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by

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RESEARCH REPORT

CHEMICAL PROCESS SYSTEMS ENGINEERING LABORATORY

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Short-Cut Operability Analysis

Part II - Estimation of f_i Detuning Parameter for Classical Control Systems

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ABSTRACT: In this paper a short-cut tuning method for multiloop PID controllers is presented. The method is an integral part of a short-cut operability analysis that provides a quick estimate of closed loop integral error. The tuning procedure provides a simple method of calculating how much multiloop controllers have to be detuned because of interaction. Tuning each loop is based on the nature and relative speeds of its interacting partners. A steady state version of the method is also presented with recommendations on its region of applicability.

Introduction

This series of articles presents a simplified method for predicting control system operability. The method provides short-cut predictions for a key measure of operability, the closed loop integral error of the controlled variable in response to a disturbance. In Part I of this series (Stanley, et al. (1985)) it was shown that the area under a multiloop error transient could be related to the area under an SISO error transient by two dimensionless parameters. The key result presented in Part I is:

$$(\int e\,dt)_{ML} = (\int e\,dt)_{SL}(f_i)(\beta_i) \tag{1}$$

where the subscripts ML and SL refer to multiloop and single loop configurations. The two dimensionless parameters are the relative disturbance gain (RDG), β_i, and the detuning parameter, f_i.

The definition and important properties of the RDG were presented in Part I. It can be calculated from steady state gains and indicates the interaction related to a specific disturbance. For 2x2 systems it is defined as:

$$\beta_i = \left(\frac{1}{1 - (K_{12}K_{21}/(K_{11}K_{22}))}\right)\left(1 - \frac{K_{Fj}K_{ij}}{K_{Fi}K_{jj}}\right) \tag{2}$$

The f_i parameter involves a ratio of controller parameters and is defined as:

$$f_i = \frac{(K_{ci})_{ML}(T_{Ri})_{SL}}{(K_{ci})_{SL}(T_{Ri})_{ML}} \tag{3}$$

where the subscript i refers to the loop under consideration. In order to calculate f_i and use it in an operability analysis, one needs a simple method to tune multiloop PID controllers.

In this paper we start out by presenting a new, short-cut approach to tuning multiloop PID controllers. A few papers on tuning multiloop PID systems have been published (Niederlinski (1971); Luyben (1986)). These tuning approaches make use of detailed dynamic models. Such information is rarely available at the design stage of a process when one would like to assess operability. In this paper we too start by assuming the availability of a dynamic model. This model is used to develop our multiloop PID tuning approach. We then address the important question of when steady state information alone is sufficient to estimate controller detuning. An answer for this question is given for an important class of 2x2 systems, namely those with first order dead-time elements in their transfer function matrix. When the product of the off diagonal time constants is less than or equal to the product of the diagonal time constants, steady state information is essentially all that is required to estimate f_i. For this class of problems steady state expressions are presented which can be used with a rough estimate of loop speed to calculate f_i for use in equation 1.

Another important result for 2x2 systems involves the magnitude of f_i as related to the steady state relative gain (RGA). We have found that systems with steady state RGA's less than 1.0 require more detuning and therefore have larger f_i's than systems with steady state RGA's greater than 1.0.

Before presenting the tuning approach the perspective taken in its development should be mentioned. We are not interested in a method to determine the best set of PID parameters, but rather one that gives reasonable estimates of tuning parameters. The goal of the paper is to provide a quick method of estimating f_i in equation 1 so that it can be used to assess process operability. In the proposed method the tuning of one loop is based on the effects of its interacting partners. The tuning method is applicable to any process; specifically, it does not require the process to be diagonally dominant. The method provides a ratio of the controller settings for the multiloop case with respect to single loop settings, allowing the use of SISO tuning charts on multiloop systems. In principle the tuning approach applies to nxn systems. A study of a large number of 2x2 cases has shown that the method produces well behaved transient responses. After a detailed discussion of the tuning method, the steady state results for 2x2 systems are presented.

II. Tuning Procedure

Marino-Galarraga, et al. (1984) presented a method for tuning 2x2 PID controllers used in a multiloop SISO structure. In this section their approach is generalized to an nxn system, shown in Figure 1. For this system the output, y, is related to the manipulated variable, m, by:

$$y = Gm \tag{4}$$

In developing the tuning procedure we focus on a particular loop in the system:

$$\Lambda_i = G_{ii}C_i \tag{5}$$

Three special cases for this loop are defined. First, Λ_i is assumed to be much slower than any other loop in the system (Case 1). Second, Λ_i is assumed to be much faster than any other loop (Case 2). Last, all loops are assumed to have identical Λ_i's and therefore equal response speeds (Case 3). For these three cases proportional only control is used and the stability limit for the i^{th} loop is calculated. An interpolation is carried out among the results of the three cases to arrive at final controller settings for the actual system. The three cases are discussed below and then the interpolation scheme is presented.

Case 1: Λ_i is the slowest loop.

If Λ_i is significantly slower than the other loops in the system, then perfect control for all y_j, $j \neq i$ can be assumed. For this case equation 4 can be written as:

$$\begin{pmatrix} o \\ o \\ \vdots \\ y_i \\ \vdots \\ o \\ o \end{pmatrix} = \begin{pmatrix} G_{11} G_{12} \cdots G_{1n} \\ G_{21} \\ \vdots \\ G_{n1} \cdots\cdots G_{nn} \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_i \\ m_n \end{pmatrix} \tag{6}$$

One way to achieve the results given by equation 6 is to use an Internal Model Controller (IMC) (Garcia and Morari (1982)) with a perfect model on all y_j, $j \neq i$. This IMC controller manipulates all m_j, $j \neq i$. Alternatively, if multiloop PID controllers are used on y_j, $j \neq i$ in an SISO arrangement and these loops are significantly faster than the Λ_i loop, then equation 6 is an excellent approximation, especially near the critical frequency of the i^{th} loop. It is only at high frquencies relative to ω_{ui} that the other PID loops would deviate from perfect control. The vector of manipulated variables can be calculated from equation 6 as:

$$m = G^{-1} y \tag{7}$$

Since all elements of y are zero except y_i, the transfer function relating y_i to m_i can be calculated from equation 7 as:

$$m_i = \hat{G}_{ii} y_i \tag{8}$$

where $\hat{G}_{ii}$ is the ii^{th} element of G^{-1}. The dynamic relative gain (Witcher and McAvoy, 1977) for an nxn system, $\lambda(s)$, is an nxn matrix that can be calculated as:

$$\lambda(s) = G \circ G^{-1} \tag{9}$$

where $\circ$ represents the element by element Hadamard matrix product

(Johnson, 1974). From equation 9 the ii^{th} element of $\lambda(s)$ is:

$$\lambda_{ii} = G_{ii}\hat{G}_{ii} \tag{10}$$

Substitution of equation 10 into equation 8 gives:

$$y_i = (G_{ii}/\lambda_{ii})m_i \tag{11}$$

If the $m_i - y_i$ loop is now closed and a proportional controller is used for C_i, the relationship that determines closed loop stability is:

$$1 + \frac{\Lambda_i}{\lambda_{ii}} = 0 \tag{12}$$

From equation 12 the ultimate gain and frequency of the slow, i^{th} loop can be calculated.

<u>Case 2</u>: Λ_i <u>is the fastest loop.</u>

If Λ_i is significantly faster than the other loops shown in Figure 1, then as an approximation these other loops can be considered as being off. Mathematically, all manipulative variables except m_i are set to 0 giving:

$$\begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & & & \vdots \\ \vdots & & & \vdots \\ G_{n1} & & \cdots & G_{nn} \end{pmatrix} \begin{pmatrix} o \\ \vdots \\ m_i \\ \vdots \\ o \end{pmatrix} \tag{13}$$

As can be seen in equation 13 y_i only depends on m_i in this case. Therefore, the stability relationship for the i^{th} loop can be written as:

$$1 + \Lambda_i = 0 \tag{14}$$

which is the traditional SISO relationship.

Case 3: <u>All loops have identical transfer functions.</u>

For this case G is replaced by G' defined by:

$$G'_{kj} = G_{kj} \tag{15}$$

$$G'_{kk} = G_{ii} \tag{16}$$

For the system shown in Figure 1 the closed loop transfer function relating y to d is:

$$y = [I + G'C]^{-1}Fd \tag{17}$$

The roots of the determinant of I + G'C determine the stability of the system in this case:

$$|I + G'C| = 0 \tag{18}$$

Equation 18 is solved for proportional only control with $s = i\omega$ to calculate the ultimate properties of the i^{th} loop. Equation 18 has multiple roots. In our studies to date we have found that the root with the lowest frequency is the best one to use to calculate controller detuning. Thus, in solving equation 18 ω is increased from 0 until the first root is found and then the calculation is terminated.

Equations 12, 14, and 18 provide explicit relationships for calculating the ultimate gain and frequency for the three special cases. These equations are solved for every loop in the system. However, most physical processes do not fall into these three limiting categories. To calculate PID tuning parameters for a general interacting system, an interpolation among the three special cases is used. This interpolation is based on relative loop speed as measured by the ultimate frequency of a loop with all other loops off, i.e., its SISO ultimate frequency. To explain the interpolation procedure it is convenient to consider a 2x2 system.

2x2 Systems:

Consider the controller gain first. The three values of the ultimate gain calculated for Cases 1-3 are normalized by dividing by K_{ui}, the SISO ultimate gain for loop i. A normalized loop speed for loop i is defined as:

$$s_i = \frac{\omega_{ui}}{\omega_{u1} + \omega_{u2}} \qquad (19)$$

A plot of normalized ultimate gain versus s_i can be made as shown in Figure 2a. The three points which result from the solution of equations 12, 14, and 18 are shown as circles. The slow case corresponds to $s_i = 0.0$, the fast case to $s_i = 1.0$, and the equal case to $s_i = 0.5$. The dotted line shown in Figure 2a can be sketched by hand or a parabola can be fitted to the three points. Alternatively, one can use linear interpolation.

Suppose that we fix Λ_1 and vary its interacting partner, Λ_2. When Λ_2 is very fast relative to Λ_1 we are at an x-axis value of 0. As Λ_2 becomes slower we move to the right. Eventually, when Λ_2 is very slow we arrive at an x-axis value of 1.0 and an SISO case for Λ_1. Since Λ_1 becomes faster and faster relative to Λ_2 along the x-axis, this axis can be viewed as the speed of loop 1 relative to the sum of the speeds of loop 1 and 2. If a gain margin technique is used to tune the controllers, then the y-axis can be viewed as the reduction in controller gain that results from interaction.

For the reset times a plot similar to Figure 2a can be constructed. From the solution of equations 12, 14, and 18 the ultimate frequency for the interacting system, ω_{uIi}, can be determined. If it is assumed that reset times are inversely proportional to ultimate frequencies, then the plot shown in Figure 2b results. Note that the x-axis is the same as in Figure 2a. The y-axis in Figure 2b can be interpreted as the change in reset time due to interaction. Again, the dashed line can be sketched in or a parabola can be fitted. If a PID controller is to be used, the y-axis in Figure 2b would also represent the change in the inverse of the derivative time due to interaction effects.

For some cases, e.g., Case 1 with a steady state relative gain greater than 1, the y-axis values in Figure 2 can be greater than 1.0. When this occurs the interaction in the system is favorable. In our tuning procedure we have chosen not to take advantage of favorable interaction and have limited the values for Cases 1-3 to 1.0 for interpolation. The rationale for this choice is that the multiloop control system should be tuned to cope with the possibility of a loop being lost due to an actuator / transmitter failure or simply being turned off. The use of more aggressive PID settings, compared to SISO values, could cause trouble in this case.

Figures 2a and 2b can be used to develop good estimates of controller parameters for 2x2 systems. The approach involves interpolating along the x-axis and reading the change in the controller parameters relative to the SISO settings from the y-axis. As a clear way of presenting these results, it is useful to consider an example to highlight the details of the interpolation procedure.

Demonstrative Example:

The controller design for an 8-tray experimental distillation tower studied by Wood and Berry (1973) is developed in this section. The column was used for methanol-water separation and was found to be well described by the transfer function model:

$$y(s) = G(s)\ m(s) + F(s)\ d(s) \tag{20}$$

The transfer function matrices were determined by experimentally pulse testing the tower (Wood and Berry, 1973):

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s + 1} & \dfrac{-18.9e^{-3s}}{21.0s + 1} \\ \dfrac{6.6e^{-7s}}{10.9s + 1} & \dfrac{-19.4e^{-3s}}{14.4s + 1} \end{bmatrix} \tag{21}$$

$$F(s) = \begin{bmatrix} \dfrac{3.8e^{-8.1s}}{14.9s + 1} \\ \dfrac{4.9e^{-3.4s}}{13.2s + 1} \end{bmatrix} \tag{22}$$

The first step is the calculation of the ultimate properties of the individual loops (Luyben, 1973), and these values are given in Table I. From the ultimate frequencies the relative speed of loop 1 is:

$$\frac{\omega_{u1}}{\omega_{u1} + \omega_{u2}} = 0.74 \tag{23}$$

and the relative speed of loop 2 is:

$$\frac{\omega_{u2}}{\omega_{u1} + \omega_{u2}} = 0.26 \tag{24}$$

The dynamic relative gain, $\lambda(s)$, for this system is given by:

$$\lambda(s) = \frac{1}{1 - \frac{0.5(16.7s+1)(14.4s+1)e^{-6s}}{(21s+1)(10.9s+1)}} \tag{25}$$

The reduction of the controller settings for the three limiting cases is calculated from equations 12, 14 and 18. The results of these calculations are given in Table I. Plots of normalized ultimate frequency and normalized ultimate gain versus relative loop speed for loops 1 and 2 are shown in Figures 2 and 3 respectively. A Lagrangian polynomial fitted to the three points is shown as a dotted line in the figure. At a relative loop speed of .74 for loop 1, the change in controller parameters relative to the SISO settings can be read from the y-axis of Figure 2. The interpolated points are represented by the closed squares, and the reduction in the ultimate properties of loop 1 are

$$\omega_{uI1}/\omega_{u1} = 0.635 \tag{26}$$

$$K_{uI1}/K_{u1} = 0.955 \tag{27}$$

If the reset time is assumed to be inversely proportional to the ultimate frequency, the reduction in the ultimate frequency given by equation 26 can be viewed as the change in the reset time resulting from interaction. That is:

$$T_{R1}/T_{RI1} = \omega_{uI1}/\omega_{u1} = 0.635 \tag{28}$$

In a similar way, equation 27 represents the reduction in the controller gain:

$$K_{cI1}/K_{c1} = K_{uI1}/K_{u1} = 0.955 \tag{29}$$

Following the same interpolation procedure for loop 2 gives the results shown in Table I. Any SISO tuning procedure or charts can be used to calculate controller parameters for the SISO loops. The tuning charts reported by Lopez (1968) are used here. The criterion he used is the ITAE (minimum integral of time multiplied by the absolute value of the error for upset response). The SISO tuning parameters and the multiloop parameters resulting from the interpolation are given in Table II.

The response of the system to a step disturbance of 0.34 lb/min in feed flow rate is shown in Figure 4. As can be seen, the tuning method developed here not only provides a systematic procedure for loop tuning, but it also results in a well behaved transient response in terms of decay ratio. Although the x_B response is sluggish it cannot be improved substantially. The sluggishness results from loop interaction when diagonal PI controllers are used. To eliminate this behavior some type of advanced control is required.

In a wide spectrum of 2x2 cases studied (Marino-Galarraga, 1985) the proposed method has been found to produce excellent PI tuning parameters. The cases studied include a model of an actual distillation tower (Luyben and Vinante (1972)), a model of a catalytic reactor (Marino-Galarraga, et al. (1984)), and distillation tower models (Toijala and Fagervik (1972); Weischedel and McAvoy (1980)). In addition 52 other cases that are discussed below were also studied. The steady state relative gain ranged from .5 to 13.7 in all the cases studied. In essentially all cases the responses obtained are well behaved, and they typically have a very good damping ratio.

2x2 Case Studies:

To test the tuning method a number of examples were generated. The process transfer function matrix, G, was chosen to have first order with dead-time elements. For 2x2 systems equation 18 becomes (McAvoy, 1983):

$$1 + 2\Lambda' + \frac{\Lambda'^2}{\lambda(s)} = 0 \qquad (30)$$

where $\lambda(s)$ is the dynamic relative gain (Witcher and McAvoy, 1977). Examination of the basic tuning equations 12, 14, and 30 shows that the only parameter that appears is $\lambda(s)$. In developing cases to be studied, we therefore chose systems with significant differences in the behavior of their dynamic relative gains. For first order with dead-time processes, $\lambda(s)$ is given by:

$$\lambda(s) = \frac{1}{1 - \dfrac{K_{12}K_{21}(T_{11}s + 1)(T_{22}s + 1)e^{-\theta s}}{K_{11}K_{22}(T_{12}s + 1)(T_{21}s + 1)}} \qquad (31)$$

where:

$$\theta = \theta_{12} + \theta_{21} - \theta_{11} - \theta_{22} \qquad (32)$$

Differences in dead-time and time constants in a process have a profound effect on $\lambda(s)$. In developing the additional cases to study two types of systems were considered: systems where the time constants cancel but there is a net dead-time, (Type I System), and systems with significantly different dead-times and time constants (Type II System).

For Type I Systems the gain ratio $(K_{12}K_{21}/K_{11}K_{22})$ and θ completely determine $\lambda(s)$. Since $K_{12}K_{21}/K_{11}K_{22}$ is related to the steady state relative gain, a knowledge of $\lambda(0)$

and θ is sufficient to tune multiloop PI controllers effectively in Type I systems. Further, we have found that θ has only a minimal effect on estimating f_i.

For Type II systems it is convenient to define a time constant ratio as:

$$R = \frac{T_{11}T_{22}}{T_{12}T_{21}} \tag{33}$$

The parameter R measures the speed of the off-diagonal elements of G relative to the diagonal elements of G. Two general categories of Type II systems are covered. One category involves R significantly greater than unity; this category involves systems with interaction that is fast relative to the diagonal dynamics. The second category with R significantly less than unity involves systems with slow interactive dynamics. For Type I systems R = 1.

Forty Type I processes ranging between fast-slow loops and equal loops were tuned using the method developed here. Table III gives the parameters for the systems. The objective of these cases is to study the performance of the tuning method for loops of different relative speeds. The time constants of the loops were taken to be equal to unity, and the different speeds were achieved by changing the dead-times. Two different values of the steady state RGA were considered, $\lambda(0) = 0.5$ and $\lambda(0) = 2$. Table IV shows the reduction in the controller settings and the f_i factors resulting from the tuning method. As in the previous examples, the SISO loops were tuned using the correlation given by Lopez (1968) for minimum ITAE. Figure 5 shows the response of one of the forty systems to a load disturbance. This response is representative of the general behavior of all the systems studied. Table IV shows an interesting result for Type I processes with the same dynamics but different steady state RGA's. Systems with steady state RGA's less than 1.0 require more detuning and therefore have larger f_i factors than those with RGA's greater than 1.0.

Twelve additional Type II systems were studied. These systems were chosen to have vast differences in the time constants of the elements. When the time constants are different and do not cancel, the value of the magnitude of the dynamic RGA can be considerably different from the steady state RGA. Table V shows the description of the twelve systems, and Table VI gives the reduction in the controller settings and the f_i factors resulting from the tuning method. As can be seen in Table VI, when R is greater than unity the controller settings have to be reduced considerably with respect to the SISO settings. The reason for this is that in this case the magnitude of the dynamic RGA is much smaller than the steady state relative gain. Conversely, when R is smaller than unity the controller settings are very similar to the SISO settings. The results of Table 6 show that for Type II systems the steady state RGA does not indicate the degree of detuning, f_i, as it does for Type I systems. Figure 6 shows the response of one of the twelve systems, namely that with the largest value of R. As can be seen the controller settings determined by the proposed tuning method result in a reasonable transient performance. This response has a smaller decay ratio than that seen in most of the other systems.

In summary, a wide spectrum of 2x2 cases has been studied, and in essentially all of them the responses produced by the tuning method are well behaved, and typically have a very good damping ratio.

3x3 and Higher Order Systems:

The tuning procedure has been applied to a limited number of 3x3 systems with success. A brief outline of its application to 3x3 and higher order systems is given here. For additional details and specific applications the reader is referred to Marino-Galarraga (1985). To explain the tuning procedure for 3x3 systems the triangle shown in Figure 7 is used. It is useful to consider this triangle as being analogous to that used for solving material balances in ternary extractions. Each vertex has a value of 1 associated with it while the side opposite a vertex has a value of 0. In extraction these values refer to mole fractions. Here the values will be the relative SISO loop speeds defined as:

$$s_i = \frac{\omega_{ui}}{\omega_{u1} + \omega_{u2} + \omega_{u3}} \qquad (34)$$

At each vertex of the triangle one loop is much faster than the other two loops. Along a side opposite its vertex a loop is much slower than the other two loops. Thus, at point 1 loop 1 is fast and along side 2-3 loop 1 is slow. First, consider tuning the three loops along side 2-3. For loop 1 equation 12 applies and the reduction in controller parameters for loop 1 apply to the entire length of side 2-3. Since loop 2 and loop 3 are fast relative to loop 1, these loops can be treated as a 2x2 system using the approach discussed above. Three cases can be solved for loop 2 along side 2-3, namely when loop 2 is fast, slow and equal to loop 3. The slow tunings apply at vertex 3, the equal tunings apply halfway between vertices 2 and 3 and the fast tunings apply at vertex 2. Now consider the case where all three loops are equal. This case corresponds to the centroid of the triangle. At this point equation 18 has to be solved three times to tune the three loops. First, Λ' is set to have diagonal elements equal to $G_{11}C$, then $G_{22}C$, and finally $G_{33}C$. Last, at vertex 1 equation 14 applies to loop 1 and equation 12 for loop 2 and loop 3. Following this tuning procedure the reduction in controller parameters for all three loops is obtained at the 7 points labeled a through g in Figure 7. To get the final reduction in settings one interpolates between these seven points.

The proposed method can be used on 4x4 and higher order systems. For 4x4 systems the equilateral triangle is replaced by a tetrahedron. Each of the four sides of the tetrahedron represents a 3x3 problem, which has just been solved, with the fourth loop being slow. At the centroid of the tetrahedron all loops are identical. Again, an interpolation based on relative SISO loop speed can be made to arrive at final settings.

III. Procedure Using Steady State RGA for 2x2 Systems

In order to relate the RDG to the area under the response curve resulting from a step disturbance, an estimate of f_i (equation 1) is necessary. However, at early stages of process design only steady state information is typically available. An important question from the point of view of process operability is the determination of when steady state information only is sufficient to estimate f_i. A steady state version of the short-cut tuning procedure for 2x2 systems is presented in this section. The procedure given here can be used to obtain first estimates of the reduction in controller settings in order to evaluate f_i. It should be noted that a significant number of industrial multiloop problems are 2x2 in nature. For these systems the results presented here are significant.

In developing the steady state procedure $\lambda(s)$ is set equal to $\lambda(0)$. Equations 12, 14, and 18 are solved and an interpolation is carried out. The three basic cases are treated below.

Case 1: Λ_1 much slower than Λ_2

For this case equation 12 becomes:

$$\Lambda_1 = -\lambda(0) \tag{35}$$

Since $\lambda(0)$ is constant, ω_{u1I} and ω_{u1} are equal in this case. The ultimate gain for the interacting system is $\lambda(0)$ times the value of the ultimate gain for the SISO system. Therefore, if we use a gain margin technique as before, the reduction in the controller gain is proportional to the reduction in the ultimate gain. The change in reset time is inversely proportional to the change in ultimate frequency:

$$T_{R1}/T_{Rs1} = 1 \qquad \text{any } \lambda(0) \tag{36}$$

$$K_{cs1}/K_{c1} = \lambda(0) \tag{37}$$

As can be seen in equation 37, if $\lambda(0) > 1$ the resulting controller gain will be higher than the SISO value. In these cases, to avoid possible stability problems if one of the loops is open, the SISO controller gain is used.

$$K_{cs1}/K_{c1} = 1 \qquad \lambda(0) > 1 \tag{38}$$

Equation 37 is used when $\lambda(0) < 1$.

<u>Case 2: Λ_1 much faster than Λ_2:</u>

The stability equation in this case is given by equation 14, as the traditional single loop equation. Therefore, for this case:

$$T_{R1}/T_{RF1} = 1 \tag{39}$$

$$K_{cF1}/K_{c1} = 1 \tag{40}$$

<u>Case 3: $\Lambda_2 = \Lambda_1$:</u> In this case equation 30 becomes:

$$\lambda_1 = -\lambda(0) \pm \sqrt{\lambda(0)^2 - \lambda(0)} \tag{41}$$

Setting $s = i\omega$, the ultimate frequency and ultimate gain are calculated from equation 41 as:

$$\angle G_{11} = \angle\left(-\lambda(0) \pm \sqrt{\lambda(0)^2 - \lambda(0)}\right) \tag{42}$$

$$K_{uE1} = \frac{\left|1 - \lambda(0) \pm \sqrt{\lambda(0)^2 - \lambda(0)}\right|}{|G_{11}|} \tag{43}$$

When $\lambda(0) > 1$ the right hand side of equation 42 is real and the ultimate frequency is unaffected by the interaction. The stability limit is determined by the positive square root, since this sign results in the greatest reduction of the ultimate gain.

When $\lambda(0) > 1$:

$$T_{R1}/T_{RE1} = 1.0 \qquad \lambda(0) > 1 \tag{44}$$

$$K_{cE1}/K_{c1} = \left| -\lambda(0) + \lambda(0)^2 - \lambda(0) \right| \tag{45}$$

From the above equations, the reset time will be equal to the SISO value, and the controller gain must be decreased up to a maximum of 1/2 its single loop value when $\lambda(0) > 1$. The value of 1/2 results when $\lambda(0) >> 1$.

If $\lambda(0) < 1$, the negative square root in equation 41 determines the stability limit (McAvoy, 1981). The ultimate frequency and ultimate gain can be calculated from equations 42 and 43 with the negative square root. If dynamic information about the system is not available, equations 42 and 43 can be approximated. For the case of a system described by a first-order with dead-time transfer function, the following approximations are derived in the Appendix:

$$T_{R1}/T_{RE1} = 1.0 - \frac{1.5}{\pi} \tan^{-1}\left\{\sqrt{1/\lambda(0) - 1}\right\} \tag{46}$$

$$K_{cE1}/K_{c1} = \sqrt{\lambda(0)}\left(1.0 - \frac{1.0}{\pi} \tan^{-1}\left(\sqrt{1/\lambda(0) - 1}\right)\right) \tag{47}$$

Thus, for $\lambda(0) < 1$, not only is the controller gain reduced from its single loop value, but the reset time is increased as well due to the interaction.

After the reductions for the slow and equal cases have been calculated, the relative speed of the loops, s_i, is used to interpolate the actual reductions for the interacting system. If the relative speed of the loops is not known and it cannot be estimated, the worst case (typically the equal case) can be used in order to obtain a conservative estimate of the reduction in the controller settings.

The results of applying the steady state tuning approach to the 40 Type I and 12 Type II processes are shown in Tables IV and VI. In calculating the f_i's it was assumed that the relative loop speeds were known. First, the results in Table IV show that for Type I processes the steady state approach is reasonably accurate in terms of estimating f_i for use in a preliminary operability analysis. It can be noted that experimentally obtained models of two actual distillation towers exhibited Type I behavior (McAvoy (1981)). One of the towers is the Wood Berry tower described by equation 21. The results of applying the steady state tuning method to this model are given in Tables I and II. The resulting transient response is shown in Figure 4. As can be seen the steady state approach results in a stable, but somewhat oscillatory response in this case. However, what we are primarily interested in is obtaining a reasonable estimate of f_i. It is shown in Part III (Marino-Galarraga, et al. 1985a) that the steady state f_i estimates in this example are accurate enough to clearly assess the operability of the control system. An interesting research area that we are pursuing is to determine which types of process control systems, particularly distillation systems, exhibit Type I behavior. For these systems the results in Table IV indicate that only steady state information is required to estimate f_i for use in assessing control system operability.

For Type II processes the results in Table VI indicate that large differences can exist between the steady state and dynamically calculated f_i's. To analyze these differences it is convenient to consider the case when $R > 1$ and $R < 1$ separately. When $R >> 1$ and $\lambda(0) > 1$ the steady state approach underestimates f_i significantly. When $\lambda(0) < 1$ and $R > 1$ the method is better but substantial errors can still exist. When $R > 1$ the off diagonal process transfer functions have faster time constants than the diagonal elements. For these processes dynamic interaction effects can be significantly different from steady state interaction effects. One example of a process with $R > 1$ is a packed bed chemical reactor discussed in Marino-Galarraga et al. (1984). The model for this reactor is:

$$\begin{pmatrix} \bar{T}_{out} \\ \\ c_{out} \end{pmatrix} = \begin{pmatrix} \dfrac{-2.265e^{-1.326s}}{0.786s + 1} & \dfrac{0.746e^{-2.538s}}{0.092s + 1} \\ \\ \dfrac{1.841e^{-0.411s}}{0.917s + 1} & \dfrac{-.654e^{-0.768s}}{0.870s + 1} \end{pmatrix} \begin{pmatrix} \bar{Q} \\ \\ \bar{T}_q \end{pmatrix} \quad (48$$

This model, which is not diagonally dominant, resulted from a simplification of a state space model originally published by Foss, et al. (1980). The control objective is the regulation of reactor outlet concentration and temperature by manipulation of quench flow and quench temperature. The value of R for this reactor is 11.0. This large value results from the fact that the time constant for thermal transients produced by changes in T_q is much smaller than the time constant for composition transients.

The two loops in this example were tuned using the dynamic model. The resulting controllers and f_i values are given in Table VII. The transient response to a step upset in Q is shown in Figure 8. The steady state tuning method was also applied to this example. The controller parameters and f_i values resulting from the steady state approach are also given in Table VII. The transient response produced by these parameters is shown in Figure 8 for comparison purposes. As can be seen in this case the steady state approach produces excellent results. However, in general, for processes with $\lambda(0) > 1$ and $R >> 1$, one cannot trust the values of f_i estimated from steady state information only.

For Type II processes with $R << 1$, the results in Table VI indicate that the steady state approach also is in error for $\lambda(0) < 1$. In this case the steady state f_i's overestimate the degree of detuning required and a simple remedy exists. For systems with $R << 1$ at least one off-diagonal element has a much slower time constant than the diagonal elements. Near the critical frequencies of the diagonal elements interaction in such systems is minimal and $\lambda(s) \cong 1.0$. If $\lambda(s) \cong 1.$, then each loop can be tuned to its SISO controller settings and $f_i \cong 1.0$. Thus, for Type II processes with $R << 1$, f_i can be approximated at the design stage as 1.0. The smaller R is then the better is this approximation. McAvoy (1983) discusses a head-box example which exhibits Type II behavior in terms of interaction effects and has an $R << 1.0$. In this example one pairs a 0. steady state RGA element. However, at high frequencies the dynamic RGA approaches 1.0 and the system is very well behaved.

In summary, we have found that for many 2x2 systems the steady state tuning procedure works very well. When the time constants cancel and R = 1 (Tables IV and V) very good results are achieved. When $R << 1.$, SISO tuning values can be used and f_i can be taken as 1.0. It is primarily when $R >> 1.0$ that the steady state method can result in large errors. These results are significant from the point of view of assessing process operability at the design stage. At this stage one typically has only steady state information about the process. It should be emphasized that what we are interested in is reasonable estimates of f_i and not <u>the</u> best estimate. The results are also significant since a number of important industrial processes involve 2x2 control systems. Although the steady state tuning method can be extended to 3x3 and higher order systems, we have only very limited results in this area. Additional work is required to determine if the steady state approach will be useful in such systems.

IV. Summary and Conclusions

A simple approach to the problem of estimating multiloop PI tuning parameters has been presented. This approach allows one to calculate how much controller parameters have to be reduced due to interaction. It can be applied to non-diagonally dominant systems. The approach allows one to extend the use of SISO tuning charts to multiloop systems. The tuning method has been found to work well on many examples, primarily 2x2 systems. For 2x2 systems where the diagonal and off-diagonal time constants cancel, it has been found that processes with steady state RGA's less than 1.0 require more detuning than those with RGA's greater than 1.0. For 2x2 systems with slow off-diagonal time constants relative to their diagonal time constants, controllers have to be detuned very little. The tuning approach can also be used on nxn systems.

A steady state version of the tuning method has been developed. For many 2x2 systems studied this steady state approach yields excellent results in terms of calculating how much controllers have to be detuned due to interaction. The steady state approach can fail when off diagonal time constants are fast compared to diagonal time constants. Since the steady state method makes use of readily available design information, it is a promising tool for assessing process operability.

Nomenclature

C	=	Diagonal controller matrix
C_i	=	Element of C
C_{out}	=	Outlet concentration
d	=	Disturbance
e	=	Error
f_i	=	Detuning factor for loop i defined by eq. 3
F	=	Diagonal matrix of upset transfer functions
G	=	Matrix of process transfer functions
G_{ij}	=	Element of G
G^{-1}	=	Inverse of G
$\hat{G}_{ij}$	=	Element of G^{-1}
G'	=	Matrix defined by equations 15 and 16
i	=	$\sqrt{-1}$
I	=	Identity Matrix
K_{ci}	=	Controller gain
K_{Fi}	=	Disturbance gain
K_{ij}	=	Process gain
m	=	Manipulative variable
Q	=	Quench flow
R	=	Time constant ratio defined by eq. 33
s	=	Laplace variable
s_i	=	Relative speed of loop i
t	=	Time
T_{ij}	=	Time constant

T_{Ri}	=	Reset time
T_{out}	=	Outlet temperature
T_q	=	Temperature of quench flow
x_B	=	Bottoms composition
y	=	Process output

Greek Letters

β_i	=	Relative disturbance gain
λ	=	Relative gain
Λ_i	=	Loop transfer function ($C_i G_{ii}$)
Λ'	=	Loop transfer function for equal case
θ	=	Dead-time
ω	=	Frequency

Subscripts

E1	=	Equal case, loop 1
F1	=	Fast case, loop 1
i	=	Index
I1	=	Interacting case
j	=	Index
k	=	Index
ML	=	Multiloop
S1	=	Slow case, loop 1
SL	=	Single loop
u	=	Ultimate

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Table 1

Ultimate Properties For Wood-Berry Tower

Single Loop

Loop Number	K_{ui}	ω_{ui}
1	2.099	1.608
2	- .422	.564

Three Limiting Interacting Cases (Full Dynamic Model)

Loop Number	Case	K_{uIi}	K_{uIi}/K_{ui}	ω_{uIi}	ω_{uIi}/ω_{ui}
1	Slow	1.355	.646	1.584	.985
1	Equal	1.855	.884	.840	.522
1	Fast	2.099	1.0	1.608	1.0
2	Slow	- .270	.640	.550	.975
2	Equal	- .219	.519	.412	.730
2	Fast	- .422	1.0	.564	1.0

Three Limiting Interacting Cases (Steady State Model)

Loop Number	Case	K_{uIi}	K_{uIi}/K_{ui}	ω_{uIi}	ω_{uIi}/ω_{ui}
1	Slow	2.099	1.0	1.608	1.0
1	Equal	1.228	.585	1.608	1.0
1	Fast	2.099	1.0	1.608	1.0
2	Slow	- .422	1.0	.564	1.0
2	Equal	- .247	.585	.564	1.0
2	Fast	- .422	1.0	.564	1.0

Table 2

Controller Tuning For Wood-Berry Tower

SISO Settings (From Lopez (1968))

Loop Number	K_{ci}	T_{Ri}
1	1.02	4.18
2	-.21	10.29

Multiloop Settings (Full Dynamic Model)

Loop Number	K_{cIi}	T_{RIi}
1	.97	6.58
2	-.11	13.14

Multiloop Settings (Steady State Model)

Loop Number	K_{cIi}	T_{RIi}
1	.660	4.18
2	-.136	10.29

Table 3
Systems With Different Relative Speeds

SYSTEMS	Steady State RGA λ	Process Dead-Times θ_{11}	θ_{12}	θ_{21}	θ_{22}
1	0.5	0.1	0.1	1.0	1.0
2	0.5	0.1	0.2	2.0	1.0
3	0.5	0.1	0.12	1.2	1.0
4	0.5	0.1	0.15	1.5	1.0
5	2.0	0.1	0.1	1.0	1.0
6	2.0	0.1	0.2	2.0	1.0
7	2.0	0.1	0.0	0.0	1.0
8	2.0	0.1	0.3	1.35	1.0
9	0.5	0.2	0.2	0.8	0.8
10	0.5	0.2	0.4	1.6	0.8
11	0.5	0.2	0.25	1.1	0.8
12	0.5	0.2	0.3	1.2	0.8
13	2.0	0.2	0.2	0.8	0.8
14	2.0	0.2	0.8	1.2	0.8
15	2.0	0.2	0.0	0.0	0.8
16	2.0	0.2	0.5	1.0	0.8
17	0.5	0.4	0.4	0.6	0.6
18	0.5	0.4	0.8	1.2	0.6
19	0.5	0.4	0.5	0.8	0.6
20	0.5	0.4	0.6	0.9	0.6
21	2.0	0.4	0.4	0.6	0.6
22	2.0	0.4	0.6	1.4	0.6
23	2.0	0.4	0.0	0.0	0.6
24	2.0	0.4	0.5	1.0	0.6
25	0.5	0.4	0.4	0.4	0.4
26	0.5	0.4	0.8	0.8	0.4
27	0.5	0.4	0.5	0.5	0.4
28	0.5	0.4	0.6	0.6	0.4
29	2.0	0.4	0.4	0.4	0.4
30	2.0	0.4	0.8	0.8	0.4
31	2.0	0.4	0.0	0.0	0.4
32	2.0	0.4	0.6	0.6	0.4
33	0.5	0.6	0.6	0.6	0.6
34	0.5	0.6	1.2	1.2	0.6
35	0.5	0.6	0.8	0.8	0.6
36	0.5	0.6	0.9	0.9	0.6
37	2.0	0.6	0.6	0.6	0.6
38	2.0	0.6	1.2	1.2	0.6
39	2.0	0.6	0.0	0.0	0.6
40	2.0	0.6	0.9	0.9	0.6

$T_{11} = T_{12} = T_{21} = T_{22} = 1$

For $\lambda = 0.5$ $K_{11} = K_{12} = K_{22} = 1$ and $K_{21} = -1$

$\lambda = 2.0$ $K_{11} = K_{12} = K_{22} = 1$ and $K_{21} = 0.5$

Table 4

Reduction in Controller Settings for Systems with Different Relative Speeds

SYSTEM	K_{cI1}/K_{c1}	T_{R1}/T_{RI1}	K_{cI2}/K_{c2}	T_{R2}/T_{RI2}	λ	f_1	f_{1s}	f_2	f_{2s}
1	0.804	0.824	0.472	0.879	0.5	1.51	1.71	2.41	2.49
2	0.822	0.814	0.584	0.623	0.5	1.49	1.71	2.74	2.49
3	0.830	0.798	0.468	0.810	0.5	1.51	1.71	2.64	2.49
4	0.913	0.771	0.490	0.727	0.5	1.42	1.71	2.81	2.49
5	0.751	1.000	1.000	1.000	2.0	1.33	1.28	1.0	1.28
6	0.878	0.841	0.833	0.939	2.0	1.35	1.28	1.28	1.28
7	1.000	0.915	0.830	0.769	2.0	1.09	1.28	1.57	1.28
8	0.965	0.825	0.828	0.966	2.0	1.26	1.28	1.25	1.29
9	0.656	0.710	0.451	0.775	0.5	2.15	2.28	2.86	2.83
10	1.000	0.599	0.615	0.546	0.5	1.67	2.28	2.98	2.83
11	0.721	0.659	0.473	0.675	0.5	2.10	2.28	3.13	2.83
12	0.769	0.642	0.495	0.640	0.5	2.03	2.28	3.16	2.83
13	0.588	1.000	1.000	1.000	2.0	1.70	1.49	1.0	1.49
14	0.976	0.713	0.700	0.879	2.0	1.44	1.49	1.63	1.49
15	1.000	0.876	0.851	0.716	2.0	1.14	1.49	1.64	1.49
16	0.726	0.797	0.692	0.932	2.0	1.73	1.49	1.55	1.49
17	0.518	0.647	0.457	0.669	0.5	2.98	2.88	3.27	3.06
18	0.839	0.487	0.711	0.480	0.5	2.87	2.88	2.63	3.06
19	0.569	0.582	0.498	0.596	0.5	3.02	2.88	3.37	3.06
20	0.620	0.550	0.541	0.557	0.5	2.93	2.88	3.32	3.06
21	0.526	1.000	0.703	1.000	2.0	1.90	1.69	1.42	1.69
22	0.630	0.722	0.584	0.787	2.0	2.20	1.69	2.18	1.69
23	0.600	0.716	0.764	0.679	2.0	2.33	1.69	1.88	1.69
24	0.553	0.823	0.563	0.877	2.0	2.20	1.69	2.03	1.69
25	0.461	0.629	0.461	0.629	0.5	3.45	3.02	3.45	3.02
26	0.686	0.461	0.686	0.461	0.5	3.16	3.02	3.16	3.02
27	0.492	0.574	0.492	0.574	0.5	3.54	3.02	3.54	3.02
28	0.538	0.530	0.538	0.530	0.5	3.51	3.02	3.51	3.02
29	0.586	1.000	0.586	1.000	2.0	1.71	1.71	1.71	1.71
30	0.559	0.743	0.559	0.743	2.0	2.41	1.71	2.41	1.71
31	0.692	0.652	0.692	0.652	2.0	2.22	1.71	2.22	1.71
32	0.538	0.847	0.538	0.847	2.0	2.19	1.71	2.19	1.71
33	0.493	0.658	0.493	0.658	0.5	3.08	3.02	3.08	3.02
34	0.855	0.485	0.855	0.485	0.5	2.41	3.02	2.41	3.02
35	0.561	0.586	0.561	0.586	0.5	3.04	3.02	3.04	3.02
36	0.610	0.556	0.610	0.556	0.5	2.95	3.02	2.95	3.02
37	0.586	1.000	0.586	1.000	2.0	1.71	1.71	1.71	1.71
38	0.592	0.751	0.592	0.751	2.0	2.25	1.71	2.25	1.71
39	0.685	0.694	0.685	0.694	2.0	2.10	1.71	2.10	1.71
40	0.550	0.852	0.550	0.852	2.0	2.13	1.71	2.13	1.71

Table 5

Systems With Different Time Constant Ratios

SYST	λ	R	θ_{11}	θ_{12}	θ_{21}	θ_{22}	T_{11}	T_{12}	T_{21}	T_{22}
1	0.5	5.00	1.0	2.0	0.8	2.4	5	2	2	4
2	0.5	10.00	1.6	2.0	0.8	3.0	8	2	2	5
3	0.5	20.00	2.0	5.0	0.8	4.8	10	2	2	8
4	2.0	5.00	1.0	2.0	0.8	2.4	5	2	2	4
5	2.0	10.00	1.6	2.0	0.8	3.0	8	2	2	5
6	2.0	20.00	2.0	5.0	0.8	4.8	10	2	2	8
7	0.5	0.20	2.0	1.0	2.4	0.8	2	5	4	2
8	0.5	0.10	2.0	1.6	3.0	0.8	2	8	5	2
9	0.5	0.05	2.0	1.6	4.8	0.8	2	10	8	2
10	2.0	0.20	2.0	1.0	2.4	0.8	2	5	4	2
11	2.0	0.10	2.0	1.6	3.0	0.8	2	8	5	2
12	2.0	0.05	2.0	1.6	4.8	0.8	2	10	8	2

For $\lambda = 0.5$ $K_{11} = K_{12} = K_{22} = 1$ and $K_{21} = -1$

$\lambda = 2.0$ $K_{11} = K_{12} = K_{22} = 1$ and $K_{21} = 0.5$

Table 6

Reductions in Controller Settings for Systems With Different Time Constant Ratios

SYSTEM	K_{cI2}/K_{c1}	T_{RI}/T_{RI1}	K_{cI2}/K_{c2}	T_{R2}/T_{RI2}	λ
1	0.335	0.675	0.393	0.737	0.5
2	0.271	0.828	0.303	0.808	0.5
3	0.275	0.722	0.343	0.793	0.5
4	0.645	0.431	0.880	0.368	2.0
5	0.446	0.382	0.487	0.389	2.0
6	0.478	0.428	0.487	0.382	2.0
7	0.923	0.798	0.999	0.816	0.5
8	1.000	0.918	0.768	0.950	0.5
9	0.824	0.952	1.000	0.978	0.5
10	0.768	0.952	0.768	0.926	2.0
11	0.854	0.928	0.963	0.906	2.0
12	1.000	0.943	0.891	0.940	2.0

f_1	f_{1s}	f_2	f_{2s}
4.42	2.63	3.45	3.00
4.46	2.79	4.08	3.05
5.04	2.68	3.68	3.01
3.60	1.61	3.09	1.61
5.87	1.66	5.28	1.66
4.89	1.61	5.38	1.61
1.36	2.99	1.22	2.61
1.09	2.99	1.37	2.61
1.27	2.99	1.02	2.61
1.37	1.61	1.41	1.61
1.26	1.61	1.15	1.61
1.06	1.61	1.19	1.61

Table 7

Controller Parameters For Packed Bed Reactor

Single Loop

Loop Number	K_{ci}	T_{Ri}
1	- .23	1.67
2	-1.48	1.19

Multiloop Tuning (Full Dynamic Model)

Loop Number	K_{ci}	T_{Ri}	f_i
1	- .11	1.77	2.22
2	- .63	1.36	2.68

Multiloop Tuning (Steady State Approach)

Loop Number	K_{ci}	T_{Ri}	f_i
1	- .12	1.67	1.92
2	- .771	1.19	1.92

List of Captions

Appendix

In the derivation that follows G_{11} is taken as a first order with dead time transfer function:

$$G_{11} = \frac{K_{11}e^{-\theta_{11}s}}{T_{11}s + 1} \qquad \text{(A-1)}$$

Equation 39 with the negative square root gives the equation that has to be solved for the equal case when $\lambda(0) < 1$:

$$\angle G_{11} = \angle\left(-\lambda(0) - \sqrt{\lambda(0)^2 - \lambda(0)}\right) \qquad \text{(A-2)}$$

Substitution of equation A-1 into A-2 gives:

$$-\tan^{-1}\omega T_{11} - \omega\theta_{11} = \tan^{-1}\left(\sqrt{1/\lambda(0) - 1}\right) - \pi \qquad \text{(A-3)}$$

Two limiting cases can be used for the $\tan^{-1}$ term in equation A-3 to estimate ω_{uE1}. First at low ω's the $\tan^{-1}$ can be approximated as:

$$\tan^{-1}\omega T_{11} \cong \omega T_{11} \qquad \text{(A-4)}$$

Substitution of equation A-4 into A-3 gives:

$$\omega_{uE1} \cong \frac{\pi - \tan^{-1}\left(\sqrt{1/\lambda(0) - 1}\right)}{T_{11} + \theta_{11}} \qquad \text{(A-5)}$$

Using equation A-4 to estimate the SISO ω_{u1} gives:

$$\omega_{u1} \cong \frac{\pi}{T_{11} + \theta_{11}} \qquad \text{(A-6)}$$

Dividing equation A-5 by A-6 gives the reduction in ultimate frequencies as:

$$\frac{\omega_{uE1}}{\omega_{u1}} = 1 - \frac{\tan^{-1}\left(\sqrt{1/\lambda(0) - 1}\right)}{\pi} \qquad \text{(A-7)}$$

Alternatively a high frequency approximation for the $\tan^{-1}$ term in equation A-3 can also be used. For high frequencies:

$$\tan^{-1} \omega T_{11} \cong \frac{\pi}{2} \qquad \text{(A-8)}$$

Substitution of equation A-8 into equation A-3 gives:

$$\omega_{uE1} \cong \frac{\pi/2 - \tan^{-1}\left(\sqrt{1/\lambda(0) - 1}\right)}{\theta_{11}} \qquad \text{(A-9)}$$

Estimation of ω_{u1} using equation A-8 gives:

$$\omega_{u1} \cong \frac{\pi/2}{\theta_{11}} \qquad \text{(A-10)}$$

Dividing equation A-9 by A-10 gives the reduction in ultimate frequencies as:

$$\frac{\omega_{uE1}}{\omega_{u1}} \cong 1 - \frac{2 \tan^{-1}\left(\sqrt{1/\lambda(0) -1}\right)}{\pi} \qquad \text{(A-11)}$$

It was found that by averaging equations A-7 and A-11 excellent results were achieved. Thus the approximation used is:

$$\frac{\omega_{uE1}}{\omega_{u1}} = 1 - \frac{1.5 \tan^{-1}\left(\sqrt{1/\lambda(0) - 1}\right)}{\pi} \qquad \text{(A-12)}$$

To calculate the reduction in controller gains the magnitude equation is used. First a low frequency approximation is used as was done to develop equation A-4. In this case:

$$|G_{11}| \cong K_{11} \qquad \text{(A-13)}$$

Substitution of equation A-13 into equation 40 gives:

$$K_{uE1} \cong \frac{\sqrt{\lambda(0)}}{K_{11}} \qquad \text{(A-14)}$$

The SISO ultimate gain can be estimated using a low frequency approximation as:

$$K_{u1} \cong 1/K_{11} \tag{A-15}$$

giving:

$$\frac{K_{uE1}}{K_{u1}} \cong \sqrt{\lambda(0)} \tag{A-16}$$

Next a high frequency approximation is made. Substitution of equation A-9 into equation 40 and taking $\omega_{uE1}\, T_{11}$ to be large gives:

$$K_{uE1} \cong \frac{\sqrt{\lambda(0)}}{K_{11}} \quad \omega_{uE1}\, T_{11} \tag{A-17}$$

The SISO ultimate gain resulting from the use of a high frequency approximation is:

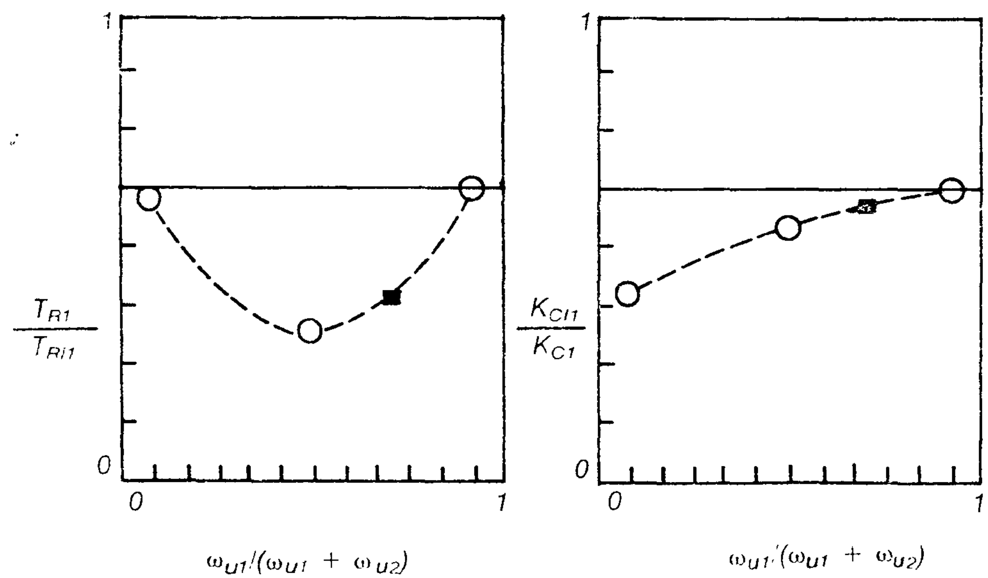
$$K_{u1} \cong \frac{\omega_{u1} T_{11}}{K_{11}} \tag{A-18}$$

Dividing equation A-17 by A-18 and substitution of equation A-11 gives:

$$\frac{K_{uE1}}{K_{u1}} \cong \sqrt{\lambda(0)} \left(1 - \frac{2\tan^{-1}\left(\sqrt{1/\lambda(0) - 1}\right)}{\pi}\right) \tag{A-19}$$

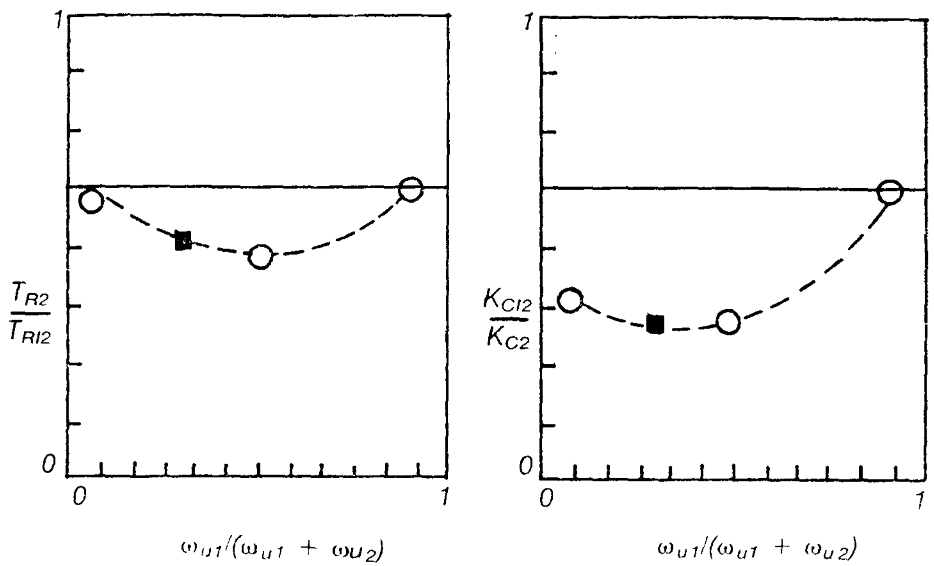
For the final approximation equations A-16 and A-19 were averaged to give:

$$\frac{K_{uE1}}{K_{u1}} = \sqrt{\lambda(0)} \left(1 - \frac{\tan^{-1}\left(\sqrt{1/\lambda(0) - 1}\right)}{\pi}\right)$$

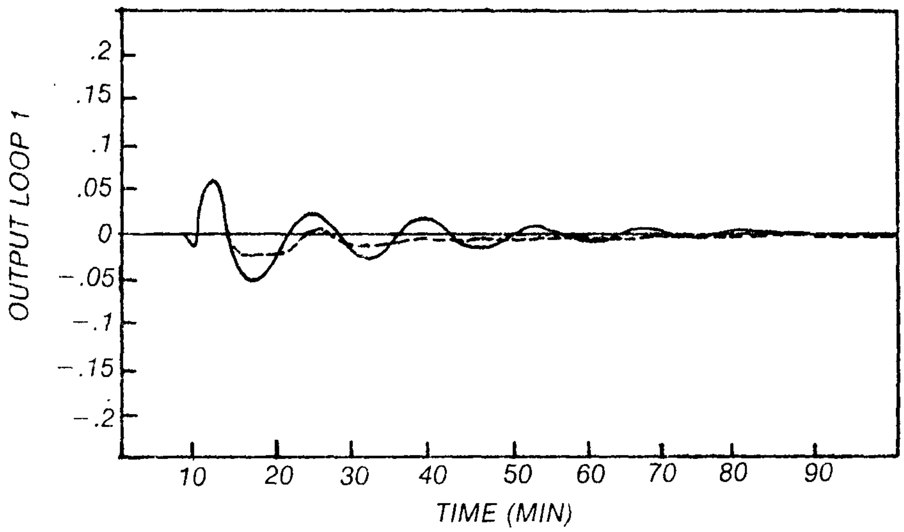


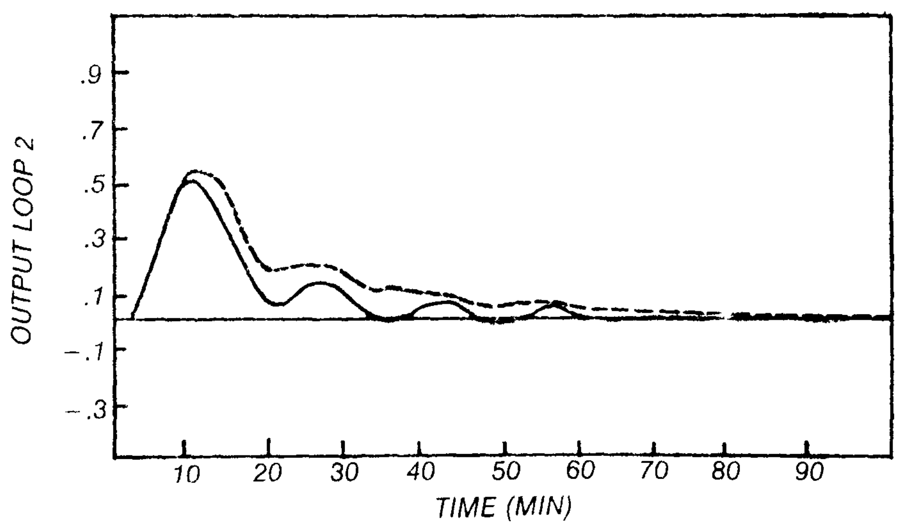
$\frac{T_{R1}}{T_{RI1}}$
1
0
0
1
$\omega_{U1}/(\omega_{U1} + \omega_{U2})$
$\frac{K_{CI1}}{K_{C1}}$
1
0
0
1
$\omega_{U1}/(\omega_{U1} + \omega_{U2})$

#2

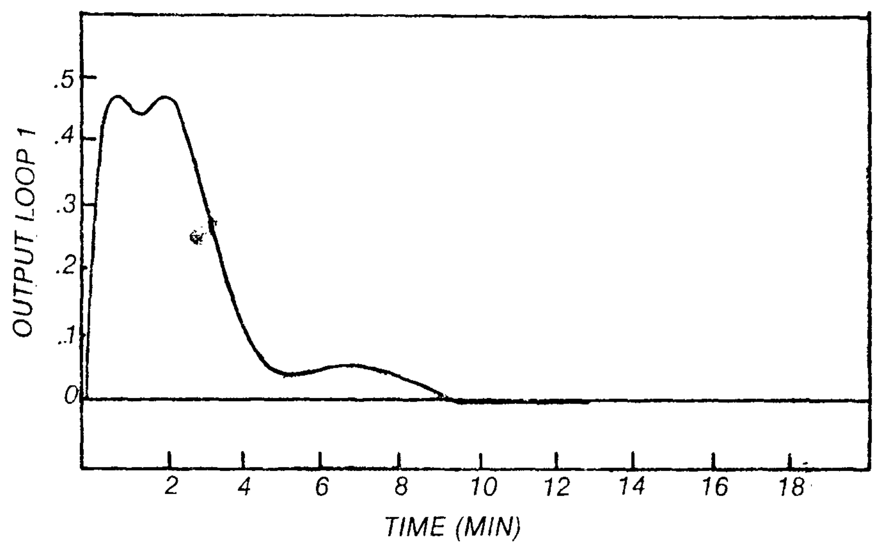

1
0
T_{R2}/T_{RI2}
0
1
$\omega_{u1}/(\omega_{u1} + \omega_{u2})$
1
0
K_{CI2}/K_{C2}
0
1
$\omega_{u1}/(\omega_{u1} + \omega_{u2})$

#3

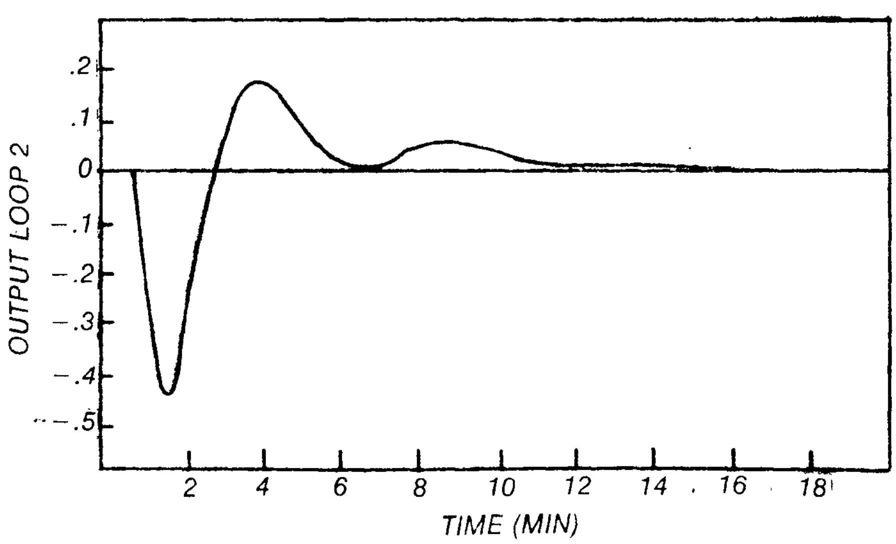

OUTPUT LOOP 1
.2
.15
.1
.05
0
-.05
-.1
-.15
-.2
10
20
30
40
50
60
70
80
90
TIME (MIN)


OUTPUT LOOP 2
.9
.7
.5
.3
.1
-.1
-.3
10
20
30
40
50
60
70
80
90
TIME (MIN)

4

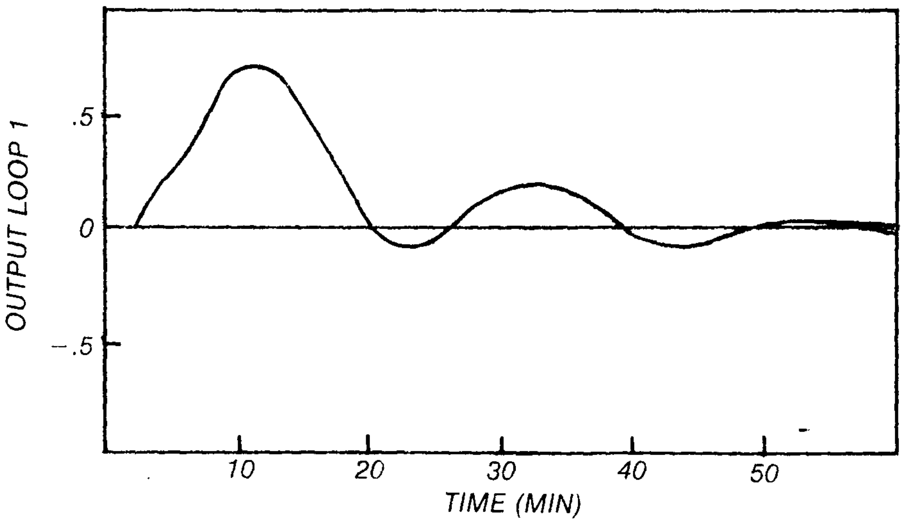


.5
.4
.3
.2
.1
0
OUTPUT LOOP 1
2
4
6
8
10
12
14
16
18
TIME (MIN)

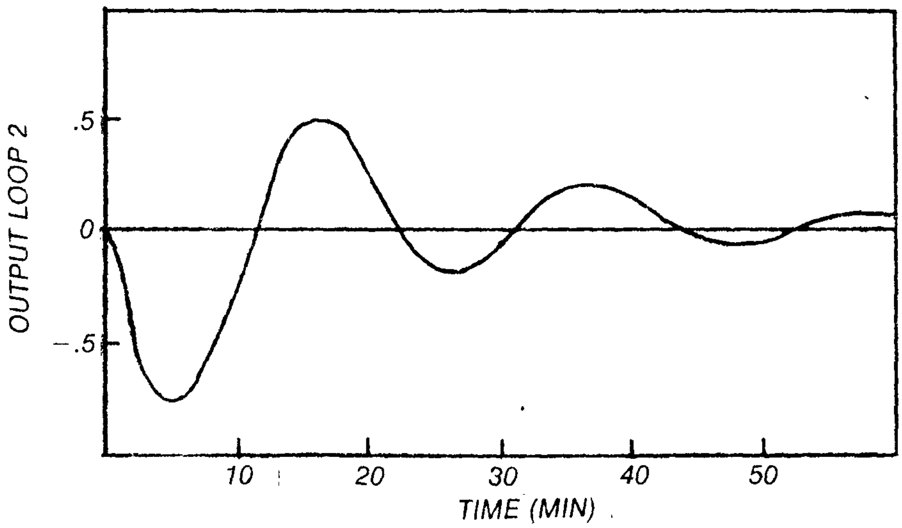


.2
.1
0
−.1
−.2
−.3
−.4
−.5
OUTPUT LOOP 2
2
4
6
8
10
12
14
16
18
TIME (MIN)

5

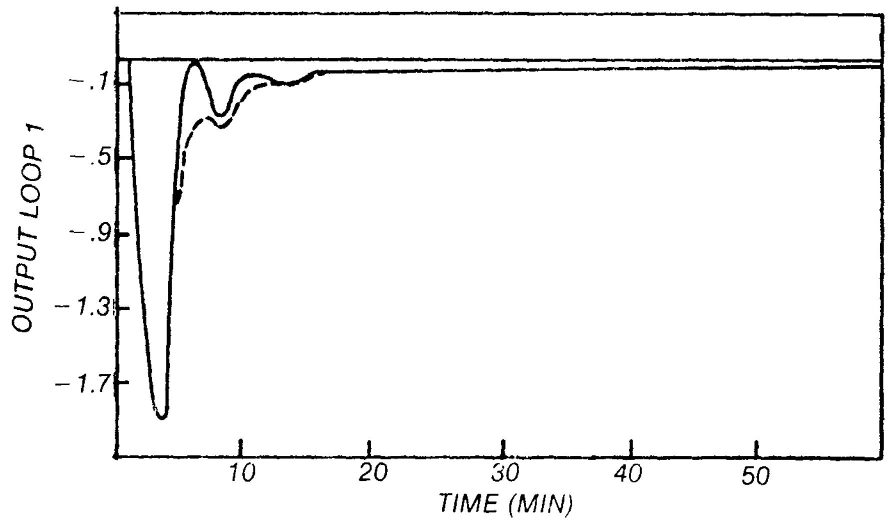


OUTPUT LOOP 1
.5
0
–.5
10
20
30
40
50
TIME (MIN)

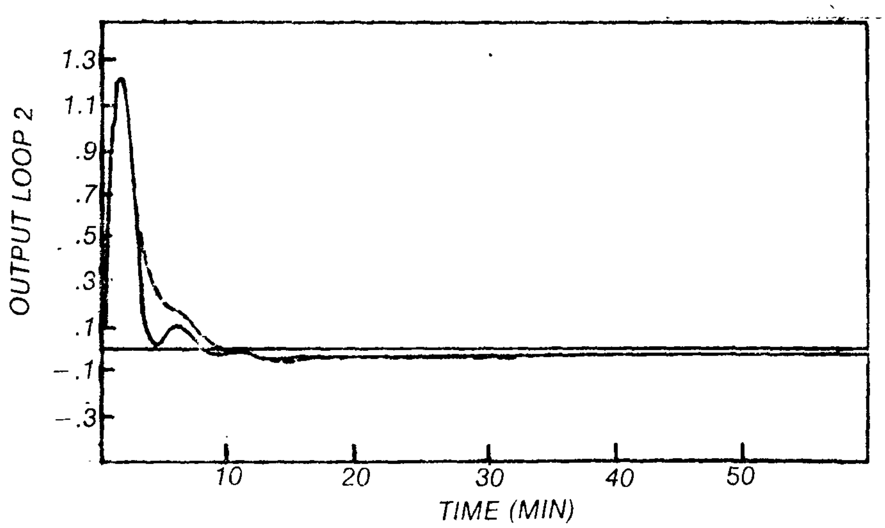


OUTPUT LOOP 2
.5
0
–.5
10
20
30
40
50
TIME (MIN)

6

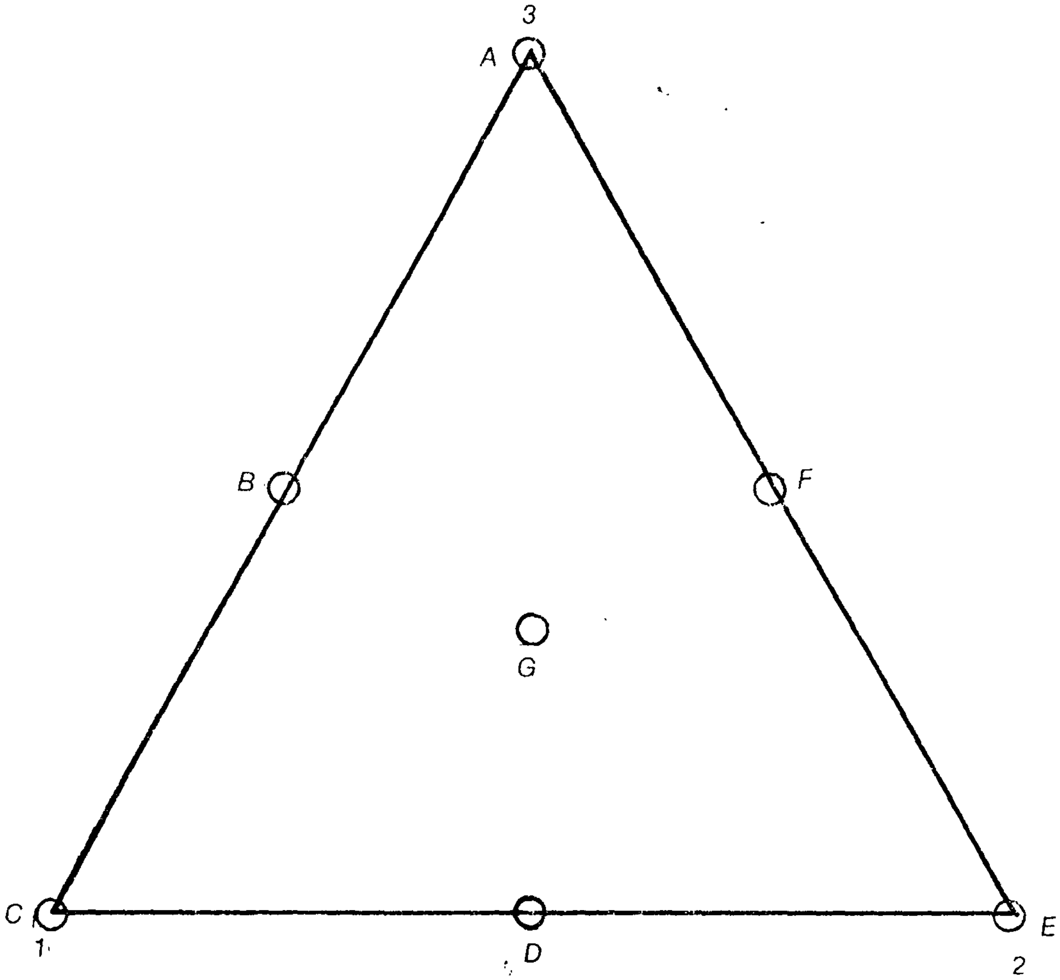


OUTPUT LOOP 1
−.1
−.5
−.9
−1.3
−1.7
10
20
30
40
50
TIME (MIN)



OUTPUT LOOP 2
1.3
1.1
.9
.7
.5
.3
.1
−.1
−.3
10
20
30
40
50
TIME (MIN)

#7



#8