, 2021).

Equation 4.6 cannot provide a unique solution for the geometry of the tube. However, it can constrain a range of morphologies that the tube must conform to, which consists of a combination of tube aspect ratios and roof-thickness-to-tube-width ratios that generate the observed ridge height and width. Inputting the dimensions of the surface ridge and the estimated width of the tube into Equation 4.6 produces the range of possible tube shapes shown in Figure 4.17.

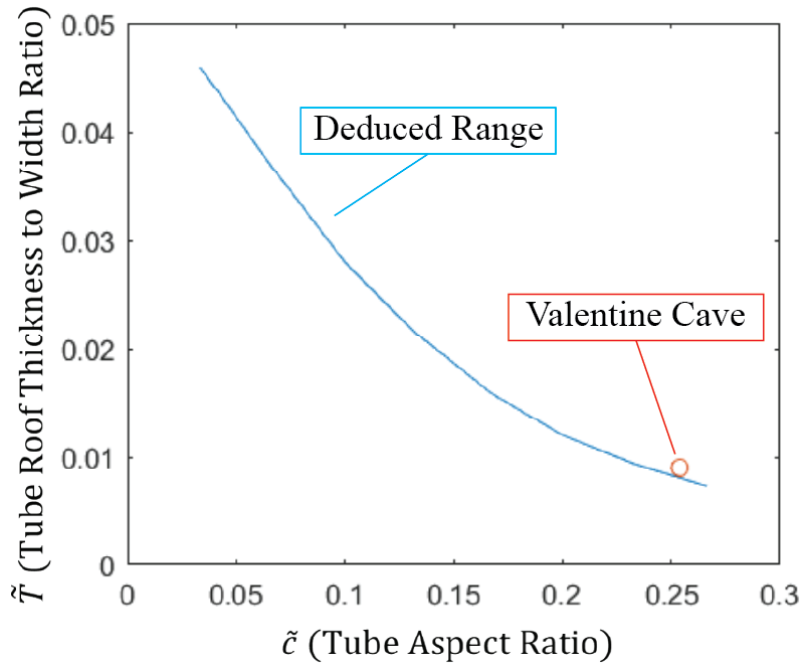


Figure 4-17: Range of possible tube shapes for Valentine Cave inferred from Equation 4.6 and observations of the surface above the tube. The red circle indicates the actual aspect ratio of Valentine Cave and the suspected roof thickness-to-width ratio at the time of inflation.

We compare this prediction to the available observations of the interior of Valentine Cave. The LiDAR data from inside the cave gives its aspect ratio as 0.254. To determine the roof-thickness-to-tube-width ratio at the time the inflation ridge formed, we must consider that the tube was still young and in the process of solidifying at the time. We found in Chapter 3 that tubes with inflation ridges must have inflated while the rock of the roof was still solidifying, resulting in a lower Young's modulus and thickness than the tube has today (Williams et al., 2022).

Using cooling relationships (Hon et al., 1994), we can estimate the likely roof thickness of the tube during its inflation. We use the current roof thickness of the tube (Whelley et al., 2020) as a reference point for calculating past roof thicknesses. We assume that the tube's aspect

ratio has not changed significantly throughout the cooling and solidification process, as cooling would have occurred from all sides, allowing us to use the aspect ratio observed in the tube today. These aspect ratio values and the roof-thickness-to-width ratio closely align with the predictions from Equation 4.6. Therefore, the strategy derived from numerical simulations of tube inflation is relevant to natural examples such as Valentine Cave.

4.5 *Discussion*

The empirical conversion from effective to apparent roof thickness and, subsequently, ridge height (Equation 4.6) allows us to constrain the size, shape, and depth of lava tubes based solely on their surface expression. This could benefit multiple aspects of tube study. On Earth, we could use this method to locate and study undiscovered tubes, examine large numbers of tubes simultaneously without needing to explore each one individually, study tubes that are inaccessible and cannot be explored, and prepare for exploration of a previously unmapped tube with maximum safety. Such studies could shed light on how the tubes and their associated flow fields formed. On other planetary bodies, this approach may be the only option available to constrain the sizes, shapes, or depths of lava tubes before landing, providing key information for planning future missions.

4.5.1 Robustness of Equation 4.6

Equation 4.6 is purely empirical. We do not have geometrical or mechanical analyses to guide this parameterization. However, we believe it is robust with regard to the physics of the tubes and represents a sound analytical solution, even though it was derived through non-analytical means. In addition to capturing our numerical results, we note that the equation can easily be transformed into a non-dimensional version (Equation 4.7). Averaging the cube of the

roof thickness to produce an effective roof thickness (Equation 4.3) is based on the well-known physical concept of the flexure of a thin elastic plate (Equation 4.1). In contrast, the appearance of tube height c and tube width W to the third power in Equation 4.6 is an empirical result. The consistency in the exponents and the natural combination of $\frac{T_e}{W}$, $\frac{T_a}{W}$, and $\frac{c}{W}$ increases our confidence in the expression obtained here.

4.5.2 Radius of Curvature

We considered that tube behavior could depend on the radius of curvature of the half-elliptical tube shape. For tube shapes as we define them here, radius of curvature is calculated by:

$$R_{curvature} = \frac{W^2}{4c} \quad (4.9)$$

However, this does not capture tube behavior or act as a replacement for Equation 4.6. As an example, we consider two tube shapes: one with height 4 m and width 12 m, and the other with height 1 m and width 6 m. We assume the same roof thickness of 1 m for each. These tubes have the same radius of curvature: 9 m. However, the first tube produces a ridge 0.062 m high; the second tube's ridge is only 0.017 m high, approximately a quarter of the first value. Additionally, radius of curvature depends only on the shape of the ellipse and cannot take into account the roof thickness; we find roof thickness to be an important factor for inflation ridge height. Since radius of curvature cannot fully capture tube inflation, this accentuates the need for a new relationship that can fully describe this process, which Equation 4.6 provides.

4.5.3 Application to characterizing lava tubes on Earth

Equation 4.6 makes it possible to constrain the internal dimensions of a lava tube by using its surface expression. The Valentine Cave test shows that our approach yields reasonable

results in a case where lava tube dimensions can be ground-truthed. However, the primary interest in this technique lies in investigating tubes where this data is unavailable. Inflation ridges may serve as indicators for locating undiscovered tubes, and Equation 4.6 can provide information about the tube dimensions based solely on the height and width of an inflation ridge. Although we do not obtain a unique solution for roof thickness and tube height, it may be possible to discuss how one of these parameters varies along a tube network if the other is known, for instance, by using a typical roof thickness related to the lava flow history, or a tube height or aspect ratio based on a few accessible examples.

Some lava tubes on Earth have been mapped, including those in the Lava Beds National Monument (Waters et al., 1990), but many others remain unmapped (Calvari & Pinkerton, 1999). Mapping these tubes can involve visual observations and measurements or LiDAR scanning. Regardless of the method, these activities are relatively time-intensive and necessitate personnel and equipment to enter and navigate the tube. When attempting to map lava tubes across a large flow system containing numerous tubes, this task can become prohibitively difficult and time-consuming. The approach of using inflation ridges to outline lava tubes and their dimensions only requires topographic information above the tubes. In addition to being fast and efficient, this method allows for mapping without needing to visit every site individually. Resources such as the NSF's OpenTopography Facility (<https://www.opentopography.org/>) and the USGS's 3DEP LidarExplorer (<https://apps.nationalmap.gov/lidar-explorer/#/>) provide access to databases of LiDAR scans. LiDAR can be collected from aircraft to cover large areas quickly. Future research could apply our method to study lava flow fields in these areas without requiring a field team to visit the site. This kind of survey can help prioritize follow-up exploration activities.

In addition to being able to cover large areas quickly, the approach of using inflation ridges to constrain the characteristics of lava tubes allows for the study of tubes that are physically impossible to explore. Some of the openings into lava tubes may be too small for access. One example is Sunshine Cave in the Lava Beds National Monument. Waters et al. (1990) noted that Sunshine Cave continues into a wide cave past the point they were able to map, but the passage leading to this cave is too narrow for anyone to pass through. We confirmed this during our visit to the same cave.

This method is also applicable to tubes that have not emptied and are still filled with solidified lava. Such tubes would be, by their nature, impossible to explore. However, they remain relevant for understanding lava flow systems, as they facilitated lava transport when the flow was active. Knowing the dimensions of these filled tubes would help understand how lava is channelized in a flow field and where and how preferred pathways formed. The dimensions of these filled pathways could be challenging to detect using other geophysical methods since there is no void space to contrast with the surrounding rock. However, inflation ridges are vestiges of events occurring when the tube was active and can serve as probes of the structure of a solidified flow.

Refining the shape and dimensions of a tube examined with the intermediary of an inflation ridge is possible if additional information about the flow is available. For instance, one may evaluate the roof thickness using a collapse pit and deduce the original height of the tube, even if a rubble pile now obscures the floor of the tube. Inflation likely occurred early in tube formation, before the roof had fully formed (Williams et al., 2022). Consequently, current roof thickness values should be considered an upper limit on the roof thickness at the time of inflation. Other geophysical methods, such as seismic surveys and ground-penetrating radar, can

sometimes provide insights into the structure of a tube's roof, although they may not offer a complete picture of its exact morphology. Our approach of utilizing the dimensions of inflation ridges can complement these methods to narrow down the ranges of possible tube shapes and estimate, for example, the location of the tube's floor, which is otherwise difficult to image (Esmacili et al., 2020).

Even if it is possible to measure the internal dimensions of a tube, the inflation ridge can provide information on the pressure and material properties during inflation (e.g., Chapter 3). For example, if there is a range of possible Young's modulus values for a lava flow, but we find that only a small subset allows a tube with the observed dimensions to produce the observed ridge height, we would constrain Young's modulus better than before. By studying multiple points along a tube system and discovering that different areas require varying pressures, we can better understand how the tube system developed and where pressure accumulated while it was active.

New information about the dimensions of tubes in a tube system could inform our understanding of how that system formed. Especially when applied to a large flow field with many tube segments, using inflation ridges as indicators of the tubes' shapes allows for finding the largest flow pathways. Observing patterns in the shapes and roof ages within a tube system could shed light on the tube formation mechanisms and cooling history. For example, channelized lava flows sometimes form rafts of floating crust on their surfaces. At a sharp turn in the channel, these rafts can jam and fuse into a short span of roof. Roof formation will then propagate upstream and downstream from such a blockage (Peterson et al., 1994). This implies that the sharp turn point has the oldest and therefore thickest roof at the time of inflation. If our roof thickness and age findings confirm this, we can infer that this particular formation mechanism was present and trace its origin.

4.5.4 Application to characterizing lava tubes on the Moon and other planets

Exploring tubes on other planetary bodies requires dedicated missions that have not yet been conducted; however, these features hold significant interest and importance. Our inflation ridge method for studying tubes can reveal information about planetary tubes from surface observations alone, which are currently available for certain bodies via remote sensing. The technique we propose could be applied wherever lava tube ridges are observed on another body to uncover new information about otherwise inaccessible tubes. Potential sites include the western flank of Alba Mons and the southeastern Tharsis region on Mars, where ridges recognized as lava tube inflation ridges have already been documented (Zhao et al., 2017; Crown et al., 2022).

Lava tubes on other planets would be valuable locations for future exploration or habitation. Knowing more about their dimensions and shapes is important for planning how to enter and explore them. Certain tube shapes may offer more usable space for missions. Previous research indicates that the shape, size, and depth of lunar lava tubes determine the usable floor area inside (Williams & Montesi, 2025). Knowing a tube's height and burial depth is essential for safely entering it. For instance, sending an exploratory device down a hole into a tube requires understanding the vertical drop involved. Future missions may focus not on immediate entry into a tube but on using geophysical methods from the surface above to confirm its existence and conduct studies. This applies to the proposed Lunar Leaper mission to the Marius Hills Hole on the Moon (Kolvenbach et al., 2024). In this scenario, having pre-existing constraints on tube morphology remains valuable. Selecting appropriate geophysical methods, such as the frequency of ground-penetrating radar or the sensitivity of a magnetometer, would benefit greatly from knowing the tube's approximate depth and size in advance.

Using the relationships we discuss requires values of E , P , and ν . For this work so far we have chosen values suitable for terrestrial tubes. Tubes on other planets would likely require different values of these parameters. However, the relationships we found would still be applicable; the only modification needed would be the inputs to Equation 4.1, which do not impact Equation 4.6. The pressure head could give an estimate of the internal pressure in a planetary tube, like in the terrestrial cases, if the location of the vent is known. This value may not be exact, but would be a useful approximation. The value of the Young's modulus could be informed by our current understanding of extraterrestrial lava flow compositions and if they differ substantially from those on Earth. The Poisson ratio ν varies little between rock types and has only a small effect on Equation 4.1. Therefore, it is likely acceptable to use a default value of ~ 0.3 , so the factor $(1 - \nu^2)$ will always be relatively close to one. The effects of gravity (Chapter 2) are very small compared to the forces needed to bend the rock of the roof, so the different gravitational strengths on other bodies would likely not affect the results significantly. If it becomes important to include the effects of different gravities, only Equation 4.1 needs to be modified. Therefore, any variables that need to be changed between planetary bodies are included only in Equation 4.1, while the more complex Equation 4.6 remains valid.

4.6 Conclusions

We have discovered and quantified a relationship between the surface ridge formed by an inflated lava tube and the dimensions and morphology of the tube itself. By using this equation, remote observations of the surface over a tube could reveal otherwise inaccessible information about the tube's internal geometry. Although the relationship cannot uniquely identify the tube shape from surface observations alone, it can constrain a range of tube shapes within which the actual tube must fit. Incorporating other observations, such as collapses, could further refine the

constraints on the tube shape. This constitutes a new method that could be applied to previously unexplored tubes on Earth and other bodies like Mars or the Moon. Our method could aid in planning future planetary tube exploration missions and enhance our understanding of lava flow fields on Earth.

Chapter 5 Candidate Inflation Ridges and Lava Tubes

Detection in the Lava Beds National Monument

5.1 *Abstract*

Lava tubes, a type of sinuous cave formed by lava flows, are a volcanic feature of interest on Earth and other bodies. Surface features such as skylights and collapses can reveal the presence of lava tubes, but not their internal dimensions. Many tubes have been explored in person and mapped. However, this is not always a feasible solution to find a tube's internal morphology, such as if the tube is inaccessible or filled. We have developed a method to find lava tubes and constrain their internal shapes using only inflation ridges above them, which are observable in surface topography data. We apply this technique to the Lava Beds National Monument to test its efficacy over a large dataset. Searching near known tubes and collapses, we find eighteen new candidate lava tube inflation ridges. Our method gives a range of possible tube shapes for each. The results support our ideal roof thickness value from Chapter 3 as a valid approximation. Most shape curves fall in a continuous range, while the ridge chain we name Double Collapse Ridge consistently falls above the others; this suggests a different tube shape and formation conditions. The more information on the tube is entered into our inflation ridge method, the more tightly it can constrain tube shape. We conducted fieldwork over our most promising tube candidate, named Sunshine Ridge, and estimate its current roof thickness using ground-penetrating radar. Combining this with our method gives a minimum tube height of 6.5 m at the time of inflation and a minimum tube aspect ratio of 0.27. One ridge is above a mapped tube, Bearpaw Cave; using the tube's current observed aspect ratio, we find a 0.9 m roof thickness at the time of inflation.

5.2 Introduction

Lava tubes are sinuous caves created when a lava flow creates a solid crust over its top surface, this crust becomes strong enough to remain stable without support, and the interior lava drains out (Peterson & Swanson, 1974; Sakimoto & Zuber, 1998; Huff & Owen, 2013; Gregg, 2017). Tubes are important for the development of lava flow fields and understanding their histories (Greeley, 1987; Gregg & Fornari, 1998; Keszthelyi & Self, 1998; Zimbelman, 1998; Michael et al., 2024). Tubes can also support unique fauna (Taylor and Krejca, 2006; Howarth et al., 2007; Eberhard and Howarth, 2021). On other planetary bodies such as the Moon and Mars, tubes are potentially crucial targets for exploration and habitation (De Angelis et al., 2002; Léveillé & Datta, 2010; Haruyama et al., 2012; Boston et al., 2014; Sauro et al., 2020; Schörghofer, 2022; Horvath et al., 2022).

As they are by definition underground features, lava tubes can be difficult to locate and study without *in situ* exploration. Finding a tube remotely relies on observable surface features that the tube produces. Collapse chains are a distinctive feature often used to detect lava tubes. If a tube's roof becomes unstable and fails, it can collapse into the tube. The collapse may be complete, leaving only a trench, or there could be a hole through an otherwise intact tube roof. A sinuous chain of collapses is a strong indicator of a lava tube under the surface (Halliday et al., 2012; Larson, 1992; Okubo & Martel, 1998; Wagner & Robinson, 2022). Intact tube segments are often found between the collapses of such chains (i.e., Waters et al., 1990). One lava tube system is often broken up into multiple different caves by collapsed segments and areas where the tube is blocked by remaining lava (Waters et al., 1990; Atkinson et al., 1975; Greeley, 1987). Skylights are a similar feature to collapses, where a hole exists in the roof of a lava tube. Instead of forming through collapse, skylights can occur when a tube roof never entirely forms (Calvari

and Pinkerton, 1998; Peterson et al., 1994). These features can, in some circumstances, be difficult to distinguish; for the purpose of identifying tubes, any series of holes or trenches following a sinuous shape strongly suggests a tube, regardless of the holes' origins.

Although skylights and collapse chains are useful for finding lava tubes, they cannot provide detailed information about the dimensions of these tubes. In many cases skylights or collapse pits do not cover the entire width of a tube (Waters et al., 1990). As such, trying to estimate tube widths from them alone would give underestimations. Skylights and collapse pits may give estimates of the roof thickness or tube height (i.e., Wagner & Robinson, 2014). However, estimating roof thickness in this way requires angled imagery, which is not always available. Roof thickness at such a spot may be uncharacteristically thin, as the initial collapse that formed the pit may also have triggered lower layers of the tube roof to break off and fall into the tube. Measuring the height of a tube in a collapse pit could be biased, as the collapse could leave a large pile of debris on the tube floor. Without knowing the height of this pile of collapse debris, the true tube height cannot be precisely measure. This would not be a problem for a skylight, as there would be no debris; however, it is difficult to distinguish between a collapse pit and a skylight visually. Furthermore, even if estimated correctly, the dimensions around a collapse pit may not be representative of the entire tube. Possible a less structurally stable tube shape, such as a thinner roof, contributed to the collapse of the tube in this spot.

When a tube fully collapses, the resulting depression is clearly visible from the surface and can be measured. However, the collapse destroys much of the original data pertaining to tube shape. The roof thickness is completely lost. The width and height could be estimated from the width and depth of the collapse, but other factors such as sliding of nearby material into the trench should also be considered.

Additionally, tubes may not have clearly visible skylights, collapse pits, or collapse chains. Some known tubes have long intact stretches without any such features (Waters et al., 1990; Atkinson et al., 1975; Greeley, 1987). Fully intact tubes may exist under the surface and be entirely unknown to us. For these reasons, it is important to have another way of finding lava tubes and determining their dimensions.

We have developed a method using inflation ridges, another surface feature produced by some lava tubes (Chapter 3), to identify undiscovered tubes and constrain their dimensions without requiring *in-situ* exploration (Chapter 4). Here, we apply this method to a lava flow field in Lava Beds National Monument (LBNM), California, which hosts numerous lava tubes. We demonstrate how inflation ridges can be used to map lava tubes and provide constraints on their geometrical characteristics.

5.2.1 Studying Inflation Ridges

In Chapter 4, we presented a method to constrain a lava tube's shape and dimensions from surface observations alone. This method relies on the identification of an inflation ridge, a topographic feature that can form when a lava tube is pressurized while active. If the pressure of lava inside the tube increases early enough in the tube's history, when its roof is still relatively thin and soft, the tube inflates, causing its roof to deform upwards and develop a ridge. The ridge can then become more solid over time and remain in place even after the tube has drained. Inflation features, including inflation ridges, have been documented in many locations on Earth. (Guest et al., 1984; James et al., 2012; Orr et al., 2015; Whelley et al., 2017; Bernardi et al., 2019; Voigt et al., 2021; Sauro et al., 2020; Tomasi et al., 2022).

Through numerical and analytical modeling, we have shown that the dimensions of an inflation ridge reflect the shape and dimensions of the tube that caused it through predictable relationships:

$$\left(5 - \frac{3}{2}\pi\right) T c^2 + \left(3 - \frac{3}{4}\pi\right) T^2 c + T^3 = T_a^3 \left(1 + 46 \frac{T_a^3}{W^3}\right) \quad (5.1)$$

where T , the roof thickness of the tube at its center and c , the height of the tube at its center are regarded as the tube shape parameters. T_a is the apparent roof thickness deduced from the inflation ridge height H and width W :

$$T_a^3 = \frac{P(1 - \nu^2)}{32E} \frac{W^4}{H} \quad (5.2)$$

E and ν are the Young's modulus and Poisson ratio of the lava, and P is the internal lava pressure, which we evaluate from the elevation difference between the flow vent and the suspected tube location. We assume that, at the time of inflation, the width W of the tube was approximately equal to the width of the ridge.

Using these relationships on an observed inflation ridge can yield new information about the underlying tube. We have tested this method on Valentine Cave, a lava tube in LBNM (Chapter 4). The curve in Figure 5.1 represents the range of possible shapes of the Valentine Cave tube inferred from its inflation ridge dimensions. This range of solutions includes the actual shape of Valentine Cave (red circle). External constraints, such as estimates of roof thickness from observations of collapse pits, could be used to further refine the inference of lava tube shape.

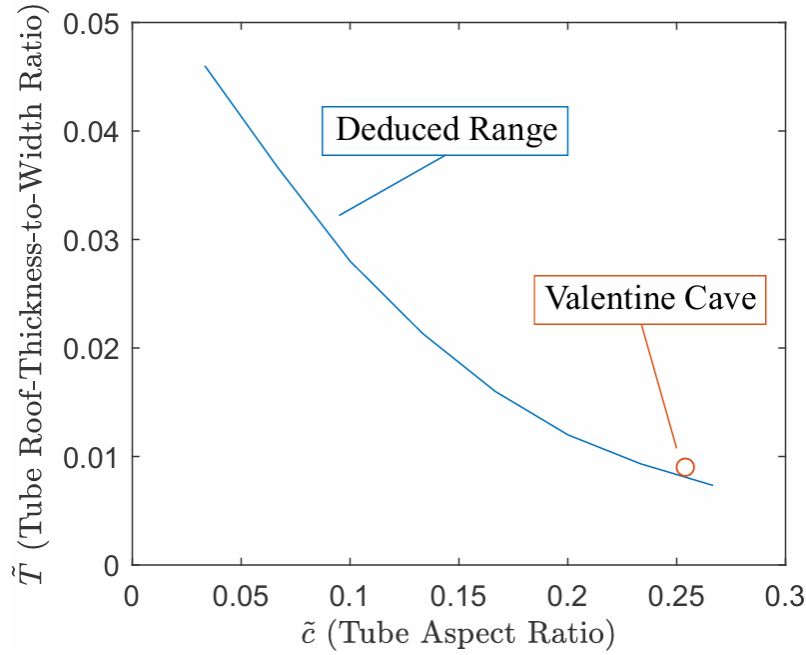


Figure 5-1: Valentine Cave results using our inflation ridge technique. The blue curve shows the range of possible tube shapes, deduced from ridge observations only. The red circle indicates the likely values for Valentine Cave at the time of inflation. Taken from Chapter 4.

This method could be very useful for studying inaccessible tubes, such as those in difficult environments or even those on other planets. This method can also study many tubes at once remotely, as opposed to exploring each one individually, provided that topographic data is available. After testing this technique on Valentine Cave in Chapter 4, we apply it in this chapter to unexplored tubes in LBNM.

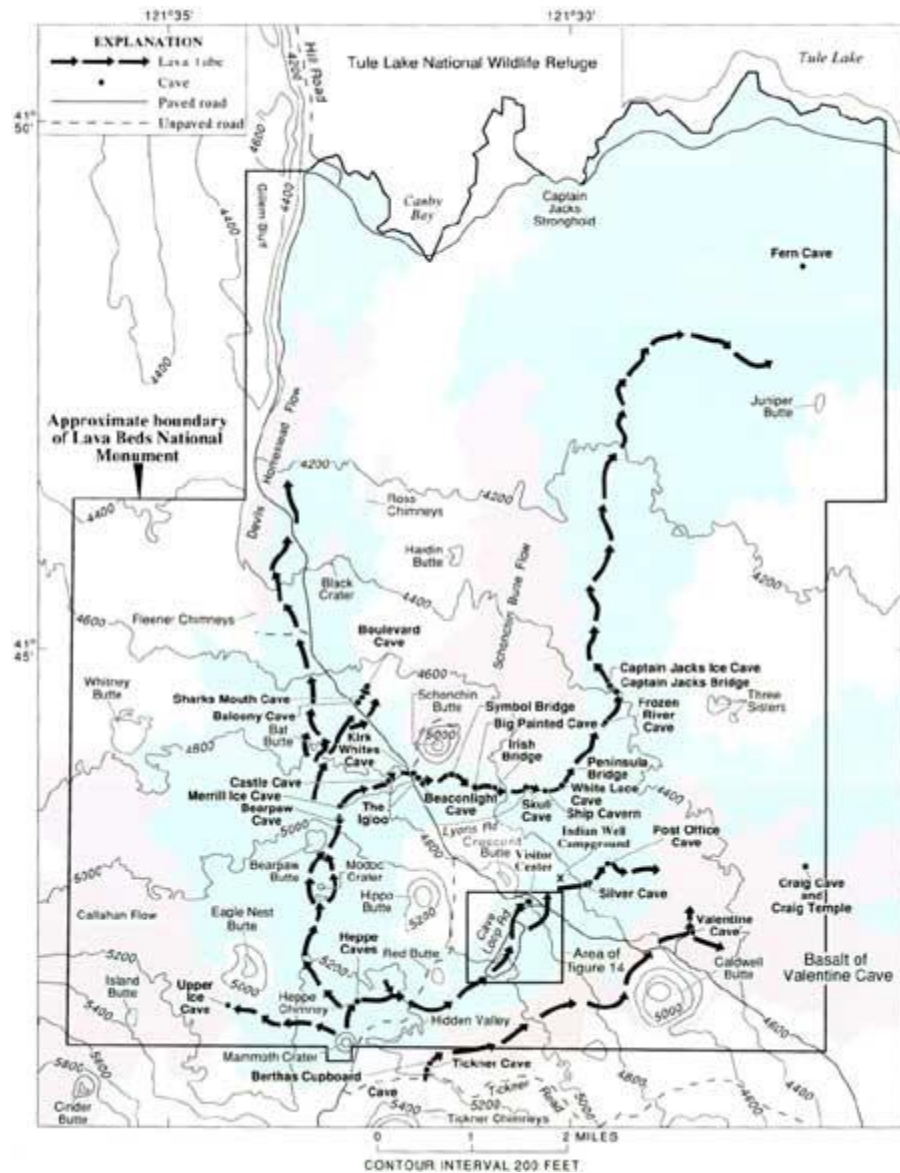


Figure 5-2: Map of the Lava Beds National Monument (LBNM). Light blue shows the basalt of Mammoth Crater. Image credit: USGS

5.2.2 Lava Beds National Monument (LBNM) as a Study Site

The Lava Beds National Monument (LBNM), California, is an ideal site to apply our inflation ridge technique to unexplored lava tubes. The LBNM has the highest concentration of lava tube caves in the continental US, featuring nine different lava tube systems, making it a

valuable study site (Larsen, 1992; DeDecker, 2014). Many of these caves are included in publicly available maps (Waters et al., 1990). Some of the tubes are accessible via roads and paths. The flows in this area are primarily composed of basalts. Tubes have also been investigated using near-surface geophysics techniques (Esmaeili et al., 2020; Bell et al., 2022; McCall et al., 2022; Magaña et al., 2025) that complement the topography-based analysis presented here. Based on the magnetic signatures of the flows, the emplacement of most of the basalts likely occurred over approximately ten years (Waters et al., 1990). The basalt of Mammoth Crater covers half of the monument and includes most of the tubes and collapse chains (light blue in Figure 5.2).

Measuring the dimensions of an inflation ridge above a tube can be difficult or impossible to do by eye. While they can reach meter-scale heights, these ridges are very broad, making them difficult to identify visually. Unrelated surrounding topography can make them even harder to locate and quantify, and they can be obscured by vegetation or ground roughness. The ridge above Valentine Cave, although a relatively clear example, was not studied until a LiDAR scan of the surface captured it and its dimensions (Whelley et al., 2017). Therefore, topographic scans such as LiDAR are necessary to document and study more ridges. We submitted a proposal to NCALM, the National Center for Airborne Laser Mapping, to map a 40 km² area of the LBNM. This proposal was accepted, and NCALM provided a LiDAR dataset over the requested area. We used this dataset to search for and study the candidate ridges discussed in the remainder of this paper. This dataset is now publicly available (OpenTopography, 2024). Figure 5.3 shows a hillshade visualization of this LiDAR dataset.

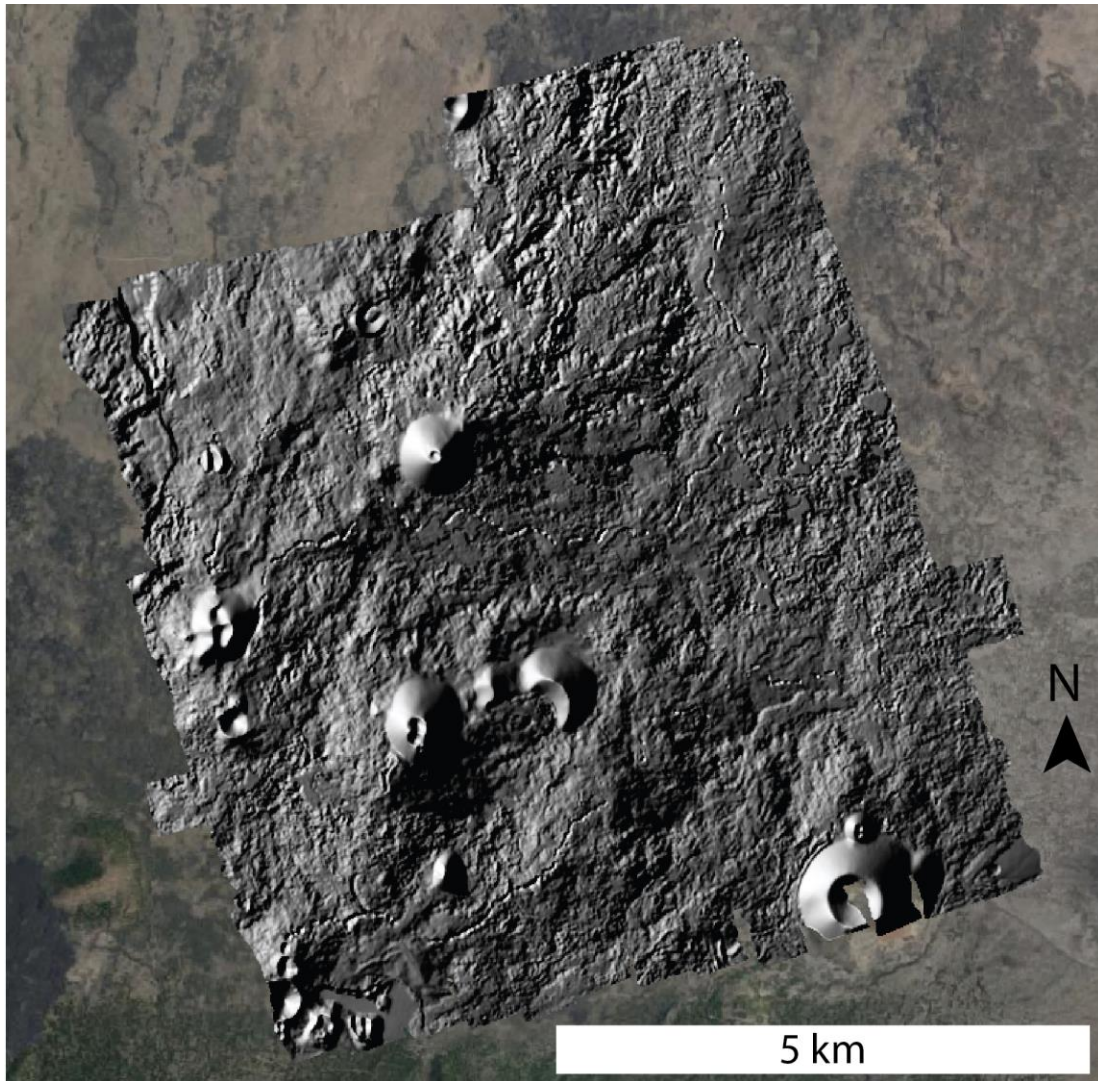


Figure 5-3: LiDAR dataset provided by NCALM over the Lava Beds National Monument (Williams, 2024). The data is shown as vertically exaggerated hillshade topography over imagery. Image center: $-121.515^{\circ}, 41.730^{\circ}$. NAIP imagery data (EROS, 2017) visualized through QGIS.

5.3 *Finding and Documenting Inflation Ridges in the LBNM*

5.3.1 Locating Inflation Ridges in Topographic Data

Inflation ridges above lava tubes can be challenging to detect. They are shallow in relation to their widths and can be altered or obscured by unrelated topographic features, or lost in the overall landscape of the region. For this reason, we employed several techniques to assist in our search.

We analyzed the NCALM LIDAR data using the QGIS mapping software. To highlight elevation rises in the data, we combined two methods (Figure 5.4): elevation contours and hillshade visualization. We used vertical contours at intervals of 0.25 m to highlight elevation features. Hillshade visualization simulates light and shadow on the landscape, given a specified location and angle of the sun. We applied a vertically exaggerated topography for hillshade to make shallow topographic features stand out more clearly. We used a low sun angle of 70° (20° from the horizon) but varied the azimuth so as not to miss features due to their orientation.

When choosing which sites to examine using the above methods, we took advantage of prior knowledge of the location of lava tubes based on maps of the area. Waters et al. (1990) mapped numerous tube systems in the region. We selected some sites based on the locations of these tubes. For example, where separate caves have been documented along roughly the same path, but no tube is known to connect them, we investigated the area between them for a potential inflation ridge. If a known tube terminated, we examined the area beyond the terminus to determine if it could extend into a tube with an inflation ridge. For this purpose, we considered both intact and collapsed cases of known tubes; we looked for ridges both at the end of mapped, intact tubes and at the end of collapse chains.

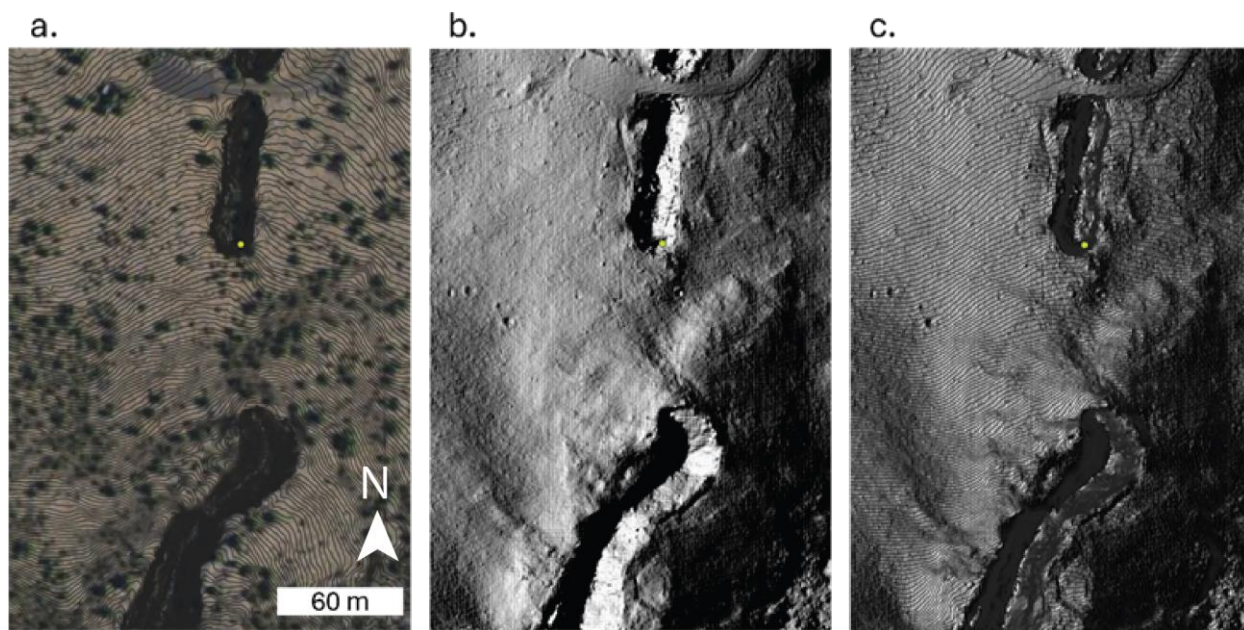


Figure 5-4: A candidate lava tube segment in the Lava Beds National Monument, with topography from the LIDAR dataset shown as (a) 0.25 m elevation contours, (b) vertically exaggerated hillshade with a 288° sun azimuth, and (c) both combined. The candidate tube segment connects the Northern and Southern collapse trenches shown. Image center: - $121.549^\circ, 41.727^\circ$. NAIP imagery data (EROS, 2017) visualized through QGIS.

We also surveyed the data for other promising inflation ridge candidates that were not associated with any additional features. We included sites that were not near any documented tubes when a strong ridge candidate appeared in the LIDAR data.

Although our goal was to locate and study unexplored tubes, we also applied our method to known tubes in some cases for reference. Some mapped tubes, such as Bearpaw Cave, exhibit possible ridges above them in the topographic data. By analyzing these ridges, we may be able to compare our findings with the tube dimensions reported on the maps, with the caveat that the

tube shape likely changed between the time the inflation ridge formed and the time the tube stopped being active.

5.3.2 Documenting Ridge Dimensions and Uncertainties

We gave each ridge an informal name, such as “Snail Ridge”, “Donut Ridge”, “Shortlink Ridge”, etc. We extracted several topographic profiles from each ridge to capture its dimensions and variability. The number of profiles per ridge varied, as longer features typically had more profiles than shorter ones; a particularly short ridge may have only one or two profiles, while most have three or more (Figure 5.5). From each profile, we recorded the vertical and horizontal positions of both edges of the ridge and the ridge peak. From this, we calculated the ridge heights and widths.

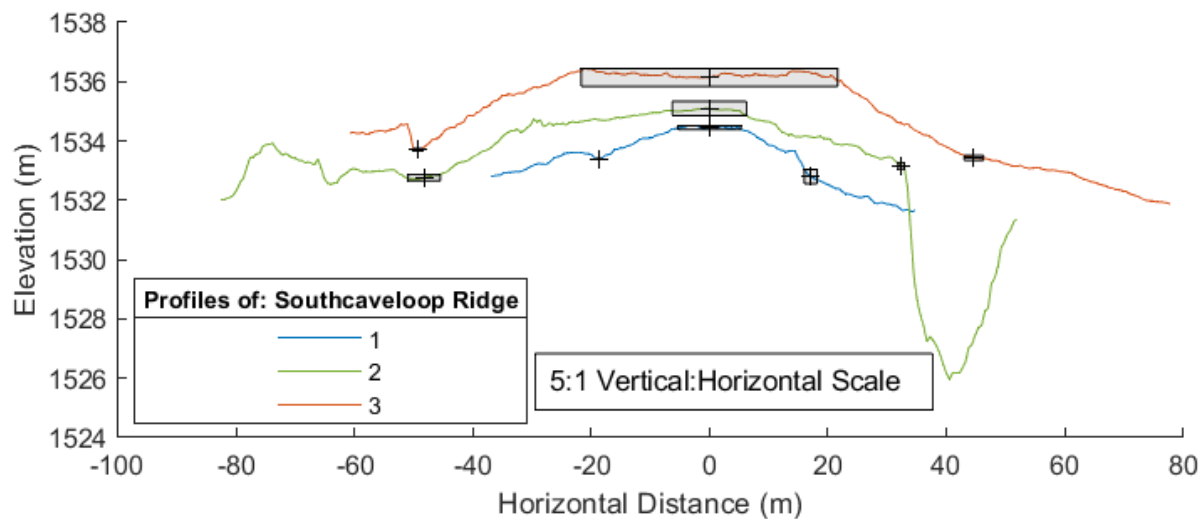


Figure 5-5: Topographic profiles extracted across the South Cave Loop ridge. The width and height of the ridge vary down-slope.

We examined each profile and recorded the left and right horizontal bounds surrounding the peak and each edge point of the ridge. We recorded two points on each side of the ridge, bracketing where we interpreted the ridge edge to be, rather than documenting a single edge point. This approach enables us to estimate an uncertainty in the measurement. In most cases, the transition from the ridge to the surrounding topography is neither sharp nor clearly defined, and it possesses some measurable uncertainty. We marked the farthest points left and right of where we thought each tube edge could be, to evaluate this uncertainty (Figure 5.6).

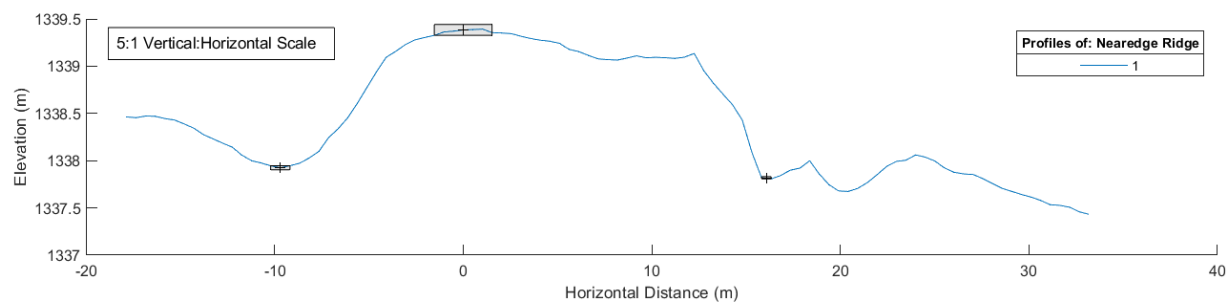


Figure 5-6: Example of one profile across Near Edge Ridge, illustrating measurement uncertainties. The gray boxes show the vertical and horizontal limits of each ridge edge and the ridge peak. The crosses represent the midpoints of these bounds, taken as the actual values.

To identify the edge of each ridge, we primarily looked for changes in slope. In some cases, when this change was sharp, determining the edge of the ridge was straightforward. However, we still chose to include one point on either side of the sharp change in slope to account for the inherent uncertainty of the data. More commonly, this slope was at least somewhat gradual. In these cases, we examined the topography of the ridge and the surrounding area, gauging the slope of the ground without the tube, and selected the point where the slope leading down from the ridge faded into the surrounding topography. We then identified the left- and right-most points that we could consider valid locations for this and recorded them as the

bounds. For example, when choosing bounds for the right edge of a tube, any points to the left of the left-most bound are on the slope leading up to the ridge and are visibly affected by the existence of the ridge. Any points to the right of the right-most bound are on the surrounding ground, independent of the ridge. For the points between the left-most and right-most bounds, it is unclear whether they are on the surrounding ground or the slope up to the ridge.

Marking the edges of each ridge requires judgment and cannot be done by a strict algorithm. We examine the surrounding topography in QGIS to estimate the location of each ridge and how it intersects with the profile. Some profiles contain multiple topographic rises, all of which appear as ridges in the profile data. The surrounding topography helps determine which of these are small bumps or topographic features unrelated to the examined ridge, and which are part of the ridge in question.

We take the true horizontal location of each ridge edge as the midpoint between its left and right bounds. The elevation at this midpoint is taken as the vertical coordinate of the ridge edge. We assume that the bounds cover a 95% confidence interval of the location of the tube edge. In a Gaussian distribution, 95% of the sample falls within 2σ of the central value covers. Therefore, we consider that our bounding boxes for the ridge edge include a central value its width is $2\sigma_x$. Therefore, the uncertainty of the horizontal position, σ , one-quarter of the distance between the bounds of the tube edge's location.

To calculate the location and horizontal uncertainty of each ridge peak, we locate the highest point on each ridge, observe the area around it where the points have an elevation comparable to the peak itself, and designate the bounds to the left and right of this area. The purpose of this is to make sure that a region of comparable high-elevation points is recorded instead of a single value, so that a small feature, such as an individual boulder on top of the

ridge, does not yield an erroneously high result. Some ridges have a dome-like shape, resulting in a relatively small peak horizontal uncertainty. Others are plateau-shaped, leading to very large horizontal bounds on the peak location but not necessarily on its elevation. We take the midpoint between the two bounds as the horizontal coordinate of the peak. The elevation of the peak is given by the elevation at this midpoint.

To calculate the vertical uncertainties of any point, such as the edge of a ridge or the ridge peak, we record the elevation difference between the midpoint and the lowest or highest point within the bounds, whichever gives a larger difference. This is typically the left or right extreme of the bounds. We consider this elevation difference to be $2\sigma_y$. We mirror this above and below the midpoint, often including some empty space above or below the ridge to account for the possibility that the topography may have eroded downwards or been slightly added to over time. This creates bounds as seen in Figure 5.6. We record σ_y as the uncertainty in this point's elevation.

Calculating the tube's width and its uncertainty is done simply by taking the distance along the profile between the left and right edges of the tube and propagating the uncertainties. To determine the height of the tube, we average the elevations of the left and right edges of the ridge to establish an elevation baseline and calculate the difference between the peak height and this baseline. Likewise, the uncertainty in height arises from propagating the uncertainties of the other variables.

We also calculate the average width and height for each ridge by combining the data from all profiles. The uncertainties in these average values are propagated from the individual uncertainties in the profile values.

5.3.3 Estimating Internal Lava Pressure

Equation 4 requires estimates of the internal lava pressure of the tube, as well as the Young's modulus and Poisson's ratio of the rock, in addition to the ridge dimensions. We used representative values of 10 GPa for the Young's modulus that take into consideration our inference that the lava was not completely cooled and still relatively soft at the time of inflation (Chapter 3), and 0.3 for the Poisson's ratio (Schultz, 1995; Lavalley & Kendrick, 2021). We use pressure head to estimate the pressure in each case:

$$p = \psi \rho g \quad (5.5)$$

where p is the pressure, ψ is the elevation difference between the source and the site of interest, ρ is the density of the material, and g is the acceleration of gravity. We use $\rho = 2900 \text{ kg. m}^{-3}$ as the density of basalt (Philpotts and Ague, 2009). The elevation difference between each ridge and the vent from which it originated is measured on the topographic LiDAR data. All the ridges that we study are in the flow unit that Larson (1992) calls "Basalt of Mammoth Crater". This flow system covers approximately two-thirds of the LBNM and contains six of the LBNM's nine major lava tube systems (Larson, 1992), so it is unsurprising that it also contains our candidate ridges. We use Mammoth Crater as the vent location for all of our ridges. Therefore, we calculate ψ for each tube by subtracting the elevation at the ridge from the elevation of Mammoth Crater.

This estimate may be slightly inaccurate, as the true vents from which the lava flowed are on an extension of the Gillem Fault near Mammoth Crater, which is now buried (Larson, 1992). However, Mammoth Crater is the best approximation available. As the ridges ultimately have the same source vent, if this is a slightly inaccurate assumption, it would change the pressure by the same amount in each case and not significantly affect our findings.

5.4 *Results*

5.4.1 Ridge Examples

We present two example ridges: Clean Ridge (Figure 5.7) and Snail Ridge (Figure 5.8). Snail Ridge extends from the edge of a lava tube collapse but does not lead into any skylights or collapses of its own. Clean Ridge connects two collapses in a chain.

Clean Ridge maintains a relatively consistent width and height, likely due to its short length. The slope on the right side of the ridge is steeper than that on the left, possibly as a result of modifications made to this side later in the flow's history.

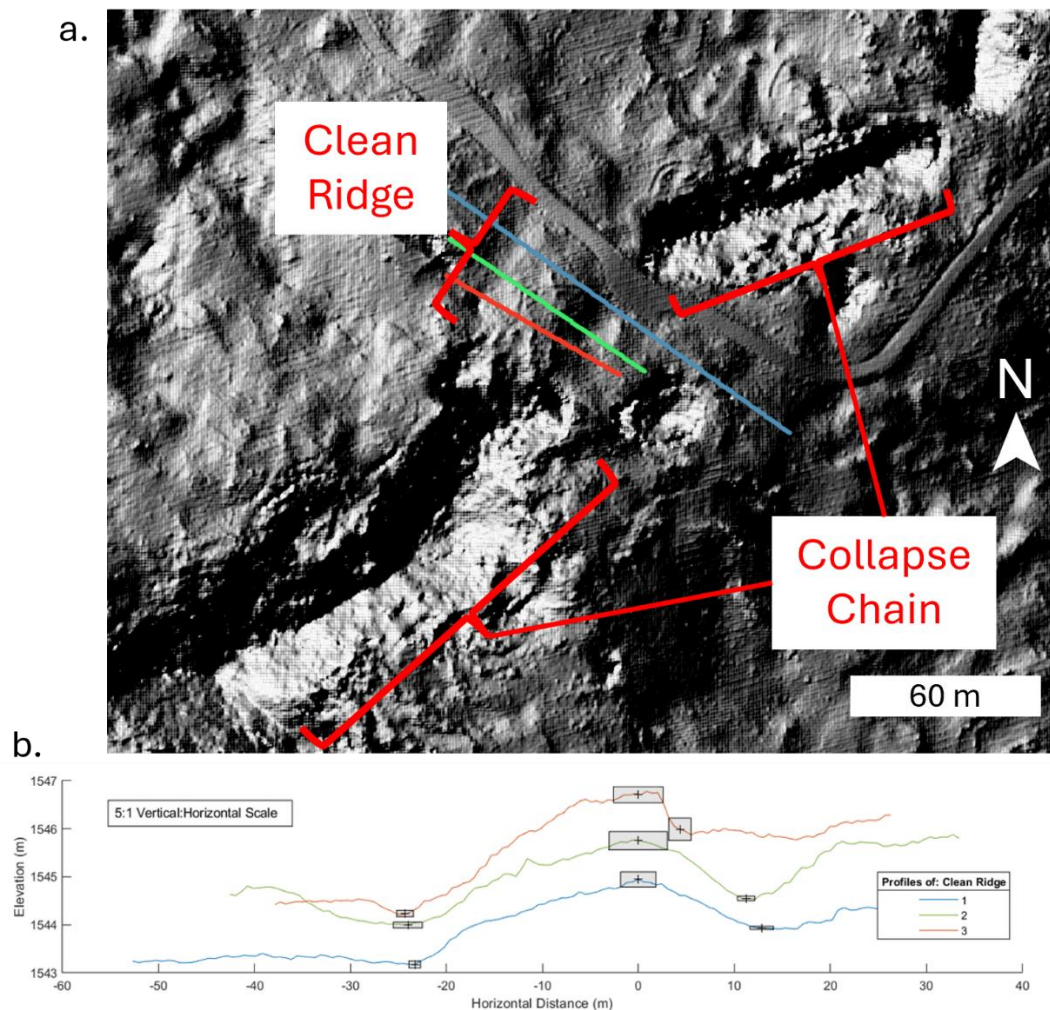


Figure 5-7: (a) Vertically exaggerated hillshade map of Clean Ridge and its surroundings, and (b) elevation profiles across Clean Ridge. Image center: -121.519° , 41.702° . The three colored lines over Clean Ridge in (a.) are the traces of the profiles in (b.)

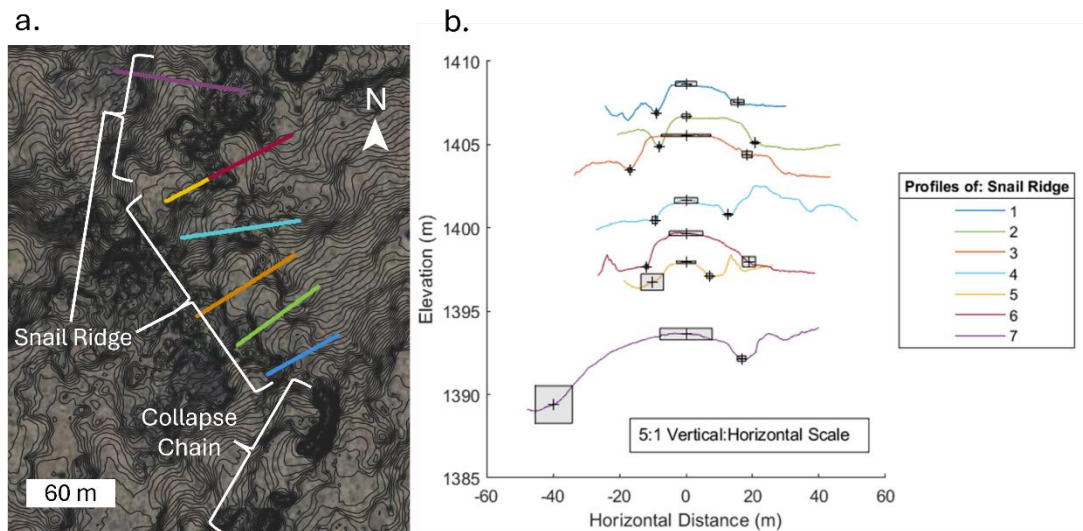


Figure 5-8: (a.) Surface imagery with 0.25 m elevation contours for Snail Ridge and its surroundings, and (b.) elevation profiles across Snail Ridge. Image center: -121.556° , 41.748° . NAIP imagery data (EROS, 2017) visualized through QGIS. The seven colored lines over Snail Ridge in (a.) are the traces of the profiles in (b.)

Snail Ridge is significantly longer than Clean Ridge, resulting in considerable variation in its shape and size downslope. The ridge features a plateau-like profile with a flat top at its beginning, transitioning to a more domed profile toward the far end. The very small Profile 6 was taken across a lobe extending out from the ridge, rather than from the main body of the ridge itself. The width of the ridge increases at its end; possibly this feature was inflated but never

drained, and the final profile captures where the last buildup of lava stagnated at the end of the flow.

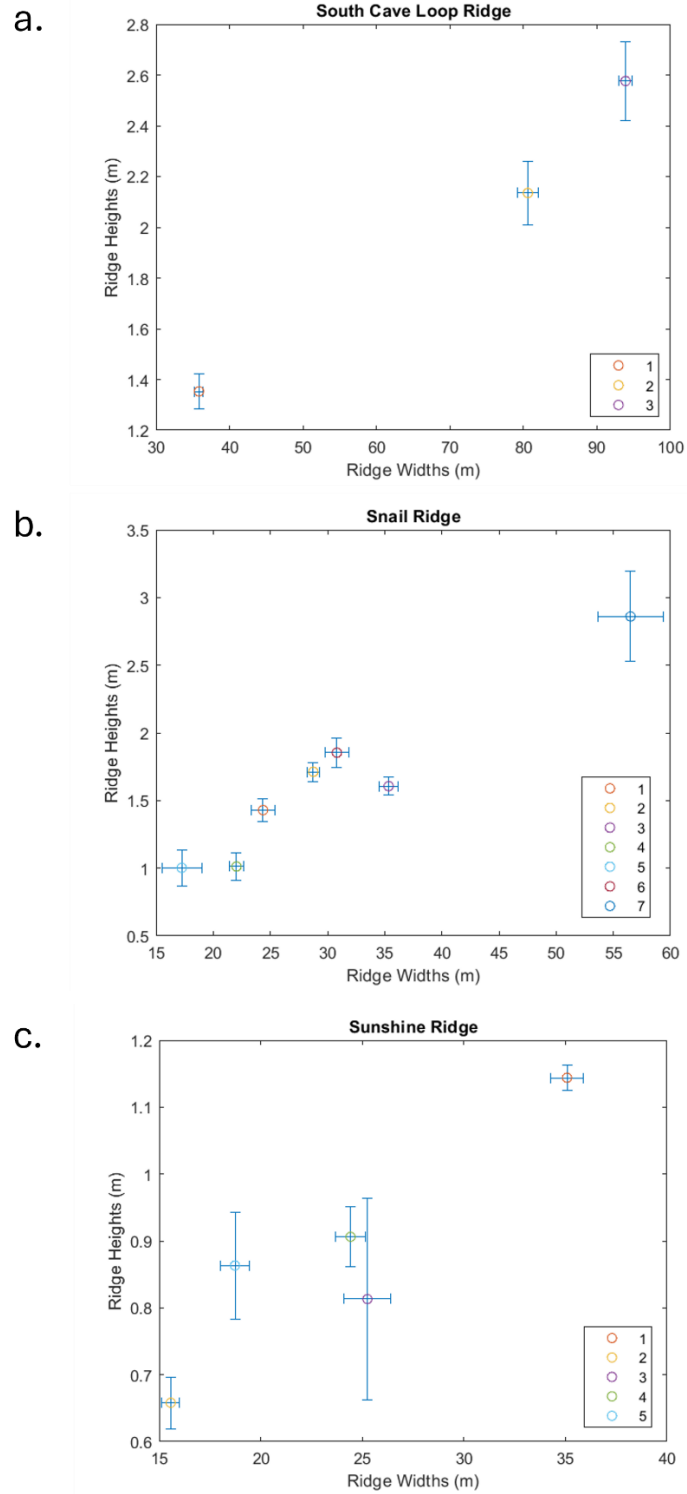


Figure 5-9: Three examples of ridges where ridge height is correlated with width along their strikes.

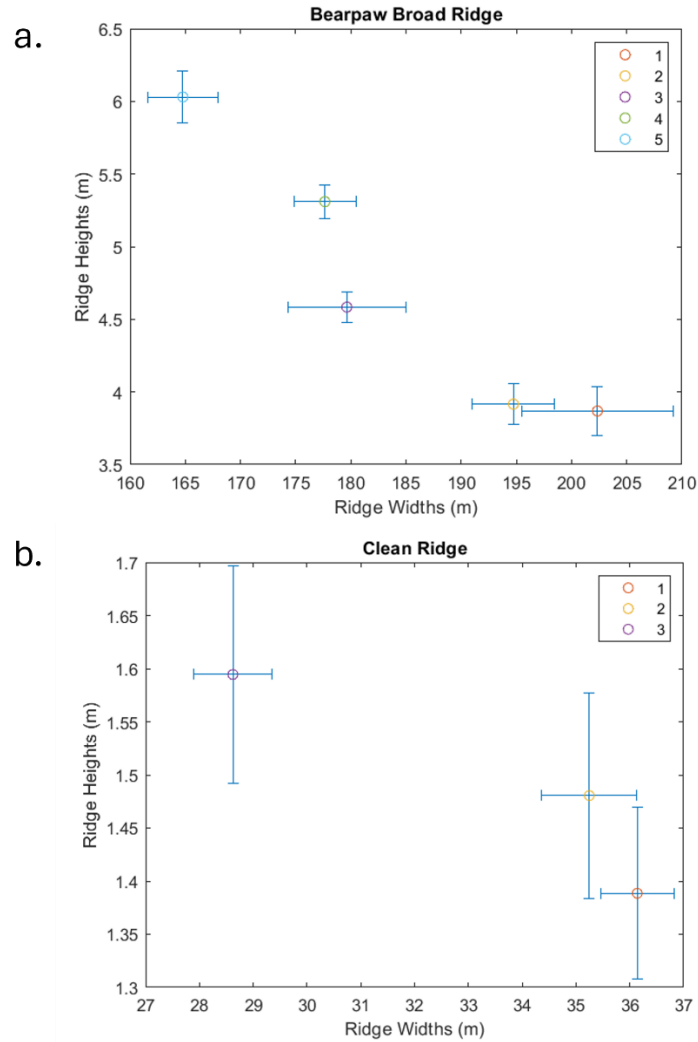


Figure 5-10: The two examples of ridges where height decreases with width.

5.4.2 Ridge Dimensions: Along-Ridge Trends

Most ridges exhibit increasing height with increasing width. Figure 5.9 provides three examples of this trend, which holds true for 16 out of 18 ridges. The only ridges to which this does not apply are Bearpaw Broad Ridge and Clean Ridge, which display the opposite trend; ridge height increases as width decreases (Figure 5.10). This may indicate that these two ridges or the underlying tube formed differently from the other LBNM ridges.

5.4.3 Ridge Dimensions: Comparison of All Ridges

We identified 18 candidate inflation ridges in the LBNM from the LiDAR data. For each one, we calculate the average width and height from the combined profiles (Figure 5.11). One ridge, Bear Paw Broad, is unusually broad and tall and underlies several others. Therefore, we treat it separately from the others. The dimensions of the other ridges vary widely; their width ranges from 19 to 104 m, with an average of 48 meters, while their heights ranges from 0.4 to 2.6 m, with an average of 1.4 meters (Table 5.1). Valentine Cave's inflation ridge falls within this range. The average aspect ratio of these ridges ranges from 0.014 to 0.059, with an average of 0.031. This is close to the ratio between the average ridge height and ridge width, 0.028. Although width and height vary, the aspect ratio has a relatively small range. The overall low aspect ratios corroborate the qualitative finding that ridges are very broad compared to their heights, making them difficult to detect without LiDAR data.

Table 5-1: Summary of dimensions for the 18 documented candidate inflation ridges. † Tube height and aspect ratio calculated assuming a minimum roof thickness $T=25$ cm. ‡ Tube height and roof thickness calculated assuming a tube aspect rate $c/W=1/3$.

Ridge Name	Coordinates	Ridge Width	Ridge Height	H/W	Ta (m)	Tube	Tube aspect	Tube	Roof
		(m)	(m)			height c †	ratio $c/W^\dagger$	height $c^\ddagger$	thickness (m) [‡]
Sunshine	-121.5191°, 41.7023°	23.8±0.36	0.88±0.04	0.037	1.58	7.2	0.30	7.9	0.21
Parking Lot	-121.5427°, 41.7413°	42.9±1.05	1.11±0.07	0.026	3.99	30.0	0.70	14.3	0.96
Snail	-121.5555°, 41.7478°	30.7 ±0.55	1.64±0.06	0.053	2.27	12.6	0.41	10.2	0.37
Trident C	-121.5449°, 41.6986°	40.6 ±1.37	0.84±0.05	0.021	2.61	15.5	0.38	13.5	0.32

Trident B	-121.5449°, 41.6986°	47.7 ±1.64	1.44±0.07	0.030	2.71	16.4	0.34	15.9	0.27
Trident A	-121.5449°, 41.6986°	53.6±2.17	1.63±0.11	0.030	3.05	19.7	0.37	17.9	0.30
Donut	-121.542°, 41.7386°	50.0 ±1.12	1.60±0.05	0.032	4.27	33.1	0.66	16.7	0.89
Clean	-121.5186°, 41.7024°	33.3 ±0.44	1.49±0.05	0.045	2.01	10.4	0.31	11.1	0.22
Near Edge	-121.4705°, 41.7209°	25.8±0.29	1.51±0.03	0.059	1.99	10.3	0.40	8.6	0.35
South Cave Loop	-121.5166°, 41.7037°	70.1±0.59	2.02±0.07	0.029	5.46	47.8	0.68	23.4	0.96
Shortlink	-121.5137°, 41.7086°	19.1±0.34	0.59±0.05	0.031	1.42	6.1	0.32	6.4	0.23

Double Collapse	-121.4891°,								
C	41.7185°	26.2±0.41	0.70±0.02	0.027	2.58	15.5	0.59	8.7	0.68
Double Collapse	-121.4891°,								
B	41.7185°	27.0±0.43	0.37±0.02	0.014	3.34	23.5	0.87	9.0	1.26
Double Collapse	-121.4891°,								
A	41.7185°	66.0±0.68	1.15±0.03	0.017	8.27	92.4	1.40	22.0	3.18
	-121.5494°,								
Bearpaw SW	41.7264°	76.7±1.06	1.18±0.06	0.015	8.05	87.1	1.14	25.6	2.36
	-121.5494°,								
Bearpaw N	41.7264°	103.7±5.11	2.19±0.13	0.021	10.65	132.6	1.28	34.6	3.02
	-121.5494°,								
Bearpaw SE	41.7264°	75.1±0.88	2.60±0.08	0.035	5.73	51.4	0.68	25.6	0.98
	-121.5494°,								
Bearpaw Broad	41.7264°	183.8±2.07	4.74±0.06	0.026	N/A	N/A	N/A	61.3	N/A

All ridges except	N/A								
Bearpaw Broad		47.8±23.6	1.35±0.59	0.031	4.12	36.0	0.64	15.9	0.97

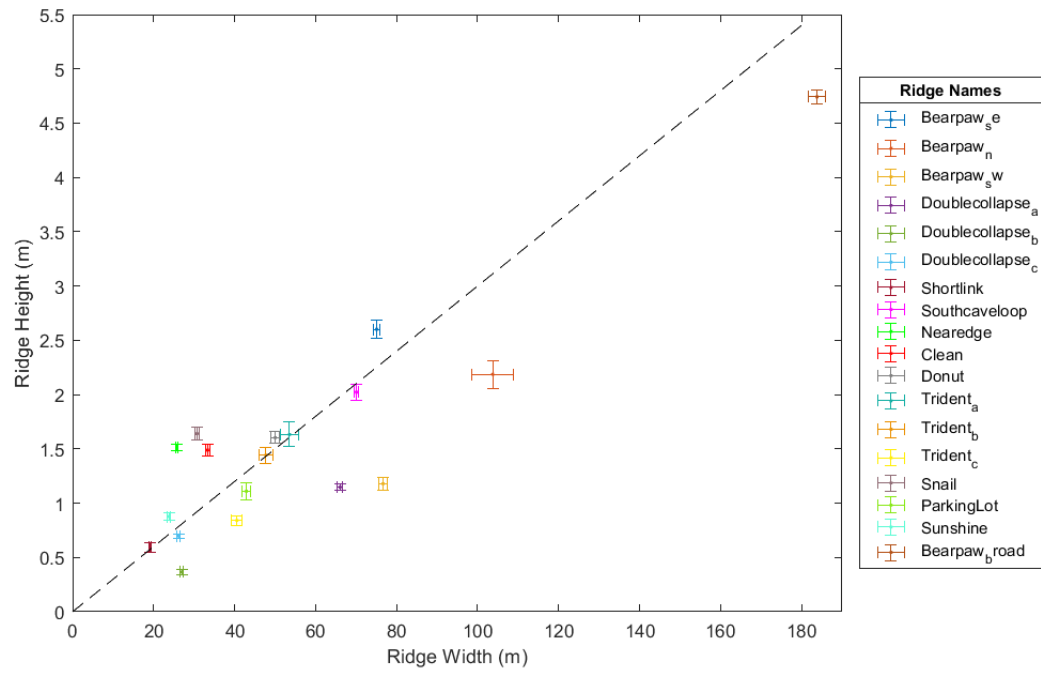


Figure 5-11: Dimensions of 18 observed ridges in the LBNM. The black dashed line shows the average ridge height-to-width ratio of 0.03.

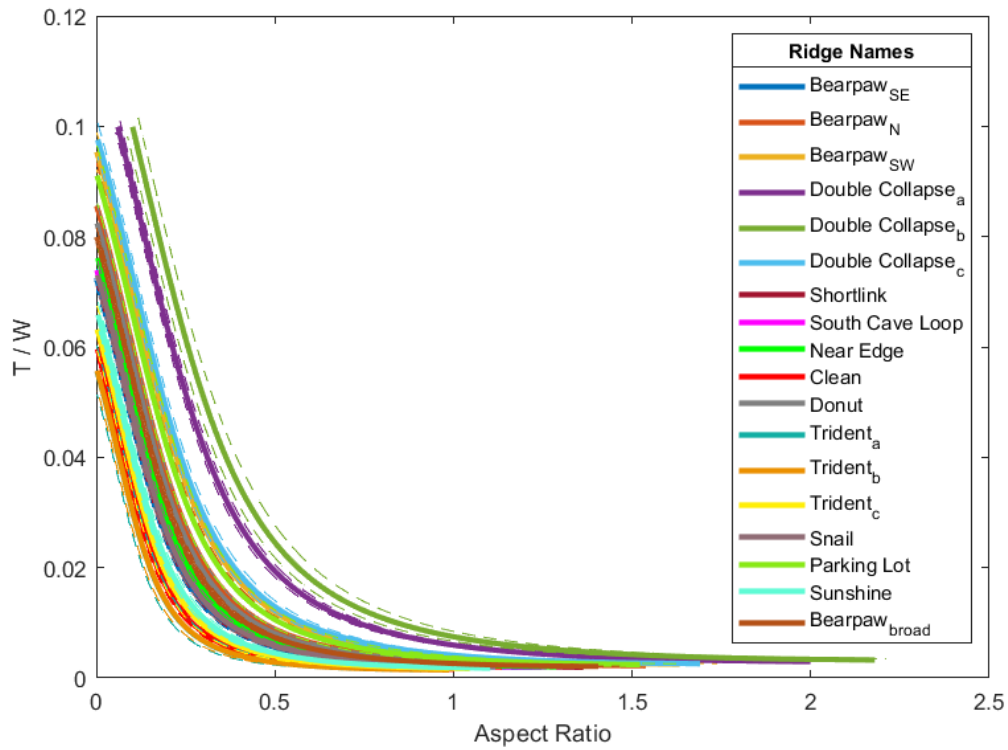


Figure 5-12: Ranges of possible tube shapes based on each ridge. Aspect Ratio is the tube height, c , divided by tube width W .

5.4.4 Tube Shape

Using the ridge dimensions shown in Figure 5.11 along with Equation 4 gives a range of possible tube shapes for each ridge. We refer to these as shape curves (Figure 5.12). Most of the shape curves fall close together and typically overlap when the result uncertainties are taken into consideration. Even the exceptionally wide and tall Bearpaw Broad Ridge follows this trend. The two exceptions are Double Collapse a and b, which lie distinctly above the others. Double Collapse c is also the highest of the other results. This indicates that the tubes belong to the same family of shape and depth, which is expected as they developed in the same flow. However, the

actual tube height and roof thickness vary significantly when the results are expressed in dimensional quantities (Figure 5.13).

Equation 5.4 can only give a range of possible roof thicknesses and tube heights or aspect ratios (Figure 5.13). Several possible values are quite unrealistic given standard information about lava flow morphology. For example, a minimum roof thickness of several meters is unlikely, even if that corresponds to a small $\frac{T}{W}$.

In Chapter 3, we found that the roof thickness of the Valentine Cave lava tube at the time of inflation was approximately 0.25 m. to help comparison, we report in Table 5.1 the tube height obtained assuming that every ridge in the LBNM formed when the roof thickness was 25 cm (Dashed black line in Figure 5.13), although we recognize that natural variations are likely.

Figure 5.13b zooms in on the low tube height and roof thickness portion of Figure 5.13a. It only shows tube heights less than 30 m, as anything larger would exceed what we expect to see for terrestrial tubes. For reference, the central height of Skull Cave, a very large tube in the LBNM, reaches a maximum of 20 m (Waters et al., 1990).

For the same roof thickness, tube height increases with increasing width; this is an expected trend from the shape of Figure 5.12. As W increases, $\frac{T}{W}$ decreases. Following a single curve on Figure 5.12, $\frac{c}{W}$ increases, and therefore c increases. As there is little differences between the curves obtained for different ridges on Figure 5.12, the trend of increasing c with W also applies to all the ridges.

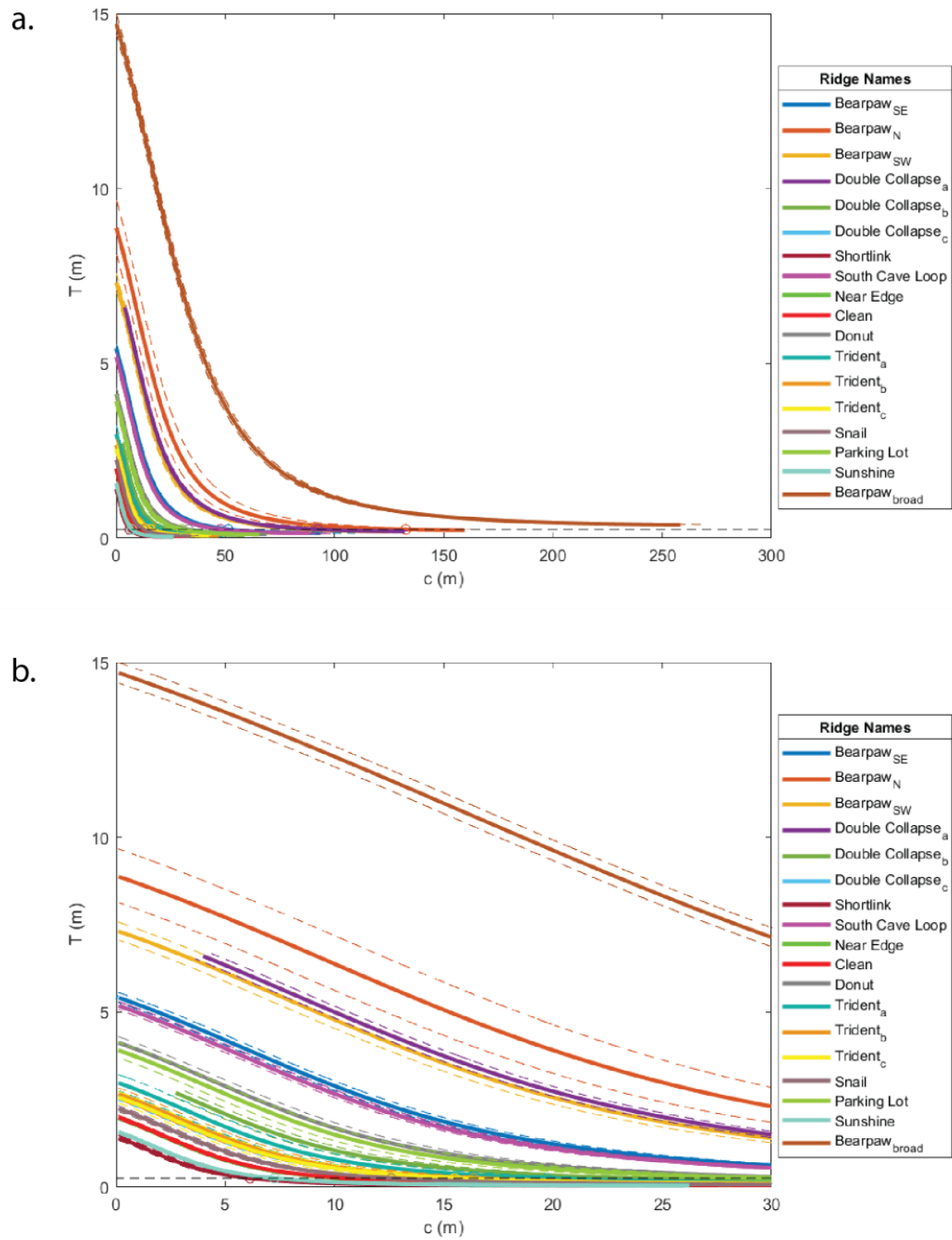


Figure 5-13: Ranges of possible tube shapes based on each ridge. (a.) shows the entire range, while (b.) shows an abbreviated range more realistic to terrestrial tubes. The black dashed line shows a roof thickness of 0.25 m.

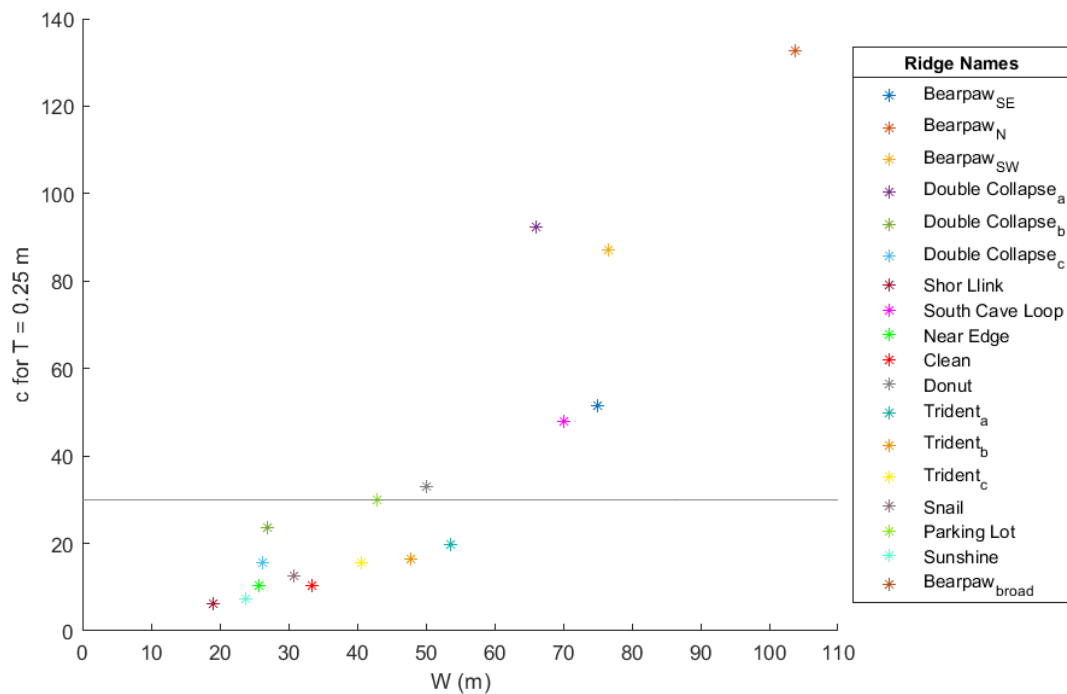


Figure 5-14: Tube heights c implied by a roof thickness T of 0.25 m of each ridge. The gray line shows $c = 30$ m, an approximate upper limit for observed lava tubes on Earth.

Most of the ridges with widths less than 60 m imply tube heights less than 30 m. These results form a cluster on Figure 5.14, with c increasing slightly with W . However, Parking Lot Ridge, Donut Ridge, South Cave Loop, Double Collapse A, and all Bearpaw Ridges require a tube height of 30 m or more if we assume a roof thickness of 0.25 m (Figure 5.14). For tubes wider than 60 m, the correlation between c and W has a steeper slope and the points are less closely clustered. The cluster of smaller tubes is closer to the dimensions we typically expect for terrestrial lava tubes, and may be more likely to be actual tubes instead of another inflation feature. The trend of tube height to tube width for these small ridges is close to $1/3$. This is significant because a $1/3$ height to width ratio is a good approximation for the overall shape of

terrestrial tubes, and is sometimes used as an idealized terrestrial lava tube shape in the literature (Blair et al., 2017; Theinat et al., 2020).

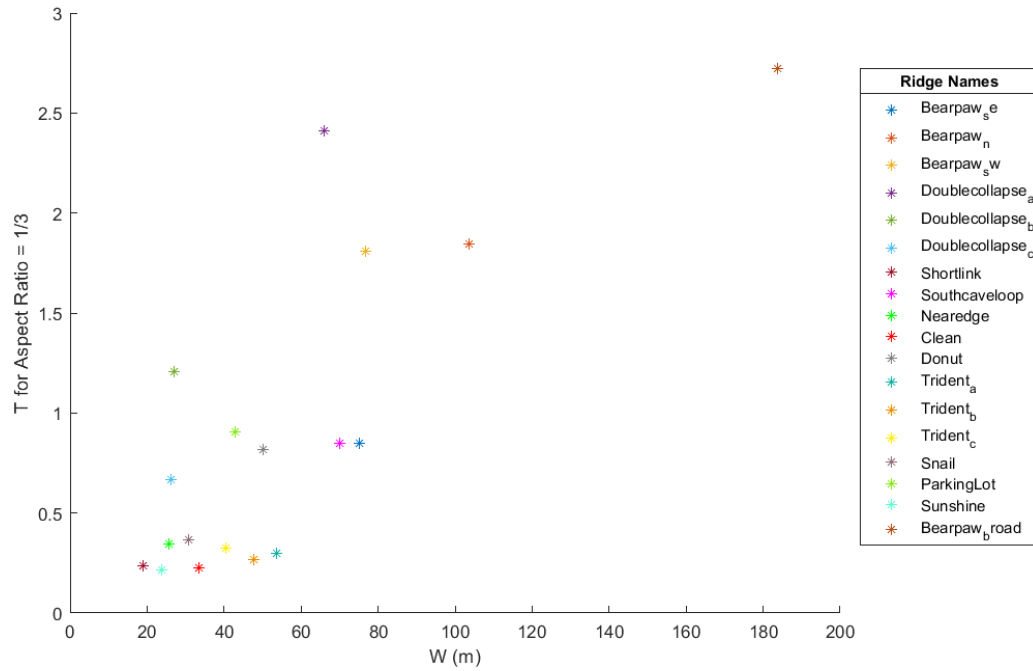


Figure 5-15: Roof thicknesses that an aspect ratio of 1:3 implies for each ridge.

If, instead of fixing roof thickness, we impose a 1:3 aspect ratio, it becomes possible to infer a minimum roof thickness for each studied ridge (Figure 5.15). Approximately half of the ridges cluster together at low T values. This tight cluster requires a roof thickness of about 0.3 m, regardless of width. Another four points correspond to a roof thickness of approximately 0.9 m, and only a small number of wide ridges demand roof thicknesses higher than 1.5 m.

5.5 *Discussion*

5.5.1 Along-Ridge Trends

Typically, ridge height and width are correlated along a single ridge (Figure 5.9). If tube width serves as a first proxy for the tube's overall dimensions, this relationship may suggest that a wide tube also has a thinner, more flexible roof. This would be consistent with the idea that progressive cooling simultaneously narrows the tube and thickens the roof. Alternatively, the tube may be wide because of high lava pressure, which not only results in a tall ridge but also forces rapid lava flow that erodes the tube walls both mechanically and thermally. In either interpretation, the relationship between tube height and width reflects the formation and evolution of the tube.

Bearpaw Broad and Clean are the only ridges that do not have increasing width with height along the ridge (Figure 5.10). There may have been another factor at play when these two ridges were forming; for instance, if the flow that created Clean Ridge was forced into a narrow pre-existing topographic feature, the tube's width may have been limited but pressure increase at the constriction, resulting in a taller ridge even as it became narrower. Since Bearpaw Broad is much wider than the other ridges, it could be an unrelated feature and not caused by inflation.

5.5.2 Comparison of All Ridges

The height-to-width ratio of all observed ridges remains relatively constant, even as the size of the tubes varies dramatically (Figure 5.11). This suggests that all the ridges formed in approximately the same way, and with approximately the same conditions.

The shape curves we deduce for concealed lava tubes below inflation ridges are consistent across the LBNM (Figure 5.12). Almost all of the curves cluster together and often

overlap one another. There is no trend in the $\frac{T}{W}$ vs tube aspect ratio $\frac{c}{W}$ shape curves based on ridge size; small and large ridges are mixed together in the same spectrum of results. This implies that even tubes of greatly different sizes have similar shapes. If a tube has an aspect ratio of 0.5, it likely has a $\frac{T}{W}$ ratio between 0.002 and 0.014. This suggests that all ridges result from the same kind of inflation process with the same general conditions, regardless of size.

The exception to this rule is Double Collapse Ridge. The three highest shape curves are for Double Collapse Ridges a, b, and c, with two of these markedly separated from the rest of the ridges. This suggests that the shape of Double Collapse Ridge differs from that of the other ridges. As all three sections of Double Collapse Ridge have higher shape curves than the others, this implies that this is a consistent characteristic of this ridge overall, rather than a measurement error in one specific ridge segment. Double Collapse Ridge may be caused by a different formation mechanism or formation conditions. For instance, it may have formed in a depression that allowed it to expand vertically but not laterally, leading to an increase in c without changing W . This ridge may also have been older than the others at the time of inflation, resulting in a thicker roof. It could be that the downward erosion explaining the higher aspect ratio of this tube led to a gap between the lava level and the roof, allowing the roof to cool more than in other tubes before inflation took place.

5.5.3 Asymptotic Behavior of Shape Curves

The asymptotic behavior of the shape curves (Figures 5.12 and 5.13) is that the aspect ratio becomes infinite when roof thickness approaches zero. Our MATLAB code solving Equation 5.4 cannot find a solution when the aspect ratio is too high. This asymptotic behavior is physically reasonable, as a tube with roof thickness of zero would be physically impossible and

unsolvable. At the other end of the shape curves, when the aspect ratio approaches zero, the roof thickness remains bounded. A tube with a tube height of zero would be a perfectly flat plane buried underground, with all of the internal pressure directed only upwards and not outwards. This problem is solvable and becomes effectively the problem of roof thickness for a simple deforming plate, T_a , as described by Equation 5.2. In the limit $c \rightarrow 0$, Equation 5.3 becomes $T_i^3 = T^3$ and Equation 5.4 becomes $T^3 = T_a^3(1 + 46 T_a^3/W^3)$. This equation approximately simplifies to $T = T_a$ as long as T is significantly smaller than W . Equation 5.2 also leads to

$$\frac{T_a^3}{W^3} = \frac{P(1 - \nu^2)}{32E} \frac{W}{H} \approx \frac{P(1 - \nu^2)}{E} \quad (5.6)$$

where we take advantage of the observation that $\frac{H}{W} \sim 0.03$ (Table 5.1). Therefore, $\frac{T_a}{W}$ can be safely neglected as long as the lava pressure is much less than the Young's modulus of the flow, which is a reasonable assumption.

5.5.4 Additional Constraints on Tube Shape

Given surface ridge dimensions, Equation 5.4 provides a range of possible tube shapes as presented above. However, it cannot yield a unique solution. Any information regarding the tube's actual shape helps better constrain the results presented above and offers a more precise idea of the tube's morphology. Table 1 includes results assuming either a roof thickness of 25 cm or a tube aspect ratio of 1/3. In the cases of Bearpaw and Sunshine, we can add constraints from direct observations.

If we assume a roof thickness at the time of inflation of 0.25 m, eight out of the eighteen ridges require a tube height above thirty meters (Figure 5.14). This is very large compared to what we typically observe in terrestrial tubes. This suggests that this assumed roof thickness may

not be representative of all the LBNM lava tubes. These eight tubes may have had older, thicker roofs at the time of inflation. Alternatively, these ridges may be caused by another mechanism.

Twelve of the ridges have a width of less than 60 m. Such widths, while reasonable, are large compared to known terrestrial tubes. However, these measurements represent tube width at the time of inflation, which may differ from the width when the tube drains. For these smaller tubes, assuming our preferred roof thickness value of 0.25 m implies reasonable tube heights and a tube aspect ratio close to 1/3, which is typical of accessible terrestrial tubes. These ridges seem to exhibit many characteristics consistent with what we expect to see in typical terrestrial tubes. Assuming the expected aspect ratio of 1/3 yields a roof thickness of 0.3 m for small-width tubes, regardless of width (Figure 5.15). This is close to our preferred value of roof thickness from previous work. However, a few tubes appear to have a roof thickness close to 1 meter, which might indicate inflation later in the tube's history. For comparison, the thickness of the Valentine Cave roof is currently 1 meter, although we estimated earlier that the ridge formed when the tube roof was significantly thinner.

We propose that these small ridges are the most representative of actual lava tubes, instead of any other kind of inflation feature. If we assume the expected aspect ratio of a lava tube, it requires a roof thickness that agrees with previously found values (Chapter 3). This implies that these are lava tubes, that their aspect ratio is close to 1:3 as we expect, and that the previously deduced roof thickness of 0.25 is a good approximation of their roof thickness at the time of inflation.

5.5.5 Bearpaw

Bearpaw Cave is a known tube connecting two collapse pits in a collapse chain. Four of our ridges are situated above or near it. Bearpaw North, Southeast, and Southwest are all located

within the same discontinuity of the collapse chain; although Southeast and Southwest do not appear to follow the path of the cave, Bearpaw North lies partially over the cave (Figure 5.16). Bearpaw Southwest and Southeast may represent two thick flows from the Southern collapse, partially covering a lava tube inflation ridge that is only visible as Bearpaw North. Bearpaw Broad underlies the entire area. Bearpaw Cave was mapped by Waters et al. (1990). While we do not have a LiDAR scan to provide its precise internal dimensions, the map includes some information about its shape.

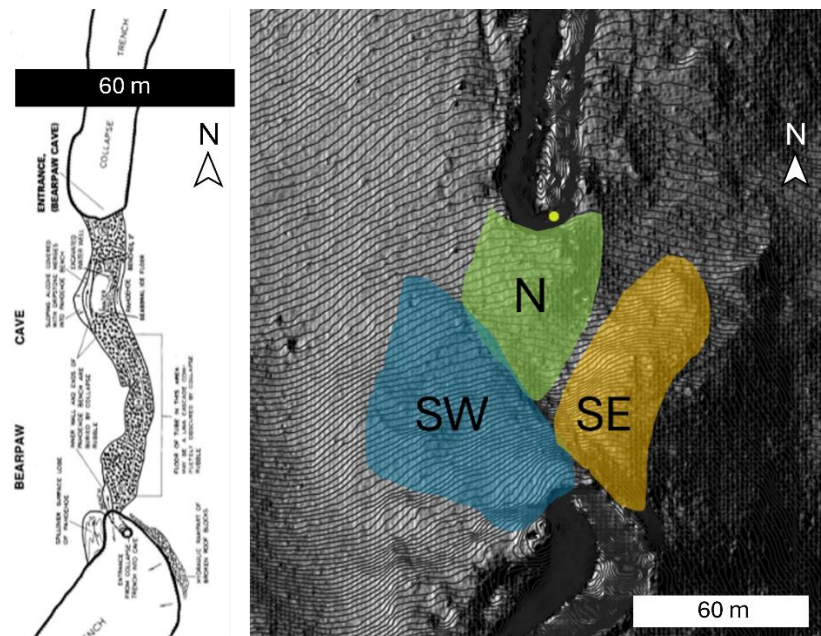


Figure 5-16: Comparison with the mapped Bearpaw Cave, left (modified from Waters et al., 1990), and the three ridges in the LiDAR dataset covering the same collapse chain discontinuity, right. Image center (both images): -121.549° , 41.727° .

Waters et al. (1990) reported tube heights of 6.7 m, 4.3 m, and 7.6 m at various points along it. We disregard the value of 18.3 m, as this measurement was taken very close to where the floor had been intentionally excavated in the past and may artificially exaggerate the natural

tube height. This results in an average tube height of 6.2 m. Estimating the width of the tube from the scale of the map provides an average tube width of approximately 13 m. The resulting aspect ratio is 0.48. Assuming that the tube's aspect ratio was the same at the time of inflation as it is now, we apply this to the shape curve of Bearpaw North Ridge. We limit this analysis to Bearpaw North because it overlies part of the tube. This gives a $\frac{T}{W}$ ratio of 0.009. Using the current width of Bearpaw North Ridge to estimate the tube width during inflation, the roof thickness at that time must have been 0.93 m. The inferred tube is about eight times larger than the observed cave. This analysis implies that lava tubes undergo significant modification after inflation. This finding is consistent with our analysis of Valentine Cave in Chapter 3, where the roof thickness at the time of inflation was only one quarter of the current roof thickness. Additionally, the width of this particular ridge may be overestimated because it sits atop the wider Bearpaw Broad Ridge and distinguishing the respective edges of these two features is difficult (See 5.5.7). As such, the difference between the current size of Bearpaw Cave and its inferred size during inflation may not be as much as eight times.

5.5.6 Sunshine Ridge

Of the multiple inflation ridges documented here, Sunshine Ridge is the one associated with the most promising tube candidate. Sunshine Cave is a lava tube mapped out by Waters et al. (1990), stretching approximately 150 m. Our topographic data shows an apparent ridge at the point where the mapped portion of Sunshine Cave ends, extending to the northeast (Figure 5.17). This ridge is 23.8 ± 0.4 m wide and 0.88 ± 0.04 m high. Adding to the interest of this location, the mappers inside Sunshine Cave noted that it leads into a large void space, although the opening leading into it is too small to allow human access or further exploration. We confirmed this ourselves when visiting the site. Based on the limited observations possible through the

narrow opening, it appears that it leads to a relatively large space, likely wider than Sunshine at that point. Sunshine Cave may therefore continue into a large tube that is reflected on the surface as the inflation ridge. The combination of the observed ridge and the confirmed void space at almost exactly the same location made this our most promising candidate site.

As part of a GEODES team field expedition in the summer of 2024, we had the opportunity to travel to the LBNM and conduct fieldwork at one site, with the aim of discovering a new tube and constraining its dimensions. We selected the area near Sunshine Cave to determine whether the known cave extends into a previously unknown one.

The GEODES team conducted ground-penetrating radar (GPR) scans, a magnetometry survey, and a by-eye surface survey of the area above Sunshine Cave and our proposed unexplored tube (Figure 5.17). We used 80 MHz and 450 MHz GPR to take scans intersecting where we knew Sunshine Cave to be and where we suspected the undiscovered tube would be. The magnetometry and surface surveys similarly covered the known extent of Sunshine Cave, the expected location of the other tube, and some of the surrounding area. We also used a hand-held inertial measurement unit (IMU) while walking the full explorable extent of Sunshine Cave to record its path underground.

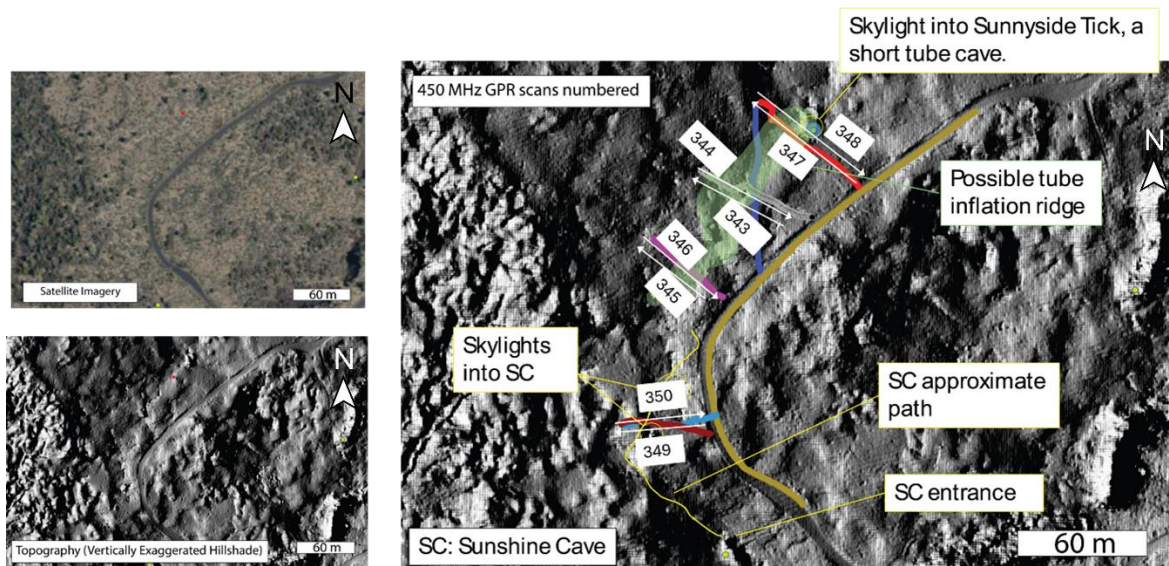


Figure 5-17: Summary image of the surface near Sunshine Cave and where we took GPR data to survey the subsurface. Image center: -121.519, 41.704. NAIP imagery data (EROS, 2017) visualized through QGIS.

The GPR 450 MHz scans reveal features of the subsurface to a depth of approximately 5 meters. We expect that a tube signature in this data would exhibit some or all of the following characteristics: a strong arcuate reflector from the roof, a weak reflectance zone from the void space below the roof, a floor reflector weaker than that of the roof, the roof reflector aligned with a nearby boundary layer, and other boundary layers abruptly stopping when intersecting the tube (layers cannot continue into the void space). Noise in the data, along with attenuation and reflectance from other nearby features, can make tube signatures harder to distinguish, so we do not expect all of these characteristics to appear in every case. Also, the tube floor may be at a greater depth than the 5-meter limit of our GPR dataset. The strong arch-shaped roof reflector is a key indicator of a tube and can be more clearly visible than the others. Therefore, this is the

characteristic we focus on most in our data. We avoid arch shapes that are too steep, as they could be artifacts from point reflectors instead of reflections from an arch-shaped surface.

We detect possible tube signatures meeting some of these requirements in several locations within our data. There are apparent arch-shaped reflectors at approximately 0.7 m depth in Scans 346 and 343, both of which intersect the proposed tube (Figure 5.18). The reflectors in Scan 346 are situated over weak reflectance zones, another indicator of tube presence. These characteristics can be ambiguous in many cases, and detecting lava tubes in GPR data is fundamentally challenging due to various factors obscuring the signal, making these signatures primarily indicative of tubes. Nevertheless, we still consider them to be tentatively promising indicators of tube presence.

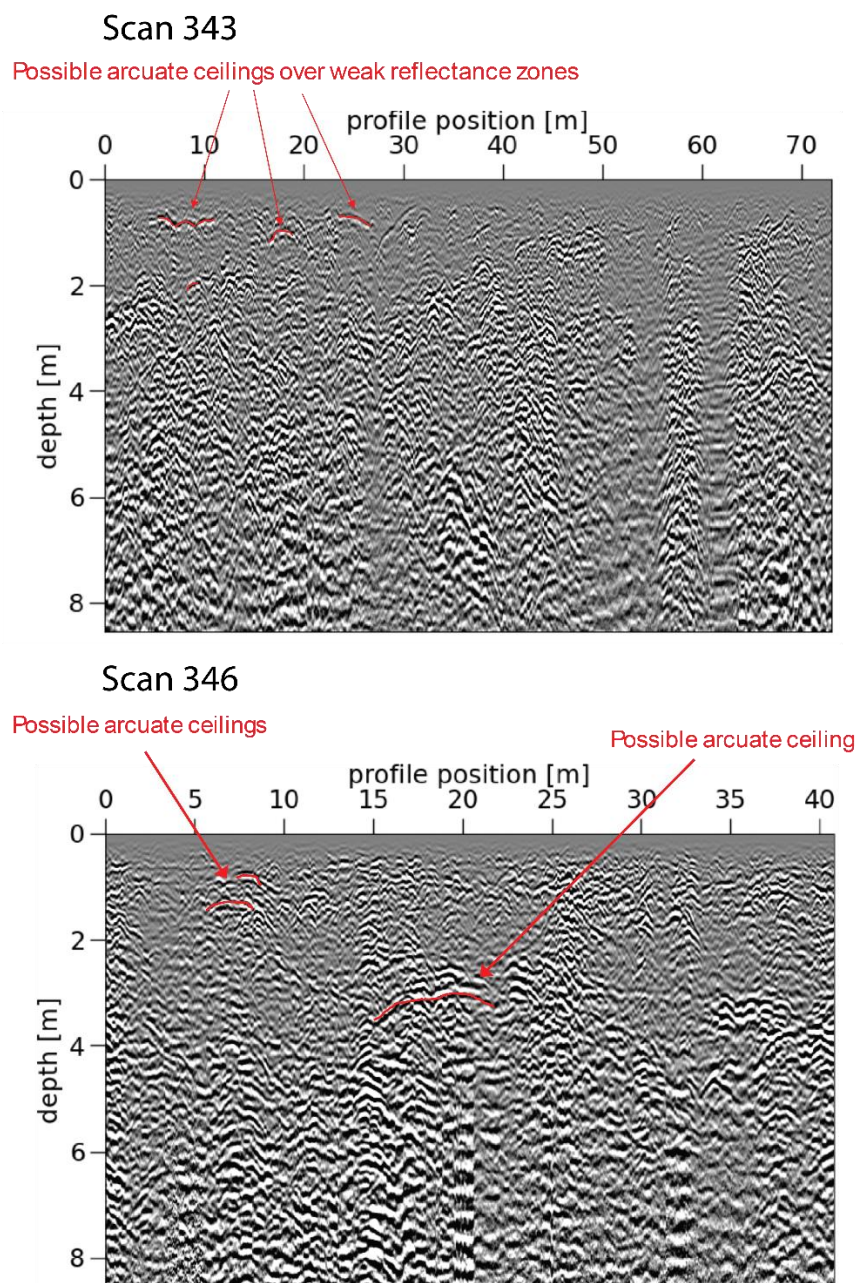


Figure 5-18: Two 450 MHz GPR scans above the candidate tube near Sunshine Cave. Possible lava tube signatures are marked.

Using the cooling relationships discussed in Chapter 3 with the current roof thickness, we find a maximum effective roof thickness at the time of inflation of Sunshine Ridge of 0.3 m. By applying this constraint to the shape curve we found for this ridge, we determine that the height

of Sunshine Cave must have been 6.5 m or greater at the time of inflation. This establishes a minimum aspect ratio of 0.27 (Figure 5.19).

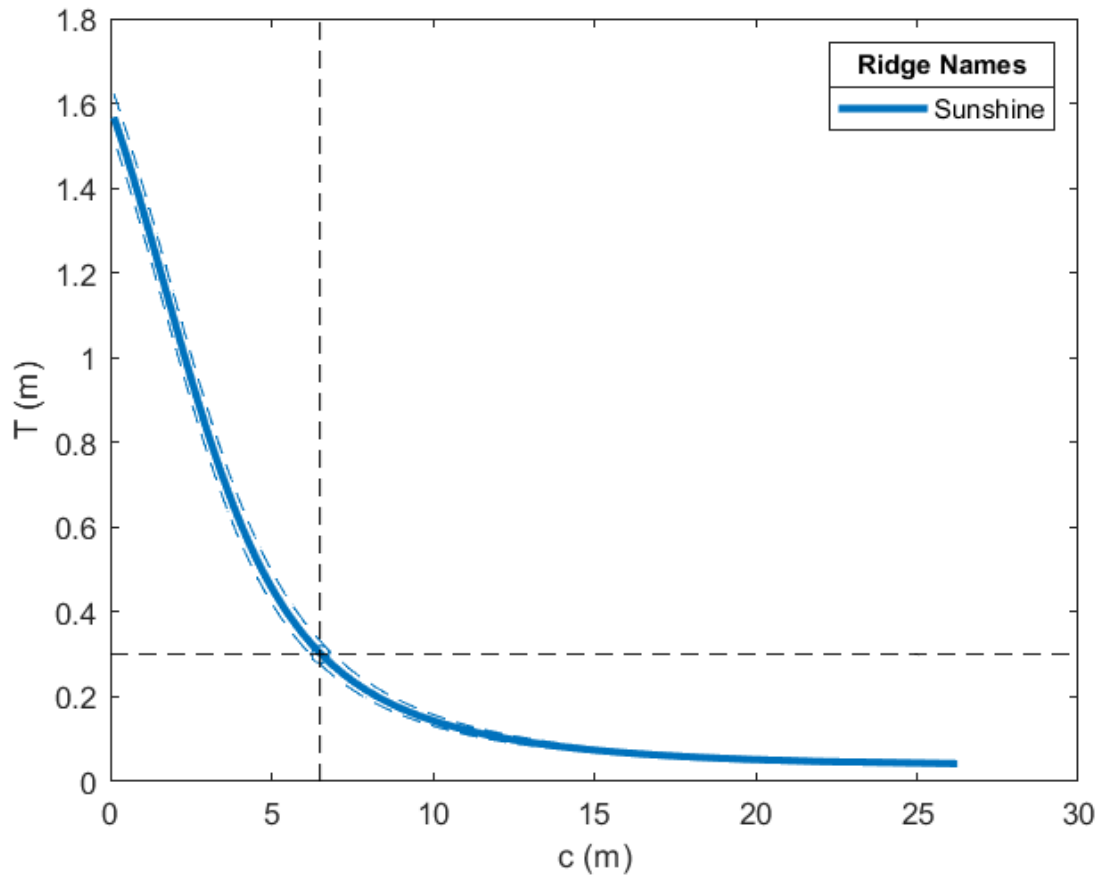


Figure 5-19: Shape curve for Sunshine Cave. The horizontal black line shows the upper limit on roof thickness of 0.3 m, deduced from GPR data. The vertical black line shows the corresponding minimum ridge height of 6.5 m.

Our strongest tube candidate signature is at 25 m in Scan 350. It features a strong arcuate roof reflector, a weak reflection zone underneath it, a nearby boundary layer aligned with the roof, other boundary layers that terminate when intersecting the possible void space, and even a possible floor reflector. This scan intersects the known path of Sunshine Cave. However, this

signature is approximately 15 m to the East of the cave. We therefore may have detected an unexpected void space parallel to Sunshine Cave. Additionally, the lack of such a signature at Sunshine Cave itself demonstrates how easily tube signatures can be lost or obscured in GPR data; this is a location where we are certain there is a tube, yet no clear signature is present.

During the surface survey, we noticed an opening on the surface approximately 0.4 m wide (Figure 5.20). This hole emitted air at a temperature unrelated to the outside environment; while the outside temperature at the field site was warm, we felt cold air near and inside the hole. This suggests that the hole is connected to a larger void space that is deep enough to remain thermally stable. The hole may therefore link to our suspected lava tube.



Figure 5-20: Hole found during a surface survey above the candidate tube near Sunshine Cave.

Image taken by Edward Williams, 2024.

5.5.7 Sources of Error in Ridge Dataset

Searching for ridges in the LBNM LiDAR dataset presents several complicating factors that hinder confident identification of a ridge's dimensions. The surrounding and underlying topography often complicates the definition of the ridge. Nearby topographic features can obscure parts of the ridge, making it unclear which topographic signatures belong to the ridge and which are unrelated, thus posing challenges in identifying the ridge dimensions.

The Bearpaw ridges present a particularly strong illustration of these difficulties. Bearpaw Broad, with a width of almost 200 m, underlies the entire area. Smaller ridges on the

surface merge smoothly into Bearpaw Broad in some instances without a single sharp change in slope. The edges of these ridges can be very hard to determine. The width estimates of Bearpaws Southwest, Southeast, and North probably include some of the width of Bearpaw Broad by mistake, inflating their values.

Ridges may have been modified by processes after they formed, such as erosion or having new flows form on top of them. Some may also have been influenced by underlying topography. We expect ridges to be relatively symmetrical, but some are not, implying that for example erosion may have worn away one side of the ridge preferentially or that the ground tilted after ridge formation.

In all cases, we attempted to draw topographic profiles that would cross the ridge perpendicularly. If these were not exactly perpendicular, the widths could be exaggerated. Ridge height is less impacted by taking profiles at an angle to the ridge's cross-section plane.

5.5.8 Ridge Shape Change over Time

These findings reflect the tube dimensions at the time of inflation, not the present. A tube's dimensions likely change after forming a ridge. If the tube continues to cool from the outside inwards while active, more lava could solidify onto the walls over time, causing the tube shape to become smaller and the roof thickness to increase. We expect the tube to cool inwards at an approximately equal rate from all directions, so the aspect ratio would remain relatively unaffected by the change in tube size.

5.5.9 Links with lava tubes

Some ridges we document are wider than any known terrestrial lava tubes. However, even if these ridges are too big to realistically be above empty lava tubes, they could be caused

by inflation features that never drain. A lava tube can form an inflation ridge even if the tube does not drain, and while a very wide lava tube would collapse if emptied, a very wide lava conduit that remains filled would be stable. For ridges such as Snail where we do not see skylights or collapses, there is no evidence that the candidate lava tube drained at any point.

We assume that all topographic ridges are caused by inflated lava tubes. However, some could be from other features. In cases such as Snail, where the ridge does not connect to skylights or collapses downstream, this could indicate a thick lava flow that is not inflated, causing the topographic high instead of an inflated tube. Even if all the ridges discussed in this Chapter are caused by inflated lava conduits, they may not all lie over lava tubes; some may be closer to inflated flows. However, since our equation does not specify a tube shape, it should still apply to a wide inflated flow as well as it does to a narrow lava tube. Lava plateaus, however, would not be captured by our model. The roof of these plateaus is supported isostatically and does not form a well-defined ridge. Although it is tempting to refer to the broadest features we've seen, such as Broadpaw Broad, as lava plateaus, we note that they exhibit a peaked topography that can be described as a ridge. This is a complication that would not be expected in a plateau.

5.6 Conclusions

We have identified 18 candidate inflation ridges across Lava Beds National Monument. We applied our inflation relationships to these ridges to determine the possible shapes of the lava tubes that may have formed them. All candidate tubes exhibit a similar range of shapes, except for Double Collapse Ridge. This suggests that, with the exception of this tube, all others formed

in approximately the same manner. The tube inferred underneath Double Collapse may have had an older, thicker roof or experienced different formation conditions.

If we assume a general aspect ratio of 1:3 for all tubes, we can estimate their roof thicknesses at the time of inflation; most small tubes then agree on a roof thickness of 0.3 m. This value is close to our preferred roof thickness values from Chapter 3 and supports it as a valid estimate.

Bearpaw Cave has been mapped, and we can apply its observed aspect ratio to narrow down the range of possible tube shapes that could create Bearpaw North Ridge. Assuming that the aspect ratio is the same now as it was during inflation, we can thus estimate that the roof thickness of Bearpaw Cave must have been 0.9 m when it inflated.

GPR data over Sunshine Cave gives a possible current roof thickness. From this, we can estimate that the height of the tube must have been 6.5 m or more, and the aspect ratio 0.27 or more, when it was inflated.

Chapter 6 Summary and Outlook

6.1 *Overview*

Lava tubes are features of interest on Earth and other bodies. It is difficult to determine any information about their shapes, sizes, or internal conditions without exploring inside them. We have found two new methods to find lava tubes and determine their dimensions and morphologies, only requiring observations of surface features. These methods do not require designated geophysical instruments, instead relying on surface topography and imagery. The two techniques are independent and rely on different types of surface feature: cracks in one case, and topographic ridges in the other. We have used these methods to investigate lava tubes on Earth and the Moon, and present them as novel tools for future lava tube study.

6.1.1 Surface Cracking Reveals the Dimensions of Lunar Lava Tubes

Through numerical modeling of large lunar tubes, we discovered that they should create visible surface cracks. The locations of these cracks reflect the dimensions of the tubes by a relationship we recorded. Cracking patterns near a tube can therefore be used not only to find the tube but to constrain its dimensions. Cracks like this near lunar tubes have not been specifically looked for or studied before. This is therefore a novel technique to find lava tubes on the Moon and their dimensions.

Applying these methods to actual lunar data, we have found three new candidate tube sites. These are in Rima Sharp, Rima Mairan, and Rima Marius. Aligned depressions in the case of Rima Mairan, and discontinuities in the rille paths of Rimae Sharp and Marius, support that these are lava tubes. We deduce that the tube in Rima Mairan is approximately 1.5 km wide, the

tube in Rima Sharp is between 1.5 and 2.0 km wide, and the tube in Rima Marius is 0.32 km wide.

6.1.2 Cracking Inside Lunar Lava Tubes

Large lava tubes under lunar conditions would develop large regions of failure inside them. Most of this failure is tensile floor cracking, but tensile roof failure and compressive wall failure would also occur. This would significantly impact the usability of these caves for exploration or habitation, creating fields of fissures and fallen debris that would interrupt the tube floor. Large tubes would only have a small region of pristine floor, unaltered by cracks or debris, on each side near the tube wall. The sizes of these usable regions depend on the dimensions of the tube. We estimate that the candidate lava tubes in Rima Mairan and Sharp would have usable regions no larger than 130 m across. The tube in Rima Marius would have one 320 m wide usable floor region, as it is too small for this internal failure to occur. These same methods could be vital for any future missions to explore or utilize lunar lava tubes, informing mission planning how much of the tube floor will be safe and how difficult it will be to access.

6.1.3 An Inflation Ridge Reveals the Formation History of Valentine Cave

We numerically modeled the inflation of Valentine Cave to find what combination of formation conditions can produce the ridge that is observed above it. The height of an inflation ridge depends linearly on pressure and Young's modulus, but nonlinearly on roof thickness. We quantified this dependence on roof thickness and produced the equation in Chapter 3. This equation relates all these parameters, so constraints on one can better constrain the others. Therefore, the current roof thickness, realistic ranges of Young's moduli, pressure estimated

from pressure head, and the observed ridge height all help us constrain the tube's conditions during inflation. We found that inflation can realistically produce Valentine Cave's ridge, but only if it occurred within the first few hundred hours after the roof formed when the rock of the roof was relatively thin and soft. We apply this relationship to two other inflation examples and find similar results; in all cases, inflation must have occurred within the first 400 hours of roof formation.

6.1.4 Inflation Ridges as a Tool to Constrain Lava Tube Shapes

The relationship between ridge height and tube roof thickness in Chapter 3 was found for Valentine Cave, and assumes Valentine Cave's specific shape. This limits its wider applicability, and means that while it can give insights on roof thickness it cannot inform other aspects of tube shape like tube height.

We developed a new equation to predict the ridge height of any inflated tube of any shape, given in Chapter 4. This equation was found empirically from a combination of numerical and analytical models but has units that agree, even from the empirically found components, supporting its validity. This equation is a potentially powerful tool to learn about lava tubes from their surface expressions. From observations of a lava tube ridge, this equation produces a shape curve of possible shapes the lava tube could have. This method is widely applicable, and could be used to study inaccessible tubes on Earth or tubes on other planets and moons. Additionally, this technique can also be applied to tubes that have not drained. Applying this method could give useful insights into specific tubes or patterns across flow fields. If any other data is available from other sources, such as roof thickness estimates from skylights, this can be inputted into our shape curves to further constrain other aspects of tube shape.

6.1.5 Inflation Ridges and Candidate Tubes in the Lava Beds National Monument

We apply our inflation relationships to ridges in the Lava Beds National Monument. To accomplish this we study aerial LiDAR topography data over a wide area of the park. From this data, informed by the locations of known tubes and collapse chains, we identify eighteen candidate lava tube inflation ridges. We calculate the width and heights of these ridges from this dataset. Applying our inflation relations, we find shape curves for each possible ridge. Most of the candidate tubes have overall agreement in their ranges of possible shapes, falling along the same curves and with their uncertainties often overlapping. One ridge, which we name Double Collapse Ridge, has a distinctly different range of shapes from the others on all three of its segments. We propose that this tube had different formation conditions from the others; possibly it formed in a preexisting narrow depression, giving it a greater height and lesser width, or was older and had a thicker roof at the time of inflation. Otherwise, all tubes seem to have formed with similar formation mechanisms and conditions. Assuming a general aspect ratio of 3:1 for these tubes, most small tubes require approximately the same roof thickness of 0.3 m.

Two specific examples, which we name Sunshine Ridge and Bearpaw North Ridge, have additional constraints we can combine with our shape curves. Bearpaw North Ridge sits above the known and mapped Bearpaw Cave. From maps of this cave we estimate its aspect ratio, add this information to our shape curve, and find that the roof thickness of the tube must have been 0.9 m when it inflated.

Sunshine Ridge lies over a possible unexplored tube extension from Sunshine Cave. We conducted fieldwork here as part of a GEODES field expedition, including ground-penetrating radar and magnetometry. Ground-penetrating radar signatures suggest a possible lava tube roof

at a depth of approximately 0.7 m. Combining this with our shape curves, we estimate that the tube height was 6.5 m or more and the aspect ratio 0.27 or more at the time of inflation.

6.2 *Future Applications*

6.2.1 Further Terrestrial Applications for Inflation Relations

6.2.1.1 New Ridges

The OpenTopography database provides topographic datasets from regions around the world. Some areas covered are volcanic and likely include lava tubes, although not in the same abundance as the LBNM. Our inflation technique could give insights on tubes if they have inflation ridges visible in this data. Further study of the NCALM dataset for the LBNM could also reveal new ridges that we did not find.

6.2.1.2 Additional Constraints from Fieldwork

Any further constraints on the sizes, shapes, or depths of our candidate tubes would let us glean more insight from our lava tube shape curves. For instance if a ground-penetrating radar or seismic survey above one of our candidate tubes yielded a roof thickness, we could better constrain the height and aspect ratio of the tube as we did near Sunshine Cave.

6.2.1.3 Sunshine Cave Candidate Tube

We have more data from fieldwork near Sunshine Cave that we have not been able to analyze in the course of this project. This data includes more GPR scans and magnetometry surveys covering Sunshine Cave itself, the possible tube extending from it, and some of the surrounding area, as well as two inertial measurement unit paths mapping the exact path of Sunshine Cave. Analyzing this data fully could confirm if a tube is present here and give more information on its depth and dimensions.

There is an opening at the bottom of Sunshine Cave that seems to lead into this candidate tube; however, human exploration is impossible because the hole is too small to allow access. Looking through the hole as much as possible, one can see an open area much wider than the hole but cannot make out its full size or shape. The hole is approximately one meter wide and 0.3 m tall, potentially large enough for something to be passed through. Putting a camera and powerful light source, or if possible a miniature LiDAR scanner, through this opening on a long pole could give new information on this unexplored space and confirm if this is a continuing lava tube or an isolated chamber.

6.2.2 Extraterrestrial Applications for Inflation Relations

Our method of using inflation ridges to study tubes requires only topographic data, and is therefore applicable to tubes on any planetary body where this data is available. An excellent study site for future applications of this method is Mars. Clear sinuous ridges have been documented on the Eastern and Southeastern Tharsis Plains and the Western flank of Alba Mons. These are not isolated ridges but complex systems in lava flow fields. Some ridges lead directly into and from sinuous depressions. These are likely inflated Martian lava tubes that have partially collapsed (Bleacher et al., 2017; Zhao et al., 2017; Crown et al., 2022).

These ridges would be an ideal site to apply the inflation ridge relationships we have developed. These features are much clearer than the ridges we have studied on Earth. This is partially due to their much larger size; the ridges have kilometer-scale widths (Crown et al., 2022). There is also less surrounding topography of the same scale to interfere with them. Ridge paths are easily distinguishable without ambiguity. The fact that these ridges were noticed in orbital imagery, while ridges in the Lava Beds National Monument cannot be clearly seen without LiDAR data, emphasizes the vast difference in clarity between the two. Our method

could therefore be applied to Martian ridges with more confidence and less relative uncertainty in the measured dimensions.

The large sample size of interconnected ridges would make it easier than on Earth to study the flow field. In our study of the LBNM, the ridge segments we found were short relative to the flow field. The Martian ridges are much longer, more continuous, and more prevalent. This would give much more data and allow the data to be more clearly compared, to find patterns such as change in lava tube dimensions up- or down-slope or in different branches of the flow field.

Our inflation method benefits from being combined with any other source of information about the tube shapes, which can help constrain our tube shape results. In the case of these Martian ridges, this is possible by using the rille segments. Studying these features in detail could give insights into which likely were never roofed over and which are collapsed lava tube segments. The morphology of a collapse could yield information on the size of the tube and thickness of its roof when it was intact. Part of a future project could be to firstly model complete lava tube collapse, to find relationships between the size and shape of the resulting trough and the size and shape of the original tube. Then, depressions and ridges along the same tube path would provide two independent constraints on the morphology of the same tube. These methods would complement each other, and combining them could constrain the tube shape much more precisely than either method could alone.

Our equations for inflation could be quite easily modified to apply to another body such as Mars. All the factors that could change on another planet, such as Young's modulus and pressure which takes into account density and gravity, are combined in the term T_a . To apply this method to another planet this term would need to be changed, but the more complicated

relationships between tube shape and this term would not. Our main findings would therefore still be usable.

Any insights that this method can give on these Martian tubes would be highly valuable, as essentially no information is currently available on the internal morphologies of lava tubes on Mars. Even limited constraints on tube shape could thus be very informative. A 3:1 aspect ratio has been used as a general idealization of terrestrial lava tube shapes, based on observations (Blair et al., 2017). Models of lava tube stability on other bodies sometimes use this for their tube shapes. However, this general shape may not apply to Martian or lunar tubes. If the results from Martian inflation ridges indicate that a 3:1 tube aspect ratio is not reasonable, this could significantly impact stability models and what sizes of tube we believe could exist. This would also require discussion of why lava tube shapes differ between the Earth and Mars, and what difference in formation mechanisms causes it.

6.2.3 Cracking Near More Planetary Lava Tubes

Our models of lunar lava tubes predict that linear cracks form on the surface near the tubes. The distances of these cracks reflect the dimensions of the tubes. Our models were for lunar tubes; however, Martian tubes may produce the same features. It would be relatively simple to adjust our models to use approximate Martian conditions and confirm if these cracks still form. If so, a future project could search the Martian surface near candidate tubes for these cracks. If they exist, they would give estimates of tube widths. Combining them with roof thicknesses measured from collapse pits would make these estimates more precise. This method could not, however, be combined with our inflation ridge method on inflated Martian tubes. Since these cracks are caused by a tube roof bowing down into the void, and the roof of an

inflated tube is already raised up above the void, we do not expect that the same cracks would form around an inflated tube.

Chapter 7 Appendix

7.1 Supplemental Material 1 (Pertaining to: Chapter 2)

7.1.1 Introduction

Text S1 describes the model setup and how models were used. Text S2 discusses how the uncertainties in the sizes of pristine floor regions were determined.

Table 7.1.S1 summarizes the observations conducted of sinuous rilles on the lunar surface using the Lunar Reconnaissance Orbiter Camera (LROC) Quickmap tool in more detail than in the main text. The table includes descriptions of the three sinuous rilles with candidate tubes discussed in the text, the rille containing the Marius Hills Hole (MHH), and other sinuous rilles examined.

Figures 7.1.S1 to 7.1.S2 support Text S1 by visualizing aspects of the modeling process. Figures 7.1.S3 and 7.1.S4 support Text S2 by showing how the effects of changing mesh size were tested. Figures 7.1.S5 to 7.1.S12 show examples of features seen on the lunar surface in LROC imagery, as discussed in Table 7.1.S1.

7.1.2 Text S1. Modeling Setup and Process

The bottom boundary of the model is fixed and cannot move in any direction, while the side boundary far from the tube is a roller that can move vertically but not horizontally. This simulates the block as surrounded by other rock that provides normal forces only, preventing it from horizontally expanding as it is compressed downwards under its weight. The left-hand boundary that intersects the tube is symmetrical, so the half-tube functions as an entire tube with a decreased computational cost.

Models are built using a 2D plane strain approximation, so deformation in the out-of-plane direction is not allowed, and the model effectively acts like a 2D slice through a larger 3D shape. Each modeled block is 20 km tall by 20 km wide, wide enough to avoid edge effects. Tube width, roof thickness, and the aspect ratios of the roof and the floor of the tube are varied over the series of models. Most models use a half-ellipse tube shape with a 3:1 width-to-height ratio and rounded corners.

Figure 7.1.S1 visualizes the progress of the model, from initialization to results. Setup parameters such as the range of widths and roof thicknesses to use and the strength of the rock are established in MATLAB scripts. MATLAB code directs COMSOL Multiphysics® to create and run the models based on these specifications using the Livelink™ for MATLAB feature of COMSOL Multiphysics®. COMSOL Multiphysics® builds a mesh and solves for the deformation and stress field on that mesh. After the models have run, MATLAB extracts stress and deformation values from the models at the desired points, especially on the internal surfaces of the tube and the top surface of the block. To make sure points on these surfaces are sampled and to simplify issues with the complicated connectivity of the COMSOL Multiphysics® mesh (needed for solution accuracy), MATLAB creates an evenly spaced grid of points across the model block and then reshapes it to fit the outline of the tube (Figure 7.1.S1). Data is interpolated from the COMSOL Multiphysics® mesh onto the reshaped grid and saved into MATLAB. Once the data has been extracted, MATLAB code uses Eq. 3 and Eq. 4 to determine where failure occurs.

The top region of the model is divided into separate horizontal domains to facilitate mesh generation. The small domains surrounding the tube are assigned finer meshes than the large domains making up most of the block. Fine meshes are advantageous near the tube where the

stress fields are more complex, and differences over small areas are important to the final results. Coarser meshes are preferable for the regions of the block farther from the tube and make model analysis more computationally efficient. To achieve different mesh densities in different areas as desired, dividing the region into distinct domains is more computationally feasible than creating one continuous mesh with significantly varying densities.

The mesh parameters used for the models were those of the “Extremely Fine” COMSOL Multiphysics® mesh setting for this geometry: maximum element size of 0.2 km, minimum element size of 4×10^{-4} km, maximum element growth rate of 1.1, curvature factor of 0.2, and resolution of narrow regions 1. Figure 7.1.S2 shows the mesh for an example tube model.

The results given by the model setup described here are robust with regard to changes in model parameters. The sensitivity of this model setup to COMSOL Multiphysics® mesh density, block size, and MATLAB sampling grid density was tested and gave a maximum uncertainty in pristine floor extent of 1.6% of the tube width (see Text S2). For the finer mesh used in this uncertainty test, the mesh parameters were unchanged except for the maximum element size, which was decreased from 0.2 km to 0.02 km.

7.1.3 Text S2. Pristine Floor Extent Uncertainty Calculations

To determine the reasonable error in the pristine floor extents, we documented for a sample subset of models (Width: 3.5, 6.5; Roof Thickness: 0.65, 0.95) how modifications of model setup changed the size of the pristine floor regions. We examined the effects of using a finer COMSOL Multiphysics® mesh, a wider modeled block, or a finer MATLAB sampling grid to interpolate the data points.

For a test subset of models with these modifications, we calculated the changes in usable floor extents as a fraction of tube width. This method of representation does not significantly

over-represent the errors in any test model run, whereas reporting error as a distance would put too much emphasis on the models with the largest tubes and reporting the error as a fraction of the pristine floor size in the nominal models would over-emphasize the variation in the smaller tube where that size is very small. The results using this calculation method also remain relatively consistent across different test methods and dimensions, with most results being close to 1%.

For each model modification, we reported the largest uncertainty value given by the subset of models tested. The maximum errors introduced by these methods were 1.3% of the tube width (using a finer MATLAB sampling grid), 0.8% of the tube width (using a finer COMSOL mesh), and 1.6% of the tube width (using a wider block size). We chose the maximum of these values, 1.6%, to represent the uncertainty overall.

For the example of the mesh density changes, the change in surface topographic features is also shown in Figure 7.1.S3. Example meshes using the two different mesh densities are also shown for comparison in Figure 7.1.S4.

Table 7.1.S1: Summary of observations for lunar sinuous rilles studied. Observations were made using LROC imagery, accessed through the LROC Quickmap tool

Rille Name:	Examples of Predicted Surface Features Observed	Other Comments and Possible Features
Rima Marius	Rille discontinuity followed on either side by linear surface depressions (14.6°N, 48.6° W) near the southeast end of the rille. Discussed in the main text (Figure 2.12).	
Rima Sharp	Rille discontinuity followed to either side by linear cracking (41.75°N, 50°W). Discussed in the main text (Figure 2.11).	Other discontinuities were observed, but none associated with cracks. Lineaments on the rille edges may be evidence of cracking, material detaching from the rille walls, or stratification in the surrounding lava. Example at 48.07°N, 49.76°W (Figure 7.1.S5).

At 48.45°N, 48.56°W, the rille splits. At this location, some linear features, possibly cracks, seem to follow the path of the rille on one side (Figure 7.1.S6).

A possible linear cracking feature following the rille path to one side is also present near the eastern end of the rille, at 47.69°N, 47.49°W. This feature is most clearly visible in LROC NAC Large Incidence Angle East Illumination imagery (Figure 7.1.S10).

At other locations, possible linear cracking features run parallel to the rille on both sides. These features are not as clearly visible as the cracks around the discontinuity discussed in the text and are not associated with places where the rille path is discontinuous. Examples are shown in Figures 7.1.S7, 7.1.S8, and 7.1.S9.

Rima Mairan	Discontinuity at the southern end of the rille; the rille ends in two large aligned depressions connected by lines of cracking (36.7°N, 46.45°W). These features are consistent with the predicted surface features that would be produced by a tube. The depressions in this location might have formed by collapse into a tube. Discussed in the main text (Figure 2.1).	No other such clear examples of rille discontinuities accompanied by lines of cracking are present along the rille path. However, there are some other locations where a short rille discontinuity is accompanied by possible cracking features, or where cracks following the rille path are seen alongside an intact stretch of rille (Figure 7.1.S11). These possible surface features are seen most clearly between 39.3°N, 47.1°W and 39.7°N, 47.3°W.
Rille containing the Marius Hills Hole (MHH)	None	In some locations possible linear cracks are present inside the trough of the rille, possibly indicating material on the sloping surface detaching and sliding into the rille or the stratification in the mare deposit.

Rima
Herigonius

None

At the Western end of the rille, where the rille path disappears, a curving linear feature is visible (14.1°N, 57.4°W) following the approximate path of the rille (Figure 7.1.S12). The nature of this feature is unclear. It may be a crack tracing the rille path, possibly caused by a concealed lava tube extending from the rille. Alternatively, this feature may be the last visible part of the edge of the rille as it disappears.

Near the curving portion of the rille containing the MHH, some possible linear features are faintly visible to the Southwest of the rille path (around 14.05°N, 56.85°W). These features are present at various distances away from the rille and are faint and short, with lengths of approximately 500 meters. These features may be related to a tube or may be caused by unrelated processes.

High-resolution data for some southern portions of the rille were not available in LROC Quickmap, so some features could have been obscured.

		However, these portions only represent a small percentage of the rille length. No evidence of cracks parallel to the rille. The high sinuosity of the rille may make it unlikely for the predicted stress fields to form and cause cracking.
Rima Hadley	None	A very short rille discontinuity forming a bridge is seen near the Apollo 15 landing site, but with no evident associated surface features.
Vallis Schröteri	None	Many short linear surface features are present near the source of the large rille, generally aligned with the rille path; however, these are not associated with rille discontinuities, do not perfectly follow the rille path, and may be associated with other nearby features instead of the rille.
Rima Milichius	None	No evidence of cracks parallel to the rille.

Rima Krieger

None

Relatively short rille extending from Krieger crater. The rille disappears at 29.5°N, 46.5°W. Two other rille paths terminate in the same approximate region; one or both may be connected to Rima Krieger by lava tubes or by a stretch of rille that has been covered over or erased. If Rima Krieger extends into a lava tube connected to one or both rilles, no obvious surface features associated with the tube are present; however, it is difficult to determine where the tube path would be to search for such features.

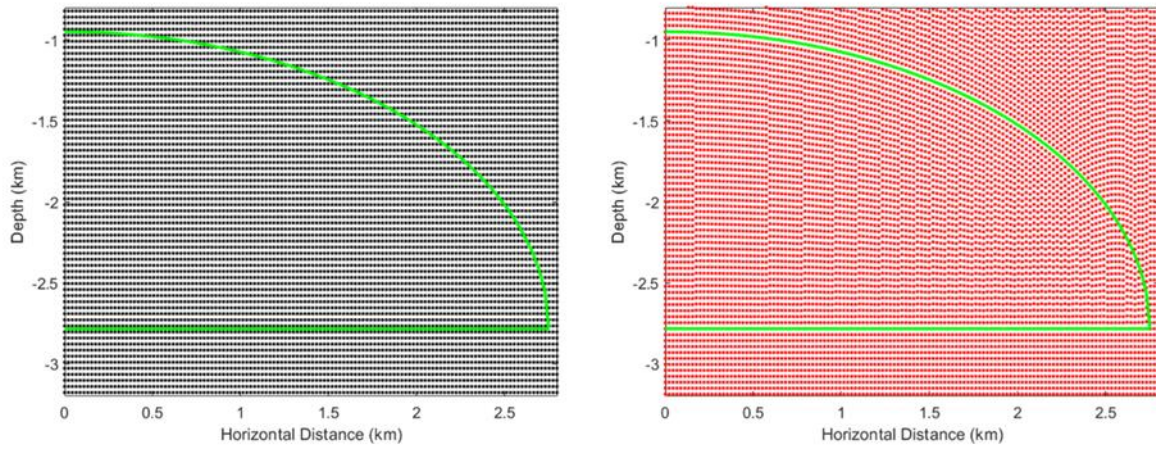


Figure 7.1.S1: MATLAB interpolation grid before (left) and after (right) reshaping to fit the tube outline.

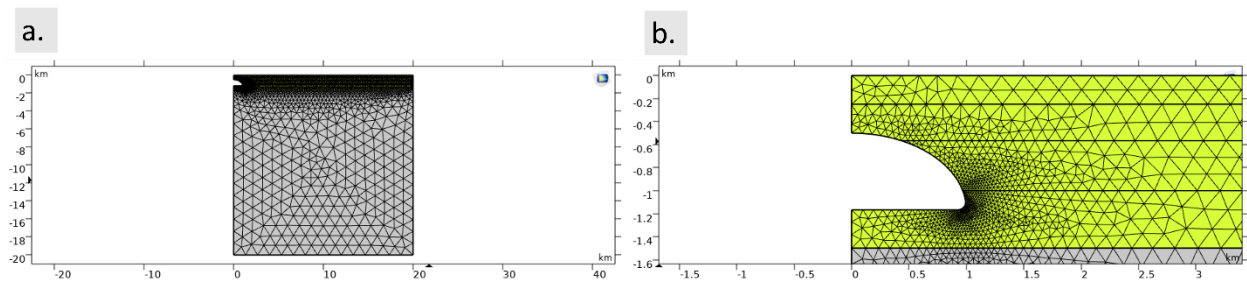


Figure 7.1.S2: Mesh setup for an example tube model, showing the whole block (a.) and a smaller region around the tube (b.)

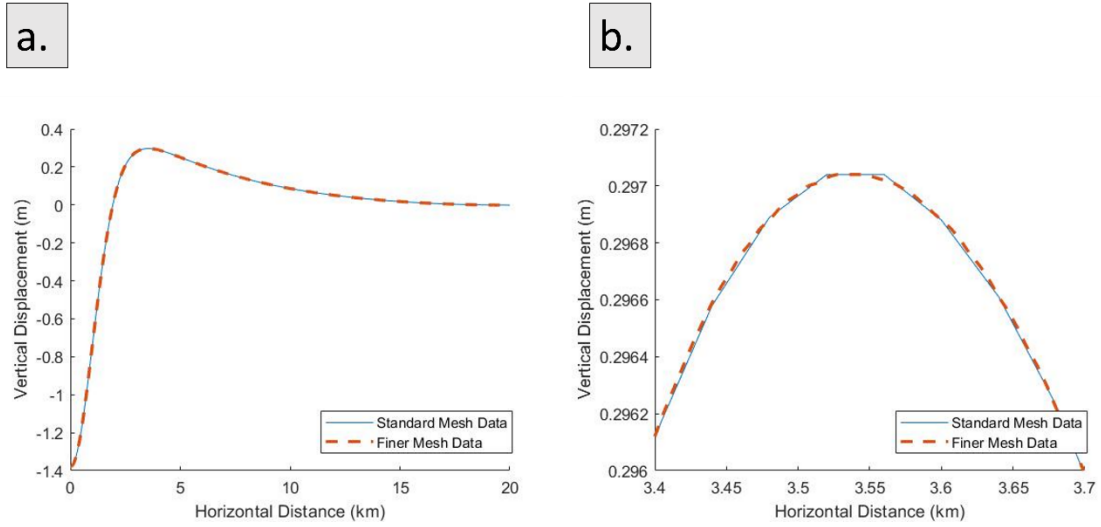


Figure 7.1.S3: Effects of different mesh setups on surface deformation profiles. The two models are the same except that one uses the original mesh (blue line), and one uses the test finer mesh (orange dashed line). (b.) is included to show the small variations between the two datasets in a zoomed-in region. Horizontal units are in kilometers, and vertical units are in meters.

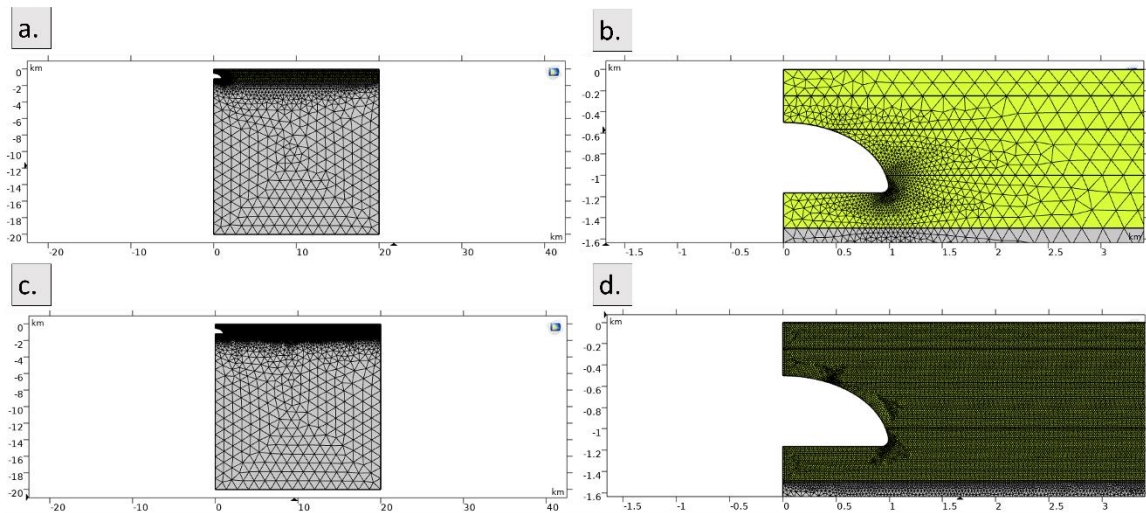


Figure 7.1.S4: Comparison of the original mesh (a. and b.) against the finer mesh used to test the effects of changing mesh parameters (c. and d.). (a.) and (c.) show the full block; (b.) and (d.) show the same smaller region around the tube.

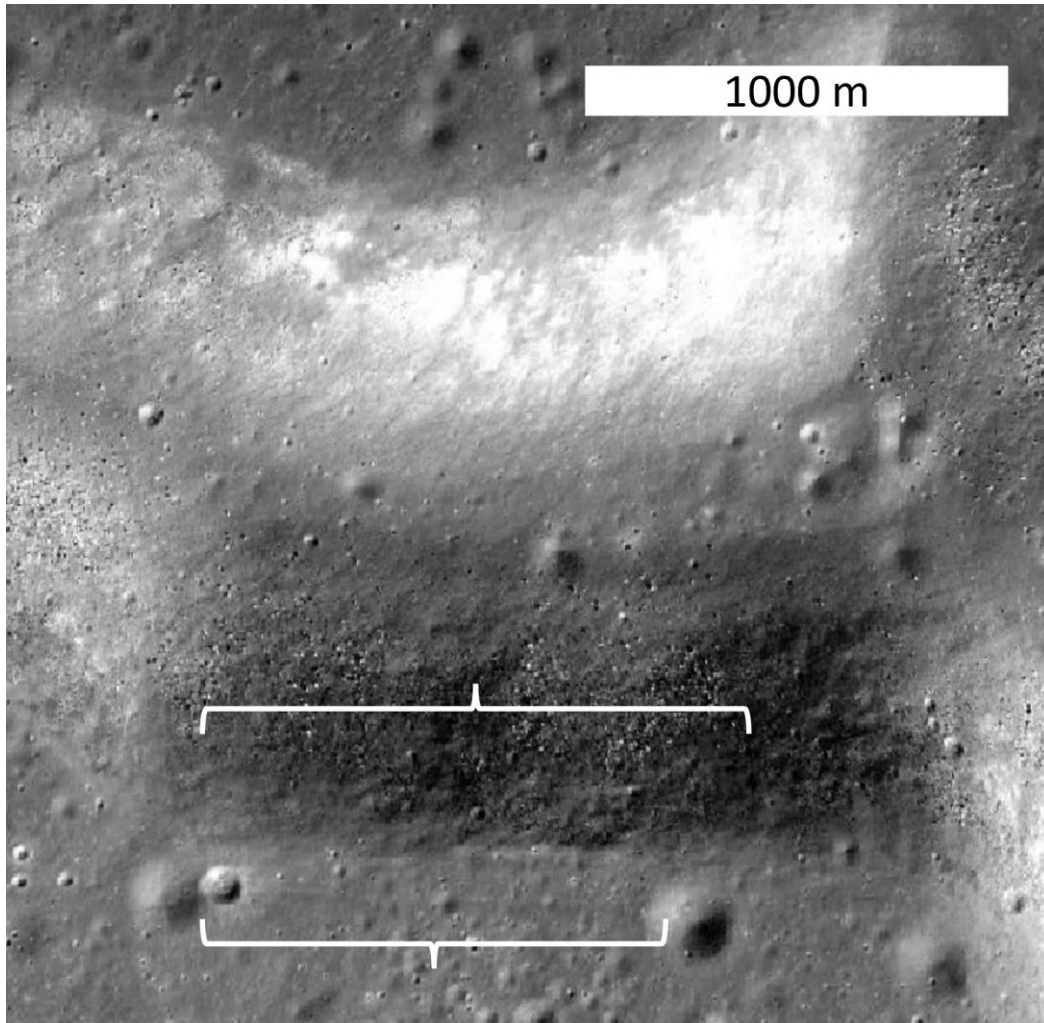


Figure 7.1.S5: Linear patterns on the walls of Rima Sharp, possibly caused by wall material detaching and sliding into the rille or indications of layering in the surrounding mare basalts.

Image source: LROC NAC imagery obtained through LROC Quickmap. Center of image: 48.07°N, 49.76°W.

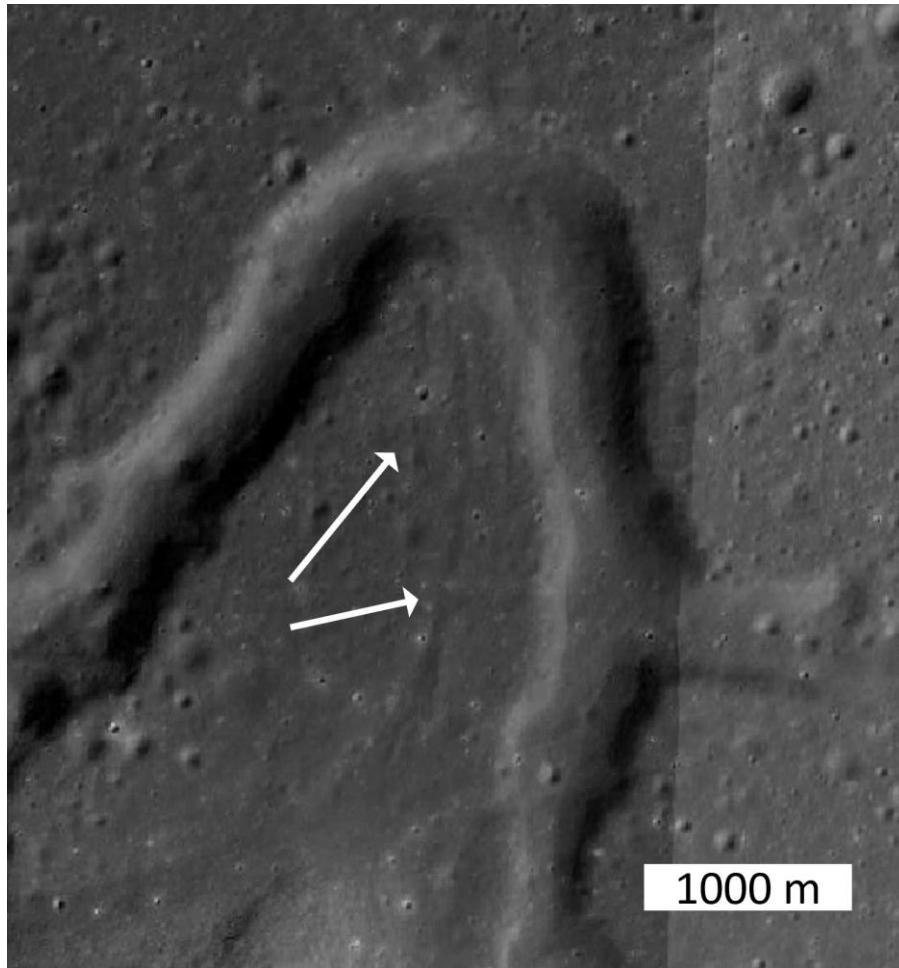


Figure 7.1.S6: Possible linear cracking features following the path of Rima Sharp on one side, near where the rille splits. Image source: LROC NAC imagery obtained through LROC Quickmap. Center of image: 48.49°N, 48.58°W.

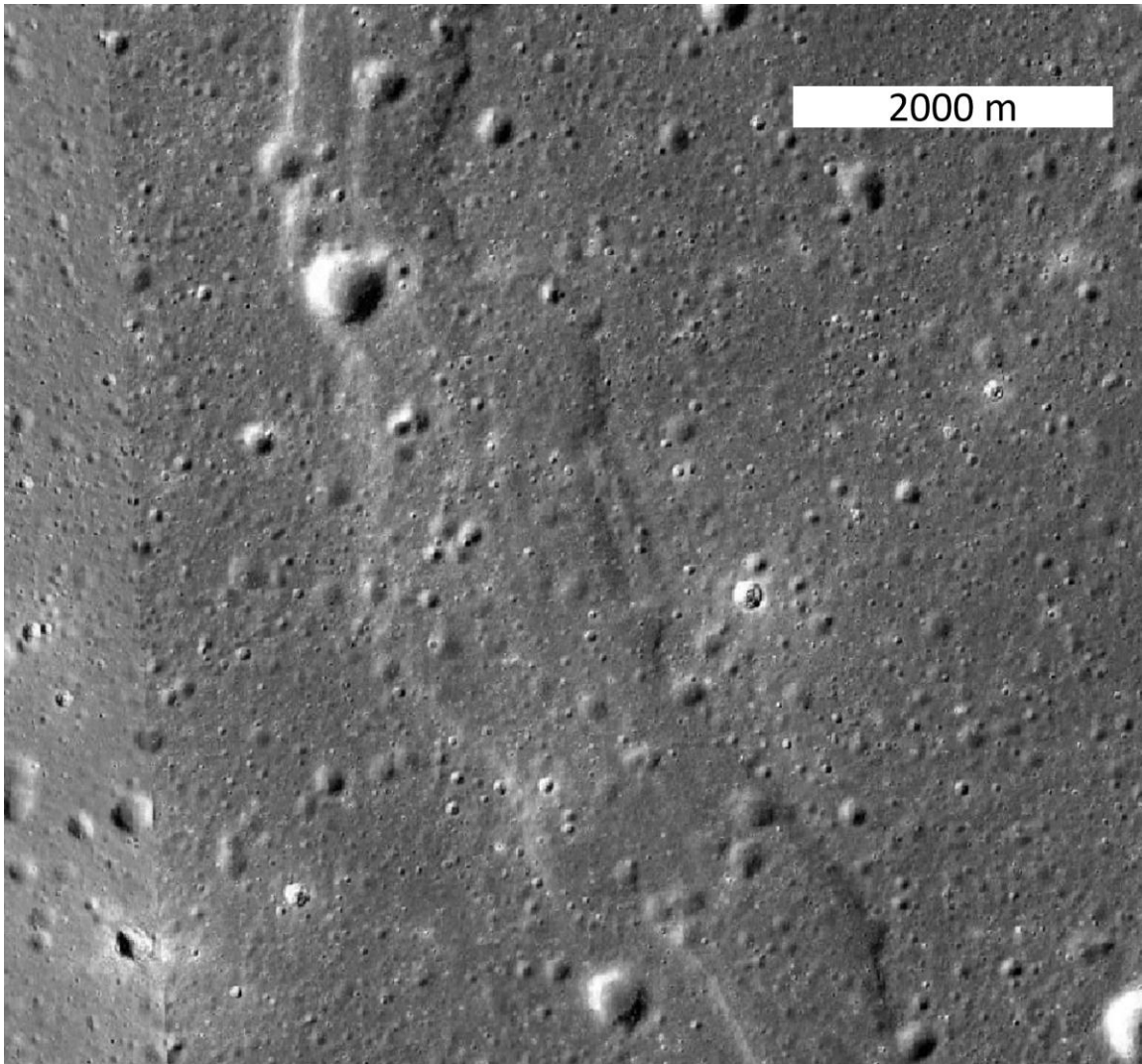


Figure 7.1.S7: Possible linear cracking features following the path of Rima Sharp, Example 1.

Image source: LROC NAC imagery obtained through LROC Quickmap. Center of image:

41.35°N, 49.9°W.

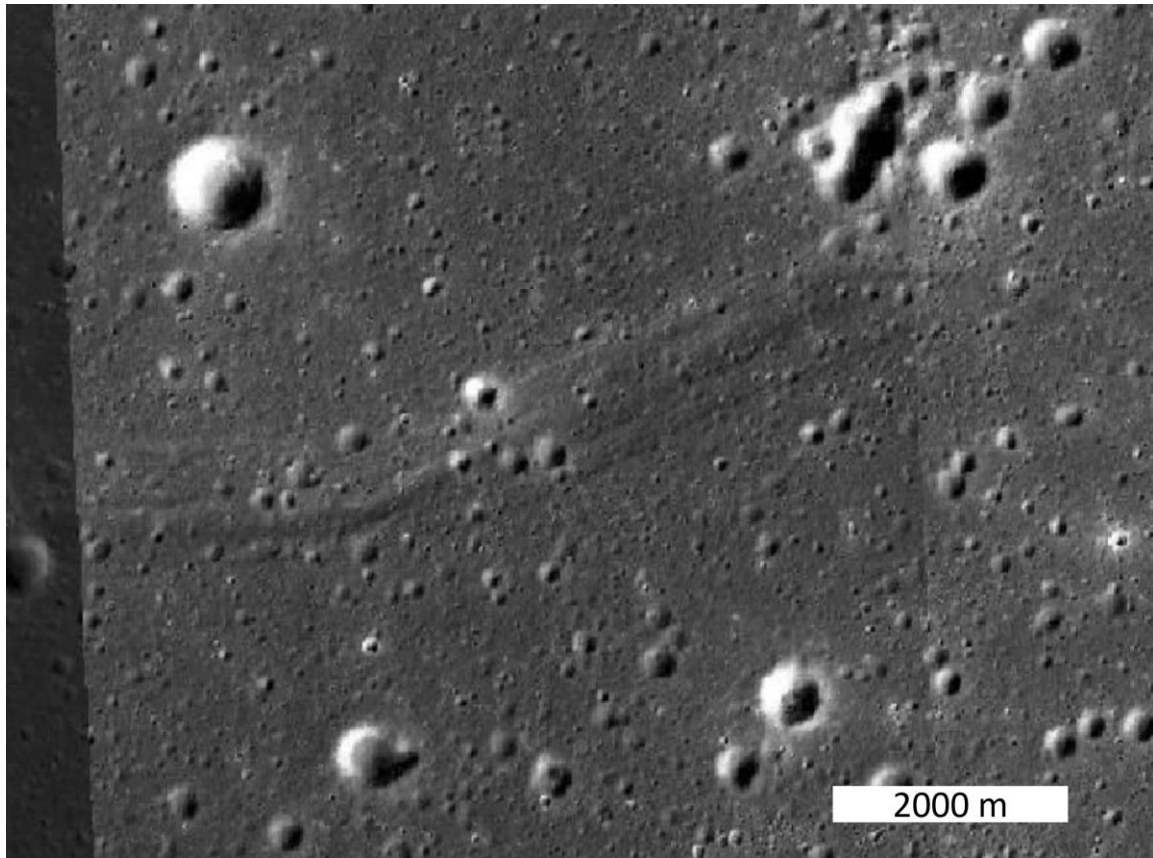


Figure 7.1.S8: Possible linear cracking features following the path of Rima Sharp, Example 2.

Image source: LROC NAC imagery obtained through LROC Quickmap. Center of image:

40.4°N, 48.6°W.

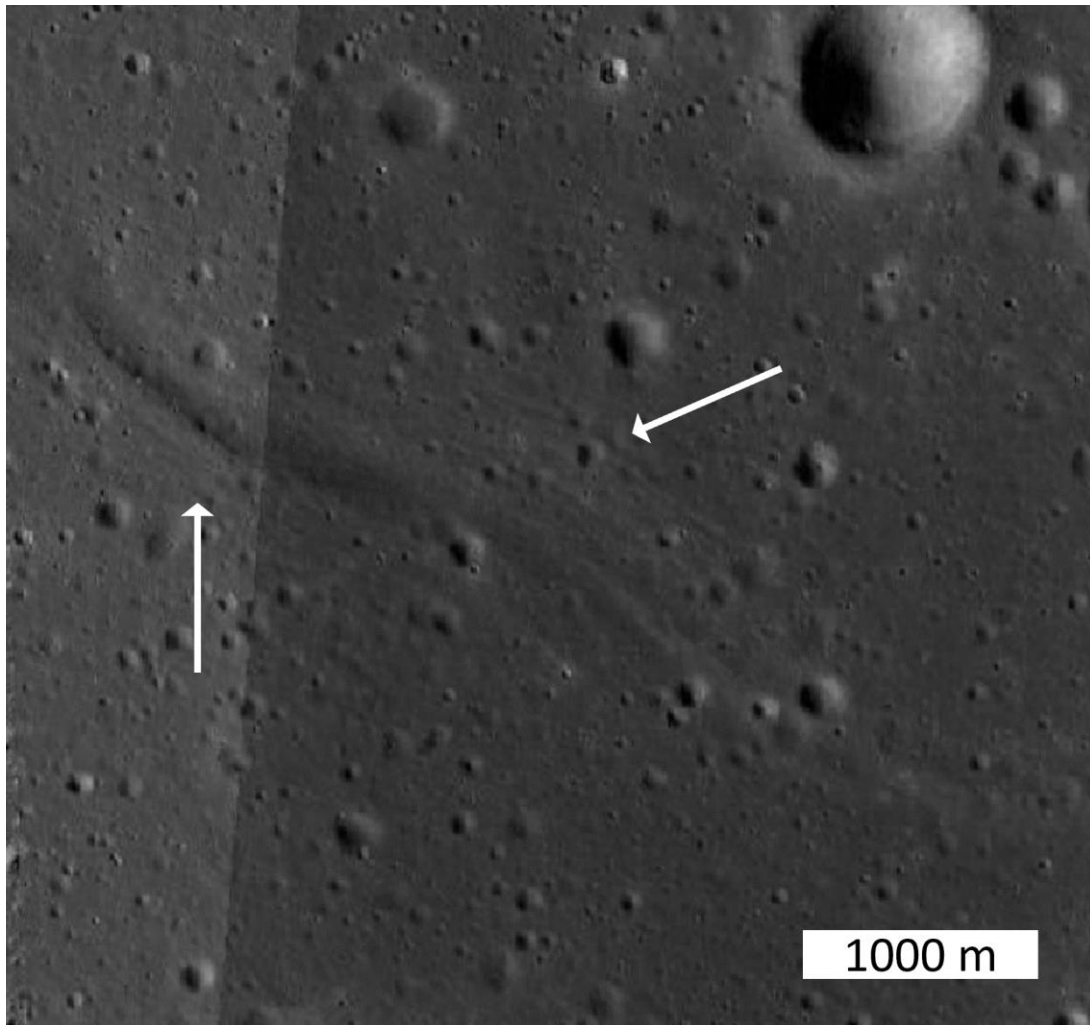


Figure 7.1.S9: Possible linear cracking features following the path of Rima Sharp, Example 3.

Image source: LROC NAC imagery obtained through LROC Quickmap. Center of image:

40.21°N, 48.01°W.

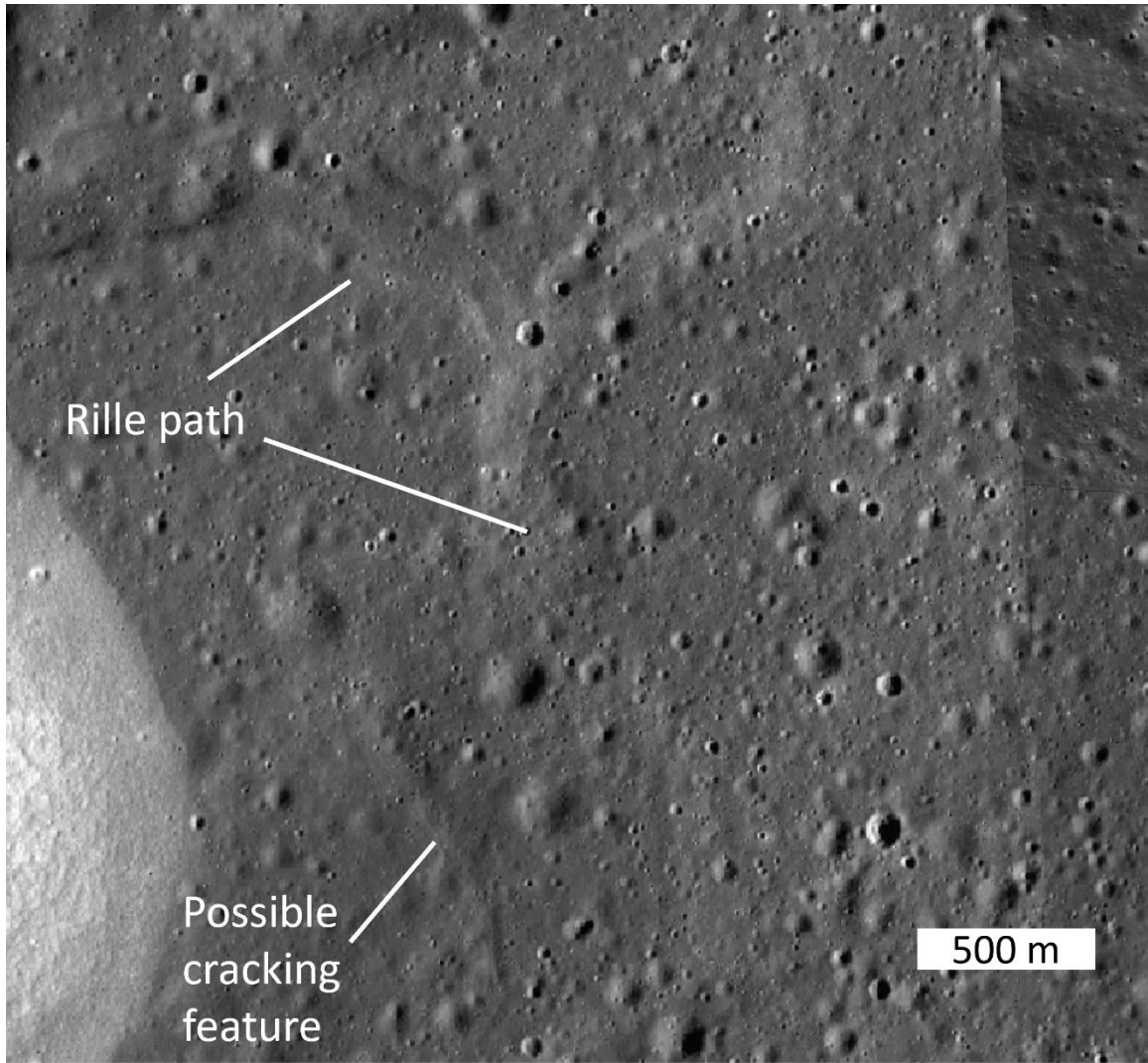


Figure 7.1.S10: Possible linear cracking feature following the path of Rima Sharp as the rille disappears. Image source: LROC NAC Large Incidence Angle East Illumination imagery, through LROC Quickmap. Center of image: 47.69°N, 47.49°W.

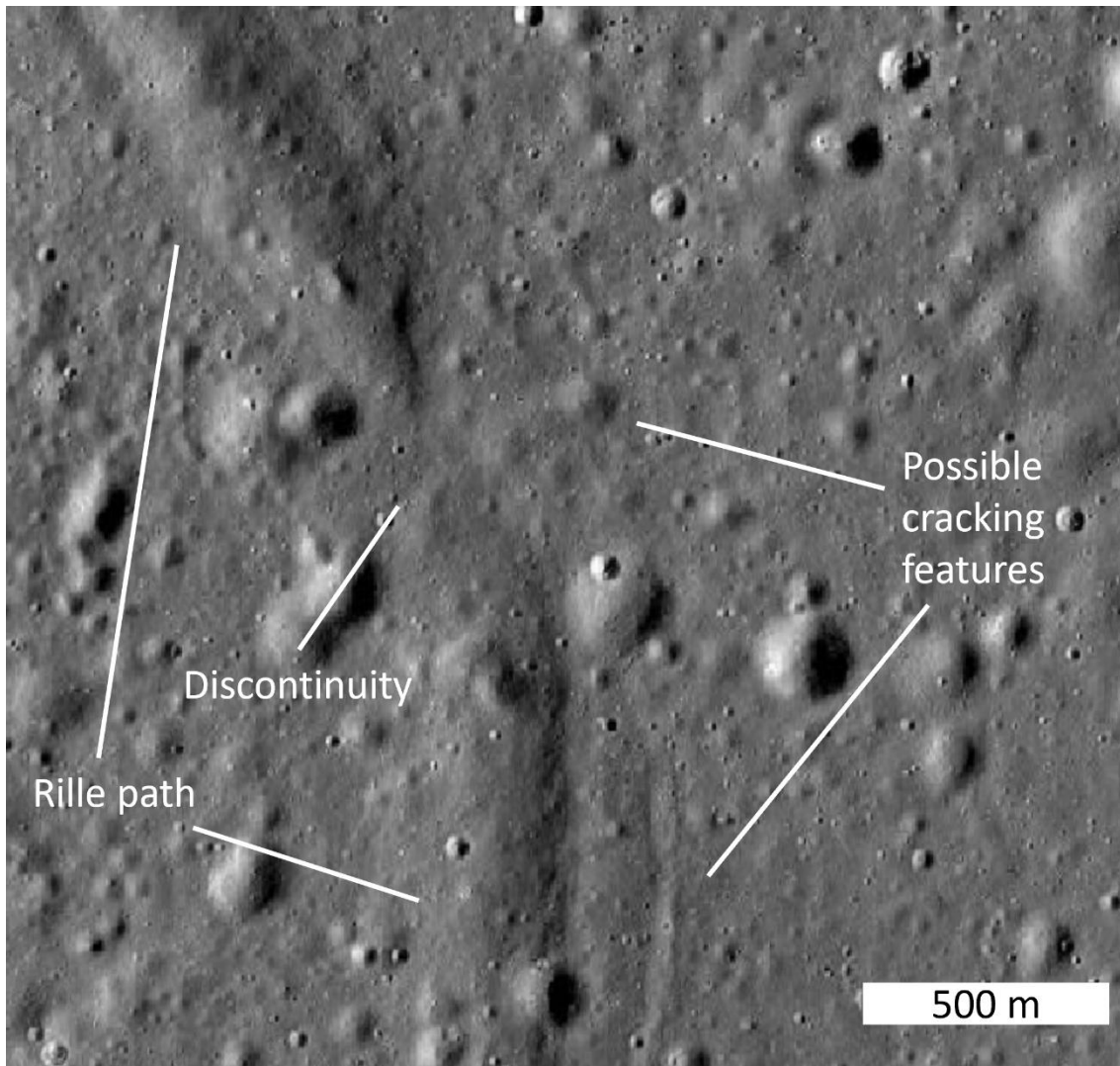


Figure 7.1.S11: Possible cracking features following the path of Rima Mairan on one side. Possible linear cracks are also present near a discontinuity in the rille. Image source: LROC NAC imagery obtained through LROC Quickmap. Center of image: 39.6°N, 47.2°W.

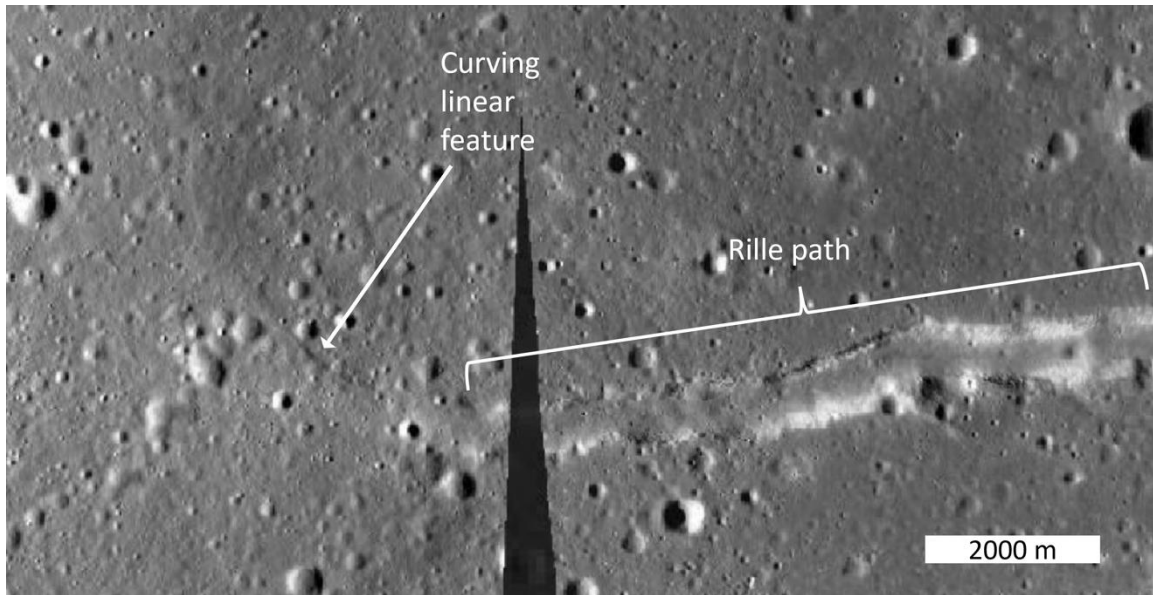


Figure 7.1.S12: Curving linear feature that may be a line of cracking or a rille edge, extending from the rille containing the Marius Hills Hole (MHH). Image source: LROC NAC imagery obtained through LROC Quickmap. Center of image: 14.15°N, 57.29°W.

7.2 Supplemental Material 2 (Pertaining to: Chapter 3)

These documents present the measurements performed on selected model runs and how they are used to document the scaling relation $L = 170 T_{ve}^{-1.4}$ (Eq. 3.3 in the main text).

7.2.1 Model result fitting

We summarize here three series of models that illustrate the scaling relations summarized in the main text.

The three series can be seen as deviations from the reference model with a Young's Modulus $E = 10$ GPa and a pressure $P = 7.1$ MPa. By trial and error, we find that the pressurized lava tube develops a 2-meter-high ridge ($H = 2$ m) when the roof thickness at the top of the tube is $t_{ve} = 0.136$ m. The associated inflation length scale is $L \equiv HE/P = 2817$ m.

The first two model series illustrate the validity of the inflation length scale concept. In Series 1, we change Young's Modulus E , keeping t_{ve} constant, and determine by trial-and-error the pressure that results in $H = 2$ m. The results (Supporting Information Table 7.2.S1) all give the same value of L . In Series 2, we change P , keeping E and T_{ve} constant, and report the inflation ridge height (Supporting Information Table 7.2.S1). Here again, all models return the same value of L . These results are a confirmation that the model behaves linearly, as expected from the physics of elasticity.

Series 3 probes a less predictable behavior, the effect of roof thickness on ridge height. Keeping $P = 7.1$ MPa, we vary the Young's Modulus from 0.5 to 20 MPa and determine by trial-and error the roof thickness that develops a 2-meter-high ridge. In an additional model, we

set $T_{ve} = 0.4165$ m and determine by trial-and-error the Young's Modulus that results in a 2-meter-high ridge. These results are summarized in Supporting Information Table 7.2.S1

The results of Series 3 show a clear anticorrelation between T_{ve} and L (Supporting Information Figure 7.2.S1a). Reporting these results on a log-log plot reveals a nearly linear relation (Supporting Information Figure 7.2.S1b), implying a power-law relationship between these two quantities. We fit the Series 3 data using a linear least-square method to

$$\log(L) = a + b \log(T_{ve})$$

$$a = 5.13 \pm 0.11$$

$$b = -1.39 \pm 0.06$$

$$Cov(a, b) = 0.0063$$

Where both L and T_{ve} are in meters.

The data shows a moderate curvature in log-log space, but we ignore it because of the unresolved importance of model design parameters (tube shape, mesh discretization, computational domain dimensions, etc.) Still, one should exert caution before extrapolating this fit to higher or lower values of T_{ve} . Note that the largest values of T_{ve} are ruled out by the examples analyzed in the main text.

The uncertainty here is based on an *a-posteriori* measurement error estimate based on the linear fit in a log-log space. To do this, we first fit the data using an arbitrary uniform standard deviation $\sigma = 1$ (its exact value does not change the best-fit line, as it is uniform) and record a

first estimate of the covariance matrix C_1 . Then, we estimate the *a-posteriori* standard deviation according to

$$\sigma^2 = \frac{\sum_i [y_i - (a + b x_i)]^2}{N - M}$$

Where $x_i = \log\left(\frac{L_i}{[\text{m}]}\right)$, $y = \log\left(\frac{T_{\text{ve}i}}{[\text{m}]}\right)$, i denotes a specific model run, N is the number of runs (here 6), and M is the number of fit parameters (here 2). The correlation matrix becomes

$$C = \sigma C_1$$

The fit is then expressed directly as a function of L and T_{ve}

$$L = L_0 T_{\text{ve}}^b$$

$$L_0 = \exp(a) = 169 \pm 19$$

$$\sigma_{L_0} = |L_0 \sigma_a|$$

We simplify this equation to $L = (170 \pm 20) T_{\text{ve}}^{-1.4 \pm 0.1}$, where L and T_{ve} are in meters.

7.2.2 Supplementary Table and Figures

Table 7.2.2: Results of numerical model runs. Quantities bolded are the unknown quantities either observed in the model solutions (ridge height) or determined by trial and error. The inflation length scale is calculated after each run.

	Young's Modulus E [GPa]	Pressure P [MPa]	Roof thickness T_{ve} [m]	Ridge height H [m]	Inflation length scale $L \equiv \frac{HE}{P}$
Series 1	5	3.55	0.136	2	2817
	10	7.1	0.136	2	2817
	15	10.65	0.136	2	2817
	20	14.2	0.136	2	2817
Series 2	10	3.55	0.136	1	2817
	10	7.1	0.136	2	2817
	10	14.2	0.136	4	2817
	10	21.3	0.136	6.01	2817
Series 3	0.5	7.1	1.019	2	140.8
	2.34	7.1	0.4165	2	659.2
	5	7.1	0.239	2	1409
	10	7.1	0.136	2	2817
	15	7.1	0.0955	2	4225
	20	7.1	0.0737	2	5634

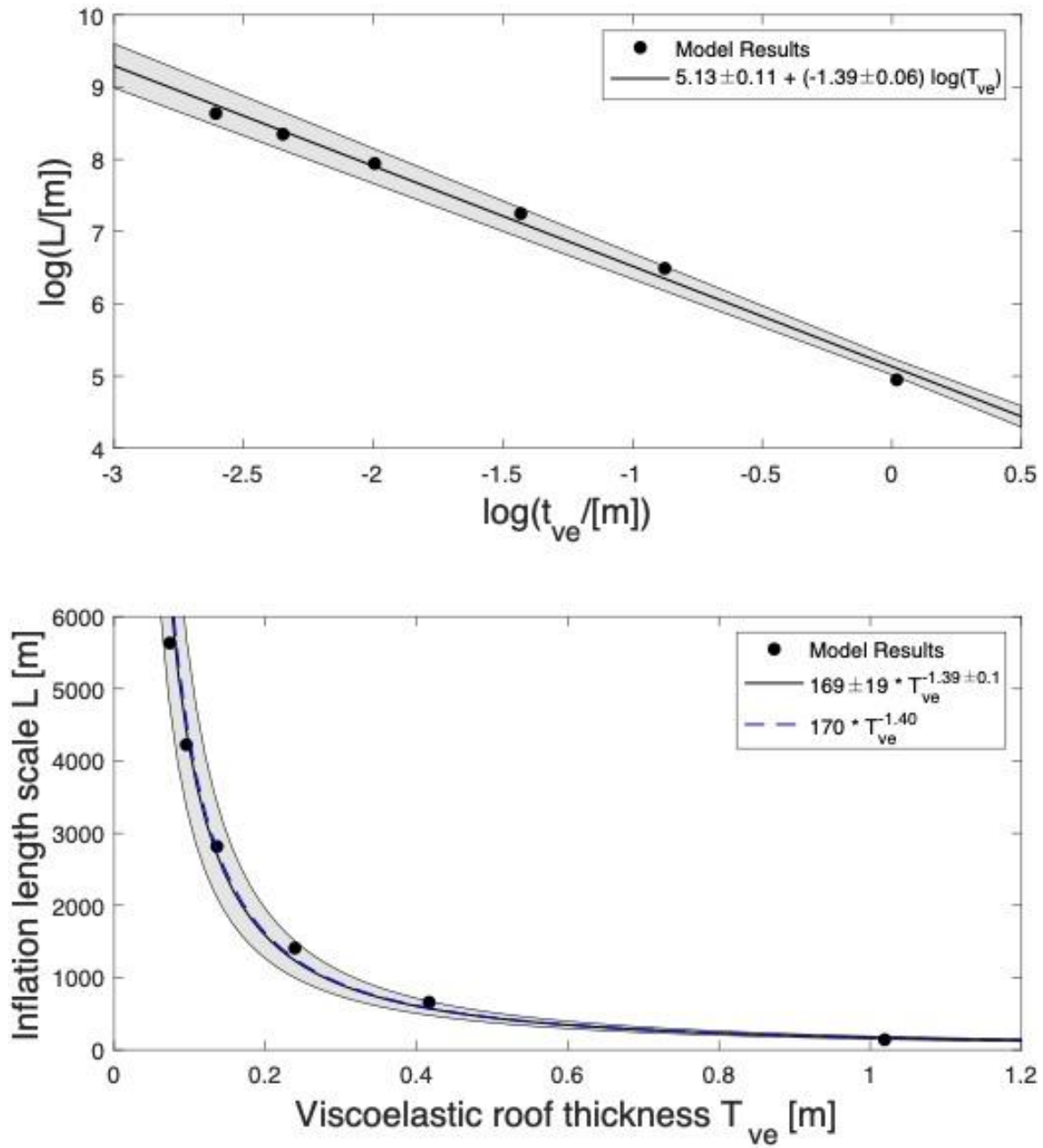


Figure 7.2.S1: Model Series 3 results (dots) and fits in a log-log (top) or lin-lin (bottom) space.

The fit is done in the log-log space and converted into a power law (black line) simplified as $L = 170 t_{ve}^{-1.40}$ (blue dashes).

References

- Ahn, U. S., & Hwang, S. K. (2009). Study on source of lava flows forming the Manjanggul lava tube. *The Journal of the Petrological Society of Korea*, 18(3), 237-253. 1226-2471(pISSN).
- Angelis, D. G., Wilson, J. W., Cloudsley, M. S., Nealy, J. E., Humes, D. H., & Clem, J. M. (2002). Lunar lava tube radiation safety analysis. *Journal of radiation research*, 43(Suppl), S41-S45.
- Balme, M., & Greeley, R. (2006). Dust devils on Earth and Mars. *Reviews of Geophysics*, 44(3). <https://doi.org/10.1029/2005RG000188>
- Bell, E. Jr., Schmerr, N., Young, K., Esmaili, S., Garry, W. B., Jazayeri, S., et al. (2022). Field mapping and modeling of terrestrial lava tube magnetic anomalies as an analog for lunar lava tube exploration and prospecting. *Journal of Geophysical Research: Planets*, 127, e2021JE007140, doi:10.1029/2021JE007140
- Bernardi, M. I., Bertotto, G. W., Jalowitzki, T. L., Orihashi, Y., & Ponce, A. D. (2015). Emplacement history and inflation evidence of a long basaltic lava flow located in Southern Payenia Volcanic Province, Argentina. *Journal of Volcanology and Geothermal Research*, 293, 46-56.
- Bernardi, M. I., Bertotto, G. W., Ponce, A. D., Orihashi, Y., & Sumino, H. (2019). Volcanology and inflation structures of an extensive basaltic lava flow in the Payenia Volcanic Province, extra-Andean back arc of Argentina. *Andean geology*, 46(2), 279-299. doi.:10.5027/andgeoV46n2-3180

- Bickel, V. T., Aaron, J., Manconi, A., & Loew, S. (2021). Global drivers and transport mechanisms of lunar rockfalls, *Journal of Geophysical Research: Planets*, 126(10), e2021JE006824. <https://doi.org/10.1029/2021JE006824>
- Blair, D. M., Chappaz, L., Sood, R., Milbury, C., Bobet, A., Melosh, H. J., Howell, K. C., & Freed, A. M. (2017). The structural stability of lunar lava tubes, *Icarus*, 282, 47-55. <https://doi.org/10.1016/j.icarus.2016.10.008>
- Blamont, J. (2014). A roadmap to cave dwelling on the Moon and Mars. *Advances in Space Research*, 54, 2140–2149, <https://doi.org/10.1016/j.asr.2014.08.019>
- Bleacher, J. E., Greeley, R., Williams, D. A., & Neukum, G. (2006). Comparison of effusive volcanism at Olympus, Arsia, Pavonis, and Ascraeus Montes, Mars from lava flow mapping using Mars Express HRSC data. In *37th Lunar and Planetary Science Conference*, Abstract 1182.
- Borghi, A., Renard, P., Fournier, L., & Negro, F. (2015). Stochastic fracture generation accounting for the stratification orientation in a folded environment based on an implicit geological model. *Engineering Geology*, 187, 135-142. <https://doi.org/10.1016/j.enggeo.2014.12.019>
- Boston, P.J., Frederick, R.D., Welch, S.M., Werker, J., Meyer, T.R., Sprungman, B., Hildreth-Werker, V., & Thompson, S.L. (2004), Extraterrestrial subsurface technology test bed: Human use and scientific value of Martian caves, *AIP Conference Proceedings*, 699, 1007– 1018. <https://doi.org/10.1063/1.1649667>
- Calvari, S., & Pinkerton, H. (1998), Formation of lava tubes and extensive flow field during the 1991–1993 eruption of Mount Etna, *Journal of Geophysical Research*, 103(B11), 27291–27301, <https://doi.org/10.1029/97JB03388>.

- Calvari, S., & Pinkerton, H. (1999). Lava tube morphology on Etna and evidence for lava flow emplacement mechanisms. *Journal of Volcanology and Geothermal Research*, 90(3-4), 263-280. doi:10.1016/S0377-0273(99)00024-4
- Carr, M. H., Greeley, R., Blasius, K. R., Guest, J. E., & Murray, J. B. Murray. Some Martian volcanic features as viewed from the Viking orbiters, *Journal of Geophysical Research*, 82, 3985–4015, <https://doi.org/10.1029/JS082i028p03985>
- Carrer, L., Pozzobon, R., Sauro, F., Castelletti, D., Patterson, G. W., & Bruzzone, L. (2024). Radar evidence of an accessible cave conduit on the Moon below the Mare Tranquillitatis pit. *Nature Astronomy*, 8(9), 1119-1126. <https://doi.org/10.1038/s41550-024-02302-y>
- Chappaz, L., Sood, R., Melosh, H. J., Howell, K. C., Blair, D. M., Milbury, C., & Zuber, M. T. (2017). Evidence of large empty lava tubes on the Moon using GRAIL gravity, *Geophysical Research Letters*, 44(1), 105-112. <https://doi.org/10.1002/2016GL071588>
- Coombs, C. R., & Hawke, B. R. (1992). A search for intact lava tubes on the Moon: Possible lunar base habitats, In *The Second Conference on Lunar Bases and Space Activities of the 21st Century*, NASA Conference Publication 3166, pp. 219-229.
- Costello, E. S., Ghent, R. R., & Lucey, P. G. (2018). The mixing of lunar regolith: Vital updates to a canonical model, *Icarus*, 314, 327–344. <https://doi.org/10.1016/j.icarus.2018.05.023>
- Crameri, F. (2018). Scientific colour maps. Zenodo. <https://doi.org/10.5281/zenodo.1243862>
- Crown, D. A., Scheidt, S. P., & Berman, D. C. (2022). Distribution and morphology of lava tube systems on the western flank of Alba Mons, Mars. *Journal of Geophysical Research: Planets*, 127(6), e2022JE007263.
- Cruikshank, D. P., & Wood, C. A. (1972). Lunar rilles and Hawaiian volcanic features: Possible analogues. *The Moon*, 3(4), 412-447. <https://doi.org/10.1007/BF00562463>

- Cushing, G.E., Titus, T.N., Wynne, J.J., & Christensen, P.R. (2007). THEMIS observes possible cave skylights on Mars, *Geophysical Research Letters*, 34(17), L17201, <https://doi.org/10.1029/2007GL030709>
- Daga, A. W., Daga, M. A., & Wendel, W. R. (1992). Evolving concepts of lunar architecture: The potential of subsele development, In *The Second Conference on Lunar Bases and Space Activities of the 21st Century*, NASA Conference Publication 3166, pp. 281-291.
- De Angelis, G., Wilson, J. W., Cloudsley, M. S., Nealy, J. E., Humes, D. H., & Clem, J. M. (2002). Lunar lava tube radiation safety analysis. *Journal of Radiation Research*, 43(1), S41-S45. <https://doi.org/10.1010.1269/jrr.43.S41>
- De Hon, R. A. (2022). Alcoves as havens from a harsh Martian environment. *Journal of Geophysical Research: Planets*, 127(8), e2021JE007022. <https://doi.org/10.1029/2021JE007022>
- DeDecker, J. (2015). *Evaluation of lava tube formation mechanisms using three-dimensional mapping and viscosity modeling: Lava Beds National Monument, California* (Master's thesis, The University of North Carolina at Chapel Hill).
- Deschamps, A., C. Grigné, M. Le Saout, S. A. Soule, P. Allemand, B. Van Vliet Lanoe, and F. Floc'h (2014), Morphology and dynamics of inflated subaqueous basaltic lava flows, *Geochemistry Geophysics. Geosystems*, 15, 2128–2150, [doi:10.1002/2014GC005274](https://doi.org/10.1002/2014GC005274).
- Dietterich, H. R., Diefenbach, A. K., Soule, S. A., Zoeller, M. H., Patrick, M. P., Major, J. J., & Lundgren, P. R. (2021). Lava effusion rate evolution and erupted volume during the 2018 Kīlauea lower East Rift Zone eruption. *Bulletin of Volcanology*, 83, 1-18. [doi.:10.1007/s00445-021-01443-6](https://doi.org/10.1007/s00445-021-01443-6)

- Ding, C., Xiao, Z. & Su, Y. (2021). A potential subsurface cavity in the continuous ejecta deposits of the Ziwei crater discovered by the Chang'E-3 mission. *Earth Planets Space* 73, 53. <https://doi.org/10.1186/s40623-021-01359-7>.
- Donnelly-Nolan, J.M. (2010). Geologic map of Medicine Lake volcano, northern California. U.S. Geological Survey Scientific Investigations Map 2927, pamphlet 48 p., 2 sheets, scale 1:50,000, <https://pubs.usgs.gov/sim/2927/>.
- Dragoni, M., Piombo, A., & Tallarico, A. (1995). A model for the formation of lava tubes by roofing over a channel. *Journal of Geophysical Research: Solid Earth*, 100(B5), 8435-8447.
- Earth Resources Observation and Science (EROS) Center. (2017). National Agriculture Imagery Program (NAIP) [Dataset]. U.S. Geological Survey. <https://doi.org/10.5066/F7QN651G>
- Eberhard, S. M., & Howarth, F. G. (2021). Undara lava cave fauna in tropical Queensland with an annotated list of Australian subterranean biodiversity hotspots. *Diversity*, 13(7), 326.
- Esmacili, S., Kruse, S., Jazayeri, S., Whelley, P., Bell, E., Richardson, J., Garry, W.B., & Young, K. (2020). Resolution of Lava Tubes with ground penetrating radar: The TubeX project. *Journal of Geophysical Research: Planets*, 125(5), e2019JE006138. <https://doi.org/10.1029/2019JE006138>
- Fagents, S. A., & Greeley, R. (2001). Factors influencing lava-substrate heat transfer and implications for thermomechanical erosion. *Bulletin of Volcanology*, 62, 519-532. <https://doi.org/10.1007/s004450000113>
- Fornari, D. J. (1986), Submarine lava tubes and channels, *Bulleting of Volcanology* 48(5), 291–298, doi:10.1007/BF01081757.

- Fornari, D., Tivey, M., Schouten, H., Perfit, M., Yoerger, D., Bradley, A., Edwards, M., Haymon, R., Scheirer, R., Von Damm, K., Shank, T., & Soule, A. (2004) Submarine Lava Flow Emplacement at the East Pacific Rise 9°50'N: Implications for Uppermost Ocean Crust Stratigraphy and Hydrothermal Fluid Circulation. In *Mid-Ocean Ridges*, Geophysical Monograph 148, ed. C. German, J. Lin, and L. Parson, pp. 187-218.
<https://doi.org/10.1029/148GM08>
- Forti, P., Galli, E., & Rossi, A. (2003). Minerogenesis of volcanic caves of Kenya. *International Journal of Speleology*, 32(1), 1.
- Goode, P. (Ed.). (2009). *The Oxford Companion to Architecture*. Oxford University Press, Oxford. 10.1093/acref/9780198605683.001.0001
- Gornitz, V. (1973), The origin of sinuous rilles. *The Moon*, 6(3-4), 337-356,
- Greeley, R. (1971). Lava tubes and channels in the lunar Marius Hills. *The Moon*, 3(3), 289-314.
- Greeley, R. (1987). The role of lava tubes in Hawaiian volcanoes. *US Geol. Surv. Prof. Pap*, 1350(2), 1589-1602.
- Greeley, R., Fagents, S. A., Harris, R. S., Kadel, S. D., Williams, D. A., and Guest, J. E. (1998), Erosion by flowing lava: Field evidence, *Journal of Geophysical Research*, 103(B11), 27325–27345, <https://doi.org/10.1029/97JB03543>.
- Gregg, T. K. (2017). Patterns and processes: Subaerial lava flow morphologies: A review. *Journal of Volcanology and Geothermal Research*, 342, 3-12.
<https://doi.org/10.1016/j.jvolgeores.2017.04.022>
- Guest, J. E., Wood, C., & Greeley, R. (1984). Lava tubes, terraces and megatumuli on the 1614–24 pahoehoe lava flow field, Mount Etna, Sicily. *Bulletin Volcanologique*, 47, 635-648.
doi.:10.1007/BF01961232

- Halliday, W.R., Favre, G., Stefansson, A., Whitfield, P., & Banks, N. (2012), Occurrence and absence of lava tube caves with some other volcanic cavities; a consideration of human habitation sites on Mars. *Lunar and Planetary Science Conference, 43*, Abstract 1613.
- Haruyama, J., Hioki, K., Shirao, M., Morota, T., Hiesinger, H., van der Bogert, C. H., ... & Pieters, C. M. (2009). Possible lunar lava tube skylight observed by SELENE cameras. *Geophysical Research Letters*, 36(21). <https://doi.org/10.1029/2009GL040635>
- Haruyama, J., Morota, T., Kobayashi, S., Sawai, S., Lucey, P. G., Shirao, M., & Nishino, M. N. (2012). Lunar holes and lava tubes as resources for lunar science and exploration. In *Moon* , ed. V. Badescu, (pp. 139-163). Springer, Berlin, Heidelberg, <https://doi.org/10.1007/978-3-642-27969-0>
- Head, J. W. (1976). Lunar volcanism in space and time. *Reviews of Geophysics*, 14(2), 265-300. <https://doi.org/10.1029/RG014i002p00265>
- Heap, M. J., Villeneuve, M., Albino, F., Farquharson, J. I., Brothelande, E., Amelung, F., ... & Baud, P. (2020). Towards more realistic values of elastic moduli for volcano modelling. *Journal of Volcanology and Geothermal Research*, 390, 106684. doi.:10.1016/j.jvolgeores.2019.106684
- Hon, K. E. N., Kauahikaua, J. I. M., Denlinger, R., & Mackay, K. (1994). Emplacement and inflation of pahoehoe sheet flows: Observations and measurements of active lava flows on Kilauea Volcano, Hawaii. *Geological Society of America Bulletin*, 106(3), 351-370. doi.:10.1130/0016-7606(1994)106<0351:EAIOPS>2.3.CO;2
- Horvath, T., Hayne, P. O., & Paige, D. A. (2022). Thermal and illumination environments of lunar pits and caves: Models and observations from the Diviner Lunar Radiometer

- experiment. *Geophysical Research Letters*, 49(14).
<https://doi.org/10.1029/2022GL099710>
- Hörz, F. (1985). Lava tubes-potential shelters for habitats. In *Lunar bases and space activities of the 21st century*, Ed. W.W. Mendel, Lunar and Planetary Institute, Houston TX., pp. 405-412.
- Howarth, F. G., James, S. A., McDowell, W., Preston, D. J., & Imada, C. T. (2007). Identification of roots in lava tube caves using molecular techniques: implications for conservation of cave arthropod faunas. *Journal of Insect Conservation*, 11, 251-261.
- Hróarsson, B., & Jónsson, S. S. (1991, August). Lava caves in the Hallmundarhraun lava flow, Western Iceland. In *Proc 6th Int Symp Vulcanospeleol* (pp. 85-88).
- Huff, W.D., & Owen, L.A. (2013). 5.8 Volcanic Landforms and Hazards. In *Treatise on Geomorphology*, Vol. 5, Tectonic Geomorphology, Ed. L.A. Owen, Elsevier, pp.148-192, <https://doi.org/10.1016/B978-0-12-374739-6.00089-0>
- Hulme, G. (1982). A review of lava flow processes related to the formation of lunar sinuous rilles, *Geophysical Surveys*, 5, 245-279, <https://doi.org/10.1007/BF01454018>
- Hurwitz, D. M., Head, J. W., & Hiesinger, H. (2013). Lunar sinuous rilles: Distribution, characteristics, and implications for their origin. *Planetary and Space Science*, 79, 1-38. <https://doi.org/10.1016/j.pss.2012.10.019>
- James, M. R., Applegarth, L. J., & Pinkerton, H. (2012). Lava channel roofing, overflows, breaches and switching: insights from the 2008–2009 eruption of Mt. Etna. *Bulletin of Volcanology*, 74, 107-117. doi.:10.1007/s00445-011-0513-9
- Kaku, T., Haruyama, J., Miyake, W., Kumamoto, A., Ishiyama, K., Nishibori, T., ... & Howell, K. C. (2017). Detection of intact lava tubes at Marius Hills on the Moon by SELENE

- (Kaguya) Lunar Radar Sounder. *Geophysical Research Letters*, 44(20), 10-155.
<https://doi.org/10.1002/2017GL074998>
- Kauahikaua, J., Cashman, K. V., Mattox, T. N., Heliker, C. C., Hon, K. A., Mangan, M. T., and Thornber, C. R. (1998), Observations on basaltic lava streams in tubes from Kilauea Volcano, island of Hawai'i, *Journal of Geophysical Research* 103(B11), 27303- 27323, <https://doi.org/10.1029/97JB03576>
- Kempe, S. (2019). Volcanic rock caves. In *Encyclopedia of caves* (pp. 1118-1127). Academic Press.
- Keszthelyi, L. (1995). A preliminary thermal budget for lava tubes on the Earth and planets. *Journal of Geophysical Research: Solid Earth*, 100(B10), 20411-20420.
- Kiefer, W. S., Macke, R. J., Britt, D. T., Irving, A. J. & Consolmagno, G. J. (2012). The density and porosity of lunar rocks. *Geophysical Research Letters*, 39(7).
<https://doi.org/10.1029/2012GL051319>
- Kolvenbach, H., Mittelholz, A., Stähler, S. C., Church, J., Arm, P., Bickel, V., ... & Hutter, M. (2024). LunarLeaper-A Mission Concept to Explore the Lunar Subsurface with a Small-scale Legged Robot. In *IAC 2024 Conference Proceedings*. International Astronautical Federation.
- Larson, C. V., & Larson, J. (1990). *Lava Beds Caves*. ABC Pub.
- Larson, C.V. (1992) Nomenclature of lava tube features. In: *6th International Symposium of Vulcanospeleology, Hilo, Aug 1991*, Ed: G.T. Rea, National Speleological Society, pp. 231-248.
- Lavallee and Kendrick (2021). *Forecasting and Planning for Volcanic Hazards, Risks, and Disasters*. Elsevier. Chapter 5, Pages 153-238.

- Léveillé, R. J., & Datta, S. (2010). Lava tubes and basaltic caves as astrobiological targets on Earth and Mars: a review. *Planetary and Space Science*, 58(4), 592-598.
<https://doi.org/10.1016/j.pss.2009.06.004>
- Marinos, P., & Hoek, E. (2000). GSI: a geologically friendly tool for rock mass strength estimation. *Paper presented at the ISRM international symposium*. 19 pp.
- Meglich, T. M., Williams, M. C., Hodges, S. M., & DeMarco, M. J. (2003). Subsurface geophysical imaging of lava tubes, Lava Beds National Monument. CA, in *3rd International Conference on Applied Geophysics, Orlando*, 13 pp.
- Miyamoto, H., Haruyama, J., Kobayashi, T., Suzuki, K., Okada, T., Nishibori, T., Showman, A. P., Lorenz, R., Mogi, K., Crown, D. A., Rodriguez, J. A. P., Rokugawa, S., Tokunaga, T., & Masumoto, K. (2005). Mapping the structure and depth of lava tubes using ground penetrating radar. *Geophysical Research Letters*, 32(21), L21316.
<https://doi.org/10.1029/2005GL024159>
- Modiriasari, A., Theinat, A. K., Melosh, H. J., & Bobet, A. (2019, June). Stability Analysis of Lunar Lava Tubes for Permanent Extraterrestrial Habitation. In ARMA US Rock Mechanics/Geomechanics Symposium (pp. ARMA-2019). ARMA.
- Modiriasari, A., Theinat, A., Bobet, A., Melosh, H.J., Dyke, S.J., Ramirez, J., Magharez, A., and Gomez, D. (2018). Size and structural stability assessment of lunar lava tubes, *Lunar and Planetary Science Conference*, 49, Abstract 2860.
- Modiriasari, A., Theinat, A., Bobet, A., Melosh, H.J., Dyke, S.J., Ramirez, J., Magharez, A., and Gomez, D. (2018). Size and structural stability assessment of lunar lava tubes, *Lunar and Planetary Science Conference*, 49, Abstract 2860.

- Nesnas, I. A. D., Kerber, L., Sellar, G., Balint, T., Denevi, B., Parness, A. J., Kornfeld, R. P., Smith, M., McGarey, P., Brown, T., Sunada, E., Gonter, K. A., Hockman, B., Hayne, P., Horvath, T., Hopkins, J. B., Johnson, A. E., Wagner, R. V., Cheng, Y., Curtis, A.G., Zacny, K., Paton, M., & Sherrill, K. V. (2023). Moon Diver: Exploring a pit's exposed strata to understand lunar volcanism. *Acta Astronautica*, 211, 163-176, <https://doi.org/10.1016/j.actaastro.2023.05.042>
- Oberbeck, V. R., Quaide, W. L., & Greeley, R. (1969), On the origin of lunar sinuous rilles, *Modern Geology*, 1, 75–80.
- Okubo, C. H., & Martel, S. J. (1998). Pit crater formation on Kilauea volcano, Hawaii. *Journal of Volcanology and Geothermal Research*, 86 (1-4) 1-18. [http://doi.org/10.1016/s0377-0273\(98\)00070-5](http://doi.org/10.1016/s0377-0273(98)00070-5)
- Orr, T. R., Bleacher, J. E., Patrick, M. R., & Wooten, K. M. (2015). A sinuous tumulus over an active lava tube at Kīlauea Volcano: Evolution, analogs, and hazard forecasts. *Journal of Volcanology and Geothermal Research*, 291, 35-48. doi.:10.1016/j.jvolgeores.2014.12.002
- Pedersen, G. B. M., Höskuldsson, A., Dürig, T., Thordarson, T., Jonsdottir, I., Riishuus, M. S., ... & Schmith, J. (2017). Lava field evolution and emplacement dynamics of the 2014–2015 basaltic fissure eruption at Holuhraun, Iceland. *Journal of Volcanology and Geothermal Research*, 340, 155-169. doi.:10.1016/j.jvolgeores.2017.02.027
- Peterson, D. W., & Swanson, D. A. (1974). Observed formation of lava tubes during 1970-71 at Kilauea Volcano, Hawaii. *IP Articles*. 8210. https://digitalcommons.usf.edu/kip_articles/8210

- Peterson, D. W., Holcomb, R. T., Tilling, R. I., & Christiansen, R. L. (1994). Development of lava tubes in the light of observations at Mauna Ulu, Kilauea Volcano, Hawaii. *Bulletin of Volcanology*, 56, 343-360. doi.:10.1007/BF00326461
- Pipan, T. & Culver, D. C. (2019). Chapter 107 - Shallow subterranean habitats, in *Encyclopedia of Caves*, 3rd Ed., Ed. W.B. White, D.C. Culver, & T. Pipan, Academic Press, pp. 896-908, <https://doi.org/10.1016/B978-0-12-814124-3.00107-2>
- Roberts, C.E., & Gregg, T.K.P. (2019) Marius, the Moon: Formation of lunar sinuous rilles by constructional and erosional processes, *Icarus*, 317, 682–688.
<https://doi.org/10.1016/j.icarus.2018.02.033>
- Robinson, M. (2011). LRO MOON LROC 5 RDR V1.0 [Data set]. *NASA Planetary Data System*. <https://doi.org/10.17189/1520341>
- Robinson, M. S., Ashley, J. W., Boyd, A. K., Wagner, R. V., Speyerer, E. J., Hawke, B. R., Hiesinger, H., & Van Der Bogert, C. H. (2012). Confirmation of sublunarean voids and thin layering in mare deposits. *Planetary and Space Science*, 69(1), 18-27.
<https://doi.org/10.1016/j.pss.2012.05.008>
- Rossi, M. J., & Gudmundsson, A. (1996). The morphology and formation of flow-lobe tumuli on Icelandic shield volcanoes. *Journal of Volcanology and Geothermal Research*, 72(3-4), 291-308. doi.:10.1016/0377-0273(96)00014-5
- Sakimoto, S. E. H., & Zuber, M. T. (1998). Flow and convective cooling in lava tubes. *Journal of Geophysical Research: Solid Earth*, 103(B11), 27465-27487.
<https://doi.org/10.1029/97JB03108>
- Sakimoto, S. E. H., & Zuber, M. T. (1998). Flow and convective cooling in lava tubes. *Journal of Geophysical Research: Solid Earth*, 103(B11), 27465-27487.

- Sauro, F., Pozzobon, R., Massironi, M., De Berardinis, P., Santagata, T., & De Waele, J. (2020). Lava tubes on Earth, Moon and Mars: A review on their size and morphology revealed by comparative planetology. *Earth-Science Reviews*, 209, 103288.
<https://doi.org/10.1016/j.earscirev.2020.103288>
- Sauro, F., Pozzobon, R., Santagata, T., Tomasi, I., Tonello, M., Martínez-Frías, J., ... & Massironi, M. (2019). Volcanic caves of Lanzarote: A natural laboratory for understanding volcano-speleogenetic processes and planetary caves. *Lanzarote and chinijo islands geopark: From earth to space*, 125-142.
- Schörghofer, N. (2022). Statistical thermodynamics of surface-bounded exospheres. *Earth, Moon, and Planets*, 126(2), 1–14. <https://doi.org/10.1007/s11038-022-09547-5>
- Schubert, G., Lingenfelter, R. E., & Peale, S. J. (1970). The morphology, distribution, and origin of lunar sinuous rilles. *Reviews of Geophysics*, 8(1), 199-224.
<https://doi.org/10.1029/RG008i001p00199>
- Schultz, R. (1995). Limits on strength and deformation properties of jointed basaltic rock masses. *Rock Mechanics and Rock Engineering*, 28(1), 1-15.
<https://doi.org/10.1007/BF01024770>
- Senthil Kumar, P., Sruthi, U., Krishna, N., Lakshmi, K. J. P., Menon, R., Amitabh, , Gopala Krishna, B., Kring, D. A., Head, J. W., Goswami, J. N., et al. (2016), Recent shallow moonquake and impact-triggered boulder falls on the Moon: New insights from the Schrödinger basin, *Journal of Geophysical Research Planets*, 121(2), 147– 179,
<https://doi.org/10.1002/2015JE004850>.
- Smith, D. S., & Scalo, J. (2007). Solar X-ray flare hazards on the surface of Mars. *Planetary and Space Science*, 55(4), 517-527. <https://doi.org/10.1016/j.pss.2006.10.001>

- Smith, R., Sammonds, P. R., & Kilburn, C. R. (2009). Fracturing of volcanic systems: Experimental insights into pre-eruptive conditions. *Earth and Planetary Science Letters*, 280(1-4), 211-219. doi.:10.1016/j.epsl.2009.01.032
- Sood, R., Chappaz, L., Melosh, H. J., Howell, K. C., & Milbury, C. (2016, March). Detection of buried empty lunar lava tubes using GRAIL gravity data. *47th Annual Lunar and Planetary Science Conference*, Abstract 9016.
- Spudis, P. D., Swann, G. A., & Greeley, R. (1988). The formation of Hadley Rille and implications for the geology of the Apollo 15 region. In *Proceedings of the 18th Lunar and Planetary Science Conference Proceeding*, pp. 243-254.
- Tanaka, K. L., & Golombek, M. P. (1989) Martian tension fractures and the formation of grabens and collapse features on Valles Marineris. In *Proceedings of the 19th Lunar and Planetary Science Conference Proceedings*, pp. 383-396.
- Taylor, S. J., & Krejca, J. J. (2006). A biological assessment of caves in Lava Beds National Monument. *Technical Report BIOD 2006 (06)*.
- Theinat, A. K., Modiriasari, A., Bobet, A., Melosh, H. J., Dyke, S. J., Ramirez, J., Choi, J., Maghareh, A., & Gomez, D. (2019). Geology Explorations of Lava Tubes in the National Beds Lava Monuments. In *50th Annual Lunar and Planetary Science Conference*, Abstract. 3232.
- Theinat, A. K., Modiriasari, A., Bobet, A., Melosh, H. J., Dyke, S. J., Ramirez, J., Maghareh, A., & Gomez, D. (2020). Lunar lava tubes: Morphology to structural stability. *Icarus*, 338, 113442. <https://doi.org/10.1016/j.icarus.2019.113442>
- Theinat, A. K., Modiriasari, A., Bobet, A., Melosh, H. J., Dyke, S. J., Ramirez, J., Choi, J., Maghareh, A., & Gomez, D. (2019). Geology Explorations of Lava Tubes in the National

- Beds Lava Monuments. In *50th Annual Lunar and Planetary Science Conference*, Abstract. 3232.
- Theinat, A. K., Modiriasari, A., Bobet, A., Melosh, J., Dyke, S., Ramirez, J., Maghareh, A., & Gomez, D. (2018). Geometry and structural stability of lunar lava tubes. In *2018 AIAA SPACE and Astronautics Forum and Exposition* (p. 5185).
<https://doi.org/10.2514/6.2018-5185>
- Titus, T.N., Wynne, J.J., Malaska, M.J., Agha-Mohammadi, A., Buhler, P.B., Alexander, E.C., Ashley, J.W., Azua-Bustos, A., Boston, P.J., Buczkowski, D.L., Chiao, L., Cushing, G.E., DeDecker, K., de León, P., Demirel-Floyd, C., De Waele, J., Fairén, A.G., Frumkin, A., Harris, G.L., Jones, H., Kerber, L.H., Leonard, E.J., Léveillé, R.J., Manyapu, K., Massironi, M., Miller, A.Z., Mylroie, J.E., Onac, B.P., Parazynski, S., Phillips, C.B., Phillips-Lander, C.M., Prettyman, T.H., Sapers, H.M., Sauro, F., Schörghofer, N., Schulze-Makuch, D., Scully, J., Uckert, K., Wagner, R.V., Whittaker, W.L., Williams, K.E., & Wong, U.Y. (2021) A roadmap for planetary caves science and exploration. *Nature Astronomy*, 5, 524–525. <https://doi.org/10.1038/s41550-021-01385-1>
- Tomasi, I., Massironi, M., Meyzen, C. M., Pozzobon, R., Sauro, F., Penasa, L., ... & Martinez-Frias, J. (2022). Inception and evolution of La Corona Lava Tube System (Lanzarote, Canary Islands, Spain). *Journal of Geophysical Research: Solid Earth*, 127(6), e2022JB024056. doi.:10.1029/2022JB024056
- Torrese, P., Pozzobon, R., Rossi, A. P., Unnithan, V., Sauro, F., Borrmann, D., Lauterbach, H., & Santagata, T. (2021). Detection, imaging and analysis of lava tubes for planetary analogue studies using electric methods (ERT). *Icarus*, 357, 114244.
<https://doi.org/10.1016/j.icarus.2020.114244>

- Valerio, A., Tallarico, A., & Dragoni, M. (2008). Mechanisms of formation of lava tubes. *Journal of Geophysical Research: Solid Earth*, 113(B8).
<https://doi.org/10.1029/2007JB005435>
- Voigt, J. R., Hamilton, C. W., Scheidt, S. P., Münzer, U., Höskuldsson, Á., Jónsdóttir, I., & Thordarson, T. (2021). Geomorphological characterization of the 2014–2015 Holuhraun lava flow-field in Iceland. *Journal of Volcanology and Geothermal Research*, 419, 107278. doi:10.1016/j.jvolgeores.2021.107278
- Wagner, R. V., & Robinson, M. S. (2022). Lunar pit morphology: Implications for exploration. *Journal of Geophysical Research: Planets*, 127(8), e2022JE007328. <https://doi.org/10.1029/2022JE007328>
- Wagner, R.V. & Robinson, M.S. (2014). Distribution, formation mechanisms, and significance of lunar pits. *Icarus*, 237, 52-60, <https://doi.org/10.1016/j.icarus.2014.04.002>
- Walker, G. P. (1991). Structure, and origin by injection of lava under surface crust, of tumuli, “lava rises”, “lava-rise pits”, and “lava-inflation clefts” in Hawaii. *Bulletin of Volcanology*, 53, 546-558.
- Walker, G. P. (1992), Morphometric study of pillow-size spectrum among pillow lavas, *Bull. Volcanol.*, 54(6), 459–474, doi:10.1007/BF00301392.
- Waters, A. C., Donnelly-Nolan, J. M., & Rogers, B. W. (1990). Selected caves and lava-tube systems in and near Lava Beds National Monument, California. *U.S. Geological Survey Bulletin*, 1673, 110 pp., <https://doi.org/10.3133/b1673>
- Weider, S. Z., Crawford, I. A., & Joy, K. H. (2010). Individual lava flow thicknesses in Oceanus Procellarum and Mare Serenitatis determined from Clementine multispectral data. *Icarus*, 209, 323-336, <https://doi.org/10.1016/j.icarus.2010.05.010>

- Whelley P. L. et al. (2017) AGU 2017, Abstract T44D-04
- Whitten, J. L., & Martin, E. S. (2019). Icelandic pit chains as planetary analogs: Using morphologic measurements of pit chains to determine regolith thickness. *Journal of Geophysical Research: Planets*, 124(11), 2983–2999. <https://doi.org/10.1029/2019JE006099>
- Wilcoski, A. X., Hayne, P. O., & Elder, C. M. (2023). Thermal environments and volatile stability within lunar pits and caves. *Journal of Geophysical Research: Planets*, 128, e2023JE007758(7). <https://doi.org/10.1029/2023JE007758>
- Wilkins, H., Iliffe, T. M., Oromí, P., Martínez, A., Tysall, T. N., & Koenemann, S. (2009). The Corona lava tube, Lanzarote: geology, habitat diversity and biogeography. *Marine Biodiversity*, 39, 155-167. doi:10.1007/s12526-009-0019-2
- Williams, E. A. (2021). Deformation and Failure of Lunar Lava Tubes (Master's thesis, University of Maryland, College Park).
- Williams, E. A., & Montési, L. G. J. (2025). Crack development inside and around lunar lava tubes. *Journal of Geophysical Research: Planets*, 130, e2024JE008553, doi:10.1029/2024JE008553
- Williams, E., Montesi, L., Whelley, P., Bell., E., (2022) Abstract, *AGU Fall Meeting 2022*, Chicago, IL, 12-16 December 2022, id. V12C-0050.
- Williams, E. (2024). Lidar Survey of Lava Tubes, CA 2023. National Center for Airborne Laser Mapping (NCALM). Distributed by OpenTopography. <https://doi.org/10.5069/G9ST7N19>.
- Wynne, J. J., Titus, T. N., Agha-Mohammadi, A. A., Azua-Bustos, A., Boston, P. J., de León, P., ... & Williams, K. E. (2022). Fundamental science and engineering questions in planetary

- cave exploration. *Journal of Geophysical Research: Planets*, 127(11), e2022JE007194.
<https://doi.org/10.1029/2022JE007194>
- Wyrick, D. Y., & Buczowski, D. L. (2022). Pit crater chains across the solar system: Evidence for subterranean tectonic caves, porosity and permeability pathways on planetary bodies. *Journal of Geophysical Research: Planets*, 127(12), e2022JE007281. <https://doi.org/10.1029/2022JE007281>
- Wyrick, D., Ferrill, D. A., Morris, A. P., Colton, S. L., & Sims, D. W. (2004). Distribution, morphology, and origins of Martian pit crater chains. *Journal of Geophysical Research: Planets*, 109(E6), E06005, <https://doi.org/10.1029/2004JE002240>.
- Zhao, J., Huang, J., Kraft, M. D., Xiao, L., & Jiang, Y. (2017). Ridge-like lava tube systems in southeast Tharsis, Mars. *Geomorphology*, 295, 831-839.